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Application of π-N Scattering Results to Λ-K^0 Production*

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The appearance of numerous phase-shift analyses of pion-nucleon scattering and the tentative identification of resonances in that process lead naturally to an application of these results to processes involving other decay channels. We choose the three isospin-$\frac{1}{2}$ resonances of the π-N system on which there seems to be the widest agreement and consider their effect on the process $\pi^-+p\rightarrow\Lambda+K^0$ near 900 MeV. The background term which we use is a K^* pole modified by an arbitrary constant factor of modulus unity. A comparison with available data is given for various resonance pairs and for the three resonances together.

I. INTRODUCTION

THE process

$$\pi^-+p\rightarrow\Lambda+K^0 \tag{1}$$

has puzzled investigators for several years. Four features of this interaction are particularly noteworthy:

(1) The angular distribution of the Λ's is peaked in the backward direction relative to the incoming π.

(2) The total cross section shows a definite peak located, in terms of the laboratory kinetic energy of the π, at about 925 MeV.

(3) A fit to the expansion

$$d\sigma/d\Omega=\sum_n C_n\cos^n\theta\,,$$

in terms of the angle θ between the K^0 and π, gives surprisingly large values for C_3 and C_4 in an energy region not far from the threshold value of 768 MeV.

(4) The average polarization of the Λ's is large and negative in the resonance region.

Tiomno *et al.*[1] proposed that feature (1) be explained by the contribution of a pole due to the exchange of a K^* having a mass which is now known to be about 888 MeV. Not long afterward, Kanazawa[2] presented a model including a $P_{1/2}$ or $P_{3/2}$ resonance in order to explain both features (1) and (2). In a more recent paper Hoff[3] combined the K^* pole with a $P_{1/2}$ resonance to reproduce features (1), (2), and (4). Subsequently, the addition of an $F_{5/2}$ resonance[4] gave better agreement with feature (3).

The idea of an $F_{5/2}$ resonance is not new. Peierls[5] assigned these quantum numbers to an assumed resonance to explain the data on photoproduction and scattering of pions on protons. Tsuchida *et al.* and Gourdin and Rimpault[6] presented models for Λ-K^0 production using this resonance proposed by Peierls, which is intended to correspond to the bump in π^--p scattering at 900 MeV.[7] Thus far, however, Hoff's is the only two-resonance model to appear. (Along with an $F_{5/2}$ resonance, Gourdin and Rimpault included a $D_{3/2}$ resonance at 600 MeV, which had also been proposed by Peierls, but they neglected its width.)

Another recent work on Λ-K^0 production is a partial-wave analysis which was performed by Rimpault[8] at five different energy values. The $F_{5/2}$ partial wave was assumed to satisfy a Breit-Wigner formula,[9] and the three parameters so defined were determined by the values of the modulus of the $F_{5/2}$ amplitude at three of the energies. He obtained a position for this assumed resonance of 890 MeV (pion kinetic energy) and a width of 180 MeV.

In a paper concerning the "900 MeV π^--p Resonance," Johnson *et al.*[10] applied a one-resonance model ($F_{5/2}$ or $D_{5/2}$), with a K^* pole, to Λ-K^0 production. The resonance term was a Breit-Wigner formula (defined in Sec. II) modified by an optical absorption factor.

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[1] J. Tiomno, A. L. L. Videira, and N. Zagury, Phys. Rev. Letters **6**, 120 (1961).

[2] A. Kanazawa, Phys. Rev. **123**, 997 (1961).

[3] G. T. Hoff, Phys. Rev. **131**, 1302 (1963).

[4] G. T. Hoff, Phys. Rev. Letters **12**, 652 (1964); Phys. Rev. **139**, B671 (1965).

[5] R. F. Peierls, Phys. Rev. **118**, 325 (1960).

[6] T. Tsuchida, T. Sakuma, and S. Furui, Progr. Theoret. Phys. (Kyoto) **26**, 1005 (1960); M. Gourdin and M. Rimpault, Nuovo Cimento **20**, 1166 (1961); **24**, 414 (1962).

[7] J. A. Helland, C. D. Wood, T. J. Devlin, D. E. Hagge, M. J. Longo, B. J. Moyer, and V. Perez-Mendez, Phys. Rev. **134**, B1079 (1964). In view of the energy dependence of a_3 and a_5 in the series $d\sigma/d\Omega=\Sigma a_n\cos^n\theta$ given by these authors, one should not confuse their interpretation of an $F_{5/2}$ resonant amplitude at 900 MeV with Hoff's $F_{5/2}$ resonance at 829 MeV.

[8] M. Rimpault, Nuovo Cimento **31**, 56 (1964).

[9] In the form of the $F_{5/2}$ resonant amplitude chosen by Rimpault, the barrier penetration factors (defined in the next section) were omitted. This amounts to the assumption of an infinite interaction radius, and makes us uncertain as to how to interpret some of his results.

[10] W. R. Johnson, F. C. Smith, and P. C. DeCelles, Phys. Rev. **138**, B938 (1965).

The K^* pole term was modified in a similar manner. On the basis of some assumptions about coupling constants, the authors found that a $D_{5/2}$ assignment to this resonance fitted the data better than an $F_{5/2}$ choice.

In reports[11] of previous work, we presented a model for this process including a K^* pole, an $F_{5/2}$ resonance at 1680 MeV and a $D_{3/2}$ resonance at 1630 MeV. This appeared to us to be the best possible two-resonance model which actually incorporated the $F_{5/2}$ resonance from π-N scattering. (In Hoff's model,[4] the $F_{5/2}$ resonance was located at 1647 MeV with a width of 10 MeV, and could therefore not be thought to correspond to the π-N scattering resonance.)

It is our purpose in this paper to incorporate some results of recent phase-shift analyses of π-N scattering into the study of Λ-K^0 production. In this way we hope to provide a further check on the validity of these analyses while gaining more understanding of the Λ-K^0 process. We should emphasize that we are not so much proposing a model as examining the process in the light of plausible predictions which were obtained from studies of the elastic channel.

II. KINEMATICS AND DYNAMICS

The kinematics of this process have been given by several authors and will not be reproduced in detail; we shall adhere to the notation of Hoff.[3] We designate the four momenta of the p, Λ, π, K^0 by p, p', k, k', their masses by m, m', μ, μ', and their energies by E, E', ω, ω', respectively, in the center-of-mass (c.m.) system.

The differential cross section is given in terms of a matrix

$$M=g+h(\boldsymbol{\sigma}\cdot\hat{p}')(\boldsymbol{\sigma}\cdot\hat{p}),$$

by

$$d\sigma/d\Omega=\tfrac{1}{2}\,\mathrm{Tr}(M^\dagger M). \tag{2}$$

If we define

$$x=\cos\theta=\hat{k}'\cdot\hat{k},$$

we get the expansions of g and h:

$$g=\sum_{l=1}^{\infty}\left(f_{(l-1)}{}^{+}-f_{(l+1)}{}^{-}\right)P_l'(x),$$
$$h=\sum_{l=1}^{\infty}\left(f_l{}^{-}-f_l{}^{+}\right)P_l'(x), \tag{3}$$

where $f_l{}^{\pm}$ is the partial-wave amplitude for orbital angular momentum l and total angular momentum $j=l\pm\frac{1}{2}$.

With the definitions

$$a=g+h\cos\theta,\quad b=h$$

Eq. (2) becomes

$$d\sigma/d\Omega=|a|^2+|b|^2\sin^2\theta. \tag{4}$$

The polarization $P(\theta)$ is taken to be positive along the direction of

$$\hat{n}=\hat{p}'\times\hat{p}/|\hat{p}'\times\hat{p}|$$

in accord with the Basel convention.[12] Then we find

$$\begin{aligned}(d\sigma/d\Omega)P(\theta)&=\tfrac{1}{2}\,\mathrm{Tr}(\boldsymbol{\sigma}\cdot\hat{n}M^\dagger M)\\&=2\,\mathrm{Im}(ab^*)\sin\theta.\end{aligned} \tag{5}$$

If we examine the processes which are associated with the present one by the substitution rule, we find singularities due to resonances in the K-π system in one crossed channel and singularities from the Σ and Σ-π resonances in the other. In the third channel we find contributions from a nucleon pole and N-π resonances.

If we neglect the width of the K^* (888) resonance, we get its contribution as a pole on the real axis of $t=-(k-k')^2$. Under certain reasonable assumptions its contribution to g and h is determined to be[3]

$$\begin{aligned}g_p&=-[(E+m)(E'+m')]^{1/2}\\&\quad\times\left[2W-(m+m')+\frac{(\mu'^2-\mu^2)(m'-m)}{M^2}\right]C,\\h_p&=-[(E-m)(E'-m')]^{1/2}\\&\quad\times\left[2W+(m+m')-\frac{(\mu'^2-\mu^2)(m'-m)}{M^2}\right]C,\end{aligned} \tag{6}$$

where W is the total c.m. energy and M is the mass of the K^*; also

$$\begin{aligned}C&=\frac{\sqrt{2}}{4}\left(\frac{fg}{4\pi}\right)\frac{1}{W|\mathbf{k}|(\beta-\cos\theta)(|\mathbf{k}||\mathbf{k}'|)^{1/2}},\\\beta&=\frac{2\omega\omega'+M^2-\mu^2-\mu'^2}{2|\mathbf{k}||\mathbf{k}'|},\end{aligned} \tag{7}$$

with $fg/4\pi$ being the product of the πKK^* and $K^*\Lambda N$ coupling constants.[13] Contributions from other pole terms are discussed in the next section.

To account for the peak in the total cross section, we consider resonances in the N-π system in the energy region under consideration. We approximate a resonating partial wave by the Breit-Wigner formula[14]

$$f_l{}^{\pm}=\frac{1}{2|\mathbf{k}|}\frac{(\Gamma_{l1}\Gamma_{l2})^{1/2}}{W_r-W-i\Gamma/2}. \tag{8}$$

The partial widths $\Gamma_{l\alpha}$ have the form[15]

$$\Gamma_{l\alpha}=2k_\alpha Rv_{l\alpha}\gamma_{l\alpha},$$

[11] J. E. Rush and W. G. Holladay, Bull. Am. Phys. Soc. **9**, 385 (1964); J. E. Rush, *ibid.* **10**, 257 (1965).

[12] Helv. Phys. Acta, Suppl. IV, 436 (1961).

[13] The kinematics and dynamics of Λ-K^0 production are discussed more fully in a Ph.D. thesis submitted to Vanderbilt University (1965) by one of us (J. E. R.).

[14] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1953), p. 306.

[15] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 390.

where k_α is the (c.m.) momentum of the particles in channel α, R is an interaction radius, and the reduced width $\gamma_{l\alpha}$ is assumed to be constant, with $(\gamma_{l1}\gamma_{l2})^{1/2}$ being either positive or negative. Inclusion of the barrier penetration factors $v_{l\alpha}$ (Ref. 16) is important on physical grounds. To our knowledge, no derivation of the Breit-Wigner formula other than the usual nonrelativistic one[14] produces the form of these factors given in Ref. 16. The form for the total width of each resonance was approximated by that of the elastic width, which has the form given above with α being the incoming channel. The results were only slightly different from those obtained with constant total widths.

III. DISCUSSIONS

In the energy region near the peak in the total cross section, as Hoff points out, we find that $g_p/h_p \approx 10$; furthermore, $\beta \gtrsim 5$ in the same region. We can therefore write

$$g_p \sim 1/(\beta - x) \approx \tfrac{1}{5}(1+\tfrac{1}{5}x),$$

$$\left(\frac{d\sigma}{d\Omega}\right)_p \sim \frac{1}{(\beta-x)^2} \approx \frac{1}{25}(1+\tfrac{2}{5}x),$$

where $(d\sigma/d\Omega)_p$ is the term in $d\sigma/d\Omega$ due to the K^* pole alone. The slope of $(d\sigma/d\Omega)_p$ is found to be insufficient to account for the peaking of the differential cross section. (These approximations are made for purposes of discussion only.)

In addition, we note that $(d\sigma/d\Omega)_p$ is a monotonically increasing, slowly varying function of W, as are the contributions to $d\sigma/d\Omega$ from each of the other pole terms. In the region of the peak in the total cross section, therefore, the combined contributions of all the pole terms to $d\sigma/d\Omega$ must be small compared to the resonance contributions which produce the peak. Since the coupling constants in the pole terms are not known with any accuracy, and it would be difficult to distinguish their separate contributions empirically, we might hope to approximate all these contributions by that of the K^* pole with the quantity $fg/4\pi$ playing the role of an arbitrary constant. That is, all pole terms tend toward reasonably flat angular distributions, as compared with the data. We see no advantage to be gained by including these other terms, with their undetermined parameters, in order to obtain a slight improvement in the fit. We simply acknowledge that the background term is not so well known as one would desire.

A calculation of the neutron pole term, with reasonable values for the coupling constants, indicates that it and perhaps other terms are not necessarily small compared to the K^* pole term. We should, therefore, not interpret the value of the quantity $fg/4\pi$ (which we obtain by using the K^* pole as the only pole term) as a prediction of the value of the product of the coupling constants f and g.

Since we will not use an S-wave resonance, the only S-wave contribution which we get is from the pole, which is real. It would be naive, however, to suppose that the S-wave amplitude is real. To take account of absorption due to other open channels, we introduce a simple modification of the Born approximation, in the form of a factor $e^{i\delta}$ applied to the pole term, where δ is taken to be real and constant. This has been done previously by one of us (W. G. H.) in a model[17] for the process $\pi^+ + p \to \Sigma^+ + K^+$.

Let us now consider a model containing one real pole term and two resonance terms. We define the quantities $a_r(n)$ and $b_r(n)$ as the contributions to a and b from the nth resonance. Using quantities to be defined below, we may write the differential cross section as

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_p + \left(\frac{d\sigma}{d\Omega}\right)_r + \left(\frac{d\sigma}{d\Omega}\right)_{ip} + \left(\frac{d\sigma}{d\Omega}\right)_{ir}, \qquad (9)$$

with

$$(d\sigma/d\Omega)_p = g_p^2 + h_p^2 + 2g_ph_p\cos\theta. \qquad (10)$$

Since g_p and h_p are real,

$$\left(\frac{d\sigma}{d\Omega}\right)_r = \sum_{n=1}^{2} [|a_r(n)|^2 + |b_r(n)|^2 \sin^2\theta], \qquad (11)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{ip} = \sum_{n=1}^{2} [2(g_p + h_p\cos\theta)\,\mathrm{Re}a_r(n)\sin\theta + 2h_p\,\mathrm{Re}b_r(n)\sin\theta], \qquad (12)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{ir} = (\mathrm{Re}f_l^{\,j}\,\mathrm{Re}f_{l'}^{\,j'} + \mathrm{Im}f_l^{\,j}\,\mathrm{Im}f_{l'}^{\,j'})Q(x). \qquad (13)$$

Here j is the total angular-momentum value of the resonating partial-wave amplitude $f_l^{\,j}$ and $Q(x)$ is a polynomial in $x=\cos\theta$, the parity of which is the product of the parities of the two resonances. For $l\leq 3$, the quantities $Q(x)$ are given in Table I, while $a_r(n)$, and $b_r(n)$ are tabulated by Hoff.[3] The only change brought about by replacing g_p and h_p by $g_pe^{i\delta}$ and $h_pe^{i\delta}$ occurs in $(d\sigma/d\Omega)_{ip}$ (12). $g_p\,\mathrm{Re}a_r(n)$ is replaced by

$$[g_p\cos\delta\,\mathrm{Re}a_r(n) + g_p\sin\delta\,\mathrm{Im}a_r(n)]$$

with similar replacements of the other two terms. For further discussion of the pole terms, see Hoff.[4]

TABLE I. The polynomials $Q(x)$.

	$P_{1/2}$	$P_{3/2}$	$D_{3/2}$	$D_{5/2}$	$F_{5/2}$
$S_{1/2}$	$2x$	$4x$	$6x^2-2$	$9x^2-3$	$15x^3-9x$
$P_{1/2}$		$6x^2-2$	$4x$	$15x^3-9x$	$9x^2-3$
$P_{3/2}$			$18x^3-10x$	$12x^3$	$45x^4-36x^2+3$
$D_{3/2}$				$45x^4-36x^2+3$	$12x^3$
$D_{5/2}$					$(9/2)(25x^5-26x^3+5x)$

[16] Blatt and Weisskopf, Ref. 15, p. 361.

[17] W. G. Holladay, Phys. Rev. 139, B1348 (1965).

The polarization may be expressed as

$$P(\theta)=P_p(\theta)+P_r(\theta)\,, \tag{14}$$

where

$$\frac{1}{2\sin\theta}\frac{d\sigma}{d\Omega}P_p(\theta)=\sum_{n=1}^{2}\left[-(g_p+h_p\cos\theta)\times\mathrm{Im}b_r(n)+h_p\,\mathrm{Im}a_r(n)\right], \tag{15}$$

$$\frac{1}{2\sin\theta}\frac{d\sigma}{d\Omega}P_r(\theta)=(\mathrm{Re}f_l{}^j\,\mathrm{Im}f_{l'}{}^{j'}-\mathrm{Re}f_{l'}{}^{j'}\,\mathrm{Im}f_l{}^j)R(x)\,. \tag{16}$$

The quantities $R(x)$, given in Table II, are so chosen that $l\leq l'$; if $l=l'$, $j<j'$; Hoff[4] presents a table of $(d\sigma/d\Omega)P_p(\theta)$. In the case of a complex pole, Eq. (15) is changed. $g_p\,\mathrm{Im}b_r(n)$ becomes

$$[g_p\cos\delta\,\mathrm{Im}b_r(n)-g_p\sin\delta\,\mathrm{Re}b_r(n)]\,,$$

etc. The extension of Eqs. (11–16) to more than two resonances will be obvious.

Let us now consider contributions from one resonance and the real pole. Regardless of the resonance chosen, it is apparent from the second paragraph of this section that, in the region near $W=W_r$, $(d\sigma/d\Omega)_r$ will dominate, and $d\sigma/d\Omega$ will tend to be symmetric. To produce an asymmetry, one could reduce the relative strength of the resonance, as was done in Hoff's one-resonance model. However, a simultaneous fit to the total cross section σ_T and to $d\sigma/d\Omega$ is difficult to achieve under these circumstances; in fact, for $l>1$ it is hopeless. In both of Hoff's models, the strength of the K^* pole is such that σ_T begins to increase for values of W not much greater than W_r; this is in contrast to the data, in which σ_T decreases from about 0.75 mb to 0.2 mb and retains the latter value for a kinetic-energy range of about 1000 MeV.[18]

Another difficulty of a one-resonance model is the fact that $\mathrm{Re}f_l{}^j$ changes sign at $W=W_r$. It is this property which causes Hoff's curves for $d\sigma/d\Omega$ at energies above her $P_{1/2}$ resonance to peak in the wrong direction. It should be noted that her introduction of an $F_{5/2}$ resonance at $W_r=1647$ MeV with $\Gamma=10$ MeV has essentially no effect on either of the above difficulties.

The complex pole term can help to alleviate the above difficulties somewhat, by introducing an interference with the imaginary part of the resonating partial wave. One may thus prevent the change of sign mentioned above, and introduce more asymmetry (but not enough) at $W=W_r$. But, in actual fact, one is still unable to obtain a reasonable fit to the data.

On the basis of the above considerations, we were led[11] to the consideration of various two-resonance models. The pair of resonances which seemed to hold the most promise at that time was $F_{5/2}$-$D_{3/2}$, which gave partial agreement with the π^--p scattering predictions.[7] Using a real K^* pole, we were led to position the $F_{5/2}$ at 1680 MeV and the $D_{3/2}$ at 1630 MeV. Although we obtained fits to the cross section and angular distributions in the energy region above and below the peak, we were unable to fit the polarization with this model.

[18] J. Schwartz, D. H. Miller, G. R. Kalbfleisch, and G. A. Smith, Bull. Am. Phys. Soc. **9**, 420 (1964).

TABLE II. The polynomials $R(x)$.

	$P_{1/2}$	$P_{3/2}$	$D_{3/2}$	$D_{5/2}$	$F_{5/2}$
$S_{1/2}$	-1	1	$-3x$	$3x$	$-(3/2)(5x^2-1)$
$P_{1/2}$		$3x$	-1	$(3/2)(5x^2-1)$	$-3x$
$P_{3/2}$			$-(9x^2-1)$	$(3/2)(x^2+1)$	$-(3/2)(15x^3-5x)$
$D_{3/2}$				$(3/2)(15x^3-5x)$	$-(3/2)(x^2+1)$
$D_{5/2}$					$-(9/4)(25x^4-14x^2+1)$

It is pertinent at this point to examine the situation in the $T=\frac{1}{2}$ part of π^--p scattering. There have been several phase-shift analyses of this process performed recently, using various types of parametrization. Roper *et al.*,[19] using an energy-dependent analysis up to 700 MeV, have found considerable evidence for a $D_{3/2}$ resonance, as predicted by Peierls, and a $P_{1/2}$ resonance, suggested by Bareyre *et al.*,[20] on the basis of total-cross-section data. The latest positions given by Roper are the $P_{1/2}$ at 585 MeV (1503 MeV in the center-of-mass system) and the $D_{3/2}$ at 638 (1536) MeV. Roper *et al.* have also found some evidence that the $F_{5/2}$ dominates the $D_{5/2}$ at 900 MeV. Auvil *et al.*[21] began with partial-wave dispersion relations calculated by Donnachie *et al.*[22] for energies up to 400 MeV. Their results also indicated a $D_{3/2}$ resonance at about 620 MeV and a possible $P_{1/2}$ at 600 MeV. They, too, suggested $F_{5/2}$ as the assignment for the 900 MeV resonance. Bransden *et al.*[23] placed the $D_{3/2}$ at 612 or 630 MeV. In addition, they found evidence for an $S_{1/2}$ resonance close by, and a possible $P_{1/2}$. Their results regarding $D_{5/2}$ and $F_{5/2}$ were inconclusive.

The findings of Bareyre *et al.*[24] were also interesting. They used an energy-independent analysis, joining the solutions for different energies smoothly. There resulted a $P_{1/2}$ resonance at 600 MeV (1512 MeV total center-of-mass energy), a $D_{3/2}$ at 630 MeV, and an $F_{5/2}$ resonance at 915 MeV. The S wave indicated a possible resonant behavior, but at an energy much higher than that given by Bransden *et al.* All the previous results

[19] L. D. Roper, Phys. Rev. Letters **12**, 340 (1964); L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. **138**, B190 (1965).

[20] P. Bareyre, C. Bricman, G. Valladas, G. Villet, J. Bizard, and J. Sequinot, Phys. Letters **8**, 137 (1964).

[21] P. Auvil, C. Lovelace, A. Donnachie, and A. T. Lea, Phys. Letters **12**, 76 (1964).

[22] A. Donnachie, J. Hamilton, and A. T. Lea, Phys. Rev. **135**, B515 (1964).

[23] B. H. Bransden, P. J. O'Donnel, and R. G. Moorhouse, Phys. Letters **11**, 339 (1964); Phys. Rev. **139**, B1566 (1965).

[24] P. Bareyre, C. Brickman, A. V. Stirling, and G. Villet, Phys. Letters **18**, 342 (1965).

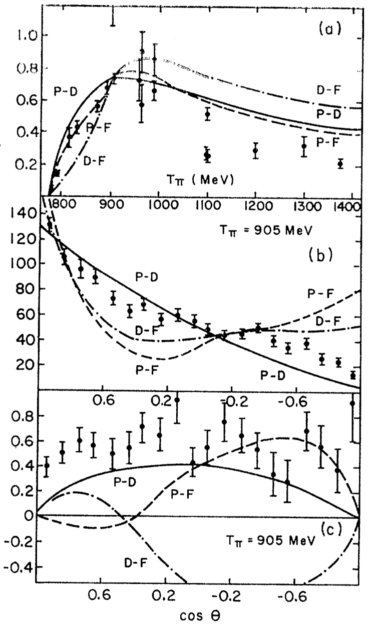

FIG. 1. (a) Typical total cross-section curves for P-D (solid line), P-F (dashed line), and D-F (dot-dashed line) resonance pairs with background, plotted as functions of the laboratory kinetic energy of the pion T_π. Data from Ref. 34. (b) and (c) Typical angular distribution and polarization curves $[\alpha P(\theta)]$ for resonance pairs with background, plotted as functions of the cosine of the K^0 production angle in the c.m. system. Data from Ref. 31.

disagreed with those of an energy-independent analysis by Cence and Cha[25] who found no resonances at all.

In addition to phase-shift analyses, there have been several more qualitative analyses which are interesting. Ogden *et al.*[26] found no necessity for two resonances between 400 and 700 MeV, nor could they rule them out. In the case of one resonance, they preferred the $D_{3/2}$. Auvil and Lovelace[27] had shown that the $D_{5/2}$ and $F_{5/2}$ phase shifts were large around 900 MeV. They argued for resonance of the $F_{5/2}$ alone at 900 MeV, as did Duke *et al.*[28] The argument for the $D_{5/2}$ by Johnson *et al.*,[10]

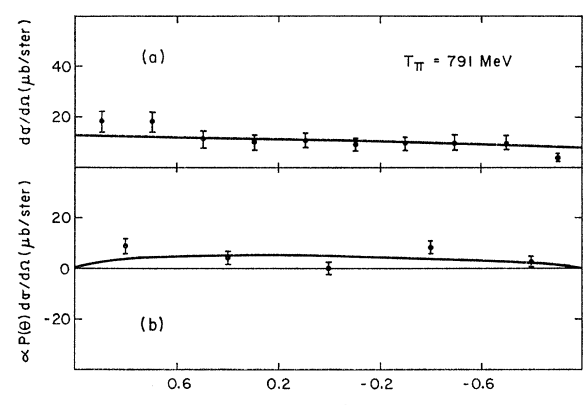

FIG. 2. (a) and (b) Angular distribution (arbitrarily normalized) and polarization $[\alpha P(\theta)d\sigma/d\Omega]$ curves for the three-resonance fit at 791 MeV. Data from Ref. 29.

[25] R. J. Cence and M. Y. Cha, Bull. Am. Phys. Soc. **10**, 528 (1965). This analysis has been extended by Cence [*ibid.* **10**, 736 (1965)] from 700 to 1000 MeV, but to date we have no report of the results available.

[26] P. M. Ogden, D. E. Hagge, J. A. Helland, M. Banner, J. F. Detoeuf, and J. Teiger, Phys. Rev. **137**, B1115 (1965).

[27] P. Auvil and C. Lovelace, Nuovo Cimento **33**, 473 (1964).

[28] P. J. Duke, D. P. Jones, M. A. R. Kemp, P. G. Murphy, J. D. Prentice, J. J. Thresher, and H. H. Atkinson, Phys. Rev. Letters **15**, 468 (1965).

based on the scattering data, depends on several simplifying assumptions not made by other authors, and we do not find their results compelling.

In our previous work,[11] we found conclusive evidence that if there are both $D_{5/2}$ and $F_{5/2}$ resonances in scattering, they do not both decay appreciably into the Λ-K^0 channel. Faced with a choice, and the possibility that there may be only one resonance at 900 MeV, we

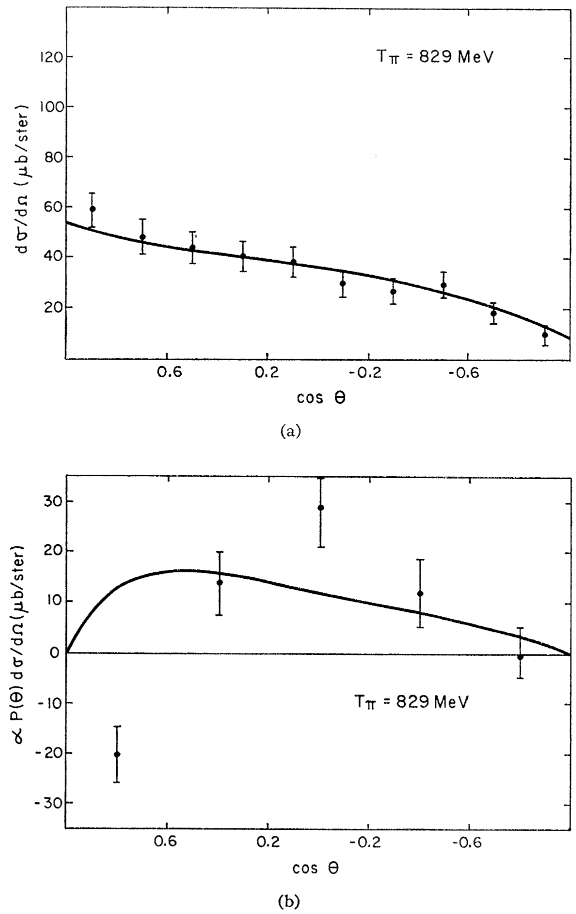

FIG. 3. (a) and (b) Angular distribution (arbitrarily normalized) and polarization $[\alpha P(\theta)d\sigma/d\Omega]$ curves at 829 MeV. Data from Ref. 29. [*Note added in proof.* In a recent unpublished report from L. B. Auerbach, D. Bowen, J. Dobbs, K. Lande, A. K. Mann, F. J. Scinlli, H. Uto, D. H. White, and K. K. Young the value of $\alpha P(\theta)$ at $\cos\theta = 0.8$ is given as 0.42 ± 0.10 in disagreement with the negative experimental value in this figure.]

are clearly led to the $F_{5/2}$. With the overwhelming evidence for the $D_{3/2}$ at about 630 MeV, we can now expect two resonances to contribute to our process. Our previous work also revealed, however, that these two resonances at the positions given, with a background, will not give a reasonable fit to the data. This is true whether we choose the $F_{5/2}$ or $D_{5/2}$ at 900 MeV, and we obtain a much better fit with the former, because it has opposite parity to the $D_{3/2}$.

Since other contributions are evidently needed for an understanding of Λ-K^0 production, we have chosen to include in our calculations a $P_{1/2}$ resonance, evidence for which was discussed in previous paragraphs. It is this combination ($P_{1/2}$,$D_{3/2}$,$F_{5/2}$) along with each of the three possible pairs, which we use in making comparisons with the data in the next section. We positioned these resonances as follows: $P_{1/2}$ (1497 MeV), $D_{3/2}$ (1534 MeV), $F_{5/2}$ (1690 MeV). The first two were based on some preliminary values given by Roper, but are not crucial, and could be moved as much as 25 or 50 MeV without affecting our results significantly. Our case for including the $P_{1/2}$ was strengthened when we discovered that it produces a peak in the Λ-K^0 total cross section at about 925 MeV, and could therefore account for much of the "hump" observed in that energy region. We feel that this fact is definitely significant.

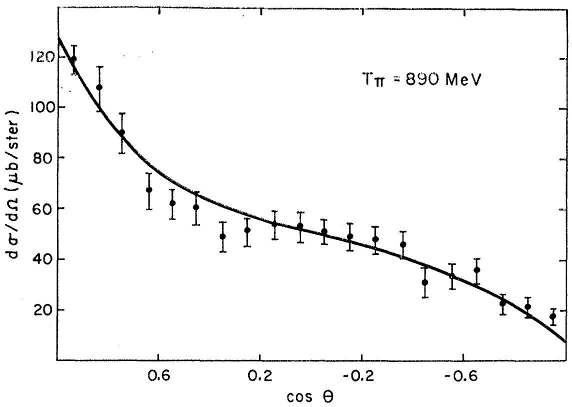

FIG. 5. Angular distribution at 890 MeV. Data from Ref. 30.

IV. COMPARISON WITH DATA AND CONCLUSIONS

We have examined Λ-K^0 production by considering, not those contributions which are most likely to fit the data well, but those which are most likely to exist. The necessity of such an approach is self evident, although both approaches have some merit. In Figs. 1–10, we

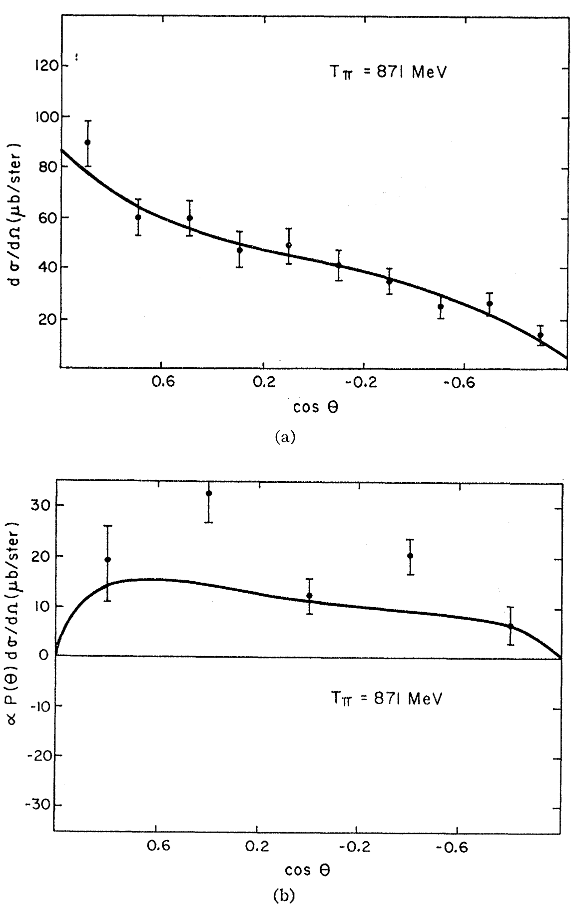

FIG. 4. (a) and (b) Angular distribution (arbitrarily normalized) and polarization [$\alpha P(\theta)d\sigma/d\Omega$] curves at 871 MeV. Data from Ref. 29.

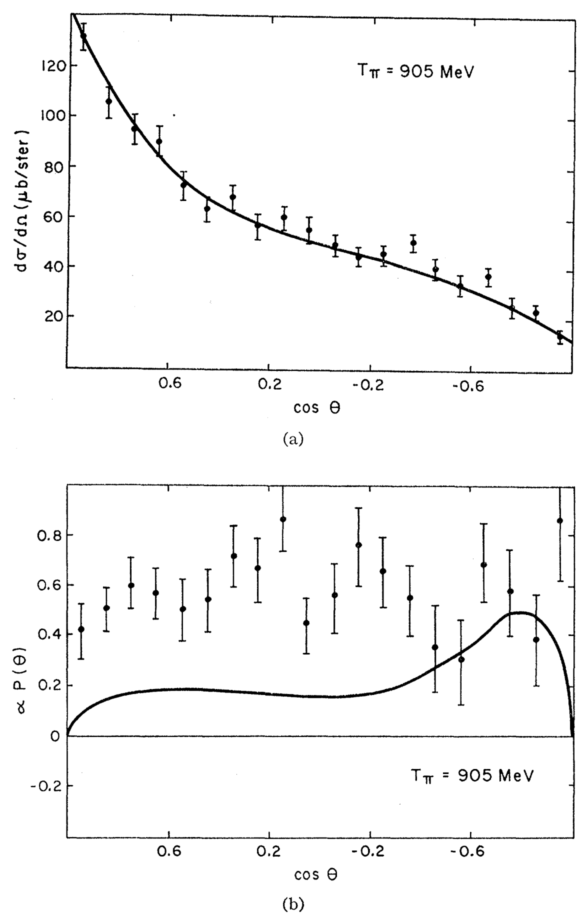

FIG. 6. (a) and (b) Angular distribution and polarization [$\alpha P(\theta)$] at 905 MeV. Data from Ref. 31. One should bear in mind that the measured value of α is -0.62 when examining the polarization data in this figure and in Fig. 7(b).

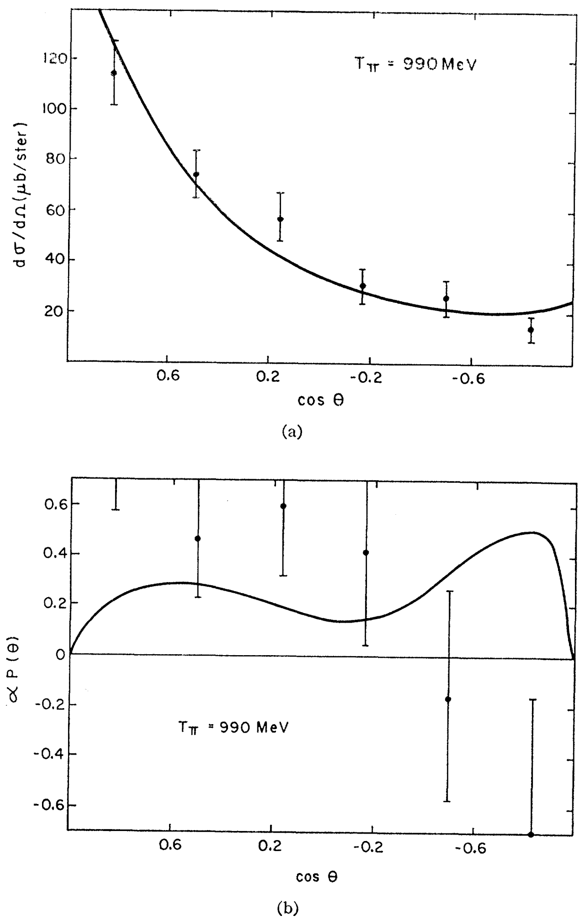

FIG. 7. (a) and (b) Angular distribution (arbitrarily normalized) and polarization $[\alpha P(\theta)]$ at 990 MeV. Data from Refs. 32 and 33.

compare our results with the data.[29–34] We plot the polarization as $\alpha P(\theta)$ or as $\alpha P(\theta)d\sigma/d\Omega$, depending on the form of the data. Here α is the decay asymmetry parameter of the Λ, which we take to be the negative of the helicity of the decay proton.[35] We are assuming the value obtained by Cronin and Overseth,[36] $\alpha=-0.62\pm0.07$.

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One is, of course, interested in determining if one of the three resonances can be neglected in seeking a fit to the data. In Fig. 1, therefore, we show typical cross section, angular distribution, and polarization curves for each pair of resonances alone (with the background). In each case the study was pursued until it was apparent that a decent fit would not be forthcoming. Notice, in particular, that the best established pair of resonances, the $D_{3/2}$ and $F_{5/2}$, yields especially poor fits to the data.

A comparison of the three-resonance fit is given in Figs. 2–10. Our analysis is based on $P_{1/2}$ and $D_{3/2}$ resonances positioned somewhat below the threshold value of 768 MeV with widths between 50 and 200 MeV, an $F_{5/2}$ around 1690 MeV with a width of about 100 MeV, and a background which is predominantly complex S wave. The position W_r, width Γ, and product of interaction radius and reduced width $R(\gamma_{l1},\gamma_{l2})^{1/2}$ for these resonances are given in Table III for the plotted curves. For the P and D states, these values are not at all critical. The value of the normalization constant $fg/4\pi$ (not to be interpreted as a product of coupling

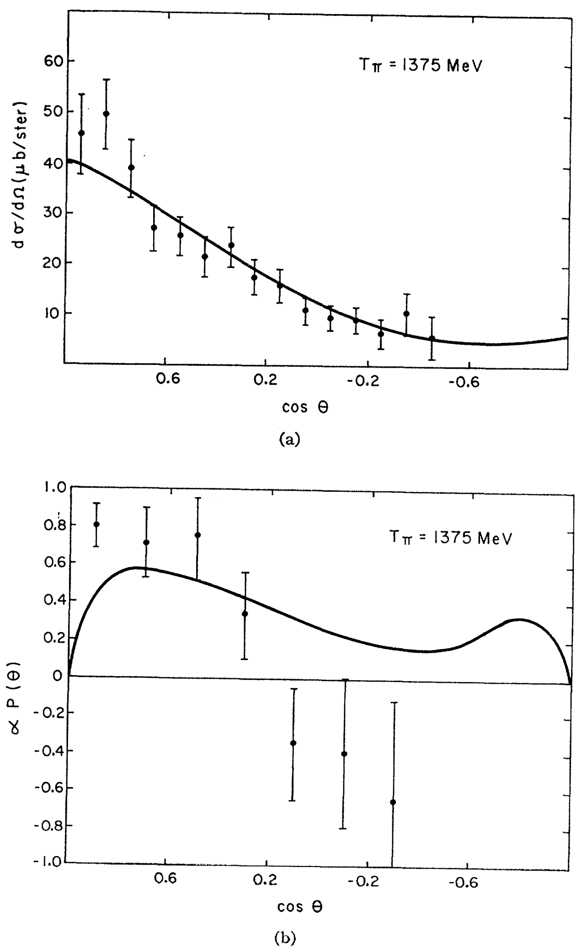

FIG. 8. (a) and (b) Angular distribution (arbitrarily normalized) and polarization $[\alpha P(\theta)]$ at 1375 MeV. Data from Ref. 34.

constants) is $fg/4\pi=-0.21$. For R we chose one pion Compton wavelength, with $\delta=43°$.

With regard to the region below 925 MeV, we should like to point out some interesting features of the existing data. If one plots least-squares fits to the angular distributions at 791, 829, 871, 890, and 905 MeV, as given by the authors of the respective papers,[29–31] one is struck by the smooth change in shape of the curves as a function of energy. This regularity is not demonstrated by the polarization data. From the fact that fits to $d\sigma/d\Omega$ only require a $\cos\theta$ series of degree 4, one might be led to expect a fit to $(d\sigma/d\Omega)\times P(\theta)/2\sin\theta$ with a $\cos\theta$ series of degree 3, but the results of such an attempt are poor. We feel that this suggests a need for more accurate polarization measurements.

Any examination of Λ-K^0 production is hampered by the scarcity of reliable data in the energy region between 925 and 1375 MeV. At present, work on this process seems to be restricted to higher energies but it is hoped that some group will undertake a very careful study of the 1100-MeV region. In particular, it would be helpful if the experimental ambiguities in the total cross section were removed in this energy region.

Despite these experimental uncertainties, it appears that our calculated total cross section does not decrease rapidly enough beyond the peak at 925 MeV. This difficulty might be remedied if absorptive effects were taken into account. It is also clear that the calculated values of $\alpha\bar{P}$ are too small. Recent evidence of an S-wave resonance at 1700 MeV might be used to improve this situation.

In spite of these difficulties, we are encouraged by the fact that the fits to the differential cross section over the energy range from threshold to $T_\pi=1400$ MeV are very good and that the general trend of all our results is qualitatively in the right direction. Analysis both of ΛK^0 production and π-P elastic scattering indicate that the $T=\frac{1}{2}$ channel has a rather complicated structure and more detailed work will be required for its full elucidation.

TABLE III. Resonance parameters used in this analysis.

Resonance	W_r (MeV)	Γ (MeV)	$R(\gamma_{e1}\gamma_{e2})^{1/2}$
$P_{1/2}$	1497	89	0.116
$D_{3/2}$	1534	105	0.063
$F_{5/2}$	1690	100	0.025

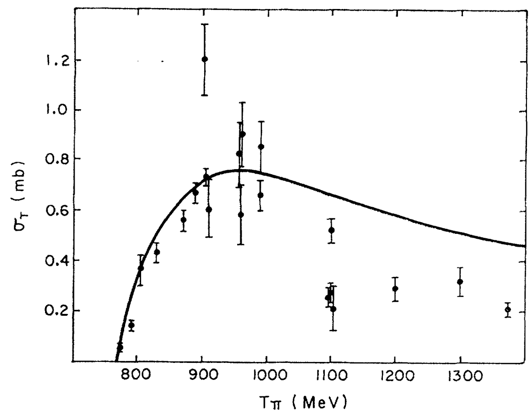

FIG. 9. Total cross section as a function of the lab kinetic energy of the pion. Data from Ref. 34.

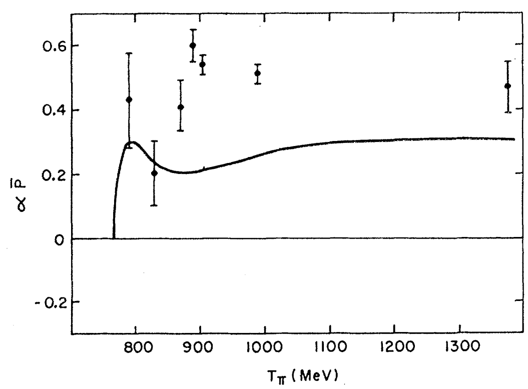

FIG. 10. Average polarization ($\alpha\bar{P}$) as a function of the lab kinetic energy of the pion. Data from Refs. 29, 31, 32, 33, and 34.

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