Meson Production from Electron-Positron Annihilation*

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The processes $e^+ + e^- \to P+P$, $P+V$, $V+V$, $P+\gamma$ (P=pseudoscalar meson, V=vector meson, γ=photon) are discussed in the context of a meson-pole model. $M(12)$ symmetry with kineton corrections [i.e., $SU(6)_W$ symmetry] is used to determine all the coupling constants. The total cross sections of all these processes are given as functions of the total c.m. energy.

I. INTRODUCTION

VARIOUS projects have been directed towards the construction of electron storage rings,[1] and hence the feasibility of electron-positron colliding-beam experiments seems imminent. The typical processes that can occur following an e^--e^+ collision are

$$\begin{aligned} &(a)\quad e^+ + e^- \to P+P,\\ &(b)\quad e^+ + e^- \to V+V,\\ &(c)\quad e^+ + e^- \to V+P,\\ &(d)\quad e^+ + e^- \to P+\gamma, \end{aligned} \qquad (1)$$

where $P(V)$ stands for a pseudoscalar (vector) meson. In the following we propose to use symmetry properties of the strong mesonic interactions to determine the cross sections of these processes.

II. FORMULATION OF THE MESON-POLE MODEL

In keeping with current ideas about the electromagnetic structure of hadrons, we shall picture the processes (1) to proceed through the following sequence

$$\begin{aligned} e^+ + e^- &\to \text{virtual } \gamma \to \text{virtual vector meson}\\ &\to \text{final states } (=PP,\, PV,\, VV,\, P\gamma) \end{aligned}$$

which is represented by the Feynman diagram in Fig. 1. p_1, p_2 are the four-momenta of the incoming electron and positron, respectively; k_1, k_2 are the four-momenta of the outgoing mesons. For the vertex A we have the well-known electromagnetic coupling between electron and photon. In the hadronic part B, we can use the $M(12)$ invariant[2] vertex function $\Gamma_{0\mu}$, which will then have to be corrected for kinetic-energy effects[3] with the kineton $\gamma\cdot K$, K being an arbitrary linear combination of the momenta k_1 and k_2. We have

$$\Gamma_{0\mu} = -\tfrac{1}{4} g m \Phi_{\mu B}{}^{A}(k) [\bar{M}_{1A}{}^{C}(k_1) \bar{M}_{2C}{}^{B}(k_2) + \bar{M}_{2A}{}^{C}(k_2) \bar{M}_{1C}{}^{B}(k_1)], \qquad (2a)$$

where

$$\bar{M}_{iA}{}^{B}(k_i) = \left[\left(1-\frac{\gamma\cdot k_i}{m_i}\right)\gamma_5\right]_\alpha^\beta P_a{}^b + \left[\left(1-\frac{\gamma\cdot k_i}{m_i}\right)\gamma\cdot\epsilon\right]_\alpha^\beta V_a{}^b, \qquad (2b)$$

m being the central mass of the meson $(6,6^*)$ supermultiplet (and is set equal to 0.6 BeV), g the dimensionless coupling constant, and $P_a{}^b$ and $V_a{}^b$ the $SU(3)$ nonet matrices for pseudoscalar and vector mesons, respectively:

$$P = \begin{pmatrix} \dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta}{\sqrt{6}} + \dfrac{X^0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta}{\sqrt{6}} + \dfrac{X^0}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\dfrac{2}{\sqrt{6}}\eta + \dfrac{X^0}{\sqrt{3}} \end{pmatrix}, \qquad (2c)$$

$$V = \begin{pmatrix} \dfrac{\rho^0+\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\dfrac{\rho^0+\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}. \qquad (2d)$$

The virtual vector-meson line in Fig. 1, including its link

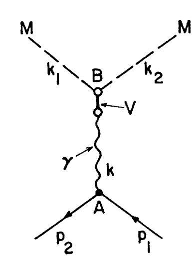

FIG. 1. Feynman diagram for electron-positron and annihilation into two-meson final states. (γ=photon; $V=\rho^0$, ω^0, or ϕ; $M=0^-$ or 1^- meson.)

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[1] Construction of storage rings is in progress in Stanford, Frascati, and Orsay. See, e.g., R. Gatto, in *Springer Tracts in Modern Physics*, edited by G. Höhler (Springer-Verlag, Berlin, 1965), Vol. 39, p. 106.

[2] B. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters **13**, 698 (1964); *ibid.* **14**, 48 (1965); B. Sakita and K. C. Wali, *ibid.* **14**, 404 (1965); R. Delbourgo, A. Salam, and J. Strathdee, Proc. Roy. Soc. (London) **A284**, 146 (1965); M. Bég and A. Pais, Phys. Rev. Letters **14**, 267 (1965).

[3] P. G. O. Freund, Phys. Rev. Letters **14**, 803 (1965); R. Oehme, *ibid.* **14**, 664, 866 (1965).

to the photon, is represented by

$$\Phi_{\mu B}{}^{A}(k)=\frac{\gamma_V}{M^2-k^2}\left[\left(1+\frac{\gamma\cdot k}{M}\right)\gamma_\mu\right]_b^a Q_\beta{}^\alpha, \qquad (2e)$$

where

$$Q=\tfrac{1}{3}\begin{pmatrix}2 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{pmatrix},$$

M is the central mass of the vector meson, and γ_V is the transition matrix element of the vector meson into the photon in the intermediate state.

At this point it is necessary to discuss the kinetic-energy effects. These correspond to kineton emission to all orders, or equivalently, to the decomposition of the $(6,6^*)$ representation of the rest symmetry $U(6)\times U(6)$ into the reducible $35+1$ representation of the collinear group $SU(6)_W$. There are four new types of terms to be added to $\Gamma_{0\mu}$:

$$\begin{aligned}\Gamma_{1\mu}&=A_1\,\mathrm{Tr}(\gamma\cdot K\Phi_\mu)\,\mathrm{Tr}(\bar{M}_1\bar{M}_2),\\ \Gamma_{2\mu}&=A_2\,\mathrm{Tr}(\gamma\cdot k_2\bar{M}_1)\,\mathrm{Tr}(\bar{M}_2\Phi_\mu),\\ \Gamma_{3\mu}&=A_3\,\mathrm{Tr}(\gamma\cdot k_1\bar{M}_2)\,\mathrm{Tr}(\bar{M}_1\Phi_\mu),\end{aligned} \qquad (3)$$

and

$$\Gamma_{4\mu}=A_4\,\mathrm{Tr}(\gamma\cdot K\Phi_\mu)\,\mathrm{Tr}(\gamma\cdot k_2\bar{M}_1)\,\mathrm{Tr}(\gamma\cdot k_1\bar{M}_2).$$

Of these $\Gamma_{1\mu}=\Gamma_{4\mu}=0$ because of Φ_μ being a pure $SU(3)$ octet. The terms $\Gamma_{2\mu}$ and $\Gamma_{3\mu}$ violate charge-conjugation invariance as they contain couplings involving three neutral vector mesons ($\rho^0\rho^0\rho^0,\rho^0\rho^0\omega,\omega\omega\omega,\omega\omega\phi,\phi\phi\phi$), and we are left with $\Gamma_{0\mu}$ alone. *We thus see that for the processes (1) the kineton corrections are identically zero and the predictions of $SU(6)_W$ are identical with those of $M(12)$ symmetry.*

We now expand (2), and separate it into three terms

$$\Gamma_{0\mu}=\Gamma_{0\mu}{}^{(1)}+\Gamma_{0\mu}{}^{(2)}+\Gamma_{0\mu}{}^{(3)}.$$

They are

(1) the vertex function for the two pseudoscalar mesons,

$$\Gamma_{0\mu}{}^{(1)}=eF_1(k^2)(k_2-k_1)_\mu, \qquad (4a)$$

where

$$F_1(k^2)=g\frac{m}{m_2}\left(1+\frac{k^2}{2m_1M}\right)\left(\frac{\gamma_V}{M^2-k^2}\right)\mathrm{Tr}(Q[P_1,P_2]);$$

(2) the vertex function for V, V,

$$\Gamma_{0\mu}{}^{(2)}=eF_2(k^2)\left\{-q_\mu(\epsilon_1\cdot\epsilon_2)\left(1+\frac{k^2}{2m_1{}^2}\right)+3\epsilon_{1\mu}(\epsilon_2\cdot k)-3\epsilon_{2\mu}(\epsilon_1\cdot k)+\frac{1}{m_1{}^2}q_\mu(\epsilon_1\cdot k)(\epsilon_2\cdot k)\right\}, \qquad (4b)$$

where

$$F_2(k^2)=g\left(\frac{m}{M}\right)\left(\frac{\gamma_V}{M^2-k^2}\right)\mathrm{Tr}(Q[V_1,V_2]),\quad q=k_1-k_2;$$

(3) the vertex function for PV,

$$\Gamma_{0\mu}{}^{(3)}=\frac{e}{m}F_3(k^2)\epsilon_{\nu\alpha\beta\mu}k_{3\nu}\epsilon_{2\alpha}k_{1\beta}, \qquad (4c)$$

where

$$F_3(k^2)=g\left(\frac{m^2}{m_1M}+\frac{m^2}{m_1m_2}+\frac{m^2}{m_2M}\right)\left(\frac{\gamma_V}{M^2-k^2}\right)\times\mathrm{Tr}(Q\{P_1,V_2\}+Q\{V_1,P_2\}).$$

The T-matrix T_X for a process X of the type (1), involving two mesons as final state in general can be written as

$$T_X=e\left[\frac{1}{(2\pi)^{3/2}}\right]^4\left(\frac{m_e{}^2}{4E_e{}^2\omega_1\omega_2}\right)^{1/2}\times\bar{v}(p_2)\gamma_\mu u(p_1)\frac{1}{k^2}\Gamma_{0\mu x}{}^{(i)}, \qquad (5a)$$

where the index X means that the corresponding term has to be extracted from the $SU(3)$ trace in (4). It follows that the differential cross section is

$$\frac{d\sigma}{d\Omega}=\frac{e^2}{(64\pi)^2}\frac{|\mathbf{k}|}{E_e{}^7}|M|^2, \qquad (5b)$$

where

$$\begin{aligned}|M|^2=\sum_{\text{pol.}}\big[&(\Gamma_{0\mu X}{}^{(i)}\cdot\Gamma_{0\mu X}{}^{(i)})(p_{1\nu}\cdot p_{2\nu}-m_e{}^2)\\&-(p_{1\mu}\cdot\Gamma_{0\mu X}{}^{(i)})(p_{2\nu}\cdot\Gamma_{0\nu X}{}^{(i)})\\&-(p_{1\mu}\cdot\Gamma_{0\mu X}{}^{(i)})(p_{2\nu}\cdot\Gamma_{0\nu X}{}^{(i)})\big],\end{aligned}$$

$\mathbf{k}$, E_e are the momentum of the outgoing meson, and the energy of the electron, respectively, in the center-of-mass system. We have averaged over the initial polarization states of both the electron and the positron; the summation in $|M|^2$ indicates sum over all polarization states in the final state. Utilizing the vertices in (4a), (4b), and (4c), we obtain the following formulas[4]:

$$\frac{d\sigma(PP)}{d\Omega}=\frac{\alpha^2}{32}|F_1(k^2)|^2\frac{k_P{}^3}{E_e{}^5}\sin^2\theta, \qquad (6a)$$

for the final state of two pseudoscalar mesons;

$$\frac{d\sigma(VV)}{d\Omega}=\frac{\alpha^2}{32}|F_2(k^2)|^2\frac{k_V{}^3}{E_e{}^5}\left\{18\left(\frac{E_e}{m_2}\right)^2(1+\cos\theta)+\sin^2\theta\left(3+\frac{4E_e{}^2}{m_2{}^2}+44\frac{E_e{}^4}{m_2{}^4}\right)\right\}, \qquad (6b)$$

[4] Some of them can be found in the paper of N. Cabbibo and R. Gatto, Phys. Rev. 124, 1577 (1961).

for the final state of two vector mesons; and

$$\frac{d\sigma(VP)}{d\Omega}=\frac{\alpha^2}{32}|F_3(k^2)|^2\frac{k_V{}^3}{E_e{}^3}\frac{1}{m^2}(2-\sin^2\theta)\,, \qquad (6c)$$

for the final state of one pseudoscalar meson and one vector meson, where $k_{P(V)}$ denote the momentum of pseudoscalar (vector) meson, E_e, the energy of the electron in the center-of-mass system, and $\alpha=e^2/4\pi=1/137$. We have neglected the terms of order m_e/E_e.

With the aid of the above formulas, we shall discuss the results under various assumptions.

III. PREDICTIONS FROM SYMMETRY AND MESON-POLE MODEL

In our pole-model scheme, $M(12)$ has shown its power by relating all processes which we consider here with only one coupling constant g even after inclusion of kinetic-energy effects [breakdown to $SU(6)_W$]. In extracting predictions from our model, we shall first consider those predictions that emerge without the use of full $M(12)$ [$SU(6)_W$] but can be obtained from $SU(3)$ and charge-conjugation invariance alone. They are

(a) $e^++e^-\to P+P$,

$$M(e^++e^-\to\pi^0+\pi^0)=0\,, \qquad (7a)$$

$$M(e^++e^-\to K_1+K_2)=0\,, \qquad (7b)$$

$$M(e^++e^-\to X+X)=0\,, \qquad (7c)$$

$$M(e^++e^-\to\pi^++\pi^-)=M(e^++e^-\to K^++K^-)\,; \qquad (7d)$$

(b) $e^++e^-\to V+V$,

$$M(e^++e^-\to\bar{K}^{*0}+K^{*0})=0\,, \qquad (8a)$$

$$M(e^++e^-\to\rho^0+\rho^0)=0\,, \qquad (8b)$$

$$M(e^++e^-\to\omega+\omega)=0\,, \qquad (8c)$$

$$M(e^++e^-\to\phi+\phi)=0\,, \qquad (8d)$$

$$M(e^++e^-\to\rho^++\rho^-)=M(e^++e^-\to K^{*+}+K^{*-})\,; \qquad (8e)$$

(c) $e^++e^-\to V+P$,

$$\begin{aligned}M(e^++e^-\to\rho^0+\pi^0)&=(1/\sqrt{3})M(e^++e^-\to\rho^0+\eta)\\&=\tfrac{1}{3}M(e^++e^-\to\omega+\pi^0)=\sqrt{3}M(e^++e^-\to\omega+\eta)\\&=M(e^++e^-\to\rho^++\pi^-)=M(e^++e^-\to K^{*+}+K^-)\\&=M(e^++e^-\to\rho^-+\pi^+)\\&=-\tfrac{1}{2}M(e^++e^-\to K^{*0}+\bar{K}^0)\\&=M(e^++e^-\to K^{*-}+K^+)\\&=-\tfrac{1}{2}M(e^++e^-\to\bar{K}^{*0}+K^0)\\&=(\tfrac{1}{4}\sqrt{6})M(e^++e^-\to\phi+\eta)\\&=(1/\sqrt{6})M(e^++e^-\to\rho^0+X)\\&=(\tfrac{1}{2}\sqrt{6})M(e^++e^-\to\omega+X)\\&=\tfrac{1}{2}\sqrt{3}M(e^++e^-\to\phi+X)\,;\end{aligned} \qquad (9)$$

TABLE I. The Clebsch-Gordan coefficients C_ρ, C_ω, and C_ϕ of the form factors and the threshold energies of all processes.

		Processes	C_ρ	C_ω	C_ϕ	Threshold (E_e) BeV
(a)		$\pi^+\pi^-$	1	0	0	0.1396
		K^+K^-	1/2	1/6	1/3	0.4938
		$K^0\bar{K}^0$	−1/2	1/6	1/3	0.4978
(b)		$\rho^+\rho^-$	1	0	0	0.7690
		$K^{*+}K^{*-}$	1/2	1/6	1/3	0.8910
		$K^{*0}\bar{K}^{*0}$	−1/2	1/6	1/3	0.8910
(c)	(1)	$\rho^0\pi^{0\,a}$	0	1/3	0	0.4543
	(2)	$\rho^0\eta$	$1/\sqrt{3}$	0	0	0.6589
	(3)	$\omega\pi^0$	1	0	0	0.4659
	(4)	$\omega\eta$	0	$1/3\sqrt{3}$	0	0.6658
	(5)	$K^{*+}K^{-\,a}$	1/2	1/6	−1/3	0.6924
	(6)	$\bar{K}^{*0}K^{0\,a}$	−1/2	1/6	−1/3	0.6944
	(7)	$\phi\eta$	0	0	$4/3\sqrt{6}$	0.7842
	(8)	ρX	$\sqrt{2}/\sqrt{3}$	0	0	0.8640
	(9)	X	$\sqrt{2}/3\sqrt{3}$	0	0	0.8709
	(10)	ϕX	0	0	$-2/3\sqrt{3}$	0.9893
(d)		$\pi\gamma$	$1/3\sqrt{2}$	$1/3\sqrt{2}$	0	0.0675
		$\eta\gamma$	$1/\sqrt{6}$	$1/9\sqrt{6}$	$-4/9\sqrt{6}$	0.2744
		$X\gamma$	$1/\sqrt{3}$	$1/9\sqrt{3}$	$2/9\sqrt{3}$	0.4795

[a] The other charged states $\rho^+\pi^-$, $\rho^-\pi^+(K^{*-}K^+,K^{*0}\bar{K}^0)$ have the same values as $\rho^0\pi^0$(or $K^{*+}K^-,\bar{K}^{*0}K^0$).

(d) $e^++e^-\to P+\gamma$,

$$\begin{aligned}M(e^++e^-\to\pi^0+\gamma)&=\sqrt{3}M(e^++e^-\to\eta+\gamma)\\&=(\sqrt{3}/2\sqrt{2})M(e^++e^-\to X+\gamma)\,.\end{aligned} \qquad (10)$$

The processes of the type $e^++e^-\to V+\gamma$ are, of course, forbidden by charge-conjugation invariance to lowest order in electromagnetic interactions and will, therefore, not be considered here. It should be remarked that forbiddenness of the processes (7a), (7c), (8b), (8c), and (8d) is due to charge-conjugation invariance. However, we shall show later that the processes (7b) and (8a), $e^++e^-\to K_1+K_2$ and $e^++e^-\to K^{*0}+\bar{K}^{*0}$, which are not forbidden by charge-conjugation invariance will be allowed[5] if we introduce the $SU(3)$ breaking splittings of the positions of the vector-meson poles.

Now we shall utilize full $M(12)$ invariance of the vertex B to calculate explicitly the cross sections of all processes (1). We shall use the physical masses of ρ, ϕ, ω and their finite widths in the vector-meson pole factors,[6] and we shall use the physical masses of mesons in the

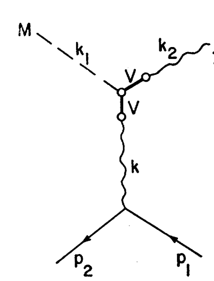
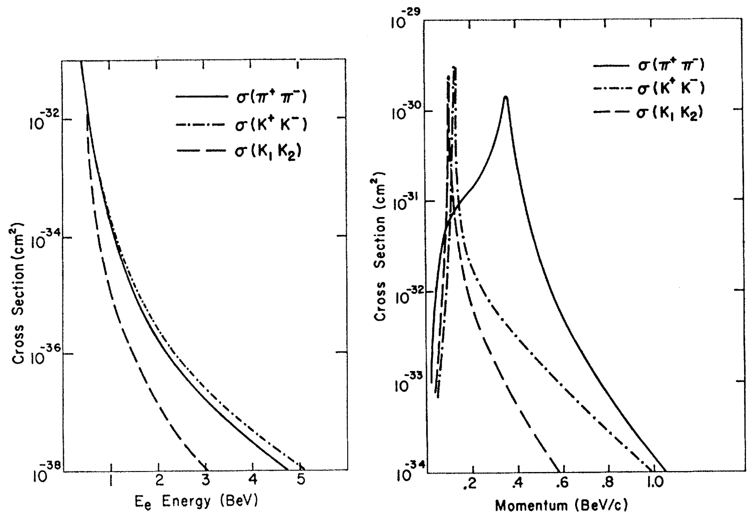


FIG. 2. Feynman diagram for electron-positron annihilation into one meson and one photon. (γ=photon; $V=\rho^0$, ω^0, or ϕ; M=03 meson.)

[5] B. Barrett and T. N. Truong, Phys. Rev. **137**, B679 (1965).
[6] M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961).



(a) (b)

FIG. 3. (a) Total cross sections for $e^+ + e^- \to P+P$ processes as a function of the *energy* of the electron in the center-of-mass system from (1–6 BeV). (b) Total cross sections for $e^+ + e^- \to P+P$ processes as a function of the *momentum* of the meson P in the c.m. system (0–1 BeV/c).

interaction and in the phase-space calculation. But we shall consider all meson masses as degenerate in evaluating the coupling constants. The form factors in (4a), (4b), and (4c) then become

$$F_i(k^2)=g_i\left[C_\rho\frac{m_\rho^2}{m_\rho^2-k^2}+C_\omega\frac{m_\omega^2}{m_\omega^2-k^2}+C_\phi\frac{m_\phi^2}{m_\phi^2-k^2}\right], \quad (11)$$

where $i=1, 2, 3$, $g_1=1$, $g_2=1$, $g_3=3$, and $m_V=m_V^0+i\Gamma_V/2$ ($V=\rho, \omega, \phi$). We use the experimental values[7] of $m_\rho^0=0.769$ BeV, $m_\omega^0=0.7827$ BeV, $m_\phi^0=1.0195$ BeV, $\Gamma_\rho=0.112$ BeV, $\Gamma_\omega=0.009$ BeV, and $\Gamma_\phi=0.0031$ BeV. The values of C_ρ, C_ω, C_ϕ for each process are listed in Table I. The conservation of charge requires the form factors for $e^+ + e^- \to \pi^+ + \pi^-$ and $e^+ + e^- \to K^+ + K^-$ processes to be normalized to one at $k^2=0$. Hence

$$F_1(0)=g(m/m_2)=g=1.$$

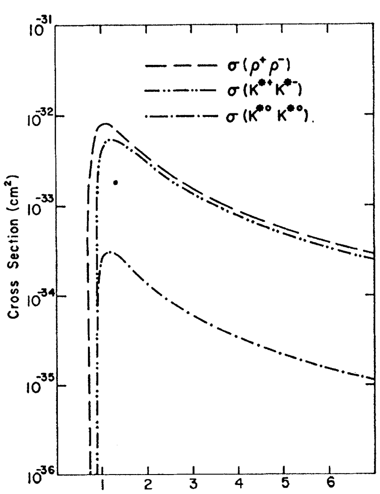


FIG. 4. Total cross sections for $e^+ + e^- \to V+V$ processes as a function of the *energy* of the electron in the center-of-mass system.

We note that it is necessary to consider masses as degenerate, $m_2=m$, in evaluating the kinematical factors appearing in the $M(12)$ expression for the coupling constants. Otherwise, one comes into conflict with the conservation of charge. This explains the simple numerical values of g_i in Eq. (11).

From (6a), (6b), and (6c) we have

$$\sigma(PP)=\frac{\pi\alpha^2}{12m_\pi^2}|F_1(k^2)\frac{k_P^3 m_\pi^2}{E_e^5},$$

$$\sigma(VV)=\frac{\pi\alpha^2}{12m_\pi^2}(\tfrac{3}{4})|F_2(k^2)|^2\frac{k_V^3 m_\pi^2}{E_e^5}\times\left(4+\frac{160}{3}\frac{E_e^2}{m_1^2}+\frac{176}{3}\frac{E_e^4}{m_1^4}\right), \quad (12)$$

$$\sigma(VP)=\frac{\pi\alpha^2}{12m_\pi^2}|F_3(k^2)|^2\frac{k_V^3}{E_e^3}\frac{2m_\pi^2}{m^2}.$$

For the $e^+ + e^- \to P+\gamma$ processes, we assume the real photon is also coupled through a vector-meson pole as shown in Fig. 2. The cross section is then

$$\sigma(P\gamma)=\left(\frac{\pi\alpha^2}{12m_\pi^2}\right)|F_4(k^2)|^2\frac{k_\gamma^3}{E_e^3}\left(\frac{2m_\pi^2}{m^2}\right), \quad (13)$$

where k_γ is the momentum of the photon in the center-of-mass system and

$$|F_4(k^2)|^2=9\left(\frac{e^2}{4\pi}\right)\Big/\left(\frac{f_{\rho\pi\pi}^2}{4\pi}\right)\times\left|C_\rho\frac{m_\rho^2}{m_\rho^2-k^2}+C_\omega\frac{m_\omega^2}{m_\omega^2-k^2}+C_\phi\frac{m_\phi^2}{m_\phi^2-k^2}\right|^2,$$

$$f_{\rho\pi\pi}^2/4\pi=2.$$

[7] A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. **37**, 633 (1965).

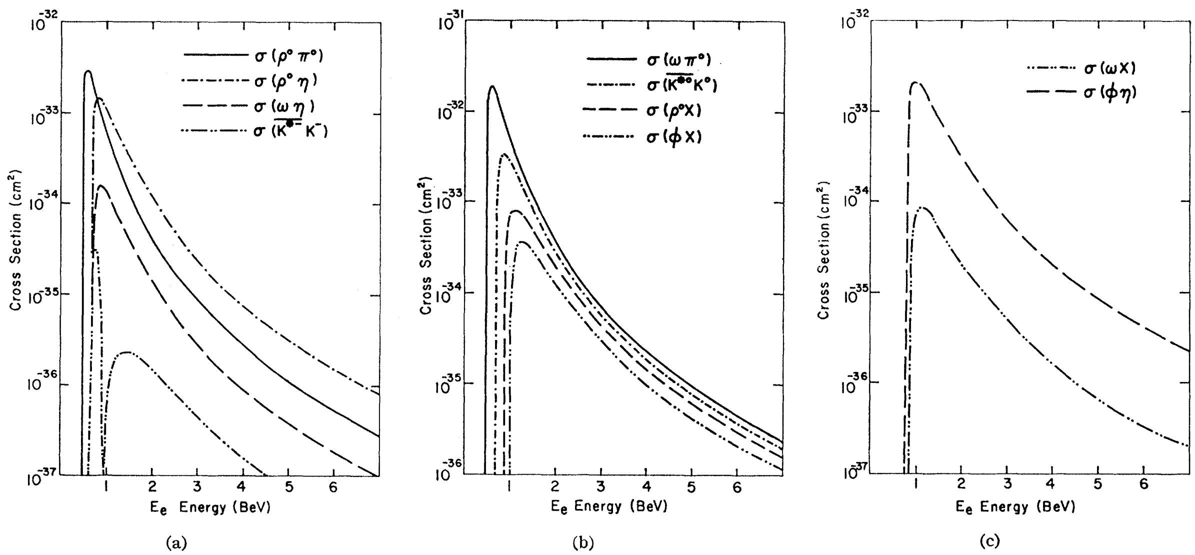


FIG. 5. Total cross sections for $e^+ + e^- \to V + P$ processes as a function of the *energy* of the electron in the center-of-mass system.

The values of C_ρ, C_ω, C_ϕ are also listed in Table I. We have plotted these cross sections for the 19 processes which we have discussed in Figs. 3–6.

We observe certain features which are peculiar to our model. For processes where the thresholds are below the masses of vector mesons (see Table I), i.e., $e^+ + e^- \to P + P$ and $e^+ + e^- \to P + \gamma$, there is a factor $m_V{}^2/\Gamma_V{}^2$ ($=50$ for ρ, 7100 for ω, 10^4 for ϕ) of enhancement in total cross sections when the total energy of the system is equal to the mass of the vector meson. At $E_e = \frac{1}{2} m_\rho$, $\sigma(\pi^+\pi^-) = 1.4$ μb, $\sigma(\pi\gamma) = 0.6$ μb, and $\sigma(\eta\gamma) = 0.2$ μb, and at $E_e = \frac{1}{2} m_\phi$, $\sigma(K^+K^-) \cong 3$ μb, $\sigma(K_1K_2) \simeq 2.5$ μb, and $\sigma(\eta\gamma) \simeq 1$ μb. At higher energy and away from the poles the cross sections drop very rapidly. For example, at $E_e = 2$ BeV $\sigma(\pi^+\pi^-)$ becomes 1.8×10^{-36} cm^2, a factor of 10^{-6} down from the peak value. We wish to emphasize particularly that by using the physical masses of the vector mesons $\sigma(K_1K_2)$ and $\sigma(K^{*0}\bar{K}^{*0})$ are no longer zero as in the exact-symmetry-limit case but have comparable values with other processes at the resonance peaks. At high energy they also drop but do not conform to the exact symmetry limit, as can easily be seen from Eq. (11).

Throughout our work the ω-ϕ mixing angle is fixed by $SU(6)$ as expressed in (2b), and the cross sections of some processes are sensitive to its values. In particular, the $\sigma(K^{*+}K^{*-})$ are suppressed by a factor of 100 with respect to the $\sigma(\bar{K}^{*0}K^0)$ because of destructive interference among ρ, ω and ϕ mesons. With a different ω-ϕ mixing angle, we would expect quite different values from those we have got. To say it in other words, the $e^+ - e^-$ colliding-beam experiments offer a chance to determine the quantities m_V, Γ_V, and the ω-ϕ mixing angle to a high degree of accuracy.

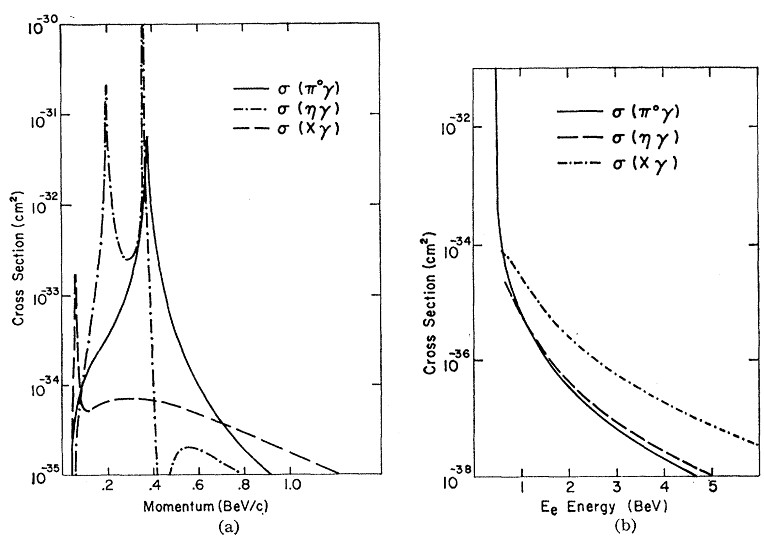


FIG. 6. (a) Total cross sections for $e^+ + e^- \to P + \gamma$ processes as a function of the *energy* of the electron in the center-of-mass system (from 1 to 6 BeV). (b) Total cross sections of $e^+ + e^- \to P + \gamma$ processes as a function of the *momentum* of the meson P in the center-of-mass system (from 0 to 1 BeV/c).

CONCLUSION

In high-energy electron-positron colliding-beam experiments the two-meson production processes offer a chance to test the $M(12)$ and $SU(6)_W$ symmetries as applied to the vector-meson pole model of the hadronic matrix elements of the electromagnetic current. The kinetic-energy effects which break $M(12)$ symmetry are included without changing the general form of the interaction vertex, and the only coupling constant, g itself, is fixed by the normalization of the form factor, $F_1(0)=1$. These is no adjustable variable in the whole formulation. Our fit is a zero-parameter fit. The cross sections are very sensitive to the masses and widths of the vector mesons. There is an enhancement of a factor m_V^2/Γ_V^2 when the total energy of the e^+-e^- pair equates to the rest masses of the vector mesons. We have neglected higher order terms in the electromagnetic interaction where spin 2^+ (e.g., f^0) resonances could possibly play a role.[8]

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[8] See Ref. 1.

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Asymptotic Behavior of the Inelasticity Parameter

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We examine the asymptotic behavior of the inelasticity parameter η under the assumption that an infinite number of 2-particle channels open up as energy becomes infinite. We investigate in detail three specific models, which can be handled analytically, and find that in general $\eta(\infty)=1$ in these models. We speculate that this may in fact be the general asymptotic behavior of η in the physical situations. We also observe that a purely imaginary elastic-scattering amplitude can be realized even as $\eta\to 1$.

I. INTRODUCTION

RECENTLY, dynamical calculations based on partial-wave dispersion relations have played an important role in many discussions on the strong interactions. In such discussions the problem of inelastic scattering is usually handled either by a coupled set of partial wave dispersion relations in the form of matrix N/D equations[1] or through the Frye-Warnock equations.[2,3] The latter procedure has the advantage that unlike the matrix N/D equations one is not limited only to two-body production channels. On the other hand the partial-wave dispersion relations of Frye and Warnock involve the inelasticity parameter η (where $\eta=e^{-2\delta_I}$) inside the integrals on the right and it becomes pertinent to require the asymptotic behavior of η at at infinity. Many calculations dealing with the solutions of the Frye-Warnock equations use the experimental values of η for known energies and then extrapolate it smoothly to some constant value between 0 and 1 (since $1>\eta>0$) at infinity.[4] The reason for doing this is based on the reasonable belief that even though the contribution to $(1-\eta)$ from an individual inelastic channel goes to zero, an infinite number of channels open up as we go to infinite energy and may yield $\eta(\infty)<1$. The purpose of this paper is to examine this statement closely. The general infinite-channel problem, of course, is much too complicated to deal with analytically and we shall look at models which are sufficiently interesting and yet simple enough to investigate. We examine three models, all of which are systems of N coupled two-body channels, where N is allowed to be an arbitrary function of energy. In particular, N may become infinite as an arbitrary power of the energy.[5] In our first model we take a factorizable form for the scattering amplitudes, and the various quantities of interest are explicitly exhibited in detail. As a specific example of this model we discuss the Zachariasen model in the Appendix. For our second model we assume only that the reaction matrix is either positive- or negative-definite (the first model is a specific example of this type). In the third model we examine the consequences of a random-phase approximation[6] to the reaction matrix.

We observe that in our model $\eta(\infty)=1$, and we speculate in conclusion that it may generally be true.

[1] J. D. Bjorken, Phys. Rev. Letters 4, 473 (1960).
[2] G. Frye and R. L. Warnock, Phys. Rev. **130**, 478 (1963).
[3] The two procedures are not always equivalent. The coupled N/D system can do more drastic things than the Frye-Warnock equations. See e.g., M. Bander, P. Coulter, and G. Shaw, Phys. Rev. Letters **14**, 270 (1965).
[4] See e.g., P. W. Coulter, A. Scotti, and G. L. Shaw, Phys. Rev. **136**, B1379 (1964); P. W. Coulter and G. Shaw, *ibid.* **138**, B1273 (1965).
[5] If we assume K fundamental thresholds (we have in mind the possibility of producing numbers of $\pi\pi$, $K\bar{K}$, $N\bar{N}$ pairs, etc.), then $N(s)$ behaves as $s^{K/2}$ as energy becomes large.
[6] See e.g., M. Bander and G. Shaw, Phys. Rev. **139**, B956 (1965).