Nonleptonic Decays of Hyperons in Broken $SU(3)\times SU(3)$

J. SCHECHTER*
The Enrico Fermi Institute for Nuclear Studies, The University of Chicago, Chicago, Illinois

AND

Y. UEDA†
Department of Physics, Indiana University, Bloomington, Indiana

(Received 11 February 1966)

We discuss the hyperon nonleptonic decays in the broken chiral $SU(3)\times SU(3)$ symmetry scheme. We obtain a new sum rule involving both the S- and P-wave amplitudes which is in rough agreement with experiment. Also by using this group, together with the generalized Sugawara-Suzuki method, we are able to prove octet dominance for these decays.

I. INTRODUCTION

IN this paper we shall investigate the nonleptonic decays of hyperons in the chiral $SU(3)\times SU(3)$ symmetry scheme.[1] Our motivation for using this particular group comes from the fact that it is generated by the weak currents and hence may be assumed to have a special significance for weak-interaction processes. Furthermore, the algebra of this group has been recently[2] used to obtain the first good estimate of the β-decay axial-vector coupling constant.

The general method of treatment will be that of perturbation theory. This would seem to be very plausible because of its success in $SU(3)$ and because of the close relationship between the algebra of currents approach and the group-theoretical approach.[3] In a previous paper[4] we have investigated the hadron currents of the leptonic decays by this method and found a generalization[5] of the Ademollo-Gatto theorem[6] which seems to be in agreement with experiment.

We shall calculate the nonleptonic hyperon decays in the first order of perturbation under which the $SU(3)\times SU(3)$ symmetry is brought down to $SU(3)$. We find that the results obtained in this way are more consistent than those obtained if the $SU(3)\times SU(3)$ symmetry is taken to be exact. Since the masses of a fundamental triplet must be zero in the exact $SU(3)\times SU(3)$ limit[7] this might be interpreted as an effect of finite masses for these supposed particles.

Two somewhat different though presumably compatible approaches will be taken. The first involves the construction of an effective Yukawa interaction having the appropriate $SU(3)\times SU(3)$ transformation property. For the most reasonable choice of baryon and meson representations, we find the following new relation:

$$A(\Lambda_-^0)-B(\Lambda_-^0)=(1/\sqrt{3})[A(\Sigma_0^+)-B(\Sigma_0^+)], \quad (1)$$

where A and B are the (S- and P-wave) coefficients in the effective Hamiltonian,

$$H_{\text{eff}}=i\bar{N}_f(A+B\gamma_5)N_i\pi. \quad (2)$$

It is interesting to note that Eq. (1) was derived from the current-current picture *without* assuming octet dominance. There is rough agreement with experiment; numerically we have[8] for Eq. (1):

$$27.3=\begin{cases}21.4\\ 8.8\end{cases}, \quad (3)$$

where there are two possibilities on the right-hand side because the fitting of the experimental data for the Σ_0^+ decay is not unique.

Our second line of approach uses the method of Sugawara and Suzuki[9] to obtain a reduced form for the nonleptonic-decay matrix element. This "reduced" matrix element is then evaluated by broken $SU(3)\times SU(3)$. In this manner we are able to show with the same choice of baryon representation needed to obtain Eq. (1) that the usual $\Delta I=\frac{1}{2}$ and Lee-Sugawara[10] relations may be obtained from the current-current picture *without* assuming octet dominance.

In Sec. II, the effective-Hamiltonian method is considered for our favored baryon representation. The Sugawara-Suzuki method is discussed in Sec. III. Finally, in Sec. IV, we investigate other choices of baryon representations.

---

* Supported by the U. S. Atomic Energy Commission.
† Supported by the National Science Foundation.

[1] M. Gell-Mann, Physics 1, 63 (1964); P. G. O. Freund and Y. Nambu, Ann. Phys. (N. Y.) 32, 201 (1965); R. E. Marshak, N. Mukunda, and S. Okubo, Phys. Rev. 137, B698 (1965).

[2] W. Weisberger, Phys. Rev. Letters 14, 1047 (1965); S. Adler, *ibid.* 14, 1051 (1965).

[3] See for example, S. Okubo, Phys. Letters 17, 172 (1965).

[4] J. Schechter and Y. Ueda, Phys. Rev. 144, 1338 (1966).

[5] This result has been independently obtained by G. S. Guralnik, V. S. Mathur, and L. K. Pandit, University of Rochester Report No. UR-875-114 (unpublished).

[6] M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1964).

[7] For example, R. E. Marshak, N. Mukunda, and S. Okubo, Ref. 1.

[8] We used (in units of $\mu^{-1/2}$ sec$^{-1/2}$) $A(\Lambda_-^0)=3.3\times10^{-7}$, $B(\Lambda_-^0)=-24\times10^{-7}$, $A(\Sigma_0^+)=-1.9\times10^{-7}$, and $B(\Sigma_0^+)=-39\times10^{-7}$ for the upper case in Eq. (3), and $A(\Sigma_0^+)=-3.8\times10^{-7}$ and $B(\Sigma_0^+)=-19\times10^{-7}$ for the lower case. These numbers were computed from the data of Rosenfeld *et al.*, Rev. Mod. Phys. 37, 633 (1965).

[9] H. Sugawara, Phys. Rev. Letters 15, 870 and 997 (1965); M. Suzuki, *ibid.* 15, 986 (1965); Y. Hara, Y. Nambu, and J. Schechter, *ibid.* 16, 380 (1966).

[10] H. Sugawara, Progr. Theoret. Phys. (Kyoto) 31, 213 (1964); B. W. Lee, Phys. Rev. Letters 12, 83 (1964).

II. THE EFFECTIVE-HAMILTONIAN METHOD

The notation and nomenclature to be used has been discussed in our previous paper[4] so will not be repeated here. The choice of baryon representation among [(3,3*),(3*,3)], [(8,1),(1,8)], and [(6,3),(3,6)] was also discussed and the first was seen to be slightly more promising. We shall use it in this section and later observe that the choice of either [(8,1),(1,8)] or [(6,3),(3,6)] would contradict experiment in first order of perturbation. As for the pseudoscalar mesons, we shall always use the nonet representation [(3,3*)-(3*,3).] There are three reasons for this choice:

(i) Nine pseudoscalar mesons are observed in nature.
(ii) The Goldberger-Treiman relation is satisfied in lowest order with this choice, as pointed out by Gell-Mann.[1]
(iii) Good agreement with the mass formula is obtained.[11]

We shall assume the current-current picture of weak interactions. Furthermore, we will also take the point of view that the current transforms as the (8,1) representation of $SU(3)\times SU(3)$. In terms of a fundamental triplet, this is of the form

$$(J_b{}^a)_\lambda = i\bar{\psi}_a\gamma_\lambda(1+\gamma_5)\psi_b\,,$$

where a and b may take on the values (1,2,3). The weak Hamiltonian transforms as the symmetric product of two currents and hence only contains interesting terms of the form (8,1) or (27,1). For the most part we shall neglect the (27,1) part in this section. This will be justified in the following section.

Let us first construct an effective nonderivative Yukawa interaction which transforms according to (8,1) and which takes account of an $SU(3)\times SU(3)$ breaking like

$$\sum_{\mu=1}^{3}(T_\mu{}^{\mu'}+T_{\mu'}{}^{\mu})\,.$$

In the previous[4] 2-component spinor notation we write for an (8,1) "Yukawa" form:

$$\begin{aligned}Y_b{}^a=\epsilon_{ndb}\epsilon^{e'l'k'}\{&R_1f_{e'}{}^n\sigma_2g_{l'}{}^dM_{k'}{}^a\\&+R_2\bar{f}_{e'}{}^n\sigma_2g_{l'}{}^aM_{k'}{}^d+R_3\bar{f}_{e'}{}^a\sigma_2g_{l'}{}^nM_{k'}{}^d\}\\&+\sum_{\mu=1}^{3}\{S_1\bar{f}_{\mu'}{}^a\sigma_2g_{c'}{}^\mu M_b{}^{c'}+S_2f_{c'}{}^a\sigma_2g_{\mu'}{}^\mu M_b{}^{c'}\\&+S_3\bar{f}_{\mu'}{}^\mu\sigma_2g_{c'}{}^aM_b{}^{c'}+S_4\bar{f}_{c'}{}^\mu\sigma_2g_{\mu'}{}^aM_b{}^{c'}\\&+S_5\delta_b{}^\mu\bar{f}_{\mu'}{}^a\sigma_2g_{k'}{}^cM_c{}^{k'}+S_6\delta_b{}^\mu\bar{f}_{k'}{}^a\sigma_2g_{\mu'}{}^cM_c{}^{k'}\\&+S_7\delta_b{}^\mu\bar{f}_{\mu'}{}^c\sigma_2g_{k'}{}^aM_c{}^{k'}+S_8\delta_b{}^\mu\bar{f}_{k'}{}^c\sigma_2g_{\mu'}{}^aM_c{}^{k'}\}\,,\end{aligned}\quad(4)$$

where the R's are arbitrary real coefficients corresponding to the unbroken case[12] and the S's are real coefficients for the broken case. In terms of the $SU(3)\times SU(3)$ meson tensors $M_d{}^{c'}$ and $M_{d'}{}^c$, the pseudoscalar-meson[13] fields $\pi_d{}^c$ are given by

$$\pi_d{}^c=(i/\sqrt{2})(M_d{}^{c'}-M_{d'}{}^c)\,.\quad(5)$$

In terms of $Y_b{}^a$ the effective nonleptonic Hamiltonian is given by

$$H_{\mathrm{eff}}=Y_3{}^2+Y_2{}^3+\mathrm{H.c.}\quad(6)$$

In four-component form the relevant part of the S_1 term, for example, may be written[4]

$$(i/2\sqrt{2})S_1\bar{N}_\mu{}^a(1-\gamma_5)N_c{}^\mu\pi_b{}^c\,.$$

Using Eq. (6), we derive Eq. (1), the six $\Delta I=\frac{1}{2}$ relations,

$$A,\ B(\Sigma_+{}^+)-A,\ B(\Sigma_-{}^-)=\sqrt{2}A,\ B(\Sigma_0{}^+)\,,\quad(7a)$$

$$A,\ B(\Lambda_-{}^0)+\sqrt{2}A,\ B(\Lambda_0{}^0)=0\,,\quad(7b)$$

$$A,\ B(\Xi_-{}^-)+\sqrt{2}A,\ B(\Xi_0{}^0)=0\,;\quad(7c)$$

and the S-wave Lee-Sugawara relation,

$$A(\Lambda_-{}^0)+2A(\Xi_-{}^-)=\sqrt{3}A(\Sigma_0{}^+)\,.\quad(8)$$

If we were to consider only the unbroken terms in Eq. (4) we would obtain the following additional results, which are in clear disagreement with the experiment:

$$A(\Sigma_+{}^+)+B(\Sigma_+{}^+)=0\,,\quad(9a)$$

$$A(\Sigma_-{}^-)+B(\Sigma_-{}^-)=0\,,\quad(9b)$$

$$A(\Lambda_-{}^0)+B(\Lambda_-{}^0)=(-4/\sqrt{6})A(\Sigma_+{}^+)\,,\quad(9c)$$

$$A(\Xi_-{}^-)+B(\Xi_-{}^-)=+(4/\sqrt{6})A(\Sigma_+{}^+)\,.\quad(9d)$$

Thus it seems that it is essential to take account of the $SU(3)\times SU(3)$ breaking interaction.

Next we look at the effects of adding a (27,1) contribution to H_{eff}. This will be of the form

$$(Y_{31}{}^{21}+Y_{21}{}^{31})+\mathrm{H.c.}\,,\quad(10a)$$

where

$$\begin{aligned}Y_{bd}{}^{ac}=\sum_{\mu=1}^{3}\{&K_1[\delta_b{}^\mu\bar{f}_{\mu'}{}^a\sigma_2g_{e'}{}^cM_d{}^{e'}+\delta_b{}^\mu\bar{f}_{\mu'}{}^c\sigma_2g_{e'}{}^aM_d{}^{e'}\\&+\delta_d{}^\mu\bar{f}_{\mu'}{}^a\sigma_2g_{e'}{}^cM_b{}^{e'}+\delta_d{}^\mu\bar{f}_{\mu'}{}^c\sigma_2g_{e'}{}^aM_b{}^{e'}]\\&+K_2[\delta_b{}^\mu\bar{f}_{e'}{}^a\sigma_2g_{\mu'}{}^cM_d{}^{e'}+\delta_d{}^\mu\bar{f}_{e'}{}^a\sigma_2g_{\mu'}{}^cM_b{}^{e'}\\&+\delta_b{}^\mu\bar{f}_{e'}{}^c\sigma_2g_{\mu'}{}^aM_d{}^{e'}+\delta_d{}^\mu\bar{f}_{e'}{}^c\sigma_2g_{\mu'}{}^aM_b{}^{e'}]\}\,,\end{aligned}\quad(10b)$$

where K_1 and K_2 are arbitrary real constants.

---

[11] R. E. Marshak, S. Okubo, and J. Wojtaszek, Phys. Rev. Letters **15**, 463 (1965).

[12] J. Iizuka and Y. Miyamoto, Tokyo University of Education report (unpublished), have discussed the *unbroken* case on the assumption of *RP* invariance. Here we investigate the *broken* case using only *CP* invariance. We neglect, as usual, the effects of final state interactions. Thus our *CP*-invariant effective amplitude will be Hermitian if the over-all phase is properly chosen.

[13] The scalar mesons are given by $S_d{}^c=(1/\sqrt{2})(M_d{}^{c'}+M_{d'}{}^c)$. Presumably their masses are too high to permit them to take part in nonleptonic decay processes.

In writing Eq. (10a) we have not assumed the existence of neutral currents. Also we have not subtracted the trace since this part was calculated in Eq. (4). With this additional contribution we find that Eqs. (7) and (8) do not necessarily hold but that, interestingly enough, Eq. (1) is still valid.

Finally, we investigate the situation when it is assumed that the effective Hamiltonian is of derivative coupling[14] type, i.e.

$$H_{\text{eff}}=i\bar{N}_f\gamma_\mu(A'+B'\gamma_5)N_i\frac{\partial}{\partial x_\mu}\pi. \quad (11)$$

We find that an appropriate $SU(3)\times SU(3)$ form cannot be constructed in the unbroken case for derivative coupling. In the broken case only the $\Delta I=\frac{1}{2}$ relations are obtained.

III. THE SUGAWARA-SUZUKI METHOD

Using the assumptions of

(i) partially conserved axial vector current,

(ii) current-current form for the weak Hamiltonian with the currents transforming as (8,1)

Sugawara and Suzuki[9] have shown that the matrix elements for nonleptonic hyperon decays,

$$(2k_0)^{1/2}\langle N'(p')\pi_b{}^a(k)|H_W(0)|N(p)\rangle$$

may be expressed in terms of matrix elements taken between *one-particle* baryon states of a quantity $H_W'(0)$ transforming under $SU(3)\times SU(3)$ according to the same representation as $H_W(0)$:

$$\langle N'(p')|H_W'(0)|N(p)\rangle.$$

It has been shown[9] that this "reduced" matrix element must be a true scalar because of CP invariance and hence contributes only to S-wave decays. The argument has been extended[15] to P-wave decays by using the method of Nambu and Shrauner.[16] The result obtained is

$$\begin{aligned}(c/\mu^2)(2k_0)^{1/2}N'\langle(p')\pi_b{}^a(k)|H_W(0)|N(p)\rangle\\ =i\langle N'(p')|H_W'(0)|N(p)\rangle\\ -i\sum_{n=\text{baryon octet}}[\langle N'(p')|\bar{B}_b{}^a|n\rangle\langle n|H_W(0)|N(p)\rangle\\ -\langle N'(p')|H_W(0)|n\rangle\langle n|\bar{B}_b{}^a|N(p)\rangle], \quad (12a)\end{aligned}$$

$$H_W'(0)=[\bar{B}_b{}^a(t=0),H_W(0)], \quad (12b)$$

where

$$\mu=\text{pion mass},$$
$$C=-2Mg_A\mu^2/\sqrt{2}g,$$
$$g_A=1.18,$$
$$g^2/4\pi=14.7.$$

$\bar{B}_b{}^a$ is the axial-vector "charge" previously[4] defined and the extrapolation of the pion mass to zero has been neglected. The first term on the right-hand side of Eq. (12a) (the original Sugawara-Suzuki term) contributes to S-wave decays and the second term to P-wave decays. Using Eq. (12) as well as the assumption of octet dominance, a reasonably good fit to all the hyperon decays in terms of four parameters has been obtained.[15] Furthermore, these four parameters can be related to other processes with fair agreement.

Here we would like to point out that in the broken $SU(3)\times SU(3)$ scheme it is not necessary to assume octet dominance in order to obtain the above results. The reason is that when the baryons are assigned to $[(3,3^*),(3^*,3)]$ and only first-order breaking of $SU(3)\times SU(3)$ is allowed the (27,1) part of either $H_W(0)$ or $H_W'(0)$ has no matrix elements between baryon states. We may see this by noting that (27,1) does not occur in the product

$$(3^*,3)\times(3^*,3)\times[(3^*,3)+(3,3^*)].$$

For the (8,1) part of $\langle N'|H_W(0)|N\rangle$ we have, to first order of perturbation,

$$(S_3{}^2+S_2{}^3)+\text{H.c.}, \quad (13a)$$

where

$$\begin{aligned}S_b{}^a=\sum_{\mu=1}^{3}\{\epsilon_{\mu nb}\epsilon^{k'm'\mu'}[C_1f_{k'}{}^a\sigma_2g_{m'}{}^n+C_2f_{k'}{}^n\sigma_2g_{m'}{}^a]\\ +C_3\epsilon_{pnb}\epsilon^{k'm'\mu'}\delta_\mu{}^a\bar{f}_{k'}{}^p\sigma_2g_{m'}{}^n\}, \quad (13b)\end{aligned}$$

where the C's are arbitrary real constants. In four-component form we have, as required,[15]

$$\langle N'|H_W|N\rangle=D(D_3{}^2+D_2{}^3)+F(F_3{}^2+F_2{}^3), \quad (14)$$

where D and F are arbitrary constants and $D_b{}^a$ and $F_b{}^a$ have been previously defined.[4]

Thus if H_W is written as

$$H_W=H_W(27,1)+H_W(8,1) \quad (15)$$

only the second term contributes on the right-hand side of Eq. (12a) and hence to the nonleptonic hyperon decays in general. The trick used in proving this statement of octet dominance was the elimination of the pion according to the Sugawara-Suzuki method. We note that this argument does not depend on the $SU(3)\times SU(3)$ assignment of the pseudoscalar mesons.

Strictly speaking, two points need to be clarified for the P-wave term of Eq. (12b). The first is whether we should sum over the baryon octet or the baryon nonet in the intermediate states. If we sum over the nonet we

[14] In comparing Eq. (11) with Eq. (2) we use the relations $A=(M_i-M_f)A'$, $B=-(M_i+M_f)B'$.

[15] Y. Hara, Y. Nambu, and J. Schechter, Ref. 9.

[16] Y. Nambu and E. Shrauner, Phys. Rev. **128**, 862 (1962).

find[15] that the amplitude for Σ_+^+ is zero. Thus we must sum over the octet. This is justified if the singlet has an appreciably different mass from the octet members of $[(3,3^*),(3^*,3)]$. The second point is whether or not we should also evaluate the matrix element $\langle N'|\bar{B}_b{}^a|N\rangle$ by $SU(3)\times SU(3)$. If we do so, we find[4] that it is only split into an arbitrary mixture of D-type and F-type terms as required in *second* order of perturbation. However, the first (or zeroth)-order result of pure D-type is not far from the experimental situation.

IV. OTHER POSSIBILITIES

In this section we shall discuss the results obtained with the baryon representations $[(8,1),(1,8)]$ and $[(6,3),(3,6)]$. Both the methods of Secs. II and III will be treated. We shall always work to first order in the $SU(3)\times SU(3)$ breaking interaction and always use the $(3,3^*)$-$(3^*,3)$ assignment for the pseudoscalar mesons. We find that these choices of baryon representation lead to a contradiction with experiment for each case treated.

First, let us consider the baryon assignment $[(8,1),(1,8)]$. We can not construct an effective Yukawa interaction in the unbroken case. In the broken case with nonderivative coupling, we have, assuming octet dominance,

$$A(\Lambda_-^0)-B(\Lambda_-^0)=-(1/\sqrt{3})[3A(\Sigma_0^+)+B(\Sigma_0^+)], \quad (16a)$$

$$A(\Sigma_-^-)=-B(\Sigma_-^-), \quad (16b)$$

as well as the $\Delta I=\frac{1}{2}$ rules and the S-wave Lee-Sugawara relation. Equation (16b) seems to contradict experiment. Under the identical circumstances with derivative coupling, we obtain in addition to the $\Delta I=\frac{1}{2}$ rules the bad relation,

$$A'(\Sigma_+^+)=B'(\Sigma_+^+), \quad (17a)$$

as well as the difference of the S- and P-wave Lee-Sugawara relations:

$$[A'(\Lambda_-^0)-B'(\Lambda_-^0)]+2[A'(\Xi_-^-)-B'(\Xi_-^-)] =\sqrt{3}[A'(\Sigma_0^+)-B'(\Sigma_0^+)]. \quad (17b)$$

If we attempt to use the baryon representation $[(8,1),(1,8)]$ in the Sugawara-Suzuki scheme we find that $\langle N'|H_W|N\rangle$ vanishes to first order of perturbation. To second order of perturbation it contains both (8,1) and (27,1).

Assignment of the baryons to $[(6,3),(3,6)]$[17] and the assumption of an effective nonderivative Yukawa interaction gives the results

$$A(\Sigma_+^+)=-\tfrac{3}{5}B(\Sigma_+^+), \quad (18a)$$

$$B(\Sigma_0^+)=A(\Sigma_0^+)-(4/3\sqrt{2})A(\Sigma_+^+), \quad (18b)$$

$$A(\Lambda_-^0)=+B(\Lambda_-^0), \quad (18c)$$

$$A(\Xi_-^-)=+B(\Xi_-^-), \quad (18d)$$

$$A(\Xi_-^-)+A(\Lambda_-^0)=-\tfrac{1}{2}\sqrt{3}A(\Sigma_0^+)+(\tfrac{1}{3}\sqrt{6})A(\Sigma_+^+), \quad (18e)$$

as well as the $\Delta I=\frac{1}{2}$ rules and the S-wave Lee-Sugawara relation. Octet dominance was assumed in deriving Eqs. (18) but they hold in both zeroth and first order of $SU(3)\times SU(3)$ breaking perturbation. (The first-order breaking does not give anything new.) Their agreement with experiment is not remarkable. Finally if we use the $[(6,3),(3,6)]$ representation in the Sugawara-Suzuki scheme we note that $\langle N'|H_W|N\rangle$ contains both (8,1) and (27,1) to first order of perturbation.

It is interesting to observe that the $[(3,3^*),(3^*,3)]$ baryon representation gives good results to first order in the methods of both Secs. II and III.

We may also consider the possibility of assigning baryons to the representations $[(b,a),(a,b)]$ instead of $[(a,b),(b,a)]$. It can be seen that the final results only differ by a reversal of sign for all the p-wave (B) amplitudes. It is amusing to note that we obtain better agreement for Eq. (1) by using the baryon representation $[(3^*,3),(3,3^*)]$. Equation (3) is then replaced by

$$20.7=\begin{cases}23.6\\13.2\end{cases}. \quad (3')$$

However, the use of $[(3^*,3),(3,3^*)]$ leads in the exact $SU(3)\times SU(3)$ limit to a β-decay weak current of the form $\bar{p}\gamma_\lambda(1-\gamma_5)n$ instead[4] of $\bar{p}\gamma_\lambda(1+\gamma_5)n$. The representations $[(8,1),(1,8)]$ and $[(6,3),(3,6)]$ lead to the more nearly correct β-decay interaction in the symmetry limit.

ACKNOWLEDGMENT

It is a pleasure to thank Professor Y. Nambu for his kind encouragement and helpful comments.

---

[17] Y. Hara, Phys. Rev. **139**, B134 (1965).