where we have used the fact that

$$V_{\rm in}|nM\rangle=V_{\rm in}^{\dagger}|nM\rangle=|nM\rangle=V_{\rm out}|nM\rangle=V_{\rm out}^{\dagger}|nM\rangle\,,$$

evident from definitions. We note also that a similar procedure applies to composite-particle states as

$$\begin{aligned}\langle(nM,ma);\,{\rm out}|(n'M,m'a);\,{\rm in}\rangle&=(\lambda n r_n)^{m/2}(m!)^{-1/2}\langle nM;\,{\rm out}|P_n(A_n)^m|(n'M,m'a);\,{\rm in}\rangle\\&=(\lambda n r_n)^{m/2}(m!)^{-1/2}\langle nM;\,{\rm in}|P_n(A_n)^m|(n'M,m'a);\,{\rm in}\rangle\\&=\langle(nM,ma);\,{\rm in}|(n'M,m'a);\,{\rm in}\rangle\\&=\delta_{nn'}\delta_{mm'}\,.\end{aligned}\qquad\text{(A2)}$$

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Peculiarities of a Free Massless Spin-3/2 Field Theory

CARL M. BENDER* AND BARRY M. MCCOY*
Harvard University, Cambridge, Massachusetts
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The properties of a massless spin-$\frac{3}{2}$ system are investigated. It is found that the energy density does not commute with itself at spacelike separation, does not transform as a tensor, and is not gauge-invariant, although the theory is shown to be relativistically invariant.

I. INTRODUCTION

MASSLESS fields of spin-1 or greater are qualitatively different from their massive counterparts. One such qualitative difference is the restriction to two helicity states. Another is the occurrence of gauge invariance in the massless spin-1 system and general coordinate invariance in the spin-2 system. Furthermore, the gravitational field (and also the linearized massless spin-2 field) exhibits features not found in the massless spin-1 field. For example, the comutator of the energy density with itself does not vanish at spacelike separation.[1,2] It is natural, then, to investigate the intermediate case of spin-$\frac{3}{2}$. In this paper it is found that the energy density neither commutes with itself at spacelike separation (is not causal) nor transforms as the 00 component of a tensor.

This investigation is based on Schwinger's action principle. The notation follows that of Johnson and Sudarshan,[3] who have studied the electromagnetic interactions of the massive spin-$\frac{3}{2}$ field. The equations of motion and commutation relations of the massless field theory will be obtained by setting $m=0$ in the Lagrange function of the massive theory. The usual procedure[4] for identifying the stress energy tensor is not applicable in the massless spin-$\frac{3}{2}$ case because the field does not transform linearly under Lorentz transformations. Instead, the energy density and its commutation relations are found by carefully taking the massless limit of the corresponding massive expressions. These limits do not exist unless the standard form of the massive spin-$\frac{3}{2}$ stress energy tensor is modified in a way which causes the massive and massless energy density to lose its causal and tensorial properties. Fortunately, this necessary modification does not destroy the relativistic invariance of either theory. Finally, it is shown that it is impossible to further modify the massless energy density to recover its covariance or causality.

II. THE MASSIVE SYSTEM

The Lagrange function for a massive, free, spin-$\frac{3}{2}$ field may be written in terms of the Rarita-Schwinger vector spinor as[3]

$$\mathcal{L}(x)=\tfrac{1}{2}i\Big[\psi^{\nu}.\beta\gamma^{\mu}\frac{1}{i}\partial_{\mu}\psi_{\nu}-\tfrac{1}{3}\psi^{\nu}.\beta\gamma_{\nu}\frac{1}{i}\partial_{\mu}\psi^{\mu}-\tfrac{1}{3}\psi^{\mu}.\beta\frac{1}{i}\gamma_{\nu}\partial_{\mu}\psi^{\nu}-\tfrac{1}{3}\psi^{\lambda}.\beta\gamma_{\lambda}\gamma^{\mu}\frac{1}{i}\partial_{\mu}\gamma_{\nu}\psi^{\nu}+m\psi^{\nu}\beta\psi_{\nu}+\tfrac{1}{3}m\psi^{\mu}.\beta\gamma_{\mu}\gamma_{\nu}\psi^{\nu}\Big].\qquad(1)$$

From this, the action principle $\delta W_{12}=G_1-G_2$ gives the

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[1] S. Deser, J. Trubatch, and S. Trubatch, Nuovo Cimento **39**, 1159 (1965).

[2] J. Schwinger, Phys. Rev. **132**, 1317 (1963).

[3] K. Johnson and E. C. G. Sudarshan, Ann. Phys. (N.Y.) **13**, 126 (1961). Their notation is used throughout with the exception $\beta^2=-1$. A dot on the line indicates fermion operator product antisymmetrization. The Lagrange function (1) is written with Johnson and Sudarshan's $W=-\frac{1}{3}$. The choice of W is irrelevant to the arguments used in this paper because it merely fixes the construction of the spin-$\frac{1}{2}$ dependent fields, as explained in their paper.

[4] J. Schwinger, Phys. Rev. **91**, 713 (1953). Note that this procedure produces a symmetric $T^{\mu\nu}$ unique up to two three-dimensional divergences since the coordinate variations considered are no more than linear.

field equations

$$\gamma^\mu \frac{1}{i}\partial_\mu\psi_\nu - \tfrac{1}{3}\gamma_\nu \frac{1}{i}\partial_\mu\psi^\mu - \frac{1}{3}\frac{1}{i}\partial_\nu\gamma_\mu\psi^\mu - \tfrac{1}{3}\gamma_\nu\gamma_\mu \frac{1}{i}\partial^\mu\gamma_\lambda\psi^\lambda + m\psi_\nu + \tfrac{1}{3}m\gamma_\mu\gamma_\nu\psi^\nu = 0. \quad (2)$$

The field equations yield

$$\gamma^0\psi^0 = 1/(m - \tfrac{2}{3}\gamma(1/i)\nabla)(1/i)\nabla_k(\psi^k + \tfrac{1}{3}\gamma_k\gamma_l\psi^l), \quad (3a)$$

$$2(1/i)\partial_\mu\psi^\mu + m\gamma_\mu\psi^\mu = 0, \quad (3b)$$

$$\partial_\mu\psi^\mu = 0. \quad (3c)$$

Inserting Eq. (3c) back into (3b) and (2) yields

$$\gamma^\nu\psi_\nu = 0, \quad (4)$$

$$(\gamma^\mu(1/i)\partial_\mu + m)\psi_\nu = 0. \quad (5)$$

In terms of the independent components Ψ^k the generator of field variations is

$$G = \tfrac{1}{2}i\int d^3x \left[\Psi\cdot\delta\Psi + \frac{1}{i}\nabla\cdot\Psi \frac{2}{3}\frac{1}{m^2 - (\frac{2}{3}\nabla)^2}\frac{1}{i}\nabla\cdot\delta\Psi\right], \quad (6)$$

where

$$\Psi^k \equiv (\delta^{kl} + \tfrac{1}{3}\gamma^k\gamma^l)\psi_l. \quad (7)$$

Inverting the generator in the space spanned by Ψ^k gives the equal-time anticommutation relations:

$$\{\Psi_k(x), \Psi_l(x')\} = [\delta_{kl} + \tfrac{1}{3}\gamma_k\gamma_l - (2/3m^2) \times (\nabla_k + \tfrac{1}{3}\gamma_k\gamma\cdot\nabla)(\nabla_l + \tfrac{1}{3}\gamma\cdot\nabla\gamma_l)]\delta^3(x-x'). \quad (8)$$

The formally Lorentz-invariant theory *is* an invariant theory because the generators of translations, rotations, and Lorentz transformations on the fields obey the required commutation relations of the Poincaré Group. Given three-dimensional invariance, relativistic invariance is assured[5] if at equal-times

$$(1/i)[T^{00}(x), T^{00}(x')] = -(T^{0k}(x) + T^{0k}(x'))\nabla_k\delta^3(x-x') + \tau(x,x'), \quad (9)$$

where

$$\tau(x,x') = -\tau(x',x), \quad (10a)$$

$$\int\int d^3x d^3x' \tau(x,x') = 0, \quad (10b)$$

$$\int\int d^3x d^3x' \tau(x,x') x'^k = 0, \quad (10c)$$

$$\int\int d^3x d^3x' \tau(x,x') x'^k x^l = 0. \quad (10d)$$

The simplest structure with the properties (10) which has the *additional* property of vanishing at spacelike separation is

$$(1/m^2)\nabla_k\nabla_l\nabla_m'\nabla_n'(f^{kl,mn}(x)\delta^3(x-x')). \quad (11)$$

Note that the m^{-2} factor is necessary to preserve the dimensions of the commutator. The only other available quantity with the proper dimensions is ∇^{-2}, but $\nabla^{-2}\delta^3(x-x') = -1/4\pi|\mathbf{x}-\mathbf{x}'|$ fails to vanish at spacelike separation. Since the above energy-density commutation relation *must* vanish at spacelike separation because all contributions to it come from the δ-function anticommutation relations, the possibility of ∇^{-2} is excluded. $T^{00}(x)$ could be interpreted as an operator representing a measurable quantity because two measurements at spacelike separation will not interfere. The purpose of this paper is to show that (a) in the massless theory the τ term no longer vanishes at spacelike separation, so that a causal interpretation of the energy density is not possible; and (b) if there is to be a *continuous* limit from the massive to the massless theory, then the energy density in the massive theory must be changed and will not have a causal interpretation.

III. THE MASSLESS CASE

The simplest way to approach the massless case is to examine the generator of field variations in the limit as $m \to 0$. To invert the coordinate dependent matrix in the generator (6) let

$$\Psi^l = \Psi^{Tl} + \Psi^{Ll} = \Psi^{Tl} + \frac{3}{2}\frac{1}{\nabla^2}(\nabla_l + \tfrac{1}{3}\gamma_l\gamma\cdot\nabla)\chi, \quad (12)$$

where

$$\nabla_l\Psi^{Tl} = 0, \quad (13a)$$

$$\nabla_l\Psi^{Ll} = \chi. \quad (13b)$$

With this unique eigenvector decomposition into longitudinal and transverse fields, the generator becomes

$$G = \tfrac{1}{2}i\int d^3x \left[\Psi^{Tl}\delta\Psi^T{}_l - \chi\frac{3}{2}\frac{1}{\nabla^2}\frac{m^2}{m^2 - (\frac{2}{3}\nabla)^2}\delta\chi\right]. \quad (14)$$

The advantage of this form is that the massless limit of G is well defined. In the limit, all reference to longitudinal fields disappears and only the transverse contribution survives. This implies that in the massless case χ has no dynamical significance. Inversion of the massless generator in the transverse subspace is simple and the transverse fields obey δ-function equal-time anticommutation relations in the space of transverse vector-spinors ($\nabla_k\Psi^{Tk} = \gamma_k\Psi^{Tk} = 0$):

$$\{\Psi^T{}_k(x), \Psi^T{}_l(x')\} = P^T{}_{kl}\delta^3(x-x'), \quad (15)$$

where $P^T{}_{kl}$, the projection operator onto transverse states, is

$$P^T{}_{kl} = \delta_{kl} + \gamma_k\gamma_l - \frac{3}{2}\frac{\nabla_k\nabla_l}{\nabla^2} - \frac{1}{2}\frac{\gamma_k\nabla\cdot\gamma\nabla_l}{\nabla^2} - \frac{1}{2}\frac{\nabla_k\nabla\cdot\gamma\gamma_l}{\nabla^2}. \quad (16)$$

[5] J. Schwinger, Phys. Rev. **130**, 800 (1963).

If the decomposition into transverse and longitudinal states is performed in the massive case then

$$\{\Psi^T{}_k(x),\Psi^T{}_l(x')\}=P^T{}_{kl}\delta^3(x-x')\,, \tag{17a}$$

$$\{\Psi^T{}_k(x),\chi(x')\}=0\,, \tag{17b}$$

$$\{\chi(x),\chi(x')\}=-\tfrac{2}{3}\nabla^2[1-(2/3m)^2\nabla^2]\times\delta^3(x-x')\,. \tag{17c}$$

This breakup of Ψ is useful because Ψ^T and χ anticommute and m^{-2} terms appear only in (17c).

The massless case may also be approached directly by taking the (well-defined) $m\rightarrow 0$ limit in the Lagrange function (1). The action principle gives equations analagous to (2) and (3) except with $m=0$. The significant difference between the massive and the massless case is that Eq. (4) cannot be derived for the massless case. Furthermore, since (4) was necessary to derive (5) all that can be derived is the massless Dirac equation for transverse spinors,

$$\gamma\frac{1}{i}\partial\Psi^{Tl}=0\,. \tag{18}$$

There is no equation of motion for χ in the massless case, so χ is dynamically undetermined and is thus a four component gauge variable representing the disappearance of the $S_z=\pm\frac{1}{2}$ states in the massless limit. A gauge transformation $\chi\rightarrow\chi'$ leaves the physical content of the system invariant.

The breakup into longitudinal and transverse parts comes independently from study of the anticommutation relations or from the equations of motion.

IV. THE MASSLESS LIMIT OF THE MASSIVE SYSTEM

For field theories of spin-0 and spin-$\frac{1}{2}$ there is no problem in letting $m\rightarrow 0$ since these systems undergo no discontinuous phenomena. The limit is attained simply by setting $m=0$ in all relevant expressions: Lagrange function, $T^{\mu\nu}$, $J^{\mu\nu}$, P^μ, etc. However, in the massless limit for systems of spin $\geqslant 1$, two transverse modes (or helicities) must decouple from the rest of the modes. Schwinger[6] has shown that such a decoupling of longitudinal and transverse modes takes place for the limit $m\rightarrow 0$ of a spin-1 field. In the spin-$\frac{3}{2}$ case the dynamical variables must decouple from the longitudinal variables in the massless limit. However, the interesting feature of the spin-$\frac{3}{2}$ system is that without a slight modification of T^{00}, this decoupling is impossible in Eq. (9).

Call expressions quadratic in Ψ^{Tl} and in χ, TT, and LL, respectively, and terms bilinear in Ψ^{Tl} and χ, TL. When $m\rightarrow 0$ there are neither anticommutation relations nor equations of motion for χ. Hence (9) decouples if it breaks up into

$$\begin{aligned}(1/i)[T^{00TT}(x),T^{00TT}(x')]+(1/i)[T^{00LL}(x),T^{00LL}(x')]\\ =-(T^0{}_k{}^{TT}(x)+T^0{}_k{}^{TT}(x'))\nabla^k\delta^3(x-x')+\tau^{TT}(x,x')\\ -(T^0{}_k{}^{LL}(x)+T^0{}_k{}^{LL}(x'))\nabla^k\delta^3(x-x')\\ +\tau^{LL}(x,x')\,.\end{aligned} \tag{19}$$

Decoupling means that there are no TL (cross) terms and the system is reducible to a direct sum in which the (nonphysical and mathematically awkward i.e., $\{\chi,\chi\}=\infty$) longitudinal part may be discarded. If Eq. (9) is to have any physical meaning whatsoever as $m\rightarrow 0$, then it must decouple and conditions for this may be listed as

(a) $\lim_{m\rightarrow 0} T^{00}=T^{00TT}+T^{00LL}$;

(b) $\lim_{m\rightarrow 0} T^0{}_k=T^0{}_k{}^{TT}+T^0{}_k{}^{LL}$;

(c) all terms in $\tau(x,x')$ with a factor of m^{-2} must become LL terms so that while m^{-2} is unbounded as $m\rightarrow 0$, interpretation of the commutator is possible since the infinity is only in the space of longitudinal variables.

The usual[4] procedure for constructing $T^{\mu\nu}$ in the massive case gives

$$T_{\mu\nu}=\frac{1}{2i}\psi^\lambda.\gamma^0\gamma_{(\mu}\partial_{\nu)}\psi_\lambda+\frac{1}{i}\partial_\sigma(\psi_{(\mu}.\gamma^0\gamma_{\nu)}\psi^\sigma)\,. \tag{20}$$

This gives

$$T_{00}{}^{TT}=\frac{1}{2i}\Psi^{Tl}\gamma^0(\gamma\cdot\nabla+im)\Psi^T{}_l\,, \tag{21}$$

and

$$\begin{aligned}T_{00}{}^{TL}=&-\frac{1}{2i}\nabla_k\nabla_l\left(\Psi^{Tl}.\gamma^0\frac{3}{2\nabla^2}\gamma^k\chi\right)\\ &-\frac{1}{i}\nabla_l\left(\Psi^{Tl}.\gamma^0\frac{3}{2\nabla^2}(m^2/im-\tfrac{2}{3}\gamma\cdot\nabla)\chi\right)\,.\end{aligned} \tag{22}$$

Since $\lim_{m\rightarrow 0} T_{00}{}^{TL}\neq 0$, condition (a) is not satisfied. Furthermore,

$$\lim_{m\rightarrow 0} T_{0k}{}^{TL}=\frac{1}{4i}\left[\nabla_l\nabla_k\left(\frac{9}{2}\Psi^{Tl}.\frac{1}{\nabla^2}\chi\right)-\tfrac{3}{4}\nabla_m\nabla_l\left(\Psi^{Tl}.[\gamma_m,\gamma_k]\frac{1}{\nabla^2}\chi\right)-\tfrac{3}{2}\Psi^{Tl}.[\gamma_k,\gamma_m]\frac{\nabla_l\nabla_m}{\nabla^2}\chi-\tfrac{3}{2}\nabla_l\left(\Psi^T{}_k.[\gamma_l,\gamma_m]\frac{\nabla_m}{\nabla^2}\chi\right)\right] \tag{23}$$

[6] J. Schwinger, *Brandeis Lectures, 1964* (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1965), p. 147.

and τ contains a TT term with a coefficiemt m^{-2}. Explicitly,

$$[T^{00}(x),T^{00}(x')]=\frac{1}{i}(T^0{}_k(x)+T^0{}_k(x'))\nabla^k\delta^3(x-x')$$
$$+\Bigg[\frac{1}{4}\nabla_k\nabla_l\nabla_m'\nabla_n'\left(\Psi^{Tl}(x).\gamma^k\frac{\delta^3(x-x')}{\nabla^2}\gamma^m\Psi^{Tn}(x')\right)-\tfrac{1}{2}\nabla_k\nabla_l\nabla_n'\left(\Psi^{Tn}(x').\gamma\cdot\nabla\frac{\delta^3(x-x')}{\nabla^2}\gamma^k\Psi^{Tl}(x)\right)$$
$$+\tfrac{1}{2}\nabla_k'\nabla_l'\nabla_n'\left(\Psi^{Tn}(x).\gamma\cdot\nabla\frac{\delta^3(x-x')}{\nabla^2}\gamma^k\Psi^{Tl}(x')\right)\Bigg]-\Bigg[\frac{1}{4}\nabla_k\nabla_l\nabla_m'\nabla_n'\left(\Psi^{Tl}(x).\gamma^k\frac{\delta^3(x-x')}{\nabla^2}\gamma^m\Psi^{Tn}(x')\right)$$
$$-\tfrac{1}{2}\nabla_k\nabla_l\nabla_n'\left(\Psi^{Tn}(x').\gamma\cdot\nabla\frac{\delta^3(x-x')}{\nabla^2}\gamma^k\Psi^{Tl}(x)\right)+\tfrac{1}{2}\nabla_k'\nabla_l'\nabla_n\left(\Psi^{Tn}(x).\gamma\cdot\nabla'\frac{\delta^3(x-x')}{\nabla^2}\gamma^k\Psi^{Tl}(x')\right)$$
$$+\left(\frac{1}{3m}\right)^2\nabla_k\nabla_l\nabla_m'\nabla_n'(\Psi^{Tl}(x).\gamma^k\gamma^m\Psi^T(x')\delta^3(x-x'))\Bigg]+\tau^{TL}+\tau^{LL}. \quad (24)$$

The terms in the first bracket in $\tau(x,x')$ arise from the ∇^{-2} in the anticommutation relation in the $[T^{00TT},T^{00TT}]$ part. The terms in the second bracket come from $[T^{00TL},T^{00TL}]$. The first bracket cancels the first three terms in the second bracket so that the entire commutator vanishes at spacelike separation.

All three decoupling conditions (a), (b), and (c) are satisfied and (24) reduces to (19) if

$$\tfrac{1}{2}i\nabla_k\nabla_l(\Psi^{Tl}.\gamma^0\tfrac{3}{2}\nabla^{-2}\gamma^k\chi)$$

is subtracted from T^{00}. The results of this procedure are as follows:

(a) The second bracket in (24) disappears so that the ∇^{-2} terms no longer cancel and the commutator no longer vanishes at spacelike separation. Causality of T^{00} has been sacrificed for the privilege of decoupling —a difficulty not encountered in lower spin systems. Thus, the energy density is not an observable. Schwinger[2] has shown that the energy-density commutator for the gravitational field is also acausal, which prohibits a local specification of energy which is not surprising in view of the correspondence principle and general relativity. However, Deser *et al.*[1] have shown that the same acausality appears in the massless spin-2 field (linearized gravitational field). The authors are led to believe that *any* field theory that is nonlocal ("nonlocal" means that there is a nonvanishing τ term in the energy density commutation relation) will have an acausal energy density in the massless limit. This does not happen in electrodynamics because that is a local system.

(b) The relativistic invariance of the theory is preserved. The massless energy density is

$$T^{00}(x)=(1/2i)\Psi^{Tl}.\gamma^0\gamma\cdot\nabla\Psi^T{}_l, \quad (25a)$$

and the momentum density is

$$T^{0k}(x)=\frac{1}{2i}\Psi^{Tl}.\nabla_k\Psi^T{}_l+\tfrac{1}{4}\nabla_m(\Psi^{Tl}\tfrac{1}{4}i[\gamma^k,\gamma^m]\Psi^T{}_l)$$
$$+\frac{1}{4i}\nabla_m[\Psi^T{}_k\Psi^{Tm}-\Psi^{Tm}\Psi^T{}_k]. \quad (25b)$$

Integrating quantities (25) over all space gives P^μ and integrating the first moment of (25) gives $J^{\mu\nu}$. The term added to T^{00} is a double 3-divergence so that integrated quantities are left unchanged. In particular, we still have $[\Psi^{Tl},P^\mu]=(1/i)\partial^\mu\Psi^{Tl}$, so this P^μ is still the generator of displacements on the field.

The spin matrix for the transverse subspace is

$$(S_k)_{lm}=\frac{1}{i}\epsilon_{klm}-\frac{1}{4i}\delta_{lm}\epsilon^{ij}{}_k\gamma_i\gamma_j. \quad (26)$$

The only eigenvalues of S_z with eigenvectors in the transverse subspace are $\pm\frac{3}{2}$, thus verifying that the transverse modes are the spin-$\frac{3}{2}$ part of the system.[7]

(c) The massless energy density no longer transforms as the 00 component of a tensor. Since

$$[T^{00}(x),T^{00}(x')]=\frac{1}{i}(T^0{}_k(x)+T^0{}_k(x'))\nabla^k\delta^3(x-x')+\tfrac{1}{4}\nabla_k\nabla_l\nabla_m'\nabla_n'\left(\Psi^{Tl}(x).\gamma^k\gamma^m\Psi^{Tn}(x')\frac{\delta^3(x-x')}{\nabla^2}\right)$$
$$-\tfrac{1}{2}\nabla_k\nabla_l\nabla_n'\left(\Psi^{Tn}(x').\left[\gamma\cdot\nabla\frac{\delta^3(x-x')}{\nabla^2}\right]\gamma^k\Psi^{Tl}(x)\right)+\tfrac{1}{2}\nabla_k'\nabla_l'\nabla_n\left(\Psi^{Tn}(x).\left[\gamma\cdot\nabla'\frac{\delta^3(x-x')}{\nabla^2}\right]\gamma^k\Psi^{Tl}(x)\right), \quad (27)$$

[7] The removal of the TL terms in the zero-mass limit does not alter $J^{\mu\nu}$ because the TL terms in T^{0k} from which J^{kl} is constructed give no contribution. This is either because they are double 3-divergences or because the antisymmetry of J^{kl} causes them to vanish.

then

$$[T^{00}(x),J^0{}_k]=\left(x^0\frac{1}{i}\partial_k-x_k\frac{1}{i}\partial^0\right)T^{00}(x)+2iT^0{}_k-\tfrac{1}{2}\nabla_l\nabla_m\int d^3x'\Psi^T{}_k(x').\left[\gamma\cdot\nabla\frac{\delta^3(x-x')}{\nabla^2}\right]\gamma^m\Psi^{Tl}(x). \tag{28}$$

Thus, $T^{00}(x)$ picks up a double 3-divergence under Lorentz transformations.

(d) In the massless case the Lagrange function is neither Lorentz- nor gauge-invariant, but the action, which is the fundamental quantity from which the entire theory is derived, is both. Since the standard procedure of constructing $T^{\mu\nu}$ from $\mathcal{L}$ is only unique up to a double 3-divergence the modification made in T^{00} did not necessitate a change in $\mathcal{L}$. The massless Lagrange function in terms of Ψ^{Tl} and χ is

$$\mathcal{L}(x)=\tfrac{1}{2}i\Psi^T{}_k.\beta\partial^\mu\frac{1}{i}\gamma_\mu\Psi^{Tk}-\tfrac{3}{4}i\nabla_k\left(\Psi^{Tk}\beta\gamma^\mu\frac{1}{i}\partial_\mu\frac{1}{\nabla^2}\chi\right). \tag{29}$$

$\mathcal{L}$ depends on χ as a 3-divergence so the action, $W_{12}=\int_{t_2}^{t_1}dt\int d^3x\,\mathcal{L}$ does not depend on χ and is gauge-invariant. To examine Lorentz transformation properties we use

$$[\Psi^T{}_l(x),J^0{}_k]=\left(x^0\frac{1}{i}\partial_k-x_k\frac{1}{i}\partial^0\right)\Psi^T{}_l+\int d^3x'(x_k-x_k')[\Psi^T{}_l(x),T^{00}(x')], \tag{30}$$

to get

$$[\Psi^T{}_l(x),J^0{}_k]=\left(x^0\frac{1}{i}\partial_k-x_k\frac{1}{i}\partial^0\right)\Psi^T{}_l+\tfrac{1}{2}i\gamma^0\gamma_k\Psi^T{}_l(x)-i\nabla_l\gamma^0\gamma\cdot\nabla\frac{1}{\nabla^2}\Psi^T{}_k$$
$$-m(\tfrac{3}{2}\nabla_l+\tfrac{1}{2}\gamma_l\gamma\cdot\nabla)\gamma^0\frac{1}{\nabla^2}\Psi^T{}_k+iP^T{}_{lk}\gamma^0\frac{3}{2\nabla^2}\frac{m^2}{im-\frac{3}{2}\gamma\cdot\nabla}\chi. \tag{31}$$

As $m\to 0$ a Lorentz transformation does not mix Ψ^{Tl} and χ, but acts on the components of Ψ^{Tl} in such a way as to maintain the transversality of Ψ^{Tl}. This is reminiscent of the vector potential A^k in quantum electrodynamics which picks up a gauge term when Lorentz transformed in order to preserve the radiation gauge $\nabla\cdot A=0$ in the new frame. Since Ψ^{Tl} and χ do not mix under Lorentz transformation, it is sufficient that the TT terms in the massless Lagrange function do not transform as a scalar for $\mathcal{L}$ itself to fail to be a scalar. Applying (31) to (29) yields

$$[\mathcal{L}^{TT}(x),J^0{}_k]=\left(x^0\frac{1}{i}\partial_k-x_k\frac{1}{i}\partial^0\right)\mathcal{L}^{TT}(x)$$
$$+\tfrac{1}{2}\nabla^l\left[\Psi^T{}_l.\gamma^0\partial^\mu\gamma_\mu\gamma^0\nabla_n\gamma^n\frac{1}{\nabla^2}\Psi^T{}_k-\nabla_n\left(\frac{1}{\nabla^2}\Psi^T{}_k.\gamma^n\partial^\mu\gamma_\mu\Psi^T{}_l\right)\right]. \tag{32}$$

Thus, $\mathcal{L}$ picks up a 3-divergence under a Lorentz transformation but W_{12} is clearly left invariant.

V. AN ALTERNATE MASSLESS THEORY

In the massless Lagrange function the χ terms appear only as 3-divergences so it is possible to omit them entirely from the theory by adding a 3-divergence to $\mathcal{L}$. This changes neither the equation of motion (18) nor the anticommutation relation (15). In this massless theory, the quantities $T^{\mu\nu}$, $J^{\mu\nu}$, etc., are just the TT terms of the massive theory. This is a relativistic invariant theory since the decoupled TT system was shown to be invariant. Suppose this $m=0$ theory could be considered separately from the massive theory and not as the limit of the massive system. Then, the massive spin-$\frac{3}{2}$ system energy density would obey causal commutation relations as usual, but the $m=0$ system *still suffers from the above-mentioned difficulties*. For the massless T^{00} to obey causal commutation relations it is necessary to find an expression quadratic in Ψ^{Tl} involving ∇_k and ∇^{-2} which, when commuted with $T^{00}(x)^{TT}=(1/2i)\Psi^{Tl}.\times\gamma^0\gamma\cdot\nabla\Psi^T{}_l$ and with itself, gives precisely the first bracket of (24). Such terms do not exist, as shown below.

Any term to be added to the energy density (25a) which neither changes the total energy operator nor the generator of Lorentz transformations must be a double 3-divergence to make T^{00} into a tensorial expression. The added term commuted with (25a) must produce the triple 3-divergence terms of τ. A fourfold 3-divergence commuted with $(1/2i)\Psi^{Tl}.\gamma^0\gamma\cdot\nabla\Psi^T{}_l$ will only produce fourfold and higher 3-divergences.

There are ten possible terms which could be added to T^{00}. They are

$$\frac{1}{i}\nabla_m\nabla_n\left(\Psi^{Tm}.\gamma^0\frac{\gamma\cdot\nabla}{\nabla^2}\Psi^{Tn}\right), \tag{33a}$$

$$\frac{1}{i}\nabla_m\nabla_n\left(\Psi^{Tj}.\gamma^0\frac{\gamma^m\gamma\cdot\nabla\gamma^n}{\nabla^2}\Psi^T{}_j\right), \tag{33b}$$

$$\frac{1}{i}\nabla_m\nabla_n\left(\Psi^{Tj}.\gamma^0\frac{\nabla_j\gamma^m}{\nabla^2}\Psi^{Tn}\right), \tag{33c}$$

$$\frac{1}{i}\nabla_m\nabla_n\left(\Psi^{Tj}.\gamma^0\frac{\gamma^m\nabla^n}{\nabla^2}\Psi^T{}_j\right), \tag{33d}$$

$$\frac{1}{i}\nabla^2\left(\Psi^{Tj}\cdot\gamma^0\frac{\gamma\cdot\nabla}{\nabla^2}\Psi^T{}_j\right), \tag{33e}$$

and five more obtained by letting ∇^{-2} act to the left.

The commutator of an arbitrary linear combination of these terms with (25a) has been evaluated. It is impossible to cancel the terms in τ causing the nontensorial behavior. Terms constructed from more derivatives do not suffice.

VI. CONCLUDING REMARKS

The notion of energy density has always suffered from a slight degree of ambiguity. A 3-divergence can always be added without affecting the total energy, a property which can be exploited to construct a symmetric $T^{\mu\nu}$ from the canonical one. Within the context of special relativity, the only coordinate transformations used are linear Lorentz transformations. Thus it would seem that there is no physical significance in the double 3-divergence that the energy density of this system picks up under Lorentz transformations.

The failure of the energy density to commute with itself at spacelike separation may have physical meaning or may be a mere curiosity of the formalism. One way to extract the physical content from the peculiarities exhibited by the energy density of this system is to couple it to the gravitational field. The consistency of such an interaction is being investigated.

Regardless of whether such an interaction is possible, the study of the massless spin-$\frac{3}{2}$ field does have a pedagogical virtue. It demonstrates that the lack of microscopic causality of the energy density in quantized gravity is not so much related to the geometrization (freedom of performing general coordinate transformations) but is more a general problem suffered by the massless spin-$\frac{3}{2}$ system as well, which is due to the high spin and massless nature of the underlying field.

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Theory of *W* Production in Nucleon-Nucleon Collisions*

FRANK CHILTON†
Argonne National Laboratory, Argonne, Illinois
AND
A. M. SAPERSTEIN
Department of Physics, Wayne State University, Detroit, Michigan
AND
E. SHRAUNER†
Department of Physics, Washington University, St. Louis, Missouri
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The cross section for $N+N\to W+N+N$ is calculated in a double peripheral model. The effects of the principle corrections such as absorption, resonant final-state interactions, and the $\pi\pi W$ form factor are considered. μ pair production, which is intrinsically related to W production through the conserved vector current, is examined as a means of testing our model and also as a source of background.

INTRODUCTION

THE possibility of a W meson, an intermediate boson related to weak interactions much as the photon is related to electromagnetic interactions, has been the subject of much work.[1] If the W exists, its coupling constant for coupling to the nucleon weak current would be $[GM_W{}^2/\sqrt{2}]^{1/2}$, where G is the Fermi coupling constant from β decay ($G\approx 10^{-5}\ M_N{}^{-2}$) and M_W is the mass of the W boson.

Theoretical estimates of W production cross sections have been made for $\pi+N\to W+N$,[2] $p+p\to W^++d$,[3] Coulomb production by leptons[4] and photons,[5] and production in e^+e^- colliding beams.[6]

If you have a reliable theory with which to make

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