Deuteron Electromagnetic Form Factors for 3 $F^{-2} < q^2 < 6\ F^{-2}$

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Two groups of measurements have been made on the elastic scattering of electrons by deuterium; in each case we observed the recoil deuteron instead of the scattered electron. In the first case the spectrometer was set at 45° so that magnetic scattering was unimportant (about 10%) and we deduced the electric form factors of the deuteron. In the second case deuterons were observed at 0°, allowing us to measure directly the magnetic form factor of the deuteron. Form factors of the neutron were deduced from these measurements for the transfer values $q^2=3$, 4, and 5 (F^{-2}). Preliminary results were given in a first paper. Here we also include a description of the experimental setup and discuss relativistic and exchange-current corrections.

I. INTRODUCTION

A PRECISE measurement of the magnetic form factor G_{Md} of the deuteron is difficult to obtain since electric scattering normally dominates the magnetic scattering, which is proportional to $\frac{2}{3}q^2/4M_d^2$. Moreover, elastic scattering is difficult to separate from the quasi-elastic scattering when only the electron is observed, especially for momentum transfers appreciably larger than $q^2 \approx 2$ F^{-2}. By detecting recoil deuterons at 0°, we were able to measure G_{Md} directly and also to make measurements at somewhat higher momentum transfers than previously possible. Preliminary results were given in a first paper.[1] We have determined deuteron (and proton) form factors using a relativistic formulation of electron-deuteron (proton) scattering. Neutron form factors were deduced using the nonrelativistic treatment developed first by Jankus[2] and later by Gourdin.[3] Experimental values of G_{Md} agree well with the ones calculated from the nonrelativistic theory without correction. All nucleon form factors are found to be equal, except G_{En} which is near zero for $3 \leq q^2 \leq 6$ F^{-2}.

II. THEORY

We have used the following formulas for the elastic scattering cross sections.

A. Relativistic Treatment

In the case of relativistic treatment, we have

$$d\sigma/d\Omega = \sigma_M G^2,$$

where

$$\sigma_M = \left(\frac{e^2}{E_1}\right)^2 \frac{(1+k)^4 \tan^2\gamma}{\cos\gamma[1+k(2+k)\sin^2\gamma]^2},$$

and G^2 is a factor describing the electromagnetic structure of the nucleus. It can be shown that a nucleus of spin J is described by $2J+1$ form factors; thus G^2 contains two form factors for the case of the proton ($J=1/2$) and three form factors for the case of the deuteron:

$$\text{proton:}\quad G^2 = \frac{G_{Ep}^2 + \eta G_{Mp}^2}{1+\eta} + 2\eta G_{Mp}^2 \tan^2(\theta/2),$$

$$\text{deuteron:}\quad G^2 = G_{Ed}^2 + (8/9)\eta^2 G_{Qd}^2 + \tfrac{2}{3}\eta(1+\eta)[1+2(1+\eta)\tan^2(\theta/2)]G_{Md}^2,$$

E_1: laboratory energy of the incident electron scattered at the laboratory angle θ,

$k = E_1/M$ with M = mass of the recoil particle,
$\eta = (\hbar c)^2 q^2/4M^2$,
γ = recoil-particle laboratory angle.

B. Nonrelativistic Treatment

The impulse approximation allows us to write the following relations between the deuteron and nucleon form factors:

$$G_{Ed} = (G_{Ep}+G_{En})f_e(q) \qquad [\text{charge}],$$

$$G_{Qd} = \frac{3\sqrt{2}}{4\eta}(G_{Ep}+G_{En})f_q(q) \qquad [\text{quadrupole}],$$

$$G_{Md} = 2[\alpha(q)(G_{Mn}+G_{Mp}) + \beta(q)(G_{Ep}+G_{En})] \qquad [\text{magnetic}].$$

C. Structure Functions of the Deuteron

If $u(r)$ and $w(r)$ are radial wave functions for the S and D states of the deuteron, we have

$$f_e(q) = \int_0^\infty (u^2+w^2) j_0\left(\frac{qr}{2}\right) dr,$$

$$f_q(q) = \int_0^\infty 2w\left(u - \frac{w}{2\sqrt{2}}\right) j_2\left(\frac{qr}{2}\right) dr,$$

$$\alpha(q) = \int_0^\infty \left[\left(u^2 - \frac{w^2}{2}\right) j_0 + \left(\frac{uw}{2} + \frac{w^2}{2}\right) j_2\right] dr,$$

$$\beta(q) = \int_0^\infty \tfrac{3}{4}(w^2 j_0 + w^2 j_2) dr.$$

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[1] D. Benaksas, D. Drickey, and D. Frèrejacque, Phys. Rev. Letters **13**, 353 (1964).
[2] V. Z. Jankus, Phys. Rev. **102**, 1586 (1956).
[3] M. Gourdin, Nuovo Cimento **28**, 533 (1963).

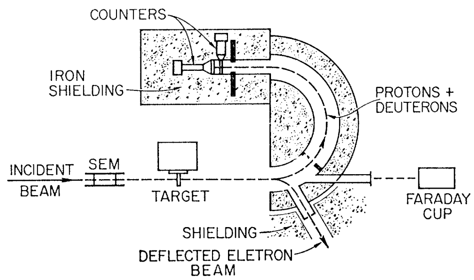

FIG. 1. General experimental arrangement.

D. Normalization

$$G_{Ep}(0)=1, \quad G_{Mp}(0)=2.793=\mu_p,$$
$$G_{En}(0)=0, \quad G_{Mn}(0)=-1.913=\mu_n,$$
$$G_{Ed}(0)=1, \quad G_{Qd}(0)=Q=24.8,$$
$$G_{Md}(0)=\mu_d=1.713.$$

III. EXPERIMENTAL SETUP

The electron beam of the Orsay Linear Accelerator was used in the energy range between 180 and 450 MeV. The energy was analyzed by a two-magnet system monitored with a nuclear-magnetic-resonance probe. In Fig. 1 we illustrate the experimental set-up for the case where the deuteron was observed at 0°. For this case the electrons passed through a special thin-walled secondary emission monitor (SEM), the liquid target, and were then bent downward so as to exit from a hole drilled in the spectrometer.

A. Thin-Walled SEM

This SEM located in front of the target was described in an earlier paper.[4] It consists of a collecting cylinder between two thin emitting foils each 3 μ thick. The absolute efficiency of this SEM was $\approx 5\%$ and was found to be stable to about 5% per day and to 1% after calibrating against a Faraday cup every 2 h.

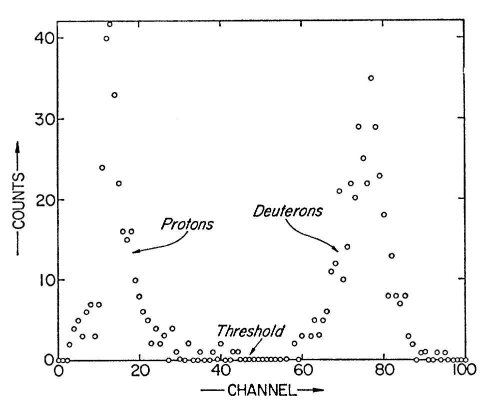

FIG. 2. Sample of coincidence spectra.

[4] D. Frèrejacque and D. Benaksas, Nucl. Instr. Methods **26**, 351 (1964).

TABLE I. Errors in cross sections (%).

Source of errors	Electron-deuteron $\gamma=45°$	Electron-deuteron $\gamma=0°$	Electron-proton $\gamma=45°$	Ratio deuteron/proton
1. Faraday-cup efficiency	0.2	0.2	0.2	0.0
2. SEM stability	0.3	1.0	0.3	0.2
3. Integrator	0.5	0.5	0.5	0.2
4. Target thickness and density	2.0	2.0	2.0	1.0
5. Detection efficiency	2.0	4.0	1.0	2.0
6. Angle	1.9	0.5	1.9	1.0
7. Energy	3.5	2.5	1.2	3.7
8. Solid angle	0.7	2.0	0.7	0.3
9. Spectrometer dispersion	1.0	1.0	1.0	0.0
10. Radiative correction	0.2	0.4	0.2	0.2
11. Counting-rate correction	0.5	0.2	0.5	0.7
12. Statistics	2.8	7.2	1.9	3.4

The error assigned to charge measurements made with this SEM was 1%.

B. Target

The target length chosen for the experiment was a compromise between the conflicting requirements of a thick target for a high counting rate and a thin target so that energy losses in the target did not excessively broaden the elastic peak. We used a 0.7-cm liquid target described earlier.[5] It is a copper frame containing three compartments: one for hydrogen, another for deuterium, and the last empty for background measurements. The windows of the target were stainless steel 12 μ thick. The entire system was capable of vertical translation so that we could remotely insert each of the three targets in the beam.

We have assigned an error of 2% to the target thickness including the error on the density. We have arrived at this error by a series of measurements on the target, both at room temperature and at liquid-nitrogen temperatures, the temperature correction to the thickness being the order of 1%. This correction was assumed to be constant between liquid-nitrogen and liquid-hydrogen temperature. A more important correction was that due to pressure. A curve deduced from measurements of the target thickness as a function of pressure was used for this correction. We have used 0.170 g/cm^3 as the density of liquid deuterium at the temperature of our target.[6]

C. Spectrometer and Solid Angle

Protons and deuterons were momentum-analyzed using a 500-MeV/c double-focusing spectrometer. The spectrometer had been previously calibrated in momentum with an accuracy of 0.5% by a floating wire measurement. Electrons and deuterons (and γ rays from bremsstrahlung) passed together into the spec-

[5] V. Round, D. Benaksas, and P. Bounin, Nucl. Instr. Methods **26**, 348 (1964).
[6] H. W. Wooley, R. B. Scott, and F. G. Brickwedde, J. Res. Natl. Bur. Std., **41**, 379 (1948).

trometer. The electrons, deflected downward, left the magnet through a specially designed hole, greatly reducing background. Because of the aperture of the electron (and γ) beam, it was not possible to define a solid angle with a slit placed before the spectrometer. To do this we placed baffles at 45° inside the spectrometer vacuum chamber. The solid angle so defined was calibrated by taking the ratio of the counting rates of protons coming from a carbon target with the baffles inside the magnet and with a geometrically defined solid angle outside the magnet (in both cases the spectrometer was set at 45°). We assigned an error of 2% to the solid angle determined by this method. The solid angle so determined was also checked by measuring the above ratio using the elastic peak from electron-proton scattering, the results being in excellent agreement with the carbon measurements. The absolute cross section so measured also agreed with our previous results.

D. Counters

The separation of protons, deuterons, and background was made using a telescope of two thin plastic scintillators (0.2 cm). The coincidence pulse (10 nsec resolving time) triggered a pulse-height analyzer which examined the signal coming from the second detector. The counters defined a band $\Delta p/p$ of about 2%. Figure 2 shows a typical counter pulse-height spectrum taken with the spectrometer set at 45°.

E. Errors

In Table I we have listed the source and value of the experimental errors, at $q^2=4$ F^{-2}.

IV. EXPERIMENTAL RESULTS

Two series of measurements were made; in the first the spectrometer was at 0°, thus measuring magnetic scattering and yielding values of G_{Md}. In the second we worked a deuteron angle of 45° in order to determine electric form factors of the deuteron $G_{Ed}^2+(8/9)\eta^2 G_{Qd}^2$; thus the angular distribution allows us to determine only a combination of the square of the two electric form factors (charge and quadrupole). In a similar experiment in hydrogen we measured proton cross sections only at 45° and have assumed for the proton that $G_{Mp}=\mu_p G_{Ep}$ as found in other experiments.[7,8] Figures 3 and 4 show samples of spectra obtained in observing the recoil deuteron; in one of them we can see π^0 production below the elastic peak. Several effects contributed to the broadening of the peak, principally

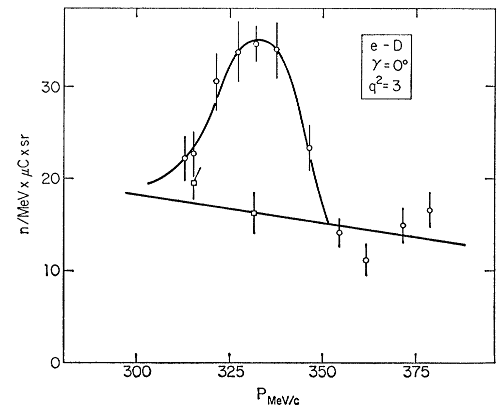

FIG. 3. Typical spectrum of electron-deuteron elastic scattering at 0°.

the finite horizontal aperture of the spectrometer and the difference of energy loss between particles arising from collisions on one side or the other of the target. Several corrections were made to the experimental results:

(i) counting rate ($\simeq$1%);
(ii) finite solid angle in the 0° experiments ($\simeq$1.2%);
(iii) radiative corrections using the calculations of Yennie and Meister[9] (5–9%).

Table II gives the absolute cross-section values measured for the proton and deuteron. Errors are also given in the same table. At 0° the errors are principally statistical. Neutron form factors are obtained by the ratio of deuteron-to-proton cross section so that the resultant errors are smaller.

A. Form-Factor Determination

From the above formulas and the cross-section measurements we have deduced the form factors

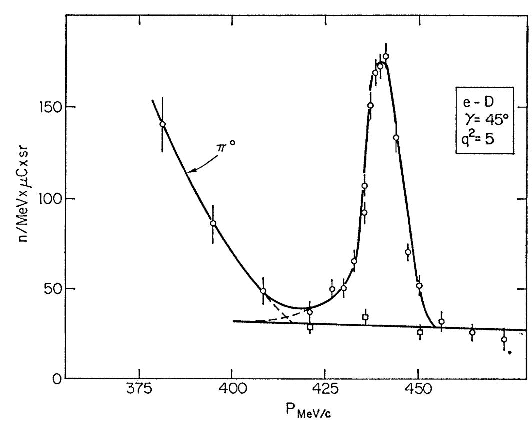

FIG. 4. Typical spectrum of electron-deuteron elastic scattering at 45°.

[7] B. Dudelzak, Ecole Normale Supérieure, Orsay, France, thesis (unpublished); B. Dudelzak and P. Lehmann, in *Proceedings of the Sienna Conference on Elementary Particles and High Energy Physics, 1963*, edited by G. Bernardini and G. P. Puppi (Società Italiana di Fisica, Bologna, 1963), Vol. I, pp. 468 and 495.
[8] T. Janssens, R. Hofstadter, E. Hughes, and M. Yearian, Phys. Rev. **142**, 922 (1966).
[9] N. Meister and D. R. Yennie, Phys. Rev. **130**, 1210 (1963).

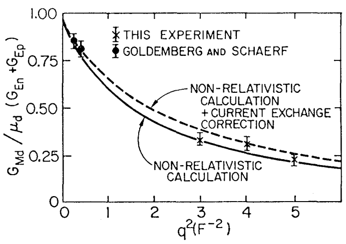

FIG. 5. The curve represents $G_{Md}/\mu_d(G_{Ep}+G_{En})$ determined from the equations of Jankus with the Hamada structure function and assuming "scaling" of the nucleon form factor. The results of Goldemberg and Schaerf are also shown. [J. Goldemberg and C. Schaerf, Phys. Rev. Letters **12**, 298 (1964)].

represented in Table III. To separate electric form factors of the deuteron we used the nonrelativistic treatment and the Hamada potential.[10]

B. Comparison between Electric and Magnetic Form Factors

For low momentum transfer it is found[7,8] that for the proton $G_{Ep}=G_{Mp}/\mu_p$. The same comparison for the deuteron, using values of the electric (charge+quadrupole) and magnetic form factors in Table IV, leads to the same conclusion within experimental error.

C. Magnetic Deuteron Form Factor

The nonrelativistic treatment gives a relation between the magnetic form factors of the nucleons and of the deuteron:

$$G_{Md}/\mu_d=2[\alpha(q)(G_{Mp}+G_{Mn})+\beta(q)(G_{En}+G_{Ep})].$$

We cannot measure G_{Mn} directly but can deduce it from the above formula if $\alpha(q)$ and $\beta(q)$ are calculated accurately. Alternatively we can assume scaling for the nucleon form factors, i.e., $G_{Mn}+G_{Mp}=(\mu_p+\mu_n)\times(G_{En}+G_{Ep})$. This assumption means, if $G_{En}=0$ (we

TABLE II. Absolute cross sections.

q^2	γ^a (deg)	$d\sigma/d\Omega$ (10^{-32} cm^2/sr)	Absolute error (in %)
		Proton	
3.00	45	46.47	3.5
3.98	45	31.75	3.75
5.015	45	23.37	4.0
		Deuteron	
3.00	45	3.61	5.4
4.02	45	1.27	6.3
5.02	45	0.573	5.8
2.97	0	0.138	12.7
3.96	0	0.090	9.4
4.93	0	0.040	12.5

[a] Angle of the recoil particle.

[10] T. Hamada. We wish to thank E. Erickson of Stanford University for supplying us with his calculations of the Hamada form factor.

TABLE III. Form factors.

q^2	G_{Ed}	$G_{Qd}/(M_d^2Q)$	G_{Md}	G_{Md}/μ_d
		Deuteron		
3.00	0.255±0.007	0.272±0.009	0.451±0.019	0.263±0.011
4.00	0.171±0.005	0.188±0.006	0.360±0.015	0.210±0.009
5.00	0.125±0.0035	0.142±0.004	0.237±0.015	0.138±0.009
6.00	0.093±0.008	0.110±0.009	...	...

q^2	$G_{Ep}=G_{Mp}/\mu_p$	$G_{En}+G_{Ep}$	G_{En}	G_{Mn}/μ_n
		Nucleon		
3.00	0.754±0.013	0.778	0.020±0.015	0.74±0.08
4.00	0.685±0.013	0.670	−0.015±0.017	0.63±0.06
5.00	0.618±0.012	0.618	−0.001±0.015	0.62±0.06
6.00	...	0.570	+0.01[a]	...

[a] For $q^2=6.00$ the value of G_{Ep} has been extrapolated from our lower data.

shall see that this is compatible with our results), and since $G_{Ep}=\mu_pG_{Mp}$, that $G_{Mn}=\mu_nG_{Ep}$. This result is consistent with several experiments, particularly with form factors deduced from deuteron electrodisintegration. In this case we can write:

$$(G_{Md}/\mu_d)/(G_{En}+G_{Ep})=2[\alpha(q)(\mu_n+\mu_p)+\beta(q)],$$

G_{Md} and $(G_{En}+G_{Ep})$ are deduced directly from experiment and the right side of the above relation depends only on the neutron-proton potential. In the ratio $G_{Md}/(G_{En}+G_{Ep})$ most common systematic errors are eliminated. Figure 5 shows the theoretical and experimental values of $G_{Md}/\mu_d(G_{En}+G_{Ep})$. There is good agreement between experimental and theoretical results, the data being insensitive to relativistic exchange effects at these low q^2.[11,12]

D. Neutron Form Factors

In this section we adopt the alternative approach using the experimental values of G_{Md} and the nonrelativistic relation and assuming always $G_{Ep}=G_{Mp}/\mu_p$, we deduce the magnetic neutron form factor G_{Mn}. Figure 6 shows the result using two different potentials, Hamada, and Glendenning and Kramer.[13] The result is

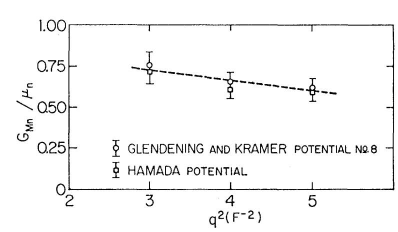

FIG. 6. Neutron magnetic form factors deduced from the magnetic scattering results. The curve represents the mean experimental value of the proton form factors G_{En} found in this experiment.

[11] R. J. Adler and S. D. Drell, Phys. Rev. Letters **13**, 394 (1964).
[12] C. Buchanan, Phys. Rev. Letters **15**, 303 (1965).
[13] N. K. Glendenning and G. Kramer, Phys. Rev. **126**, 2159 (1962).

TABLE IV. Charge and magnetic scattering.

q^2	G_{Md}/μ_d	$[G_{Ed}^2+(8/9)\eta^2 G_{Qd}^2]^{1/2}$
3.00	0.263±0.017	0.260±0.007
4.00	0.210±0.009	0.180±0.005
5.00	0.138±0.009	0.134±0.004

that $G_{Mn}/\mu_n \simeq G_{Mp}/\mu_p \simeq G_{Mp}$, which means that all nucleon form factors are found equal, within our error, except G_{En} which we find to be near zero. G_{En} is obtained by a subtraction between deuteron and proton data. Several results are represented in Fig. 7. Those of Grossetête *et al.*[14] seem slightly positive, while those of Drickey and Hand[15] show that the mean value of G_{En} is zero. Our results substantiate those of Drickey and Hand.

Now we ask the question: What is the validity of the nonrelativistic treatment in this region of momentum transfer. The Jankus calculation needs several assumptions; we discuss here in particular the relativistic and the meson exchange corrections.

E. Relativistic Corrections

It is not possible, as yet, to do a complete relativistic calculation in the three-body problem, but many people have tried to estimate corrections. We have the results of Blankenbecler,[16] Jones,[17] Gross[18] and later, more complete calculations of Tran,[19] who has generalized the Blankenbecler calculations, considering the deuteron composed of two fermions. The result is an increase of the cross section (the cross section decreases in Blankenbecler's calculation). The variation is less important for the magnetic structure function than for the electric.

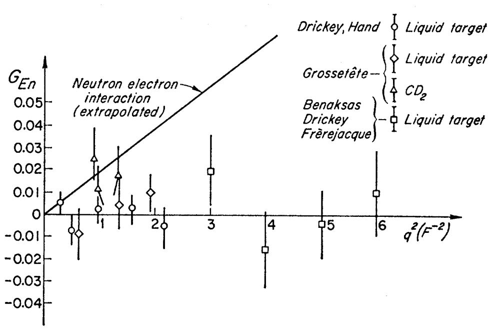

FIG 7. A compilation of G_{En} from the low q^2 results of elastic electron-deuteron scattering deduced from the nonrelativistic theory. These results show a marked deviation from the neutron-electron interaction.

[14] B. Grossetête and P. Lehmann, Nuovo Cimento **28**, 423 (1963).
[15] D. Drickey and L. Hand, Phys. Rev. Letters **9**, 521 (1962).
[16] R. Blankenbecler, Phys. Rev. **111**, 1684 (1958).
[17] H. Jones, Nuovo Cimento **26**, 790 (1962).
[18] F. Gross, Phys. Rev. **142**, 1025 (1966).
[19] T. V. Tran Thanh Van, thesis, Ecole Normale Superiéure, Orsay, France (unpublished); Nuovo Cimento **30**, 1100 (1963).

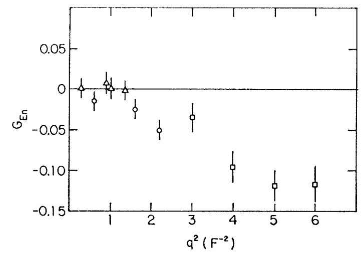

FIG. 8. Neutron charge form factor using the Tran relativistic theory. The abrupt curvature at low q^2 is further increased if this theory is valid.

The most important feature derived from Tran's calculations is that the G_{En} must decrease rapidly when the momentum transfer increases, being negative above $q^2 = 1.3$ F^{-2}. Qualitatively the opposite considerations are arrived at using the theory of Gross at least at very small q^2. Since the Tran calculation is extended to larger q^2, we have chosen to exhibit the results of this theory in Fig. 8. Such abrupt curvature at low q^2 seems extremely difficult to reconcile with any dispersion theory prediction of the nucleon form factors.

F. Current-Exchange Corrections

Recently Drell and Adler[11] have examined the contribution to magnetic scattering coming from exchange diagrams, the lightest system being a ρ and a π. They find a contribution which resolves the well-known discrepancy between the deuteron magnetic moment (0.857) and that predicted by most models with $\approx 7\%$ D state (0.835). This contribution, being independent of q^2 to first order, can give a large contribution to magnetic scattering at the larger values of q^2. Unfortunately our data are insensitive to this effect and the results are compatible with either assumption as seen in Fig. 5. We have not used this correction in extracting the nucleon form factors although later, higher q^2 data seem to indicate the need for it.[12]

The Tran relativistic calculation and the current exchange corrections tend to have the same effect, namely to increase the calculated scattering cross section. They explain the well-known anomaly in the static magnetic moment of the deuteron but within present experimental error seem best tested at higher q^2 than available in this experiment.

ACKNOWLEDGMENTS

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