PHYSICAL REVIEW VOLUME 148, NUMBER 4 26 AUGUST 1966

Electromagnetic Fields and Rotating Masses*

JEFFREY M. COHEN
Sloane Physics Laboratory, Yale University, New Haven, Connecticut
(Received 24 March 1966)

In addition to their "Machian" effects on inertial frames, rotating bodies also have "Machian" effects on electromagnetic fields. It is well known that a rotating charged shell in flat space exhibits an electromagnetic field within it. However, if this rotating charged shell is surrounded by a (concentric) slowly rotating mass shell which rotates with the same angular velocity, the electromagnetic field within the charged shell decreases as the mass of the outer shell increases. Finally, as the gravitational radius approaches the radius of the mass shell, the electromagnetic field within the rotating charged shell vanishes. If in this limit the two shells rotate with different angular velocities, one *cannot* distinguish (even with electromagnetic fields reaching *beyond* the mass shell) whether the charged shell is rotating or the mass shell is rotating in the opposite direction. Therefore, in a certain cosmological model of our universe, the electromagnetic field within a rotating charged shell vanishes when the latter rotates with the same angular velocity as the bulk of the matter in the universe.

I. INTRODUCTION

IN 1918, Thirring[1] showed that a slowly rotating mass shell drags along the inertial frames within it. Since he used the weak-field approximation to Einstein's equations,[2] Thirring's result is valid only when the induced rotation of the inertial frames is small compared to the rotation rate of the shell. Mach's principle[3] suggests that, for mass shells comprising more nearly *all* the matter in the universe than those treated by Thirring, the inertial properties of space within the shell are completely determined by the shell itself. Recently it has been shown that this is indeed the case.[4] As the mass of the slowly rotating shell increases, the inertial frames within it are dragged along more and more until, in the limit as the Schwarzschild radius approaches the shell radius, the angular velocity of the inertial frames approaches that of the shell.

Since the inertial properties of space within the mass shell are completely determined by the shell in this limit, it seems natural to ask if properties other than inertial ones are also completely determined by the mass shell. To investigate this question, we consider the electromagnetic field associated with a single uniformly charged rotating shell of negligible mass concentric with the outer mass shell. Such a calculation would also throw light on a conjecture of Schiff[5] that the electromagnetic field outside two concentric uniformly charged shells having equal and opposite charge vanishes when there is no relative rotation between the charged shells and the distant matter in our universe. Schiff made this conjecture after re-examining the well-known result that *in flat space* this electromagnetic field vanishes when the shells do not rotate relative to a Galilean coordinate system.

Today it is possible to test this conjecture using a particular cosmological model of our universe, a rotating mass shell (with radius approaching its gravitational radius[6]) which represents the matter in the universe. This model has the merit that the inertial frames within the shell cannot rotate with respect to the shell. This explains why the "fixed stars" are fixed with respect to our local inertial frames.[7]

In the spirit of Schiff's conjecture one would expect the electromagnetic field within a charged shell to vanish when it rotates with the same angular velocity as the bulk of the matter in the universe. However, if such an effect exhibits itself, we are faced with a paradox described below. If the charged shell rotates relative to an inertial observer (who remains at a constant distance from the shell) in the asymptotically flat region at infinity, this observer sees a current represented by the rotating charged shell. Since a current must be linked by electromagnetic lines of force, one would expect this observer to see an electromagnetic field within the charged shell. (This latter idea is also suggested by the well-known result that, in flat space, an electromagnetic field manifests itself within a uniformly charged shell which rotates relative to the inertial frames at infinity.) Thus we are left with the question: Does the field vanish or not?

This paradox is resolved when one takes into account the differences in the proper times of the various observers. These proper times are related by the position-dependent red-shift factor [V in Eq. (5)] of the Schwarzschild solution. As the Schwarzschild radius is approached, time advances more and more slowly (e.g.,

* This work has been supported in part by the National Aeronautics and Space Administration and by the National Science Foundation.

[1] H. Thirring, Physik. Z. **19**, 33 (1918).

[2] A. Einstein, Sitzber. preuss. Akad. Wiss. Physik. Math. 688 (1916).

[3] E. Mach, *The Science of Mechanics* (Open Court Publishing Company, La Salle, Indiana, 1902)

[4] D. Brill and J. Cohen, Phys. Rev. **143**, 1011 (1966).

[5] L. I. Schiff, Proc. Nat. Acad. Sci. **25**, 391 (1939).

[6] It has been frequently pointed out (see e.g., R. H. Dicke, J. Wash. Acad. Sci. **48**, 1959) that this relation between the mass and radius appears to hold for the actual universe to within the accuracy of the observations" (i.e. to within a factor of 100) at the present epoch. Of course, the mass distribution of the actual universe is different from that of a shell.

[7] For recent observational data see G. M. Clemence (to be published).

an operation which takes one second in the proper time of an observer at infinity may seem to take a thousandth of a second when seen by an observer at or within the mass shell). In the cosmological model considered here, the radius of the mass shell approaches its gravitational radius. Thus if the charged shell rotates with finite angular velocity in the proper time of an observer within the mass shell, it will be nonrotating in the observer frames which are inertial at infinity. Hence all observers will agree that the magnetic field vanishes.

In Sec. II we give the well-known solution for the electromagnetic field associated with a rotating charged shell in flat space. This flat-space solution is similar to the curved-space solution (Secs. IV and VI) since space within the mass shell is flat. Since the main interest is the effect of the masses of the universe *on* the field rather than vice versa, we consider an electromagnetic field of negligible stress energy. This condition simplifies the calculations by reducing the problem of solving the Einstein-Maxwell equations to that of solving only the curved-space Maxwell equations, for the metric is already known.[4] This metric for a thin slowly rotating mass shell is given in Sec. III. In Sec. IV, Maxwell's equations are solved in the curved space associated with a rotating mass shell. Boundary conditions are discussed in Sec. V. The effect of the masses in the universe on the electromagnetic field associated with a rotating charged shell is discussed in Sec. VI.

II. CHARGED SHELL IN FLAT SPACE

In flat space, the electromagnetic field (of negligible stress energy) associated with a rotating charged shell is well known. However, for completeness and to facilitate comparison with later results we give this familiar solution here in the kind of language which we will be using later in curved space; the components of the electromagnetic field are given with respect to orthonormal Cartan frames[4,8] ω^0, ω^1, ω^2, ω^3 chosen parallel to the differential forms $d\bar{t}$, $d\bar{r}$, $d\theta$, $d\phi-\bar{\Omega}d\bar{t}$, respectively. In three-dimensional flat space, components relative to these frames are the familiar r, θ, and ϕ components of vector analysis (Fig. 1). Here we are using the metric

$$\begin{aligned} ds^2 &= -(\omega^0)^2+\sum_{i=1}^{3}(\omega^i)^2 \\ &= -d\bar{t}^2+d\bar{r}^2+\bar{r}^2d\theta^2+\bar{r}^2\sin^2\theta(d\phi-\bar{\Omega}d\bar{t})^2, \end{aligned} \tag{1}$$

where $\bar{\Omega}$ is the angular velocity of the inertial frames relative to an observer.

Let h_1 and h_2 be the components of the magnetic field relative to the orthonormal frames ω^1 and ω^2. The angular dependence of h_1 and h_2 is the same for all $\bar{r}$, viz.,

$$h_1=n(\bar{r})\cos\theta;\quad h_2=p(\bar{r})\sin\theta, \tag{2}$$

with n and p being functions of $\bar{r}$ only. For a rotating uniformly charged shell of radius $\bar{r}_c$, total charge q, and angular velocity $\bar{\omega}_c$ relative to an observer, the solution of the Maxwell equations takes the form

$$\begin{aligned} &e_1=0, \\ &n=-p=2q(\bar{\omega}_c-\bar{\Omega})/3\bar{r}_c \quad \text{for} \quad \bar{r}<\bar{r}_c; \end{aligned} \tag{3}$$

and

$$\begin{aligned} &e_1=q/\bar{r}^2, \\ &n=2p=(\bar{r}/\bar{r}_c)^3 2q(\bar{\omega}_c-\bar{\Omega})/3\bar{r}_c \quad \text{for} \quad \bar{r}>\bar{r}_c. \end{aligned} \tag{4}$$

Because of the choice of frames, the above components are given relative to a nonrotating frame. Note that it is the relative velocity of the charged shell relative to the inertial frames $(\bar{\omega}_c-\bar{\Omega})$ which appears in the above equations. When this relative velocity vanishes, the electromagnetic field within the charged shell (3) vanishes also.

III. ROTATING MASS SHELL

In this paper, the main interest is in the effects of a thin rotating mass shell *on* the electromagnetic field rather than vice versa; it is reasonable, therefore, to consider an electromagnetic field of negligible stress energy. Since this electromagnetic field does not affect the geometry, the metric for a weakly charged shell within a mass shell is the same as that for the mass shell alone. This metric for a thin slowly rotating mass shell[9] of radius r_0 was found in Ref. 4

$$ds^2=\Psi^4[dr^2+r^2d\theta^2+r^2\sin^2\theta(d\phi-\Omega dt)^2]-V^2dt^2, \tag{5}$$

where

$$\begin{aligned} &V=V_0=(r_0-\alpha)/(r_0+\alpha), \quad \Psi=\Psi_0=1+(\alpha/r_0), \\ &\Omega=\Omega_0 \quad \text{for} \quad r<r_0, \end{aligned} \tag{6}$$

$$\begin{aligned} &V=(r-\alpha)/(r+\alpha), \quad \Psi=1+(\alpha/r), \\ &\Omega=(r_0\Psi_0^2/r\Psi^2)^3\Omega_0 \quad \text{for} \quad r>r_0. \end{aligned} \tag{7}$$

Here the constants have the values

$$\Omega_0=\omega_s/(1+[3(r_0-\alpha)/8\alpha(1+\beta_0)]), \tag{8}$$

$$\beta_0=\alpha/2(r_0-\alpha), \tag{9}$$

$$V_0=(r_0-\alpha)/(r_0+\alpha), \tag{10}$$

$$\Psi_0=1+(\alpha/r_0), \tag{11}$$

2α is the mass of the shell as seen by an observer at infinity, and ω_s is the angular velocity of the mass shell; the elastic stress in the shell is proportional to β_0.

[8] E. Cartan, *Les Systèmes Différentials Extérieurs* (Hermann & Cie., Paris, 1945); see also, e.g., D. Brill and J. Cohen, J. Math. Phys. **143**, 238 (1966) and the references cited there.

[9] Comparison of the exterior solution given here with that of Kerr [R. Kerr, Phys. Rev. Letters, **10**, 87 (1963)] shows that when a is sufficiently small so that terms of higher power than the first are negligible but m is allowed to be large, Kerr's exterior solution can be matched to an interior solution.

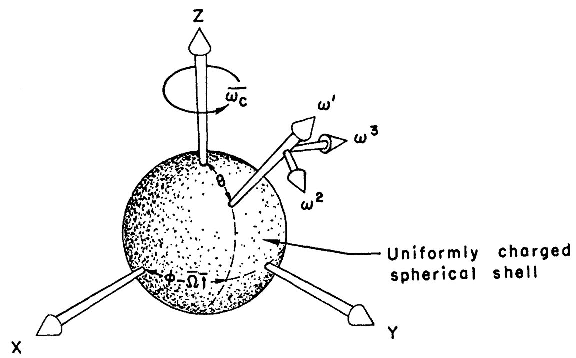

FIG. 1. Orientation of Cartan's moving orthonormal frames ω^μ relative to the uniformly charged shell which rotates with angular velocity $\bar{\omega}_c$

To facilitate the transition (later) to a particular cosmological model of our universe, it is convenient to transform the metric (5), which is the same as Eq. (1) at infinity, into a form which is the same as Eq. (1) within the mass shell. This is accomplished via the coordinate transformation

$$V_0 t=\bar{t};\quad \Psi_0^2 r=\bar{r}. \tag{12}$$

The transformed metric retains the form of Eq. (5),

$$ds^2=\bar{\Psi}^4[d\bar{r}^2+\bar{r}^2d\theta^2+\bar{r}^2\sin^2\theta(d\phi-\bar{\Omega}d\bar{t})^2]-\bar{V}^2d\bar{t}^2, \tag{13}$$

if we set

$$\begin{aligned}
&\bar{\alpha}=\alpha\Psi_0^2(\bar{r}_0,\bar{\alpha}),\\
&\bar{\Psi}=\Psi(\bar{r},\bar{\alpha})/\Psi_0(\bar{r}_0,\bar{\alpha}),\\
&\bar{V}=V(\bar{r},\bar{\alpha})/V_0(\bar{r}_0,\bar{\alpha}),\\
&\bar{\omega}_s=\omega_s/V_0(\bar{r}_0,\bar{\alpha}),\\
&\bar{\Omega}_0=\bar{\omega}_s/(1+[3(\bar{r}_0-\bar{\alpha})/8\bar{\alpha}(1+\beta_0)]),\\
&\bar{\Omega}=(\bar{r}_0\Psi_0^2/\bar{r}\bar{\Psi}^2)^3\bar{\Omega}_0\quad \text{for}\quad \bar{r}>\bar{r}_0,\\
&\bar{\Omega}=\bar{\Omega}_0\qquad\qquad\qquad \text{for}\quad \bar{r}<\bar{r}_0,\\
&\beta_0=\bar{\alpha}/2(\bar{r}_0-\bar{\alpha}).
\end{aligned} \tag{14}$$

The metric takes the same form as that of Eq. (1) in the entire space enclosed by the mass shell since space is flat there.

For the metric (13), a convenient set of orthonormal frames is

$$\begin{aligned}
&\omega^0=\bar{V}d\bar{t},\\
&\omega^1=\bar{\Psi}^2d\bar{r},\\
&\omega^2=\bar{r}\bar{\Psi}^2d\theta,\\
&\omega^3=\bar{r}\bar{\Psi}^2\sin\theta(d\phi-\bar{\Omega}d\bar{t}).
\end{aligned} \tag{15}$$

IV. CURVED-SPACE MAXWELL EQUATIONS

A. Formulation and Solution

By analogy with the flat-space solution of Maxwell's equations (2–4), we assume that the nonvanishing components of the electromagnetic field relative to the frames (15) are e_1, h_1, and h_2. Hence the electromagnetic-field tensor can be expressed as the differential form[10]

$$\begin{aligned}
f&=\tfrac{1}{2}f_{\mu\nu}\omega^\mu\wedge\omega^\nu\\
&=e_1\omega^0\wedge\omega^1+h_1\omega^2\wedge\omega^3-h_2\omega^1\wedge\omega^3
\end{aligned} \tag{16}$$

in Cartan's notation.[8,10]

To write the curved-space Maxwell equations in the language of exterior differential forms the electric-current vector must be expressed as an exterior differential form

$$J=J_\mu\omega^\mu. \tag{17}$$

The tensor expression for this vector is well known

$$J^\mu=\sigma U^\mu, \tag{18}$$

where σ denotes the electric charge density and U^μ denotes the four-velocity of an element of charge. Relative to the vectors dual to the above orthonormal frames, the nonvanishing components of the four-velocity are

$$U^\mu=(1,0,0,\bar{r}\bar{\Psi}^2\sin\theta(\bar{\omega}_c-\bar{\Omega})/\bar{V}) \tag{19}$$

to first order in $\bar{\omega}_c-\bar{\Omega}$. Here $\bar{\omega}_c$ is the angular velocity of the charged shell relative to an observer. Hence the electric-current vector can be expressed as the one form

$$J=\sigma[-\omega^0+\bar{r}\bar{\Psi}^2\sin\theta(\bar{\omega}_c-\bar{\Omega})\omega^3/\bar{V}]. \tag{20}$$

The source-free set of Maxwell equations[10,11]

$$df=0, \tag{21}$$

takes the form

$$0=[(\bar{r}\bar{\Psi}^2)^2h_1]_{\bar{r}}\sin\theta+\bar{r}\bar{\Psi}^4(h_2\sin\theta)_\theta. \tag{22}$$

Here the subscripts $\bar{r}$ and θ denote partial differentiation with respect to $\bar{r}$ and θ. The other set of Maxwell equations[10,11]

$$\delta f=J\quad\text{or}\quad d{*}f={*}J, \tag{23}$$

contains two nontrivial equations,

$$[(\bar{r}\bar{\Psi}^2)^2e_1]_{\bar{r}}=\sigma\bar{r}^2\bar{\Psi}^6, \tag{24}$$

and

$$\begin{aligned}
(\bar{r}\bar{\Psi}^2\bar{V}h_2)_{\bar{r}}-\bar{V}\bar{\Psi}^2h_{1\theta}+e_1(\bar{r}\bar{\Psi}^2)^2\bar{\Omega}_{\bar{r}}\sin\theta\\
=\sigma\bar{r}^2\bar{\Psi}^6(\bar{\omega}_c-\bar{\Omega})\sin\theta.
\end{aligned} \tag{25}$$

Integration of Eq. (24) witq respect to $\bar{r}$,

$$\int_0^{\bar{r}}[(\bar{r}\bar{\Psi}^2)^2e_1]_{\bar{r}}d\bar{r}=\int_0^{\bar{r}}\sigma\bar{r}^2\bar{\Psi}^6d\bar{r}, \tag{26}$$

[10] D. Brill, Phys. Rev. 133, 3845 (1964). The definitions of the de Rham d and δ operators used here are the same as those used by C. Misner and J. Wheeler, Ann. of Phys. 2, 525 (1957).

[11] G. de Rham, *Variétés Différentiables* (Herman & Cie., Paris, 1960).

yields the electric field

$$e_1=0 \quad \text{for} \quad \bar{r}<\bar{r}_c, \tag{27a}$$

$$e_1=q/\bar{r}^2 \quad \text{for} \quad \bar{r}_c<\bar{r}<\bar{r}_0, \tag{27b}$$

$$e_1=q/(\bar{r}\Psi^2)^2 \quad \text{for} \quad \bar{r}_0<\bar{r}. \tag{27c}$$

Here the total electric charge q is defined by

$$q=\int_-^+ \sigma\bar{r}^2\Psi^6 d\bar{r};$$

the limits of the region containing the charge are denoted by $-$ and $+$.

Using the definitions (2), we can put Eq. (22) in the form

$$0=[(\bar{r}\Psi^2)^2 n]_{\bar{r}}+2\bar{r}\Psi^4 p \tag{28}$$

and Eq. (25) in the form

$$(\bar{r}\Psi^2\bar{V}p)_{\bar{r}}+\bar{V}\Psi^2 n+e_1(\bar{r}\Psi^2)^2\bar{\Omega}_{\bar{r}}=\sigma\bar{r}^2\Psi^6(\bar{\omega}_c-\bar{\Omega}). \tag{29}$$

Elimination of p by substitution of Eq. (28) into Eq. (29) yields

$$[\bar{V}\bar{r}(\bar{r}\Psi^2)^3 n_{\bar{r}}]_{\bar{r}}=2e_1(\bar{r}\Psi^2)^4\bar{\Omega}_{\bar{r}}-2\sigma(\bar{r}\Psi^2)^4\Psi^2(\bar{\omega}_c-\bar{\Omega}). \tag{30}$$

For $\bar{r}<\bar{r}_0$ and $\bar{r}\neq\bar{r}_c$, Eq. (30) takes the form

$$[\bar{r}^4 n_{\bar{r}}]_{\bar{r}}=0, \tag{31}$$

since

$$\sigma=0 \text{ and } \bar{\Omega}_{\bar{r}}=0. \tag{32}$$

Equation (31) admits the general solution

$$n=l+l'\bar{r}^{-3} \quad \text{for} \quad 0<\bar{r}<\bar{r}_c, \tag{33}$$

$$n=l_1+l_1'\bar{r}^{-3} \quad \text{for} \quad \bar{r}_c<\bar{r}<\bar{r}_0. \tag{34}$$

Here l, l', l_1, and l_1' are integration constants. When $\bar{r}>\bar{r}_0$, we obtain

$$n=l_2+l_2'F(\bar{r})+K'(\bar{r}\Psi^2)^{-3}, \tag{35}$$

where

$$K'=[qV_0\bar{r}_0^3\Psi_0^2/2\bar{\alpha}]\bar{\Omega}_0, \tag{36}$$

$$F(\bar{r})=2\alpha(\bar{r}\Psi^2)^{-1}+4\alpha^2(\bar{r}\Psi^2)^{-2}+\ln V; \tag{37}$$

l_2 and l_2' are constants of integration.

V. BOUNDARY CONDITIONS

To determine the constants of integration, boundary conditions must be imposed. Regularity at the origin requires that

$$l'=0. \tag{38}$$

In the asymptotically flat region far from the source, the electromagnetic field must decrease as the distance from the source increases. Thus we set

$$l_2=0. \tag{39}$$

Integration of Eq. (30) across the surface $\bar{r}=\bar{r}_c$ yields the boundary condition

$$n_{\bar{r}}\Big|_{\bar{r}_c^-}^{\bar{r}_c^+}=-2q(\bar{\omega}_c-\bar{\Omega}_0)/\bar{r}_c^2. \tag{40}$$

Here $\bar{r}_c^+$ and $\bar{r}_c^-$ denote the limit $\bar{r}\to\bar{r}_c$ taken from above and below, respectively. The integration of Eq. (28) across this surface ($\bar{r}=\bar{r}_c$) yields

$$n\Big|_{\bar{r}_c^-}^{\bar{r}_c^+}=0. \tag{41}$$

To obtain Eq. (41) we assumed that p is a regular distribution (i.e., it corresponds to a function). If p were not regular (e.g. if it contained Dirac-δ functions or derivatives of δ functions), Eq. (41) might not hold. In the calculations which follow, we find that this assumption is justified.

Integration of Eq. (30) across the surface $\bar{r}=\bar{r}_0$ yields the condition

$$n_{\bar{r}}\Big|_{\bar{r}_0^-}^{\bar{r}_0^+}=0, \tag{42}$$

while integration of Eq. (28) yields

$$n\Big|_{\bar{r}_0^-}^{\bar{r}_0^+}=0. \tag{43}$$

We thus have four boundary conditions and four undetermined constants l, l_1, l_1', and l_2'.

Matching of the solutions (33), (34), and (35) across the boundaries determines these constants. Thus n is determined. One can obtain p from Eq. (28); h_1 and h_2 are determined by Eq. (2). If m approaches zero, there results the flat-space solution given in Eqs. (3) and (4), as expected.

VI. ROTATING UNIVERSE

A shell of matter with radius approaching its Schwarzschild radius has often been taken to represent the bulk of the matter in an idealized cosmological model of our universe. In this cosmological model the motion of the local inertial frames is completely determined by the matter, i.e., there cannot be a rotation of the local inertial frames within the shell relative to the "fixed stars" (the mass shell).[4]

If Mach's principle applies to more than just local inertial properties, it suggests that "nonlocal" phenomena which may not even be inertial are also completely determined by the bulk of the matter in the universe (if matter makes the chief contribution to the stress energy). The "nonlocal" field considered here is the electromagnetic field associated with a rotating charged shell within the mass shell; its electric field, for example, extends beyond the mass shell to the asymptotically flat region at infinity. If the masses of the universe determine not only inertial properties but

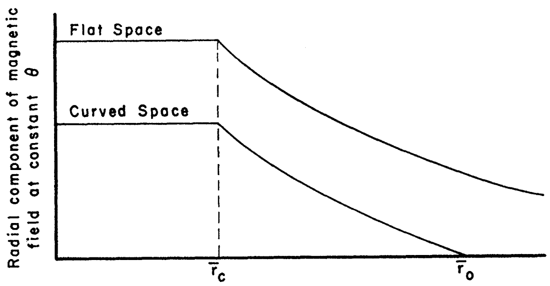

FIG. 2. Radial component of magnetic field versus radius for a rotating charged shell in flat space and in the curved space generated by a thin rotating mass shell with gravitational radius approaching its actual radius. For both curves, the difference between the angular velocity of the charged shell $\bar{\omega}_c$ and the angular velocity of the inertial frames within the mass shell $\bar{\Omega}_0$ is the same.

also the electromagnetic field within the charged shell, one might expect this field to vanish when the charged shell rotates with the same angular velocity as that of the bulk of matter in the universe $\bar{\omega}_s$.

A rotating charged shell, in this universe, generates the following components of the electromagnetic field:

$$e_1=0,\quad n=-p=(2q/3\bar{r}_c)(\bar{\omega}_c-\bar{\omega}_s)[1-(\bar{r}_c/\bar{r}_0)^3] \quad \text{for} \quad 0<\bar{r}<\bar{r}_c, \tag{44}$$

$$\begin{aligned} e_1&=q/\bar{r}^2,\\ n&=(2q\bar{r}_c^2/3\bar{r}^3)(\bar{\omega}_c-\bar{\omega}_s)[1-(\bar{r}/\bar{r}_0)^3],\\ p&=(q\bar{r}_c^2/3\bar{r}^3)(\bar{\omega}_c-\bar{\omega}_s)[1+2(\bar{r}/\bar{r}_0)^3] \quad \text{for} \quad \bar{r}_c<\bar{r}<\bar{r}_0, \end{aligned} \tag{45}$$

and

$$\begin{aligned} e_1&=q/(\bar{r}\bar{\Psi}^2)^2,\\ n&=p=0 \quad \text{for} \quad \bar{r}_0<\bar{r}. \end{aligned} \tag{46}$$

The above are the solutions given in Eqs. (33), (34), and (35) in the limit $\bar{\alpha}$ approaches $\bar{r}_0$.

We see that the electromagnetic field within the charged shell depends on the angular velocity of the charged shell relative to the bulk of matter in the universe. If this angular velocity vanishes, so does the electromagnetic field within the shell.

Outside the mass shell, the magnetic field vanishes (Fig. 2) independent of $\bar{\omega}_c-\bar{\omega}_s$. This is because the observer outside the mass shell (using the frames ω^μ) sees nonrotating shells if an observer within the mass shell sees the shells rotating with a finite angular velocity.

VII. DISCUSSION

In flat space we are used to having a magnetic field within a rotating charged shell. However, if there is enough mass around, this is not necessarily so. Namely, if a rotating charged shell is surrounded by a concentric mass shell which rotates (slowly) with the same angular velocity, the magnetic field within the charged shell decreases as the mass of the outer shell increases. This magnetic field vanishes when the mass becomes so large that the gravitational radius approaches the actual radius of the mass shell.

If, in this limit, the two shells rotate with different angular velocities, we get the "Machian" results that one cannot distinguish whether the charged shell is rotating or the mass shell is rotating in the opposite direction. This is so, despite the fact that the electromagnetic field reaches beyond the mass shell to the asymptotically flat region at infinity.

A shell of matter with radius approaching its Schwarzschild radius has often been taken to represent the bulk of the matter in an idealized cosmological model of our universe. In this cosmological model the local inertial frames are completely determined by the matter, i.e., there cannot be rotation of the local inertial frames relative to the "fixed stars" (the mass shell). Since the discussion of the preceding paragraphs applies to this cosmological model, we see that the results obtained here via the standard Einstein-Maxwell theory are consistent with Mach's principle.

ACKNOWLEDGMENTS

I am indebted to Professor Dieter R. Brill for helpful discussion and for reading the manuscript. I should also like to thank Marion D. Cohen for checking some of the calculations.