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Cosmic-Ray Modulations in the Solar System and in Interstellar Space

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The intensity of cosmic-ray protons between 1959 and 1963 (i.e., near solar maximum and near solar minimum) was modulated twice as much as the intensity of helium at the same velocity, for energies from 0.2 to 0.7 BeV/nucleon. Similar results were obtained by other authors for the modulation between 1963 and 1964 when the intensities of protons and helium nuclei were measured by many investigators. It has been shown by other authors that the ratio of helium nuclei to protons of the same velocity decreases with increasing cosmic-ray energy from about 0.3 to about 0.05 in the energy interval 0.2–10 BeV/nucleon. These observations are shown to be in agreement with Dorman's modification of Parker's solar-modulation theory for intermediate values of rigidity, i.e., with a modulation function of the form $\exp[-K(t)/R\beta]$, where R and β are the rigidity and velocity of the cosmic-ray particle, respectively, t is time, and $K(t)$ depends on various solar-wind parameters. Additional investigations show that while Dorman's expression fits the data well between rigidities of 1.5 and 15 BV, it no longer does so at lower rigidities at certain times of the solar cycle. A transition to Parker's modulation function $\exp[-K(t)/\beta]$ seems to occur. Also a purely rigidity-dependent modulation function of the form $\exp(-K(t)/R]$ fits nearly all of the experimental data. The local interstellar spectra of cosmic-ray protons and helium are investigated, assuming that these spectra are similar in shape between energy values of 1 and 10 BeV/nucleon. Using these results, such parameters of the interplanetary medium as the number of scattering centers along the cosmic-ray path and the radial extent of the cosmic-ray convection and diffusion region are evaluated, and are found to agree with those proposed by Parker. The scale factors of the scattering centers are found to be distributed between 6×10^{10} cm and 10^{12} cm, or more, provided that the magnetic field strengths of the scattering centers do not vary beyond the earth's orbit with the radial distance from the sun.

I. INTRODUCTION

VARIOUS theories attempting to describe the solar modulation of cosmic rays have been developed. An electric-field deceleration model proposed by Nagashima[1] and Ehmert[2] has recently been investigated by Freier and Waddington[3] and by Nagashima, Duggal, and Pomerantz.[4] These authors observed a satisfactory agreement between the theory and experimental results. But as a result of this theory, the electron/positron ratio should vary appreciably[3] over the solar cycle, since electrons would be accelerated toward the sun, while the positrons would be decelerated. To prevent the acceleration of slow galactic electrons toward the sun, an additional modulation is required at least at low energies.

A modulation due to a dipole field of the sun, caused by ring current systems has been proposed by Elliot.[5] This model also provides a good fit to the experimental data.[6] However, Explorer X data[7] show that in interplanetary space the field direction is not that of a dipole.

Parker has suggested a model[8–10] based on solar-wind diffusion. The modulation is described by two limiting cases according to this model: If the Larmor radius of the cosmic-ray particle is small compared to the linear scale of the plasma clouds or magnetic kinks, the modulation is velocity dependent. If the Larmor radius is large, on the other hand, the modulation is predominantly rigidity-dependent. The modulation in both cases depends also on the solar-wind velocity and the

[1] K. Nagashima, J. Geomagnetism Geoelectricity **5**, 141 (1953).
[2] A. Ehmert, in *Proceedings of the Moscow Cosmic Ray Conference* (International Union of Pure and Applied Physics, Moscow, 1960), Vol. IV, p. 142.
[3] P. S. Freier and C. J. Waddington, Space Sci. Rev. **4**, 313 (1965).
[4] K. Nagashima, S. P. Duggal, and M. A. Pomerantz, Planet. Space Sci. **14**, 177 (1966).
[5] H. Elliot, Phil. Mag. **5**, 601 (1960).
[6] W. R. Webber and F. B. McDonald, J. Geophys. Res. **69**, 3097 (1962).
[7] H. R. Anderson, Science **139**, 42 (1963).
[8] E. N. Parker, Phys. Rev. **107**, 924 (1957).
[9] E. N. Parker, Astrophys. J. **133**, 1014 (1961).
[10] E. N. Parker, *Interplanetary Dynamical Processes* (Interscience Publishers, Inc., New York, 1963), Vol. 8.

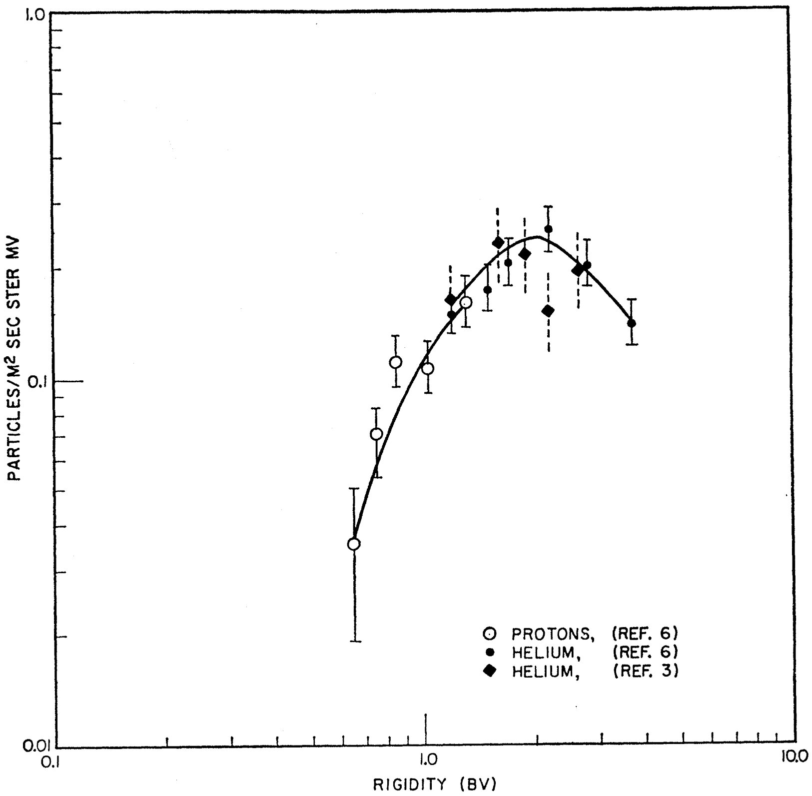

FIG. 1. The rigidity spectra of cosmic-ray protons and helium in 1959, (Refs. 6 and 3). The helium-nucleus data are multiplied by 7.2. Curves have been fitted to the data.

number of clouds or kinks along the cosmic-ray path. Dorman[11,12] has proposed an intermediate type of modulation for the case where some of the kinks are smaller and some are larger in scale than the Larmor radius of the cosmic-ray particle. According to Dorman, this model is applicable to rigidities between 1.5 and 60 BV.

In a comparison of the modulation of protons and helium for the period between 1956 and 1959, Webber and McDonald[6] came to the conclusion that between rigidities of 0.6 and 4 BV, Parker's model does not describe the modulation adequately. However, the recently discovered split in the proton and helium rigidity spectra[13–16] has been interpreted as being suggestive of a velocity dependent solar modulation. On the basis of relative modulation of protons and helium between 1961 and 1963, at a rigidity of about 1 BV, Webber concluded that solar modulation is velocity dependent.[17] Gloeckler[18] fitted the observed modulation of helium between 0.5 and 1.5 BV by Parker's high- and low-rigidity limiting cases and found the low-rigidity expression (which is velocity-dependent) to give a good fit to the data. On the basis of the above papers, and on the basis of the similarity of proton and helium energy/nucleon spectra between 0.03 and 0.6 BeV/nucleon, Balasubrahmanyan, Boldt, and Palmeira[19] also concluded in favor of Parker's velocity-dependent model at low rigidities.

Nagashima, Duggal, and Pomerantz investigated[4] the modulation between 0.8 and 3 BV, using Parker's low-rigidity model, but decided that a partial rigidity dependence yields a better fit. The expression they favor is equivalent to Dorman's modification of Parker's model. For rigidities 6–12 BV, based on satellite data[20,21] on heavy primary nuclei, they decided in favor of Parker's high-rigidity limiting case. It will be shown in Sec. III that Dorman's modification of Parker's model also fits these satellite data. (They also found that the electric-field model provides a good fit.)

[11] L. I. Dorman, in *Proceedings of the Moscow Cosmic Ray Conference* (International Union of Pure and Applied Physics, Moscow, 1960), Vol. IV, p. 320.

[12] L. I. Dorman, Progr. Elem. Particle Cosmic Ray Phys. **7**, 1 (1963).

[13] J. Ormes and W. R. Webber, Phys. Rev. Letters **13**, 106 (1964).

[14] P. S. Freier and C. J. Waddington, J. Geophys. Res. **70**, 5753 (1965).

[15] C. E. Fichtel, D. E. Guss, D. A. Kniffen, and K. A. Neelakantan, J. Geophys. Res. **69**, 3293 (1964).

[16] V. K. Balasubrahmanyan and F. B. McDonald, J. Geophys. Res. **69**, 3289 (1964).

[17] W. R. Webber, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1965), Vol. 46, No. 2.

[18] G. Gloeckler, J. Geophys. Res. **70**, 5333 (1965).

[19] V. K. Balasubrahmanyan, E. Boldt, and R. A. R. Palmeira, Phys. Rev. **140**, B1157 (1965).

[20] A. C. Durney, H. Elliot, R. J. Hynds, and J. J. Quenby, Proc. Roy. Soc. (London), **A281**, 553 (1964).

[21] M. A. Pomerantz, S. P. Duggal, and L. Witten, Space Res. **4**, 972 (1964).

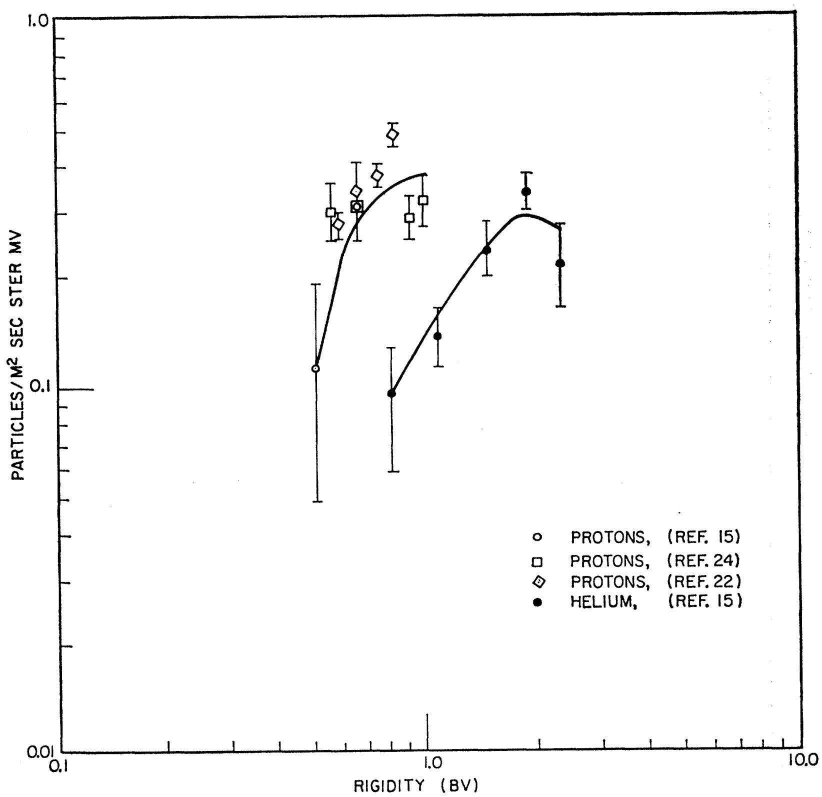

FIG. 2. The rigidity spectra of cosmic-ray protons and helium in 1961 (Refs. 15, 24 and 22). The helium-nucleus data are multiplied by 7.2. Curves have been fitted to the data.

Balasubrahmanyan *et al.*[19] have attempted to evaluate the various modulations all the way back to the cosmic-ray sources, by considering in addition to solar modulation also ionization losses in the interstellar medium and transmission efficiency at the sources themselves. A critical discussion of some of their arguments is presented here.

In Sec. II, the cosmic-ray proton and helium rigidity spectra and energy/nucleon spectra are compared over a large portion of the last solar cycle. In Sec. III, the nature of solar modulation is investigated. In Sec. IV, the local interstellar cosmic-ray proton and helium energy/nucleon spectra are discussed. In Sec. V some aspects of cosmic-ray acceleration and modulation in cosmic-ray source regions are explored. In Sec. VI, some properties of the interplanetary medium are investigated. Section VII presents the conclusions.

II. COSMIC-RAY SPECTRA

A. Rigidity Spectra of Protons and Helium Nuclei

Figures 1–3 show the proton and helium rigidity spectra obtained in 1959, 1961, and 1963, respectively. The June 1959 data come from McDonald and Webber[6] and from Freier and Waddington.[3] The data from summer 1961 come from Meyer and Vogt[22] (their data below rigidities of 0.55 BV were not utilized on the basis of arguments presented by Freier and Waddington[23]), from Bryant *et al.*,[24] and from Fichtel *et al.*[15] The data for 1963 were taken from Ormes and Webber,[13] Freier and Waddington,[14] Fichtel *et al.*,[15] Balasubrahmanyan and McDonald,[16] and the lowest rigidity data are taken from McDonald and Ludwig[25] and from Fan *et al.*[26] The helium data have been normalized to the proton data by multiplying the helium intensities by 7.2. The latter value has been used by Webber and McDonald.[6] Curves have been fitted to all the spectra.

Figure 4 presents the fitted curves of Figs. 1–3. The qualitatively striking features are: In 1959, the proton and normalized helium spectra coincide in the narrow region where they overlap (1.2–1.3 BV), both numerically, and in slope. There appears to be no splitting of the helium and proton spectra, but there are no experimental data on helium below a rigidity of 1.2 BV.

The helium spectrum of 1961 appears rather similar to the 1959 spectrum in the rigidity interval where comparison is possible (1.2–2.3 BV). The intensity has increased only slightly, by a factor of about 1.2. The

[22] P. Meyer and R. Vogt, Phys. Rev. **129**, 2275 (1963).

[23] P. S. Freier and C. J. Waddington, J. Geophys. Res. **70**, 2111 (1965).

[24] D. A. Bryant, T. L. Cline, V. D. Desai, and F. B. McDonald, J. Geophys. Res. **67**, 4983 (1962).

[25] F. B. McDonald and G. H. Ludwig, Phys. Rev. Letters **13**, 783 (1964).

[26] C. Y. Fan, G. Gloeckler, and J. A. Simpson, J. Geophys. Res. **70**, 3515 (1965).

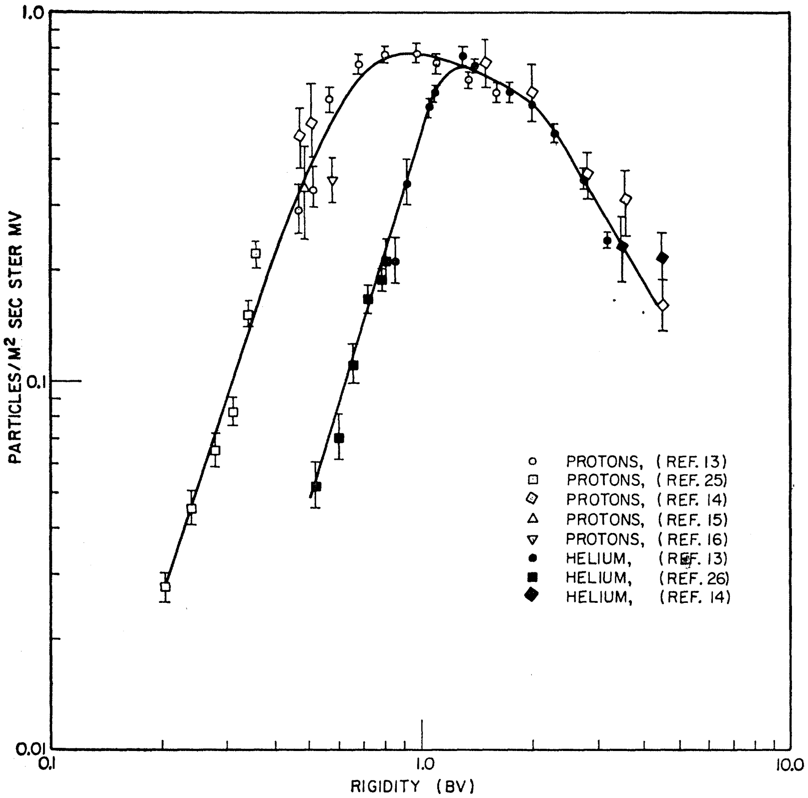

FIG. 3. The rigidity spectra of cosmic-ray protons and helium in 1963 (Refs. 13–16, 18, 26, 25). The helium-nucleus data are multiplied by 7.2. Curves have been fitted to the data.

proton spectrum (below 1.0 BV), on the other hand, has increased greatly in intensity from 1959, by a factor of about 4. The helium and proton spectra in 1961 are split in the interval where they can be compared (0.8–1.0 BV). These differences will be considered again in Sec. III.

By 1963, both spectra have increased further in intensity. The proton and helium spectra above 1.3 BV appear to be similar, while below 1.3 BV a split appears.[13–16]

B. Energy/Nucleon Spectra of Protons and Helium Nuclei

The ratio of helium to proton fluxes at a rigidity of about 15 BV was shown to be approximately 1:7 by Shapiro *et al.*[27] This ratio is independent of particle rigidity[28] between 1.5 and 20 BV, and appears to be so even at higher rigidities.[17] Furthermore, at high rigidities (approximately 1000 BV) the primary cosmic-ray rigidity (or energy/nucleon) spectra follow a power law[29] $dJ/dR \propto 1/R^{2.7}$ or[30] $dJ/dR \propto 1/R^{2.5}$. The latter case is considered in the third paragraph of Sec. IIB. The energy/nucleon ϵ of a particle is related to rigidity R by the expression

$$\epsilon = RZ/A\beta - M/A\,, \tag{1}$$

where Z is the atomic number or charge number of the particle, β its velocity in units such that the velocity of light is unity, M its mass, and A its mass number. At high rigidities, $\beta \approx 1$ and the term M/A becomes negligible. Therefore, at high rigidities, $\epsilon \approx R$ (numerically, not dimensionally) for protons, and $(1/2)R$ in case of helium. Hence, at high values of rigidity or energy/nucleon, the ratio of helium to protons is $2^{2.7}/2 = 3.2$ times smaller in an energy/nucleon interval than in a rigidity interval.

Therefore, the normalization factors of the helium and proton rigidity and energy/nucleon spectra (7 and 5.7, respectively) in the paper of Balasubrahmanyan *et al.*,[19] that fit the data below 1 BeV/nucleon, cannot both apply at high values of energy.

Assuming that the proton-to-helium ratio of 7.2 is valid at high rigidities, the normalization factor by which the helium spectra must be multiplied in an energy/nucleon representation is $7.2 \times 3.2 = 23$. This value is consistent with that given by Webber[17] at 10 BeV/nucleon. The same normalization factor of 23 is also obtained by using the proton to helium ratio of 8.0 and a power law $dJ/dR \propto 1/R^{2.5}$, as proposed re-

[27] M. M. Shapiro, B. Stiller, and F. W. O'Dell, Nuovo Cimento Suppl. **8**, 509 (1958).
[28] F. B. McDonald and W. R. Webber, J. Phys. Soc. Japan **17**, Suppl. A-II, 428 (1962).
[29] A. G. Barkow, B. Chamany, D. M. Haskins, P. L. Jain, E. Lohrmann, M. W. Teucher, and M. Schein, Phys. Rev. **122**, 617 (1961).

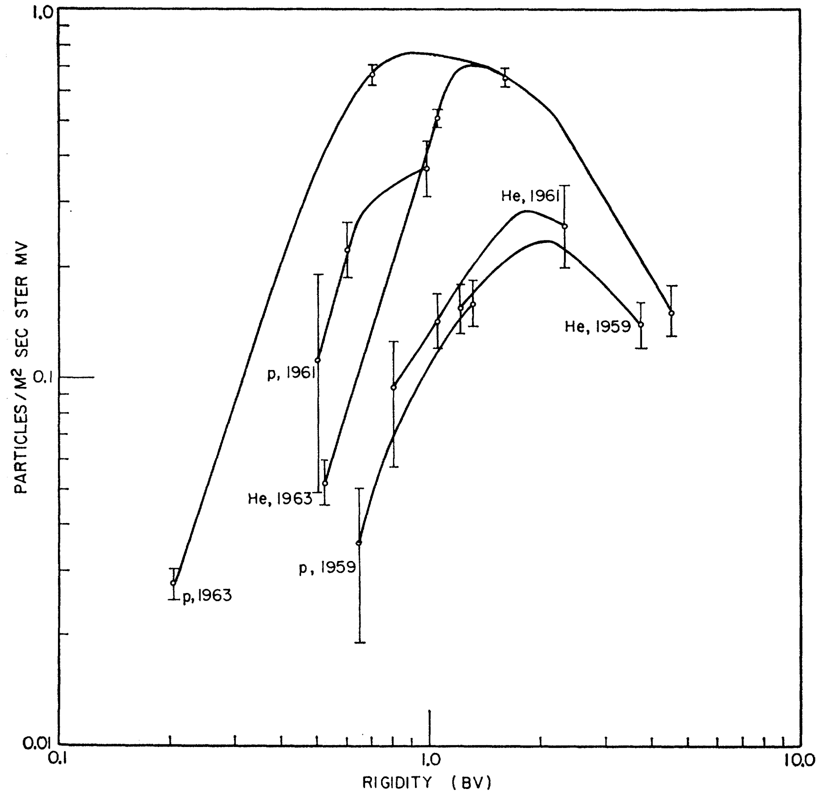

FIG. 4. The rigidity spectra of cosmic-ray protons and helium in 1959, 1961, and 1963. The fitted curves of Fig. 1-3 are presented here in a single graph. Representative magnitudes of errors have been drawn in.

cently by Webber.[30] The reasons for normalizing the spectra at *high* values of energy/nucleon or rigidity are given at the beginning of Sec. IV.

Figure 5 shows the fitted curves of the proton and helium rigidity spectra of Fig. 4 converted to an energy/nucleon scale with the helium spectra multiplied by a factor of 23. The ratio of helium to protons increases in 1959 and in 1963 as the energy/nucleon decreases from 1.0 BeV/nucleon to 0.2 BeV/nucleon. Below 0.2 BeV/nucleon, the available proton and helium spectra (obtained in 1963) appear parallel. The degree of modulation of the proton spectra between 1959 and 1963 is about two times that of the helium spectra between the same years, from 0.2 to 1.0 BeV/nucleon. These observations will be interpreted in the following section.

III. THE NATURE OF SOLAR MODULATION

The observed split and increasing separation of the proton and helium rigidity spectra between 1.3 BV and 0.6 BV is suggestive of a velocity (i.e., energy/nucleon)-dependent solar modulation, and it has been so interpreted by many authors, at least for rigidities below approximately 1.5 BV. This interpretation need not be correct, since the split can be explained to a large degree in terms of modulation effects prior to entering the solar system, e.g., at the cosmic-ray sources, and ionization losses in interstellar space. Ionization loss $\propto (Z^2/M)F(\beta)$, and is hence nearly the same for helium and protons at the same values of energy/nucleon. Figure 3 in the paper of Balasubrahmanyan *et al.*[15] shows an interstellar spectrum (curve B) that has been modulated by transmission from source, and passage through 2.5 g/cm² material. If we transform curve B to a rigidity scale for protons and helium separately, and normalize the proton curve at a rigidity of 3 BV to the helium curve, there is an increasing split between the helium and proton spectra as we proceed to lower rigidities. The magnitude of the deduced split at 0.6 BV is similar to the actual split depicted in Fig. 4.

On the basis of models[8–12] describing modulation in terms of diffusion through solar-wind clouds or magnetic kinks, we shall explore the solar modulation by means of the equation

$$I \equiv dJ/dR = (dJ/dR)_0 \exp[-K(t)f(R,\beta)]. \qquad (2)$$

Here dJ/dR is the intensity per unit rigidity interval per steradian, at time t, at the top of the earth's magnetosphere. With the subscript 0 the same expression denotes the value before solar modulation. $K(t)$ is proportional to the solar-wind velocity and the effective number of scattering centers along the path of the particle. The latter quantities depend on the solar cycle and hence on time. It is assumed that $K(t)$ does not depend on rigidity; anyhow, any rigidity dependence

[30] W. R. Webber, in *Proceedings of the Ninth International Conference on Cosmic Rays, London, 1965* (The Institute of Physics and The Physical Society), Vol. 1, p. 403.

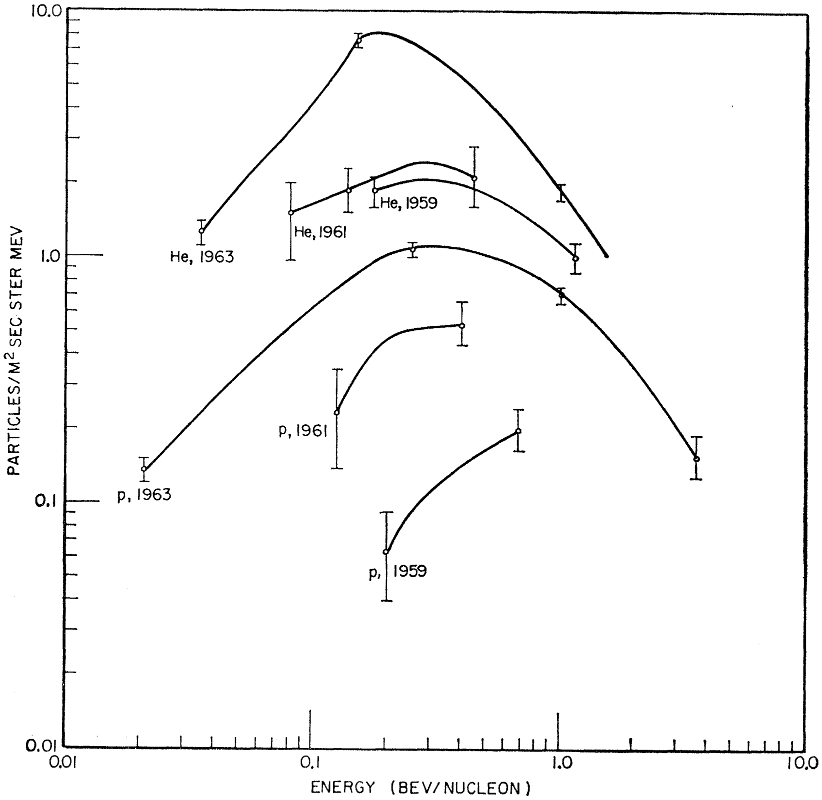

FIG. 5. The cosmic-ray proton and helium energy/nucleon spectra in 1959, 1961, and 1963. The curves are calculated from fitted rigidity-spectrum curves of Fig. 4. The helium spectrum has been normalized to the proton spectrum by multiplying by $7.2\times 2^{1.7}=23$. Representative magnitudes of errors have been drawn in.

of $K(t)$ can be transferred over to the function $f(R,\beta)$. R and β are the rigidity and velocity of the cosmic-ray particle, respectively.

We shall consider the following forms of $f(R,\beta)$: (a) $f(R,\beta)=1/(R\beta)$ which corresponds to Dorman's intermediate-rigidity case, (b) $f(R,\beta)=1/\beta$, which corresponds to Parker's low-rigidity limiting case, (c) $f(R,\beta)=1/(R^2\beta)$, which corresponds to Parker's high-rigidity case, and (d) $f(R,\beta)=1/R$, which is purely rigidity dependent.

The method of analysis is similar to that of Gloeckler.[18] Figure 6 presents values of $(I[1963]-I[1959])/I[1959]$ as a function of rigidity. The differential intensities are taken from the proton and helium curves of Figs. 1 and 3. Curves that represent models (a) to (d) have been drawn in, normalizing them to the helium data at 1.5 BV. We note that the curves representing Dorman's model and a purely rigidity-dependent modulation represent the data well over the whole range of rigidities (from 0.65 to 3.5 BV), while Parker's two limiting cases do not.

Figure 7 presents the corresponding values based on a comparison of the 1963 and 1961 spectra. Here we observe a "split" in the relative modulation of protons and helium, that fits best Parker's low-rigidity model below 1 or 2 BV. The data above 1 BV can also be fitted by Dorman's model, and the purely rigidity-dependent model.

Figure 8 presents the corresponding values already published by Gloeckler[18] on low-rigidity helium modulation in 1963–64. We observe that both Parker's low-rigidity model, and the purely rigidity-dependent model yield good fits to the data.

Figure 9 is evaluated from a comparison of proton spectra for rigidities of 1.2–15 BV, at solar maximum and solar minimum, as given by Webber.[17] His data are based on variations of ion chamber measurements over the solar cycle. Dorman's model and the purely rigidity dependent model again provide good fits to the data; Parker's high- and low-rigidity limiting cases do not.

Nagashima *et al.*[4] have analyzed the modulation of heavy nuclei observed on Ariel I[21] in 1962, and Explorer VII[20], in 1960, between 3.5 and 12 BV. They came to the conclusion that Parker's high-rigidity expression best describes the modulation between 6 and 12 BV. If, however, the satellite data at 3.5 BV are included, and the 5% normalization figure for the 1960 and 1962 data at 12 BV is changed to 10%, then Dorman's modulation function provides a better fit to the data, as shown in Fig. 10. (If the analysis is confined only to the high-rigidity interval, between 6 and 12 BV, then both Parker's high-rigidity modulation function and Dorman's function can be fitted to the data.) The 5% normalization figure for the integral flux at 12 BV, used by Nagashima *et al.*, was estimated on the basis of the records of two equatorial neutron monitors near sea level. The alternative normalization value of 10% is based on Webber's compilation of ion chamber data,[17] extrapolated to the top of the atmosphere. These show a nearly 20% modulation at a 12 BV cutoff, between solar maximum and solar minimum. Between the flights of Ariel I and Explorer VII, the change in the neutron monitor counting rate was approximately half of that between solar maximum and solar minimum—hence the 10% normalization value was chosen in the alternative analysis presented in Fig. 10.

Manzano and Winckler[31] have also attempted to evaluate the relative modulation of spectra as a function of rigidity. At 12 BV they predict the solar modula-

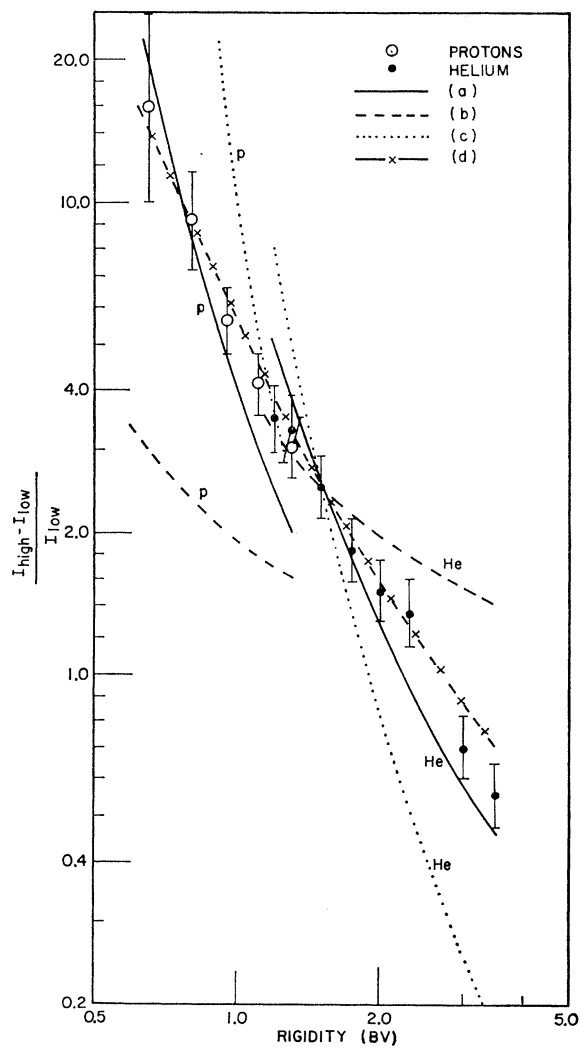

FIG. 6. The fractional change in cosmic-ray proton and helium fluxes between 1963 and 1959 as a function of rigidity. Curves *a*, *b*, *c*, and *d* represent Dorman's modulation function, Parker's low-rigidity modulation function, Parker's high-rigidity modulation function, and a purely rigidity-dependent modulation function, respectively. The data are based on Refs. 6, 3, 13–16.

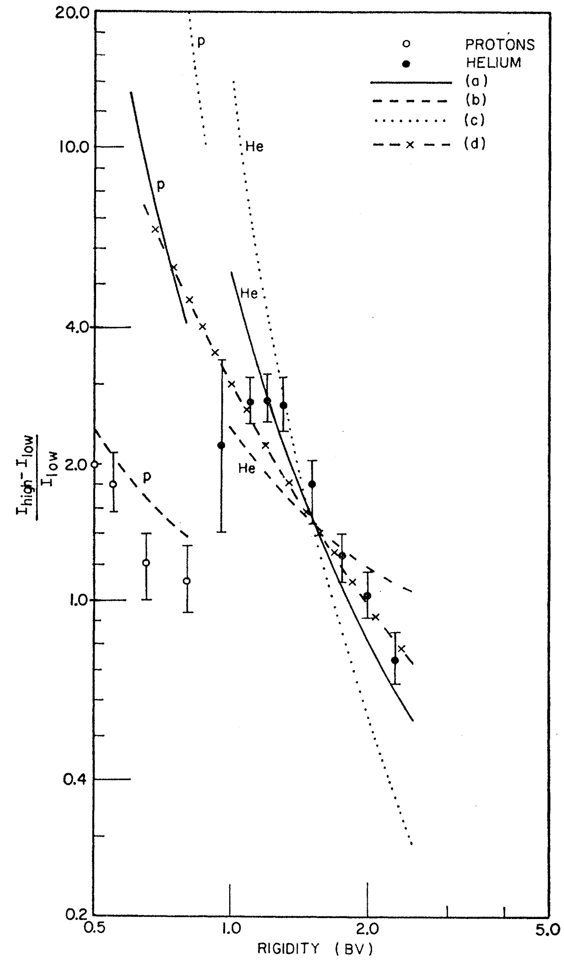

FIG. 7. The fractional change in cosmic-ray proton and helium fluxes between 1963 and 1961 as a function of rigidity. Curves *a*, *b*, *c*, and *d* represent Dorman's modulation function, Parker's low-rigidity modulation function, Parker's high-rigidity modulation function, and a purely rigidity-dependent modulation function, respectively. The data are based on Refs. 13-16, 24, and 22.

tion effect to be 4 times less than that evaluated by Webber[17] (only about 6% instead of 20%). The discrepancy appears to be due to an error in the derivation of Eq. (3) in the paper of Manzano and Winckler; the expression $(d/dP)[I-I_0]$ is not equal to $I_0(d/dP)[(I-I_0)/I_0]$, unless I_0 is a constant, which is not the case here. That their 6% figure is too low, can also be inferred from Fig. 5 of Manzano and Winckler, which shows the Mt. Norikura neutron monitor (at about 12-BV geomagnetic cutoff) to vary by 9% over the solar cycle. The variation in the primary cosmic-ray differential intensity at 12 BV has to be appreciably larger than 9%.

Ormes and Webber[32] have recently compared the 1963 and 1964 proton and helium spectra for rigidities

[31] J. R. Manzano and J. R. Winckler, J. Geophys. Res. **70**, 4097 (1965).

[32] J. F. Ormes and W. R. Webber, in *Proceedings of the Ninth International Conference on Cosmic Rays, London 1965* (The Institute of Physics and The Physical Society), Vol. **1**, p. 349.

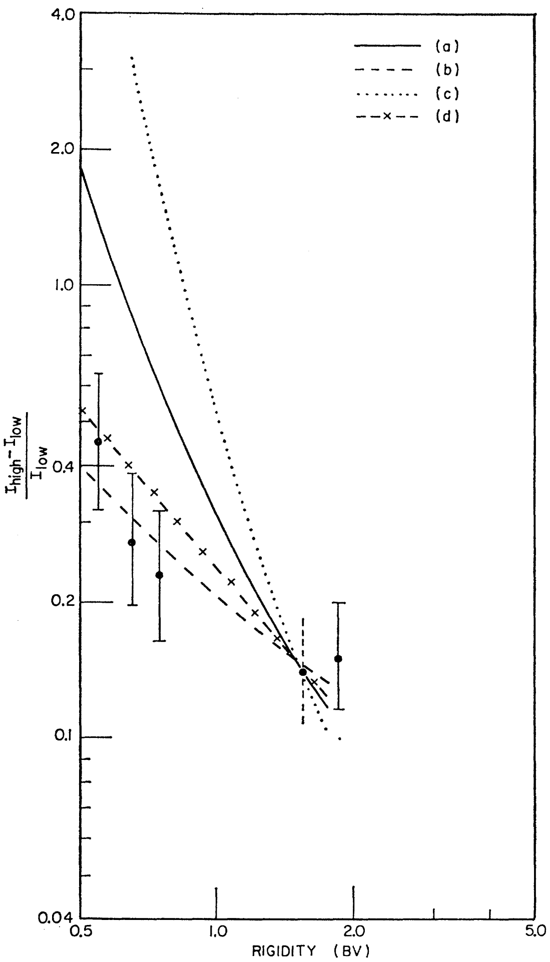

FIG. 8. The fractional change in cosmic-ray helium fluxes between a three-month period in 1964 and 1963 as a function of rigidity. Curves a, b, c, and d represent Dorman's modulation function, Parker's low-rigidity modulation function, Parker's high-rigidity modulation function, and a purely rigidity-dependent modulation function, respectively. The data are taken from Ref. 18.

of 0.6 to 4 BV. But the data could not be read off from their graphs with sufficient accuracy to present here in the form of Figs. 6–10. Their data, when treated as in Figs. 6–10, are consistent with Dorman's model and the purely rigidity-dependent model, all the way from 0.6 to 4 BV. The data do not agree with the predictions of Parker's high- and low-rigidity limiting cases.

It is seen from Fig. 5 that the separation of the helium and proton spectra in 1963 does not continue to increase with decreasing energy, below 0.15 BeV/nucleon. This is consistent with a transition to a velocity-dependent modulation, as predicted by Parker. (A proton at an energy of 0.15 BeV/nucleon has a rigidity of 0.55 BV.)

Taken together, the data of Figs. 6–10, and those of Ormes and Webber for 1963 and 1964, may be summarized as follows: In the interval 1.5–15 BV, the best fit is provided by Dorman's modification of Parker's model. Between 0.6 and 1.5 BV the best fit is sometimes provided by Parker's low-rigidity model, and sometimes by Dorman's model. The value of the transition rigidity separating the regions of applicability of the two models may vary with time, since it depends on the scale of the smallest plasma clouds or magnetic kinks, and their field strengths. These may be variable over the solar cycle.

Another set of data favors the operation of a rigidity-dependent component in the solar modulation of galactic cosmic rays, between 1.2 and 1.8 BV—the time variations in the abundance ratios of helium isotopes and of light-to-medium nuclei (compiled by Hildebrand and Silberberg[33]).

The difference in the 1959 and 1961 spectra can be explained by a transition in the limiting rigidity from less than 0.7 BV to about 1.0 BV, below which the modulation would change over from Dorman's form to Parker's velocity-dependent low-rigidity form. Such

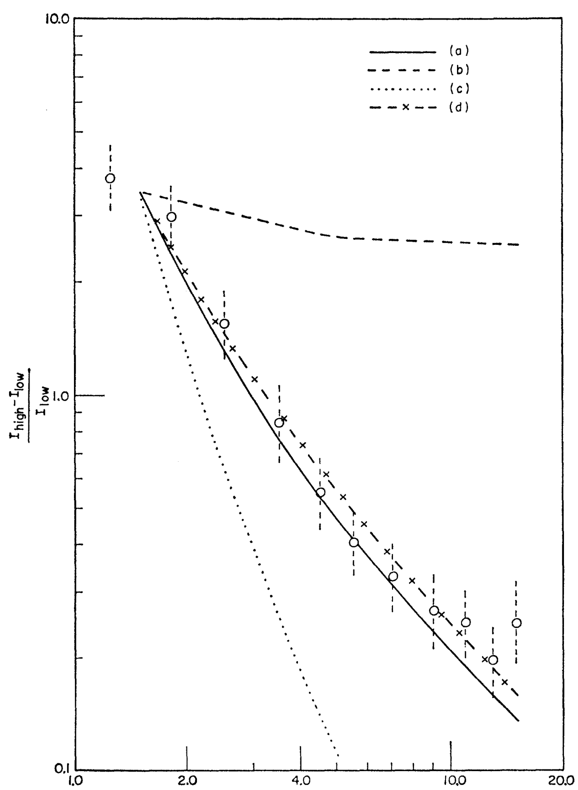

FIG. 9. The fractional change in cosmic-ray proton fluxes between solar minimum and solar maximum as a function of rigidity. Curves a, b, c, and d represent Dorman's modulation function, Parker's low-rigidity modulation function, Parker's high-rigidity modulation function, and a purely rigidity-dependent modulation function, respectively. The data are based on Ref. 17.

[33] B. Hildebrand and R. Silberberg, Phys. Rev. 141, 1248 (1966).

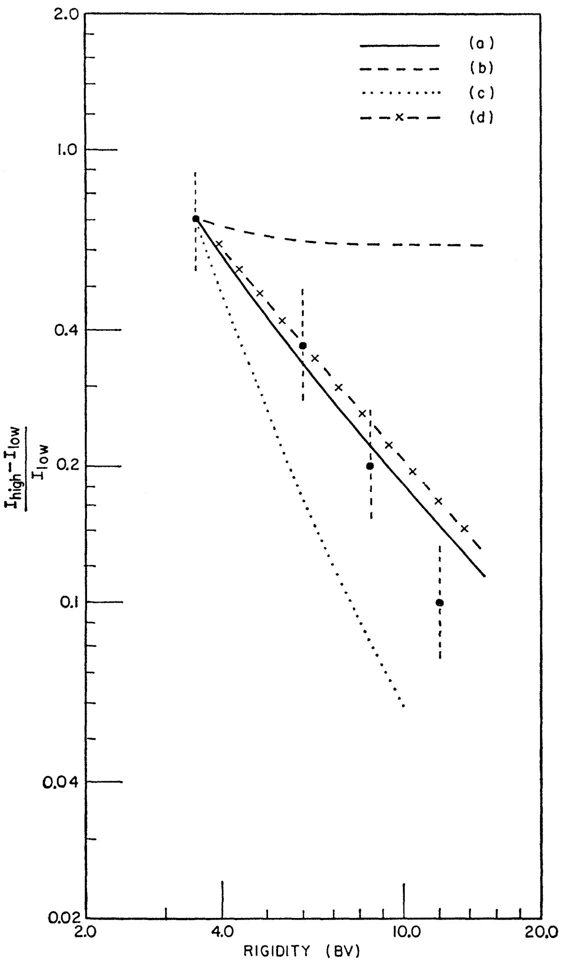

FIG. 10. The fractional change in cosmic-ray heavy nuclei fluxes between 1962 and 1960 as a function of rigidity. Curves a, b, c, and d represent Dorman's modulation function, Parker's low-rigidity modulation function, Parker's high-rigidity modulation function, and a purely rigidity-dependent modulation function, respectively. The data are based on Refs. 21 and 20.

a change can leave the flux above 1.0 BV nearly unchanged, while leading to a large increase in flux at rigidities below the limiting rigidity. The possibility that solar protons contributed appreciably in 1961 to the proton flux between 0.6–1.0 BV seems to be discounted by the form of the spectrum, which rises with increasing rigidity.

The purely rigidity-dependent model also provides a good fit in most cases, but cannot explain the difference between the 1959 and 1961 proton spectra. Parker's low-rigidity model $f(R,\beta)=1/\beta$ at low rigidities, and Dorman's model $f(R,\beta)=1/R\beta$ at somewhat higher rigidities may "simulate" the purely rigidity-dependent function $f(R,\beta)=1/R$. This comes about, because at low values of rigidity the relativistic factor $\gamma\approx 1$, hence, approximately, $R\propto\beta$, while at high rigidities, $\beta\approx 1$, hence, approximately, $R\propto R\beta$. To eliminate the possibility of a purely rigidity-dependent modulation, one could determine the relative change in the modulation of protons and helium nuclei at the same values of energy/nucleon, at low energies, where Dorman's model should no longer be applicable. For a velocity-dependent modulation, protons and helium are then modulated equally, while for a rigidity dependent modulation, with $f(R,\beta)=1/R$, protons are modulated 2 times more than helium on a logarithmic scale.

A solar modulation of the form $\exp[-K(t)/R]$ or $\exp[-K(t)/R\beta]$ (with R to the first power), leads to the relationship that at the same values of energy/nucleon, protons are modulated twice as much as helium on a logarithmic scale, since protons have half the rigidity of helium nuclei. [The phrase "on a logarithmic scale" means, e.g., that when the intensity of helium has increased by a factor of 4 as it did between 1959 and 1963 at 0.2 BeV/nucleon, the intensity of protons should increase by a factor of 16 (rather than 8, which would be the case if the change in intensity of protons were twice that of helium on a linear scale). The observed increase of 16 ± 6 in the proton intensity at 0.2 BeV/nucleon favors the modulation of protons to be twice that of helium on a logarithmic scale rather than on a linear scale.] A comparison of the proton and helium energy/nucleon spectra of 1959 and 1963 in Fig. 5 shows that protons are indeed modulated

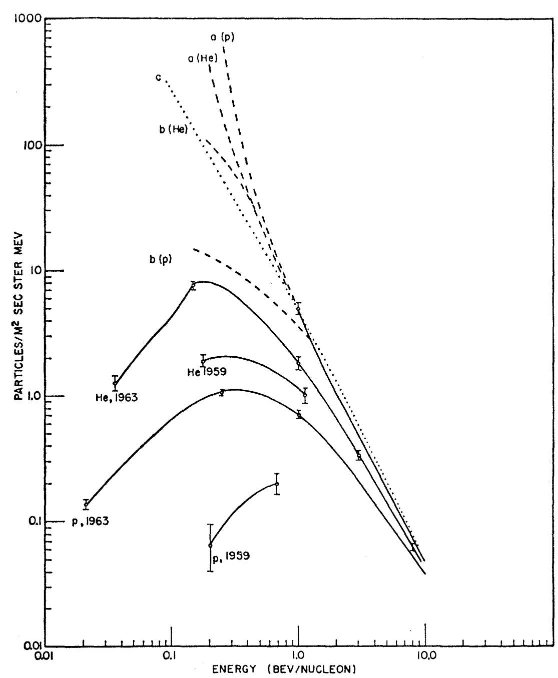

FIG. 11. The cosmic-ray proton and helium spectra in 1959 and 1963. The data of 1959 are based on Refs. 6 and 3, those of 1963 on Refs. 13-18, 25, and 26. Curves a and b represent the local interstellar spectra for particular models discussed critically in the text. Curve c represents a power-law rigidity spectrum. The helium spectra have been multiplied by 23.

about twice as much as helium. Such a difference in the relative modulation of protons and helium nuclei was also observed by Ormes and Webber[32] on comparing the 1963 and 1964 spectra at rigidities of 0.6 to 4 BV. The above form of solar modulation is also consistent with Webber's observation[17] that the ratio of helium to protons in 1963 decreases with increasing cosmic ray energy between values of 0.2 and 10 BeV/nucleon, from about 0.3 to about 0.05.

The observed relative modulations of the helium and proton spectra disagree with the theory of Axford[34] as treated by Quenby,[35] which predicts the helium and proton modulations to be similar at the same values of energy/nucleon.

IV. THE LOCAL INTERSTELLAR ENERGY/NUCLEON SPECTRUM OF COSMIC RAYS

A comparison of the energy/nucleon spectra of cosmic-ray protons and helium is helpful in determining the nature of various modulations that the cosmic rays undergo after acceleration. At the same values of energy/nucleon, the effects of ionization loss on the proton and helium spectra are almost equal, while a rigidity-dependent transmission in the solar system or in cosmic-ray source regions would cause the helium and proton spectra to diverge. These modulation effects should affect much less the high-energy cosmic rays. Hence, to determine the nature and the degree of the modulations, it is convenient to normalize the proton and helium spectra at the high-energy end.

In the rigidity interval where Dorman's form of solar modulation is applicable, the ratio of helium to protons at an energy/nucleon ϵ is given by

$$(dJ/d\epsilon)_{\mathrm{He}}/(dJ/d\epsilon)_p=[(dJ/d\epsilon)_{\mathrm{He}}/(dJ/d\epsilon)_p]_0 \times\exp[K(t)/2R\beta]. \quad (3)$$

Here $[(dJ/d\epsilon)_{\mathrm{He}}/(dJ/d\epsilon)_p]_0$ is the ratio of helium to protons in local interstellar space, and R is the rigidity of protons at an energy/nucleon ϵ.

If (1) the cosmic-ray acceleration process in source regions is energy/nucleon dependent, (2) the transmission from cosmic-ray source regions is energy/nucleon dependent, and (3) cosmic-ray acceleration in interstellar space is either velocity-dependent, or is negligible, then the factor $[(dJ/d\epsilon)_{\mathrm{He}}/(dJ/d\epsilon)_p]_0$ is constant and can be normalized to unity. In this case the difference between the helium and proton spectra in Fig. 5 is due to solar modulation. The arguments in the following section, and the rigidity-independence of the helium-to-proton ratio above a rigidity of 1.25 BV suggest, however, that the cosmic-ray acceleration process in source regions and transmission from source regions may be predominantly rigidity dependent. In this case

$$[(dJ/d\epsilon)_{\mathrm{He}}/(dJ/d\epsilon)_p]_0 =2[(dJ/dR')_{\mathrm{He}}{}^{R'=2R}/(dJ/dR')_p{}^{R'=R}]_0, \quad (4)$$

where R is again the rigidity of protons at an energy/nucleon ϵ. If the local interstellar cosmic-ray rigidity spectrum follows the power law $dJ/dR=KR^{-q}$ in the relevant rigidity interval, then Eq. (4) reduces to 2^{1-q}. In this case $[(dJ/d\epsilon)_{\mathrm{He}}/(dJ/d\epsilon)_p]_0$ is again constant and can be normalized to unity. If, however, the local interstellar cosmic-ray rigidity spectrum flattens out at lower rigidities, then the ratio of helium to protons increases with decreasing energy/nucleon. The increase in the helium-to-proton ratio with decreasing energy/nucleon, above 200 MeV/nucleon, shown in Fig. 5 for 1963, can be reproduced by assuming the local interstellar proton and helium rigidity spectra to be similar to the solar minimum spectrum (Fig. 19 in Webber's review paper[17]), and transforming them to an energy/nucleon scale.

We have seen that solar modulation reduces the intensity of helium half as much as that of the protons at the same value of energy/nucleon. This makes it possible to evaluate the slope of the local interstellar cosmic-ray spectrum from Eqs. (2), (3), and (4), provided the ratio of helium to protons is constant in local interstellar space in the relevant energy/nucleon interval. In this section we shall attempt to determine quantitatively the magnitude of solar modulation at solar minimum, as well as the local interstellar cosmic-ray energy/nucleon spectrum.

Figure 11 gives again the energy/nucleon spectra of protons and helium in 1963, but both have been extended to 10 BeV/nucleon, using the proton spectrum at solar minimum, taken from Webber,[17] and normalizing it to the 1963 spectrum at an energy of 1 BeV/nucleon. The helium spectrum above 1 BeV/nucleon has been obtained from the helium/proton ratios at different values of energy/nucleon, as given in Fig. 47 of Webber's review paper.[17] Representative magnitudes of errors have been drawn in. The error flags at 3 and 8 BeV/nucleon are based on helium-to-proton intensity ratios, and represent hence only the separation of the helium and proton intensity spectrum curves.

Assuming that the ratio of helium/protons in local interstellar space does not vary with energy/nucleon above 1 BeV/nucleon (i.e., that the local interstellar spectrum either is energy/nucleon dependent, or is rigidity dependent and follows a power-law between rigidities that correspond to energies of 1 to 10 BeV/nucleon), we calculate the solar-system transmission function and the intensity of the cosmic rays in local interstellar space from the difference of the proton and helium spectra, using Dorman's model. The calculation yields $\exp[(-2.9\pm0.3)/R\beta]$ for the transmission function in 1963, which means that at 1 BeV/nucleon, the intensity of protons in 1963 is reduced by a factor of

[34] W. I. Axford, Planet. Space Sci. **13**, 115 (1965).
[35] J. J. Quenby, in *Proceedings of the Ninth International Conference on Cosmic Rays, London, 1965* (The Institute of Physics and The Physical Society), Vol. 1, p. 180.

approximately 7 ± 2 from that in local interstellar space and helium by a factor of approximately 2.7 ± 0.5. If, however, the local interstellar cosmic-ray spectrum in the relevant rigidity interval (1) is a rigidity-dependent spectrum and (2) becomes flatter towards lower values of rigidity, then the above numerical values are smaller. Curves $a(p)$ and a(He) represent the 1963 proton and helium spectra, respectively, multiplied by the factor $\exp[2.9/R\beta]$, i.e., curves $a(p)$ and a(He) represent the local interstellar proton and helium spectra, if Dorman's expression for intermediate values of energy were applicable also at the low energies shown.

A good support and internal consistency test for the applicability of Dorman's model and for the energy-independence of the helium to proton ratio in interstellar space at *intermediate* values of energy is provided by the following observation: The curves $a(p)$ and a(He) nearly coincide above 0.7 BeV/nucleon (they are represented by a single line in Fig. 11, above an energy of 1 BeV/nucleon); i.e., the relative modulation of protons and helium nuclei in the energy/nucleon interval between 0.7 and 10 BeV/nucleon is in agreement with Dorman's theory, and with energy independence of the helium-to-proton ratio in the above energy interval in local interstellar space. Neither of Parker's transmission functions $\exp[-K/\beta]$ and $\exp[-K/R^2\beta]$ can lead to such agreement for the local interstellar proton and helium spectra. Furthermore, curves $a(p)$ and a(He) are close to curve c above an energy of 1 BeV/nucleon. Curve c represents a power-law rigidity spectrum of the form $dJ/dR=KR^{-q}$, where $q=2.4$. The latter numerical value was obtained by requiring the rigidity spectrum to coincide with $a(p)$ or a(He) at energies of 1.0 and 10 BeV/nucleon. The similarity of curves $a(p)$, a(He), and c between 1 and 10 BeV/nucleon means that if the difference of the proton and helium energy/nucleon spectra is predominantly due to the local interstellar spectrum becoming flatter with decreasing rigidity in the relevant rigidity interval, then the deviation from a power law spectrum is similar to that produced by a transmission mechanism of the form proposed by Dorman.

Below an energy of about 0.3 BeV/nucleon, curves $a(p)$ and a(He) do not appear to be good representations of the local interstellar spectra; due to ionization losses the slopes should decrease rather than increase at lower values of energy. Curves $b(p)$ and b(He) show how the curves $a(p)$ and a(He) become modified, if the modulation function changes abruptly over to the form $\exp[-K/\beta]$ below a rigidity of 2 BV.

Fermi deceleration of cosmic rays in the solar-wind clouds or kinks in magnetic field lines will also modulate[35] the energy spectra, but since Fermi acceleration is velocity dependent, both protons and helium nuclei are affected equally at the same value of energy/nucleon.

Considering ionization loss effects at energies above 0.15 BeV/nucleon to be negligible, we conclude that while above 1 BeV/nucleon the modulation is well described in terms of Dorman's model, below it the case is more complicated:

I. Either the transition from Dorman's model to Parker's low-rigidity model is gradual with decreasing energy, with the local interstellar spectra being between $a(p)$ and $b(p)$, and a(He) and b(He), possibly in the neighborhood of (c), or lower, or

II. The transition is abrupt, and the local interstellar spectra of protons and helium are in the neighborhood of $b(p)$ and b(He), respectively, or lower. In this case, the difference between the proton and helium local interstellar spectra could arise from a rigidity-dependent transmission efficiency at the source, since at the same value of energy/nucleon, protons have the lower rigidity, and hence their intensity is suppressed more than that of helium nuclei.

Furthermore, the slope of the cosmic-ray spectrum in cosmic-ray sources, at low energies, may also be energy or rigidity dependent. This introduces a further variable into consideration below energies of 1 BeV/nucleon. In light of this, no definite statement on the local interstellar energy spectra below 1 BeV/nucleon will be made at the present time.

V. SPECULATIONS ON ACCELERATION AND MODULATION IN COSMIC-RAY SOURCE REGIONS

Balasubrahmanyan *et al.*[19] assumed the cosmic-ray spectrum in the cosmic-ray source regions to be an energy/nucleon rather than a rigidity spectrum. As a possible argument for a different assumption we consider a comparison with the universal hydrogen to helium-nucleus abundance[36] of 8:1. This compares favorably with the cosmic-ray abundance ratio 7.2:1 in the rigidity interval and less favorably with the ratio 23:1 in the energy/nucleon interval. The fact that high-Z elements become progressively more abundant in cosmic rays compared to the universal abundance also favors the ratio 7.2:1, and hence a rigidity-dependent spectrum inside cosmic-ray sources.

It is seen in Fig. 11 that the separation of protons and helium does not increase below 0.15 BeV/nucleon but remains approximately constant at a value of 6. If solar modulation is purely velocity dependent below energies of 0.15 BeV/nucleon, then the constant separation of the helium and proton energy/nucleon spectra implies that these spectra differ by the same factor of six already in local interstellar space, i.e., before solar modulation (unless the separation is due to some residual rigidity-dependent solar modulation still being present at low values of energy per nucleon). Such a difference can be interpreted in terms of a rigidity (or partly rigidity)-dependent transmission efficiency at the cosmic-ray sources; since helium, having the higher

[36] F. Hoyle and R. J. Tayler, Nature **203**, 1108 (1964).

rigidity at the same value of energy/nucleon, is reduced less in intensity than protons. This conclusion on the nature of the transmission efficiency at cosmic-ray sources differs from that of Balasubrahmanyan *et al.*[19] but if their transmission efficiency is plotted as a function of $1/R$ or $1/R\beta$, instead of $1/\beta$, we obtain an equally good straight-line fit. Hence the straight-line fit alone does not provide a unique solution on the nature of the transmission efficiency at cosmic-ray sources.

VI. THE INTERPLANETARY MEDIUM

In Secs. III and IV we concluded (1) that the transition from Dorman's model to Parker's low-rigidity model occurs in the neighborhood of a rigidity of 1 BV, (2) that Dorman's model is applicable at least to a rigidity of 15 BV, and (3) that in 1963 $K(t)$ was $\leq 2.9\pm0.3$ in the interval where Dorman's model is valid. With the use of the above data and the theories of Parker[8–10] and Dorman,[11,12] various properties of the interplanetary medium can be evaluated.

Dorman's model can be used to determine the size or scale factors l of the smallest and largest plasma clouds or magnetic kinks emitted from the sun, if the field strength H of the scattering centers is given. A calculation yields: $l_{\min}\times H \simeq 3\times10^6$ G cm and $l_{\max}\times H \geq 5\times10^7$ G cm. If $H\simeq 5\times10^{-5}$ G also in the outer regions of the interplanetary medium, as it is in the neighborhood of one astronomical unit,[37] then the scale factors are distributed between $l_{\min}\simeq 6\times10^{10}$ cm and $l_{\max}\geq 10^{12}$ cm.

The modulation parameter $K(t)=(3v\nu/c)R_0$. Here v is the solar wind velocity, c is the velocity of light, ν is the number of scattering centers along the cosmic-ray path, away from the earth, and R_0 is the transition-rigidity between the regions of applicability of Dorman's and Parker's low-rigidity models. The product $v\nu=3\times10^{10}$ cm/sec. The evaluation of ν is complicated by the fact that the solar wind velocity is variable; at about 2 astronomical units, it decreases[38] by a factor of about 30, and remains then nearly constant up to 5 astronomical units. If we still use the value of $v\simeq 3.5\times10^7$ cm/sec measured by Lyon and Ness[37] in the neighborhood of one astronomical unit, then we obtain the number of scattering centers along the cosmic-ray path, $\nu\geq 800$. The above data permit the evaluation of the lower limit for the radial extent of the cosmic-ray convection and diffusion region beyond the orbit of the earth for 1963. Parker[10] gave the relationship $\nu=Nl^2(r_2-r_1)$, where N is the density of plasma clouds or magnetic kinks, r_1 is 1 astronomical unit and r_2 is the radial distance from the sun to which the scattering centers extend. The lower limit is $(r_2-r_1)=3$ astronomical units, obtained by setting $\nu=800$, $N=1/l^3$ and $l=6\times10^{10}$ cm. These values are similar in magnitude to those proposed by Parker.[10] If the modulation parameter $K(t)$ is somewhat smaller than 2.9, then the lower limits of ν and r_2 are also somewhat lower.

[37] N. F. Ness, in *Proceedings of the Ninth International Conference on Cosmic Rays, London, 1965* (The Institute of Physics and The Physical Society), Vol. 1, p. 14.

[38] J. C. Brandt and R. W. Michie, Phys. Rev. Letters 8, 195 (1962).

VII. CONCLUSIONS

(1) At rigidities of 1.5–15 BV, the relative solar modulation effects at different times of the solar cycle can be well described in terms of the modulation function $\exp[-K(t)/R\beta]$, proposed by Dorman for intermediate values of rigidity, where R and β are the rigidity and velocity of the cosmic-ray particle, respectively. Also a purely rigidity-dependent modulation function $\exp[-K(t)/R]$ fits the data.

(2) Between rigidities of 0.6 and 1.5 BV, the modulation is sometimes best fitted by Parker's expression $\exp[-K(t)/\beta]$, at other times again by the above-stated Dorman's expression, or the purely rigidity-dependent modulation function. It is possible that the limiting rigidity for the applicability of either model varies with time.

(3) The magnitude of the increase in the helium/proton ratio with decreasing energy/nucleon, below 10 BeV/nucleon, can be accounted for in terms of Dorman's modulation function.

(4) At the same value of energy/nucleon helium nuclei are modulated half as much as protons, as predictable from Dorman's model, or the above rigidity-dependent modulation function.

(5) The difference between the proton and helium energy/nucleon spectra can be used to deduce the shape and intensity of the local interstellar cosmic ray spectrum above energies of 1 BeV/nucleon, provided that the ratio of helium to protons in local interstellar space does not vary with energy above 1 BeV/nucleon. The latter condition is supported by the internal consistency argument given in Sec. IV. The modulation in summer 1963, at 1–10 BeV/nucleon, is then given by $\exp[(-2.9\pm0.3)/R\beta]$. This means that at 1 BeV/nucleon, in summer 1963, the intensity of cosmic-ray protons in local interstellar space was about 7 ± 2 of that near earth, while the intensity of helium was about 2.7 ± 0.5. These values should be regarded as upper limits, since a contribution to the observed difference between the proton and helium energy/nucleon spectra could be due to the local interstellar cosmic-ray spectrum becoming flatter at lower values of rigidity, provided that the local interstellar spectrum is rigidity-dependent.

(6) If the magnetic field strength of the scattering centers is also 5×10^{-5} G in the outer regions of the interplanetary medium, as it is near 1 astronomical unit, then the size or scale l of the scattering centers varies between $l_{\min}\simeq 6\times10^{10}$ cm and $l_{\max}\geq 10^{12}$ cm. If there is no partial breakdown of Dorman's model at energies of 1–10 BeV/nucleon, and if the ratio of

helium/protons in local interstellar space is constant in this interval, then the number of scattering centers, in 1963, along the cosmic-ray path exceeded 800. The lower limit to the radial extent of the cosmic-ray convection and diffusion region beyond the orbit of earth, in 1963, was 3 astronomical units. These values are not appreciably different from those used by Parker.[10]

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Quantization of Spin-2 Fields*

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A massive spin-2 field has been quantized using Schwinger's action principle. Lorentz invariance and physical positive-definiteness requirements have been verified.

I. INTRODUCTION

THE problem of quantization of massive spin-2 fields as well as other higher spin fields has been studied rather extensively in the past.[1] However, the question of whether the quantization of fields with spin 2 according to the techniques of the quantum action principle will lead to results which are consistent with Lorentz invariance as well as other physical requirements has not been touched. The recent experimental evidence on the existence of spin-2 particles arouses new interest in these problems. In this paper,[2] an attempt is made to study these problems. We limit our attention to a free, massive spin-2 field only. The quantization for massless spin-2 fields will be discussed in a separate publication.

II. CANONICAL FORMALISM

It is well known that a spin-2 tensor field should be represented by a symmetric tensor $h_{\mu\nu}$. In order to construct a Lagrange function which contains the gradient of the field variables linearly, we have to introduce additional field variables which transform like a third-rank tensor. Although the introduction of a symmetric tensor ${}_\lambda\Gamma_{\mu\nu}$ is more usual in the quantized gravitational field, we find that it is more convenient here to choose an antisymmetric tensor ${}_\mu H_{\nu\lambda}$ with the following symmetry properties[3]:

$$ {}_\mu H_{\nu\lambda} = -{}_\mu H_{\lambda\nu}\,, $$
$$ {}_\mu H_{\nu\lambda} + {}_\nu H_{\lambda\mu} + {}_\lambda H_{\mu\nu} = 0\,. $$

These two alternative descriptions are equivalent and they describe the same physical system. We first concentrate our attention on the second description only. We will show in the next section that these two descriptions are indeed equivalent. The Lagrange function of a spin-2 tensor field characterized by this antisymmetric tensor is given by[2]

$$ L = \tfrac{1}{2}(h_{\mu\nu}\,.\,\partial_\lambda {}^\mu H^{\lambda\nu} - {}^\mu H^{\lambda\nu}\,.\,\partial_\lambda h_{\mu\nu}) + \tfrac{1}{4}({}_\mu H_{\nu\lambda}\,.\,{}^\mu H^{\nu\lambda} - H_\lambda\,.\,H^\lambda) - \tfrac{1}{2}m^2(h_{\mu\nu}\,.\,h^{\mu\nu} - h^2)\,. \quad (1) $$

The plus and minus signs associated with the second and third terms have physical content. They are associated with the positive-definiteness requirements of this boson system. H_λ and h are shorthand notations for

$$ {}^\mu H_{\mu\lambda}\,, \quad \text{and} \quad h^\mu{}_\mu\,, $$

respectively.

The field equations follow from the principle of stationary action:

$$ \partial_\lambda {}^{(\mu} H^{\lambda\nu)} - m^2(h^{\mu\nu} - g^{\mu\nu} h) = 0\,, \quad (2) $$

$$ 2\,{}_\mu H_{\nu\lambda} - (g_{\mu\nu} H_\lambda - g_{\mu\lambda} H_\nu) - 2(\partial_\nu h_{\mu\lambda} - \partial_\lambda h_{\mu\nu}) = 0\,. \quad (3) $$

A symmetrization for the indices μ, ν in the parenthesis is understood. It is straightforward to show

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† John Parker Fellow.

[1] M. Fierz, Helv. Phys. Acta **12**, 3 (1939); M. Fierz and W. Pauli, Proc. Roy. Soc. (London) **A173**, 211 (1939); R. J. Rivers, Nuovo Cimento **34**, 386 (1964). A complete list of classical papers can be found in the bibliography of E. M. Corson, *Introduction to Tensors, Spinors, and Relativistic Wave-equations* (Blackie & Son, Ltd., Glasgow, 1953).

[2] Publication of this paper was stimulated by a recent paper of D. Adler, Can. J. Phys. **44**, 289 (1966). Throughout this paper we use the following notations: $g_{\mu\nu} = (-1, 1, 1, 1)$; all Greek indices μ, ν, $\cdots$ vary from 0 to 3 and all Latin indices i, j, $\cdots$ vary from 1 to 3. Repeated indices are to be summed over. The dots between the field operators indicate that the latter are symmetrically multiplied.

[3] Both descriptions are deduced from J. Schwinger, Phys. Rev. **130**, 1253 (1963). The Lagrange function for the Γ description was given by J. Schwinger to whom I am deeply indebted.