TABLE I. $B(E2, 2 \rightarrow 0)$ values for the 1.52-MeV transition in Ca^{42}, in units $e^2 F^4$. For the estimate in column 2, a radius $R=1.2A^{1/3}$ F was used. The value[a] given in column 3 is for an effective charge of 0.5e.

Experiment	Weisskopf estimate[b]	$(f_{7/2})^2{}_{2+} \rightarrow (f_{7/2})^2{}_{0+}$	Bertsch[c]
74±17	8.8	4.6	3.2

[a] We thank Dr. T. A. Hughes for this result.
[b] See Ref. 4.
[c] See Ref. 3.

With the value $N(E_r)=4.8\times10^{-3}N_{\rm total}$ for $\lambda=+\frac{1}{3}$, taken from Fig. 4, the average scattering observed in the two geometries leads to a width $\Gamma=(4.8\pm0.4)\times10^{-4}$ eV. If allowance is made for the range $-\frac{1}{3}<\lambda<+1$, the result of our study may be summarized as

$$\Gamma=(4.8\pm1.1)\times10^{-4}\text{ eV},$$

corresponding to a mean life of 1.4×10^{-12} sec. This experimental result is compared in Table I with various theoretical estimates.

Clearly, the 1.52-MeV transition in Ca^{42} is quite collective. If, e.g., it is attributed to the recoupling of two $f_{7/2}$ neutrons, the observed rate would imply the large value of 2e for the effective charge of an $f_{7/2}$ neutron. The transition is considerably more collective than was expected on the basis of Bertsch's calculations, and this presumably[3] calls for stronger mixing between deformed and undeformed wave functions.

ACKNOWLEDGMENTS

The authors thank Dr. G. F. Bertsch for suggesting this problem and for communicating to them the results of his calculations. They are indebted to Dr. G. L. Rogosa of the U. S. Atomic Energy Commission for arranging the loan of Ca^{42}, without which this study would not have been feasible. They gratefully acknowledge helpful discussions with Dr. S. Fallieros, Dr. T. A. Hughes, and Dr. M. Soga.

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$Si^{28}(d,p\gamma)Si^{29}$ Angular Correlations from 4 to 6 MeV*

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A study of the (p,γ) angular correlations involving the 1.28- and 2.03-MeV states in Si^{29} populated by the $Si^{28}(d,p)Si^{29}$ reaction has been carried out at five deuteron bombarding energies between 4 and 6 MeV. Angular-correlation measurements were carried out in the reaction plane and in that plane perpendicular to the reaction plane which contains the incident-beam direction. The data were analyzed by means of the distorted-wave stripping formalism. Statistical tensors, describing the orientation of the intermediate excited nuclear states, were calculated at each energy. Since the reaction-plane measurements on the 2.03-MeV state alone were sufficient for determining the statistical tensors describing the correlation over the sphere, comparison has been made between the measured perpendicular-plane correlations and the correlations predicted from these statistical tensors. In addition, a combined set of statistical tensors has been calculated utilizing data from the reaction-plane and perpendicular-plane correlations. The magnitudes of the proton polarizations have also been calculated for the 2.03-MeV excited-state reaction using the combined set of statistical tensors. The observed correlations are in agreement with the predictions of a distorted-wave theory and not with those of the plane-wave theory.

I. INTRODUCTION

THIS paper reports on the investigation of the $(d,p\gamma)$ angular correlations on Si^{28} through the first (1.28 MeV) and second (2.03 MeV) excited states in Si^{29}. These investigations are an extension to lower deuteron bombarding energies of the work of Kuehner, Almqvist, and Bromley[1] at Chalk River, Canada.

The general validity of the distorted-wave Born approximation (DWBA) in describing deuteron stripping reactions is now well accepted. The proton and deuteron wave functions used in the Born approximation are calculated from optical-model potentials whose parameters have been determined over wide energy ranges from fits to elastic-scattering angular distributions. Using these optical potentials and distorted-wave functions in describing incident deuterons and outgoing nucleons, reasonable fits to the angular distribution of the emitted nucleon in stripping reactions can be obtained. Current efforts in improving the DWBA description of stripping reactions involve the nature of the spin dependence of the deuteron and nucleon potentials. A number of theoretical investigations have been done in

* This work was supported in part by The National Science Foundation under Grant No. GP-2210.
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this area.[2–6] Experimentally, the polarization of the emitted nucleons is most sensitive to the details of the spin interaction, and to a lesser extent so are $(d,p\gamma)$ correlations. The angular distributions of the emitted nucleons do not appear to be very sensitive to the spin-dependent forces.

Huby, Refai, and Satchler,[7] hereafter referred to as HRS, have developed a formalism for the distorted-wave theory of stripping reactions in which they examine those features of the theory which are independent of the details of the distortion. The polarization of the emitted protons and the correlation function of de-excitation gamma rays are expressed as functions of statistical tensors ρ_{kq} which can be calculated given the details of the reaction. Calculations of this type have been performed at the Oak Ridge National Laboratory using the code SALLY[8] and the results are compared with the experimental quantities measured in this experiment.

The HRS formalism and also that of Tobocman and Satchler[9] are primarily concerned with those features of the DWBA theory which are characteristic of the theory in general as distinct from its details. One of the purposes of this experiment was to investigate some of those general features of the theory. Such measurements serve to test the validity of the DWBA assumptions as well as provide insight into the use of spin-dependent forces.

Although the HRS formalism assumes that the distorting potentials are spin-independent, polarization of the outgoing nucleons can still occur for all orbital angular momentum transfer $l_n \geq 1$. This polarization is due to the distorting potentials alone. It is known that the spin-independent DWBA treatment has not only failed to fit large-angle proton polarization data but predicts zero proton polarization for $l_n = 0$ transfer when considerable polarization has been measured for this case.[10–16] However, the spin-independent treatment may still have validity in the region of scattering angles on or near the main stripping peak where the various amplitudes are large.

A number of $(d,p\gamma)$ experiments have been performed[1,17–19] to check the prediction of the HRS formalism. The $Si^{28}(d,p\gamma)Si^{29}$ angular correlations from 6 to 9 MeV by Kuehner *et al.*[1] were a particularly stringent test of the HRS formalism since the complexity of the angular correlation permitted the determination of four independent parameters, the ρ_{kq}. They chose this reaction for measurement since it permits the study of two d-wave neutron capture situations, with $j = l_n \pm \frac{1}{2}$, and essentially identical kinematics. In the $j = l_n + \frac{1}{2}$ case (2.03 MeV), the emitted gamma ray to the spin $\frac{1}{2}$ ground state of Si^{29} has been measured as being a pure $E2$ multipole.[20] For the $j = l_n - \frac{1}{2}$ case (1.28 MeV) the $E2/M1$ mixing ratio has been measured[21] as 0.21 ± 0.03. Hence all the variables of the reaction were known (angular momentum transfer, spin and parity of target nucleus, the spins and parities of the intermediate excited states, and the multipolarity of the decay gamma rays). The only uncertainty in the reaction is the amount of competing compound-nucleus contribution. The HRS formalism considers only the direct stripping process.

In their determination of the statistical tensors, using the assumptions of the spin-independent distorted-wave approximation, Kuehner, Almqvist, and Bromley were unable to make unambiguous assignments of the statistical tensors over the entire energy range studied since the relationships among the theoretical parameters lead to the solution of a cubic equation. Thus, for $E_d < 7$ MeV there were two roots of possible physical significance, and hence two sets of statistical tensors, whose energy dependence was different. In fact, the energy dependence of the two sets of tensors indicated that at lower bombarding energies the predicted angular correlation would be very different for the two sets of statistical tensors. Measurements of this angular correlation at lower bombarding energies ($E_d < 6$ MeV) would thus be a sensitive test of the theory and would also serve to distinguish unambiguously between the two solutions. In addition, angular correlations measured in planes other than the reaction plane would serve to determine the correct solution unambiguously. This then was our final motivation in pursuing the $Si^{28}(d,p\gamma)Si^{29}$ angular correlation to lower energies.

II. EXPERIMENTAL APPARATUS

The beam from the Ohio State University 5.5-MeV Van de Graaff accelerator was magnetically analyzed and stabilized to 0.1% energy resolution. The beam was bent through a 7-port switching magnet into a scattering

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chamber, where the beam was collimated to a spot of 0.040 in. in diameter on target. Finally, the beam was collected in a Faraday cage at the rear of the scattering chamber and integrated by a 0.1% current integrator. Electric-field electron suppression was used on the collector.

The scattering chamber used for the correlation experiments was a 0.063-in. thick stainless-steel cylinder 10 in. in diameter with a hemispherical top. The chamber walls presented a uniform attenuating thickness to gamma rays over the upper half hemisphere. A solid-state detector which was capable of being positioned remotely was contained in the chamber for the detection of the scattered charged particles. This detector was collimated to subtend 0.012 sr. Tantalum foils thick enough to stop the high intensity deuteron groups due to the elastic and inelastic scattering from silicon and oxygen, were placed in front of the particle detector. The spectrum resolution was somewhat worse than that obtained without foils. However, the admission of the intense low-energy deuteron pulses would have led to significant pulse pile-up in the amplifiers. The gamma rays were detected by a remotely positioned 3-in.×3-in. NaI scintillator mounted on an RCA 8054 multiplier phototube. The scintillator was located external to the scattering chamber with the front face 7 in. from the target center. The positioning of this detector, as determined from the coincidences from positron annihilation, had an absolute uncertainty of $\pm 1°$.

The signals from both detectors were amplified in the control room by Cosmic double-delay-line amplifiers which are part of the Cosmic modular coincidence circuit. After amplification the signals from the proton and gamma ray counters went to separate single-channel pulse-height analyzers and into the Cosmic fast-slow coincidence modules. A separate signal from the particle-detector amplifier was delayed by an amount greater than the time resolution width of the coincidence circuits and then put into a separate fast-slow unit in coincidence with the gamma-ray detector. Consequently, one unit measured the sum of true plus accidental coincidence counts and the second unit measured accidental counts alone. The time resolution width of the two separate units was checked periodically. With a resolving time of 45 nsec, a mean ratio of true to accidental counts of 4:1 was obtained for the 2.03-MeV radiation and 6:1 for the 1.28-MeV radiation.

Thin self-supporting targets of SiO_2 were prepared from sections of a blown quartz bubble. Target energy thicknesses were determined by measuring the apparent shift of the $Li^7(p,n)$ threshold when the target was inserted into the beam. This thickness corresponded to ≈ 150 keV at 5 MeV.

III. THE HRS FORMALISM

In this section we should like to review that part of the HRS formalism which is applicable to our investigation.

The correlation function for the emission of gamma rays in the direction (θ,φ) when an excited level decays from I_B to a lower level I_C can be written in a form which is generally valid independent of the assumptions that I_B was formed by the stripping mechanism:

$$W(\mathbf{k}_p,\mathbf{k}_d,\theta,\varphi) \propto \sum_{L,L',k,q} F_k(LL'I_CI_B)(2k+1)^{-1/2} \times \rho_{kq}(I_B,I_B)C_LC_{L'}Y_k{}^q(\theta,\varphi), \quad (1)$$

where the $\rho_{kq}(I_B,I_B)$ are the statistical tensors describing the polarization of I_B after it has been formed, the F_k's are the gamma-emisson coefficients[22]; L (or L') is the gamma multipolarity, allowing for multipole mixtures, C_L being the corresponding multipole amplitude. The index k takes even values not greater than $2I_B$ or $2L_{\max}$.

When the axis of quantization is chosen in the direction $\mathbf{n}$ of $\mathbf{k}_d\times\mathbf{k}_p$, q takes on only even values. If the stripping mechanism is assumed, the $\rho_{kq}(I_B,I_B)$ are subject to further restriction and can be expressed in terms of another set of statistical tensors $\rho_{kq}(ll')$

$$\rho_{kq}(I_B,I_B)=\frac{(2I_B+1)^{1/2}}{2(2I_A+1)} \times \sum_{j,l,j',l'} \frac{\theta_{jl}\theta_{j'l'}(-1)^{l'}\eta_k(jj'I_AI_B)\rho_{kq}(ll')}{(ll'00|k0)}, \quad (2)$$

where η_k is the coefficient tabulated by Satchler.[23] The index k cannot exceed $2l_{\max}$ or $2j_{\max}$. The θ_{jl} are the "reduced width amplitudes" which contain the main nuclear structure information, and I_A is the spin of the target nucleus. The ρ_{kq} are defined by

$$\rho_{kq}(ll')=\sum_{mm'}(-1)^{l'-m'}(ll'm-m'|kq)B_{lm}B^*{}_{l'm'}, \quad (3)$$

where the B_{lm} are the amplitudes for the absorption of a neutron with quantum numbers (l,m).

For comparison with the higher energy data from Chalk River, we use the notation of Satchler and Tobocman[9] and define the normalized tensors

$$d_{kq}(ll')=\rho_{kq}(ll')\{(-1)^l(ll'00|k0)(2l+1)^{1/2}\rho_{00}(ll')\}^{-1}. \quad (4)$$

The general correlation function for the axis specified is now

$$W(\theta,\varphi)=\sum_k g_k\left\{d_{k0}P_k(\theta)+2\sum_{q>0}(-1)^q \times\left[\frac{(k-q)!}{(k+q)!}\right]^{1/2} P_k{}^q(\cos\theta)|d_{kq}|\cos q(\varphi-\alpha_{kq})\right\}, \quad (5)$$

where

$$g_k=\eta_k(j_nj_nI_AI_B)F_k(LI_CI_B). \quad (6)$$

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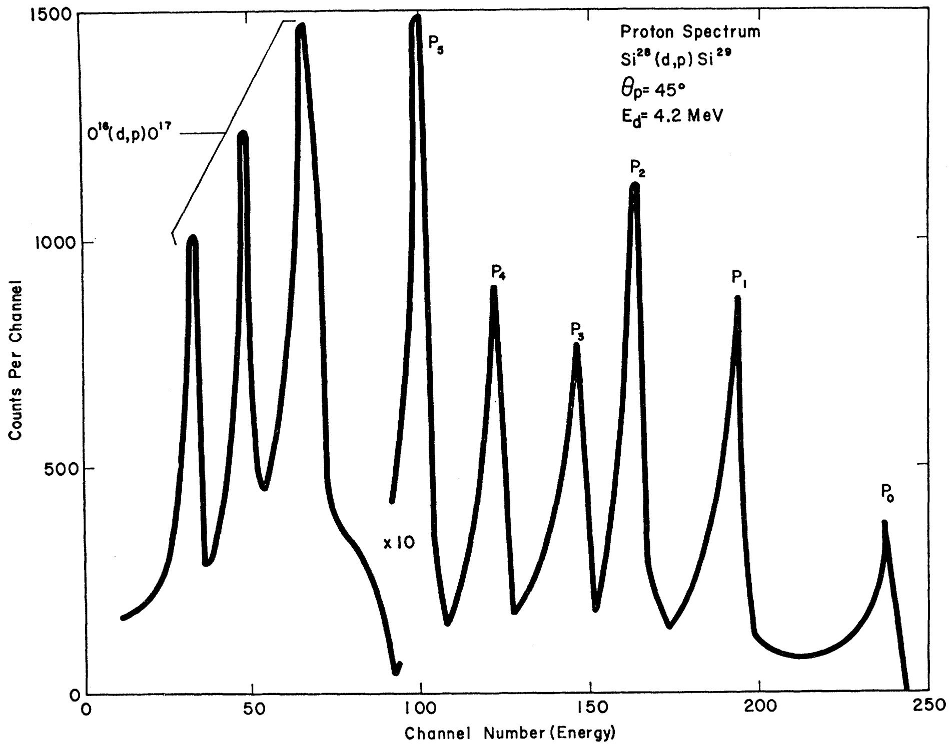

FIG. 1. Part of a spectrum of protons emitted at a laboratory detector angle of 45° in the $Si^{28}(d,p)Si^{29}$ reaction. The peaks labeled P_0, P_1, etc., refer, respectively, to the proton groups which leave Si^{29} in its ground, first, etc., excited states.

Satchler and Tobocman[9] give the following relationships between the parameters:

$$d_{42}=-\left(\frac{5}{12}\right)^{1/2}d_{22},\quad d_{40}=\frac{7}{12}+\frac{5}{12}d_{20},\tag{7}$$

and the phase angle $\alpha_{42}=\alpha_{22}+\frac{1}{2}\pi$. In addition, Kuehner *et al.*[1] have derived a fourth relationship among the d_{kq} which can be expressed as

$$\frac{72}{35}(1+d_{20})|d_{44}|^2-2\left(\frac{72}{35}\right)^{1/2}|d_{22}|^2|d_{44}|\cos4(\alpha_{22}-\alpha_{44})$$
$$-\tfrac{1}{4}(1-d_{20})(1-d_{20}{}^2)+(1-d_{20})|d_{22}|^2=0.\tag{8}$$

(A typographical error in the original equation[24] resulted in a negative sign before the final term.) Finally, the phase angles α_{kq} appearing in Eq. (5) are defined by

$$d_{kq}=|d_{kq}|\exp(-iq\alpha_{kq}).\tag{9}$$

For the general case of $l_n=2$ transfer, the angular correlation between the emitted protons and de-excitation gamma rays has $k_{max}=4$ in Eq. (5) in a favorable case. This occurs if the complexity of the correlation is not further limited by the spin of the intermediate state or the multipolarity of the emitted gamma ray, i.e., k takes even values not greater than $2I_B$ or $2L_{max}$. In such a favorable case, eight independent theoretical parameters, apart from normalization, are required to describe the angular correlation. The distorted wave stripping assumptions reduce the number of independent parameters to four (see Eqs. 7 and 8). Hence a determination of these four parameters, the d_{kq}, completely specify the angular correlation over the sphere.

[24] J. A. Kuehner (private communication).

IV. EXPERIMENTAL RESULTS

Figure 1 is the spectrum of protons measured at a laboratory detector angle of 45° and for a deuteron energy of 4.2 MeV. The angular distributions of the protons leading to the 2.03-MeV second excited state and 1.28-MeV first excited state are shown in Fig. 2. Angular distributions of the P_1 and P_2 proton groups were not analyzed but were measured in order to locate the position of the first stripping maximum. The particle detector was then set at the peak of the stripping maximum (40°) for all further angular correlation measurements. A thick target yield curve of the P_2 proton group, Fig. 3, was taken at 100-keV intervals in order to locate those energies where the correlation yield would be maximized.

For the reaction leading to the second excited state of Si^{29}, the excitation energy is 2.03 MeV, spin and parity of the state $I_B=5/2^+$, and an angular momentum transfer is $l_n=2$. The ground-state gamma-ray transition is known to be a pure $E2$ transition. The reaction-plane angular correlation can then be expressed as

$$W(\tfrac{1}{2}\pi,\varphi)=\textstyle\sum_n A_n\cos n(\varphi-\alpha_{nn})\quad\text{with}\quad n=0,2,4,\tag{10}$$

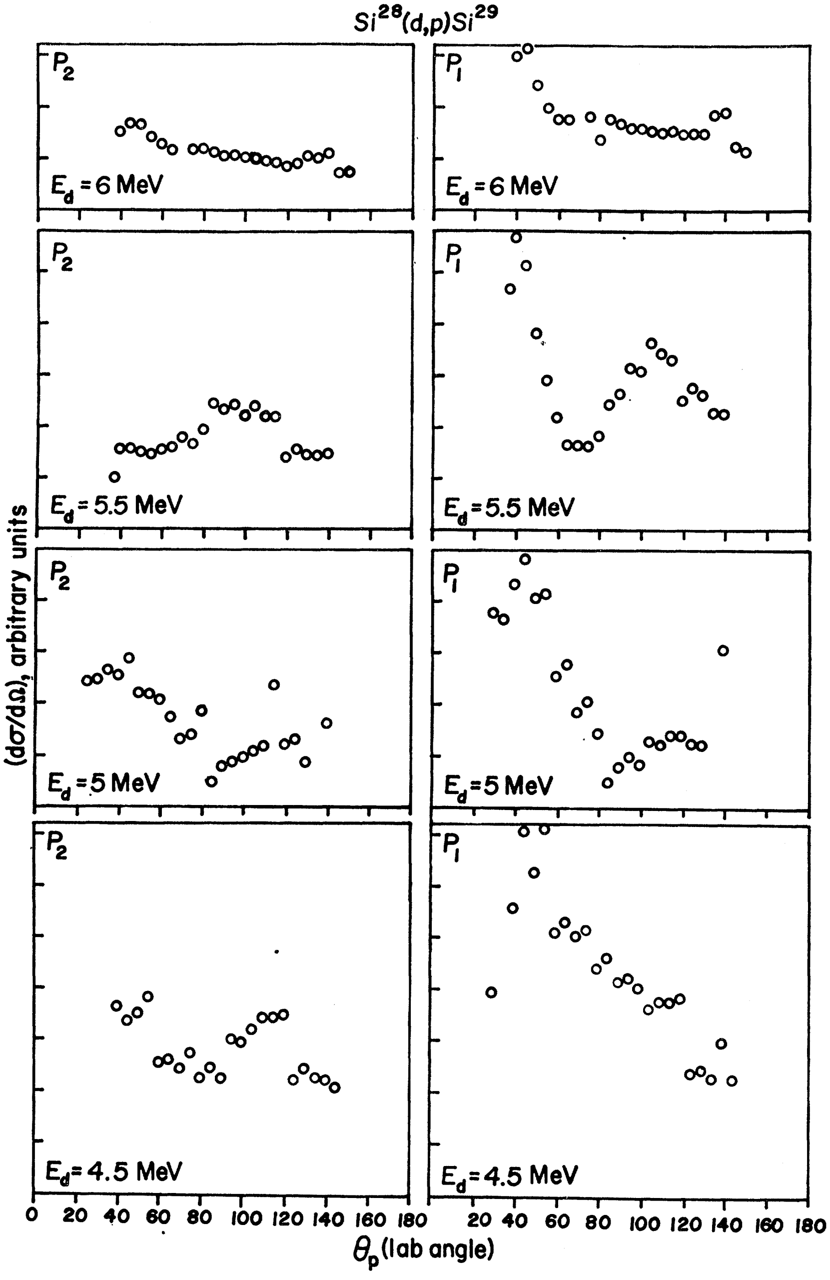

FIG. 2. The angular distributions of the protons leaving Si^{29} in its 2.03-MeV second excited state and 1.28-MeV first excited state. The distributions are normalized to the same collected beam charge.

where

$$A_2/A_0=|d_{22}|(2.13-0.915d_{20})^{-1}, \tag{11}$$

and

$$A_4/A_0=-|d_{44}|(1.46-0.626d_{20})^{-1}. \tag{12}$$

[The negative sign in Eq. (12) was omitted as a typographical error in Ref. 1.[24]]

The results of the $(d,p_2\gamma)$ reaction plane angular correlations taken at deuteron energies of 4.2, 4.8, 5.2, 5.8, and 6.07 MeV are shown in Fig. 4. The solid curves are the least-squares fits to the experimental data. The error flags on the experimental points are the standard deviations due solely to statistical fluctuations. Table I contains the experimental coefficients of Eq. (10) for the 2.03-MeV radiation. For comparison, the results of the Chalk River group are included so that the energy variation may be observed. The experimental parameters do not show large fluctuations over the complete energy range. Also, the general character of the angular correlations does not appear to change over the energy range. These observations agree qualitatively with the generally small energy dependence predicted by direct reaction mechanisms.

The results of the azimuthal angular correlations for decay through the 2.03-MeV state is shown in Fig. 5. The plane $\varphi=0$, π was chosen solely for experimental ease. The solid curves are again least-squares fits to the experimental data. Table II contains the measured coefficients for the 2.03-MeV radiation in the expansion

$$W[\theta,0(\pi)]=\sum_{n=0,2,4} D_nP_n(\cos\theta), \tag{13}$$

where

$$\frac{D_2}{D_0}=\frac{0.571d_{20}-0.078|d_{22}|\cos2\alpha_{22}+0.456|d_{44}|\cos4\alpha_{44}}{1+0.544|d_{22}|\cos2\alpha_{22}-0.319|d_{44}|\cos4\alpha_{44}} \tag{14}$$

and

$$\frac{D_4}{D_0}=-\frac{0.333+0.238d_{20}+0.466|d_{22}|\cos2\alpha_{22}+0.137|d_{44}|\cos4\alpha_{44}}{1+0.544|d_{22}|\cos2\alpha_{22}-0.319|d_{44}|\cos4\alpha_{44}}. \tag{15}$$

Figure 6 shows the results of the $(d,p_1\gamma)$ reaction-plane correlation for the proton group at 1.28-MeV excitation in Si^{29}. In this case, the angular-correlation function is limited to terms in $n=0$ and 2 by the $\frac{3}{2}$ spin of the intermediate state. The solid curves are the least-squares fits to the experimental data. The coefficient ratios for the 1.28-MeV radiation appear in Table III. Again, the results of the Chalk River group are included for comparison.

TABLE I. Parameters in the expansion $W=(\frac{1}{2}\pi,\varphi)=\sum A_n\cos n(\varphi-\alpha_{nn})$ for the 2.03-MeV radiation ($n=0$, 2, 4).

E_d	A_2/A_0	A_4/A_0	$-\alpha_{22}$ (deg)	$-\alpha_{44}$ (deg)
		OSU		
4.2	0.15±0.06	−0.20±0.05	153±16	65±6
4.8	0.24±0.07	−0.15±0.04	20±7	67±7
5.2	0.16±0.07	−0.19±0.04	72±30	70±5
5.8	0.08±0.02	−0.10±0.03	48±5	71±7
6.07	0.18±0.05	−0.15±0.05	39±20	68±9
		Chalk River		
6.07	0.16±0.03	−0.12±0.03	56±9	59±5
6.74	0.20±0.05	−0.22±0.03	70±7	69±4
6.98	0.22±0.03	−0.16±0.03	74±3	79±2
7.98	0.19±0.02	−0.24±0.03	57±5	68±2
8.56	0.18±0.04	−0.24±0.05	40±12	60±4
8.96	0.18±0.04	−0.26±0.02	75±4	62±2

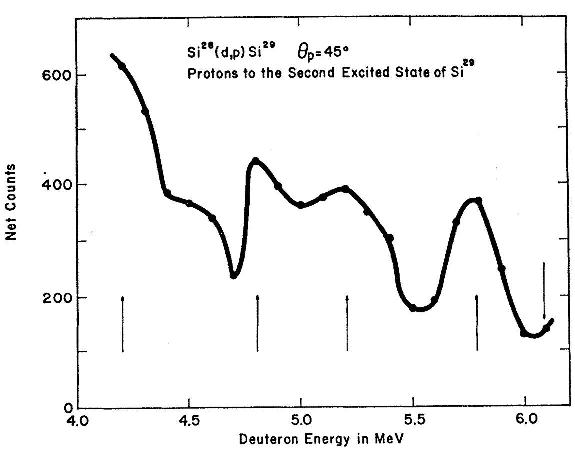

FIG. 3. A thick-target yield curve for the P_2 proton group. The arrows indicate those deuteron bombarding energies at which angular correlations were measured. The solid curve through the experimental points serves only to guide the eye.

TABLE II. Parameters in the expansion $W(\theta,0(\pi))=\sum_n D_nP_n(\cos\theta)$ in the plane perpendicular to the reaction plane.

2.03-MeV radiation ($n=0$, 2, 4)			1.28-MeV radiation ($n=0$, 2, 4)		
E_d	D_2/D_0	D_4/D_0	E_d	D_2/D_0	D_4/D_0
4.2	−0.46±0.08	0.16±0.07	4.2		
4.8	−0.28±0.11	0.06±0.07	4.8		
5.2	−0.50±0.11	0.08±0.08	5.2	0.11±0.06	0.007±0.05
5.8	−0.40±0.08	0.06±0.06	5.8	0.02±0.08	0.03 ±0.06
6.07	−0.28±0.09	0.23±0.08	6.07	0.02±0.07	0.02 ±0.06

TABLE III. Parameters in the expansion $W(\frac{1}{2}\pi,\varphi)=\sum B_n\cos n(\varphi-\alpha_{nn})$ for the 1.28-MeV radiation. ($n=0$, 2.)

E_d	B_2/B_0	$-\alpha_{22}$ (deg)
	OSU	
4.2	−0.12 ±0.02	83±3
4.8	−0.11 ±0.01	45±5
5.2	−0.13 ±0.03	55±10
5.8	−0.12 ±0.02	51±6
6.07	−0.11 ±0.03	66±4
	Chalk River	
6.07	−0.035±0.015	61±16
6.98	−0.060±0.015	31±10
7.98	−0.084±0.013	41±8
8.58	−0.110±0.029	35±10
8.96	−0.097±0.037	43±13

V. DISCUSSION

1. The $Si^{28}(d,p_2\gamma)Si^{29}$ (2.03-MeV) Angular Correlation

In the example of greatest experimental complexity, the correlation through the second excited (2.03-MeV) state, four experimental numbers are measured directly. These are A_2/A_0, A_4/A_0, α_{22}, and α_{44}. Since the DWBA approximation limits the description of the polarization

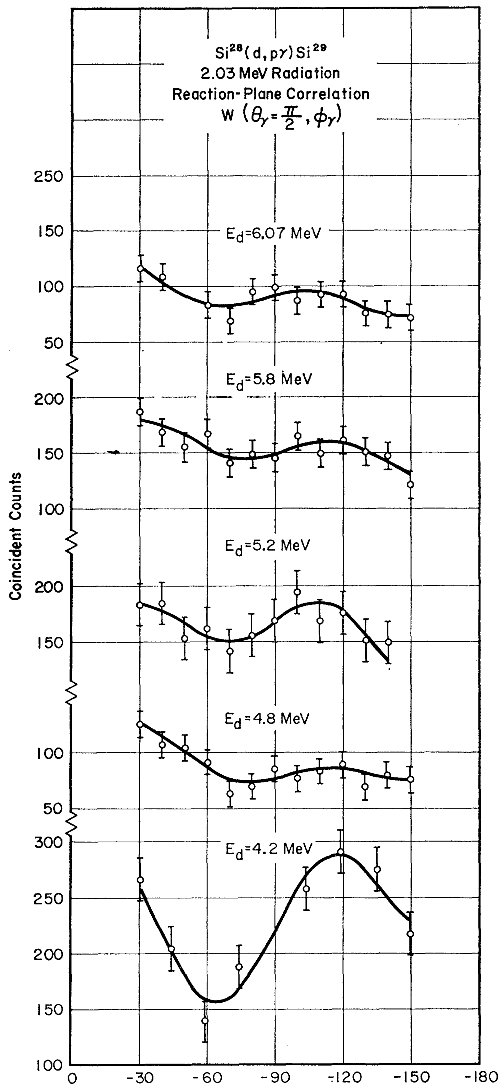

FIG. 4. Proton-gamma angular correlations in the de-excitation of the 2.03-MeV state in Si^{29}, measured in the reaction plane with the proton detector set on the observed peak of the $l_n=2$ stripping angular distribution. The solid curves are the least-squares fits to the experimental data. Error flags on the experimental points are the standard deviations due solely to statistical fluctuations. The angle ϕ is measured with respect to a right-handed coordinate system with the z axis directed along $\mathbf{k}_d\times\mathbf{k}_p$ and the x-axis directed along $\mathbf{k}_d$.

of the state I_B to four parameters, these parameters may be directly calculated from the reaction-plane correlation alone by the use of Eqs. (8), (11), and (12). Substitution of the measured numbers into Eq. (8) results in a cubic equation in d_{20}. There appears to be no *a priori* reason to indicate which of the solutions are spurious.

The Chalk River group removed the ambiguity of their multiple solutions by assuming that the statistical tensors determined from the 2.03-MeV angular correlations applied equally well to the 1.28-MeV correlations. This assumption is based upon the idea that the matrix elements B_{lm} are essentially independent of the nuclear structure. This relative insensitivity of the B_{lm}'s to the details of the final state permits the comparison of experimental data for reactions proceeding to different levels in the same nucleus provided they have the same orbital angular momentum transfer and, also provided the cases are similar kinematically. The agreement between the measured parameters of the 1.28-MeV data and the values predicted from the 2.03-MeV data were satisfactory for the Chalk River data—we shall defer until later in the paper our discussion and application of this method to our data.

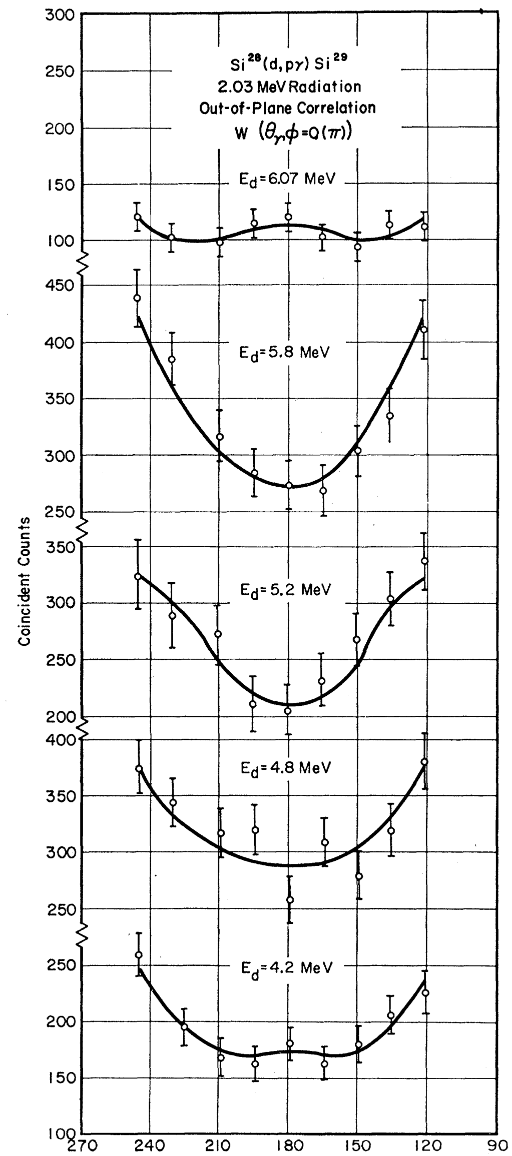

FIG. 5. Proton-gamma angular correlations in the de-excitation of the 2.03-MeV state in Si^{29}, measured in that plane perpendicular to the reaction plane which contains the incident beam direction. The geometry used is identical to that described in the caption of Fig. 4.

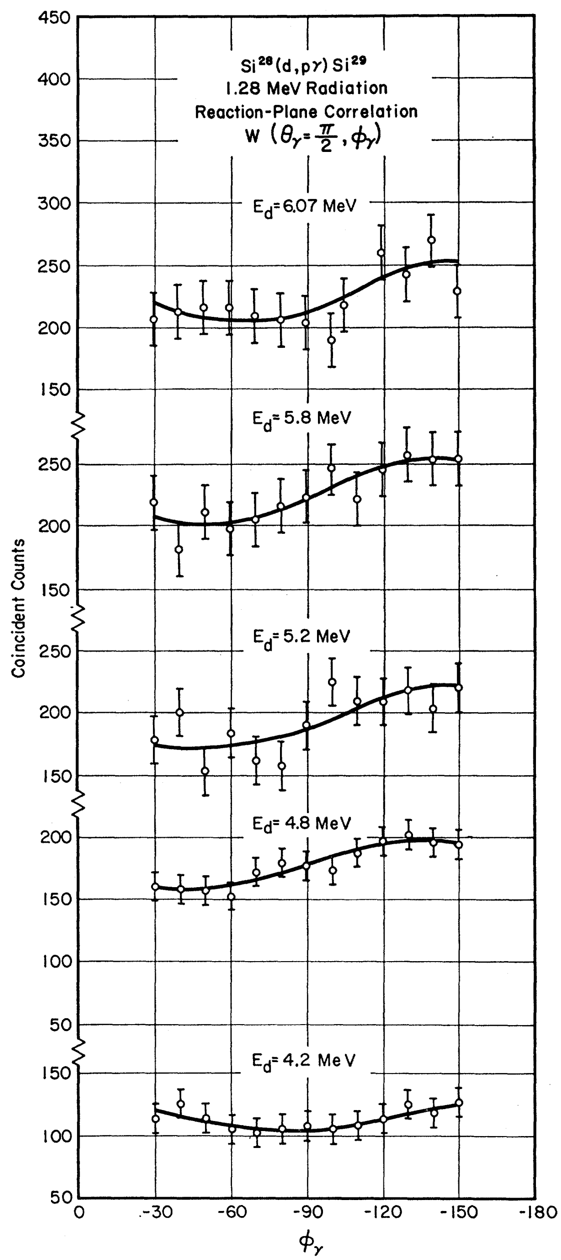

FIG. 6. Reaction plane proton-gamma angular correlations in the de-excitation of the 1.28-MeV state in Si^{29}. The geometry is identical with that described in the caption of Fig. 4.

An alternative analysis of our experimental data can be done by using both the reaction-plane data and the azimuthal-plane data to determine a "best" set of statistical tensors over the measured energy range. Although this method determines a set of parameters of the HRS formalism which best fits the experimental data, it is not necessarily a proof of the validity of the HRS approximations due to the uncertainty of compound nucleus contributions and spin-orbit effects.

In order to utilize both sets of correlation data, the solutions of Eqs. (11) and (12) for $|d_{22}|$ and $|d_{44}|$ in terms of d_{20} are substituted into Eq. (14) or (15) which leads to a single linear equation for d_{20}. Using Eq. (14), for example, leads to

$$d_{20}=\frac{C_1+eC_2}{C_3+eC_4}, \tag{16}$$

where

$$e=D_2/D_0,\ a=A_2/A_0,\ b=A_4/A_0,\ C=\cos2\alpha_{22},\ d=\cos4\alpha_{44}, \tag{17}$$

$$\begin{aligned} C_1&=0.665bd-0.166aC,\\ C_2&=0.456bd-1.16aC-1.00,\\ C_3&=0.285bd-0.071aC-0.571,\\ C_4&=0.200bd-0.498aC. \end{aligned} \tag{18}$$

Figures 7 and 8 show the results of the calculations for (ρ_{20}/ρ_{00}), $|\rho_{22}/\rho_{00}|$, and $|\rho_{44}/\rho_{00}|$ using the single linear equation for d_{20}. The Chalk River data are included for comparison. As can be seen, our solutions join smoothly onto one of the two Chalk River solutions. The other solution appears to be spurious.

For comparison, Tables IV and V contain the ρ_{kq} as calculated at two deuteron bombarding energies, 5.2 MeV and 5.8 MeV. (D_2/D_0) and (D_4/D_0) are the coefficients of the measured azimuthal angular correlation. The ρ_{kq} are calculated both by means of the reaction-plane correlations along (solutions 1, 2, and 3) and by means of the combined reaction-plane and azimuthal-plane correlations. The (D_n/D_0) can also be calculated directly from the ρ_{kq} for each of the three possible reaction-plane solutions. The predicted values of the

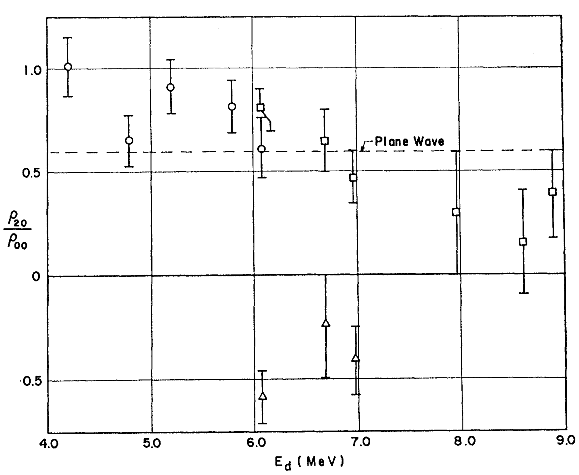

FIG. 7. Values of the statistical tensor ratios ρ_{20}/ρ_{00}, obtained from the reaction $Si^{28}(d,p_2\gamma)Si^{29}$ (2.03 MeV), are shown as a function of deuteron bombarding energy. The open circle points are the values calculated from the combined reaction-plane and azimuthal-plane angular correlations of this experiment. The square and triangular points, for the deuteron energies between 6 and 9 MeV, are taken from Ref. 1. The corresponding plane-wave prediction is shown as the dashed line. In this and succeeding figures the errors shown reflect the uncertainties in the least squares values of the correlation coefficient ratios and phase angles.

TABLE IV. Comparison of the measured azimuthal-plane coefficients, D_n/D_0, for the 2.03-MeV radiation with the coefficients calculated from the reaction-plane statistical tensors.

		$E_d=5.2$ MeV				
	Measured	Sol. 1	Sol. 2	Sol. 3	Combined (D_2/D_0)	Combined (D_4/D_0)
D_2/D_0	-0.50 ± 0.11	0.74	-0.56	0.11		
D_4/D_0	0.08 ± 0.08	-0.60	0.04	-0.29		
ρ_{20}/ρ_{00}		-1.55	0.84	-0.36	0.94 ± 0.13	1.15 ± 0.20
$\|\rho_{22}/\rho_{00}\|$		0.18	0.52	0.36	0.53 ± 0.22	0.57 ± 0.20
$\|\rho_{44}/\rho_{00}\|$		0.19	0.58	0.38	0.59 ± 0.15	0.62 ± 0.15

TABLE V. Comparison of the meaured azimuthal-plane coefficients, D_n/D_0, for the 2.03-MeV radiation with the coefficients calculated from the reaction-plane statistical tensors.

		$E_d=5.8$ MeV				
	Measured	Sol. 1	Sol. 2	Sol. 3	Combined (D_2/D_0)	Combined (D_4/D_0)
(D_2/D_0)	-0.40 ± 0.10	$+0.81$	-0.40	$+0.24$		
(D_4/D_0)	$+0.06\pm0.06$	-0.06	-0.17	-0.43		
ρ_{20}/ρ_{00}		-1.68	0.87	-0.50	0.83 ± 0.13	1.88 ± 0.50
$\|\rho_{22}/\rho_{00}\|$		0.19	0.26	0.17	0.25 ± 0.13	0.33 ± 0.13
$\|\rho_{44}/\rho_{00}\|$		0.10	0.30	0.19	0.29 ± 0.11	0.38 ± 0.12

(D_n/D_0) by solution 2 agree best with the measured values and also with the combined sets.

Figure 9 shows the energy variation of α_{22} and α_{44}. For comparison, the plane-wave values are also included in the figures. The energy variation of α_{22}, the oscillation about the plane-wave value, is reminiscent of the angular variation of α_{22} as calculated by Satchler and Tobocman.[9] Again the Chalk River data are included for comparison.

2. DWBA Calculations

Dr. G. R. Satchler of the Oak Ridge National Laboratory has kindly calculated for us the statistical tensors

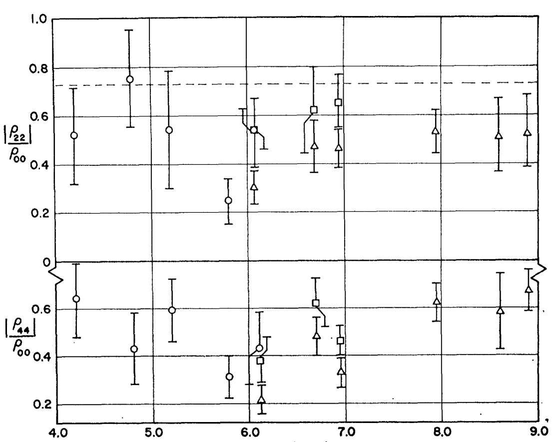

FIG. 8. Values of the statistical tensor ratios $|\rho_{22}|/\rho_{00}$ and $|\rho_{44}/|\rho_{00}$, obtained from the reaction $Si^{28}(d,p_2\gamma)Si^{29}$ (2.03 MeV), are shown as a function of deuteron bombarding energy. The open circle points are the values calculated from the combined reaction-plane and azimuthal-plane angular correlations of this experiment. The square and triangular points are taken from Ref. 1. The plane-wave predictions are shown as a dashed line.

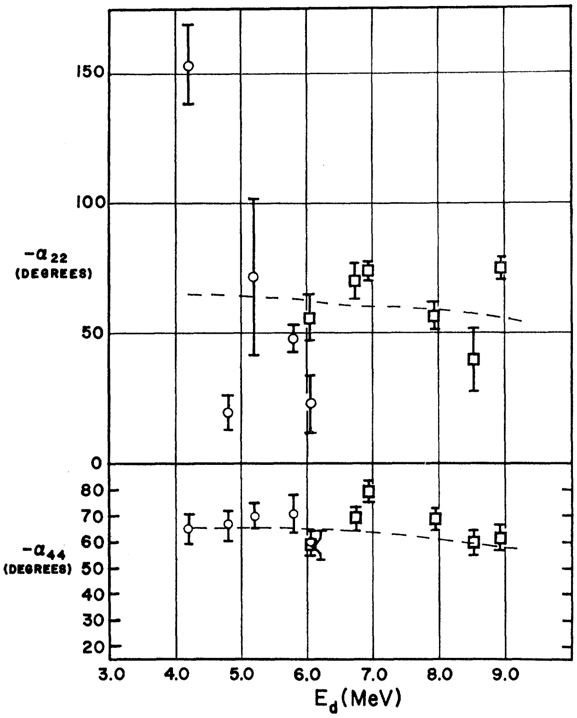

FIG. 9. The statistical tensor phase angles α_{22} and α_{44} are shown as functions of the incident deuteron bombarding energy. As in Figs. 7 and 8, the open circle points are from this experiment whereas the square and triangular points are from Ref. 1. The plane-wave prediction is shown as a dashed line.

d_{kq} for this reaction[25] using the "zero range" formalism without spin-orbit coupling, the code SALLY. The results of the calculations of the d_{kq} as a function of particle detector angle is shown in Figs. 10, 11, and 12 for incident deuteron bombarding energies of 4.2, 4.8, 5.2, 5.8, and 6.07 MeV. Our experimentally determined d_{kq} are plotted on the same curves at the single detector angle of 40°. It should be emphasized that the curves plotted in Figs. 10, 11, and 12 are *not* fits to our experimental data but rather are DWBA calculations using optical model parameters taken as averages of Si+d, Al+d, and Al+p fits.[25]

3. The $Si^{28}(d,p_1\gamma)Si^{29}$ (1.28 MeV) Angular Correlation

The reaction-plane angular correlation through the first excited (1.28 MeV) state in Si^{29} may be written

$$W(\tfrac{1}{2}\pi,\varphi)=\textstyle\sum_n B_n \cos n(\varphi-\alpha_{22}), \tag{19}$$

where n is limited to $n=0$ and 2 by the $\frac{3}{2}$ spin of the state. The value of B_2/B_0 predicted by the HRS formalism is

$$\frac{B_2}{B_0}=\frac{(0.375)^{1/2}(X^2+2\sqrt{3}X-1)|d_{22}|}{(X^2+1)-\frac{1}{4}(X^2+2\sqrt{3}X-1)d_{20}}, \tag{20}$$

[25] G. R. Satchler (private communication).

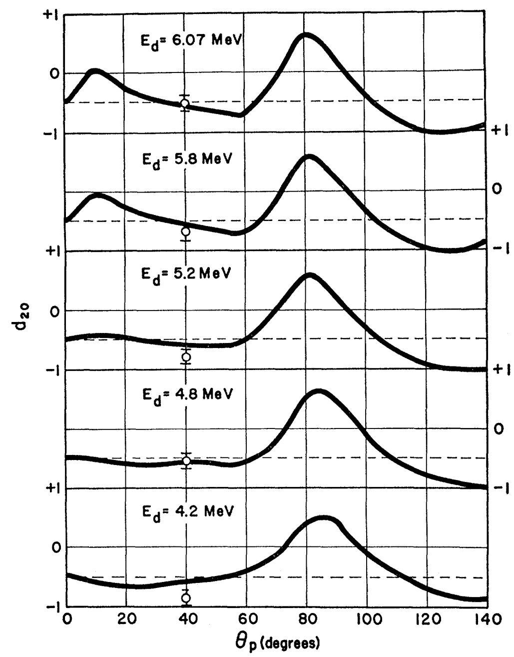

FIG. 10. The solid curves show the calculated values of the normalized statistical tensors d_{20} as a function of proton detector angle. These curves, and those in Figs. 11 and 12, were calculated using the code SALLY by G. R. Satchler of the Oak Ridge National Laboratory. The solid curves are *not* fits to the experimental data. For comparison, the experimental points shown are calculated from the combined reaction-plane and azimuthal-plane angular correlations. The plane-wave predictions are shown as dashed curves.

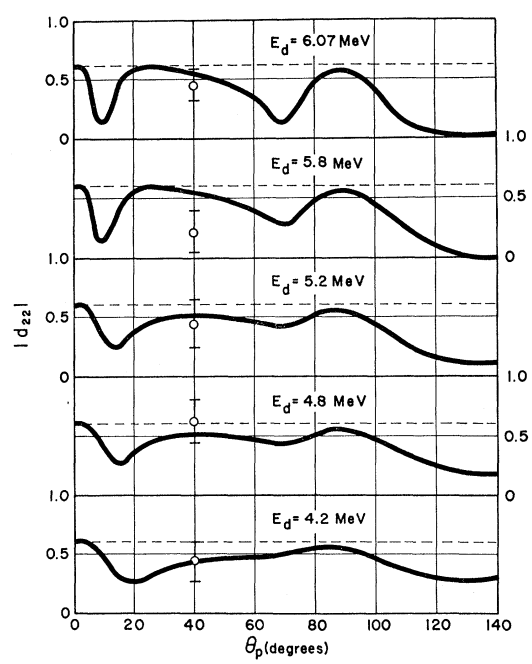

FIG. 11. The solid curves show the calculated values of the magnitude of the normalized statistical tensors d_{22} as a function of proton detector angle. The solid curves are *not* fits to the experimental data shown. (See caption of Fig. 10.) Plane-wave predictions are shown as dashed curves.

where X is the multipole amplitude ratio E_2/M_1 for the 1.28-MeV radiation. This equation disagrees with Eq. (17) of the Chalk River paper.[1] Inserting the best value for this ratio $X=0.21\pm0.03$ the expression may be written as

$$\frac{B_2}{B_0}=-\frac{(0.14\pm0.06)|d_{22}|}{1.04+0.057d_{20}}, \tag{21}$$

which agrees with Eq. (18) of the Chalk River paper.[1]

The data of Figs. 7, 8, and 9 constitute a complete set of statistical tensors obtained at each incident energy using the reaction-plane and the azimuthal-plane correlation data. Hence these tensors can be used to calculate the correlation parameters for the reaction via the 1.28-MeV state. To the extent that these calculations agree with the measured correlation parameters they may be interpreted as supporting the assumption that the matrix elements are independent of the detailed structure and total angular momentum of the final nuclear state.

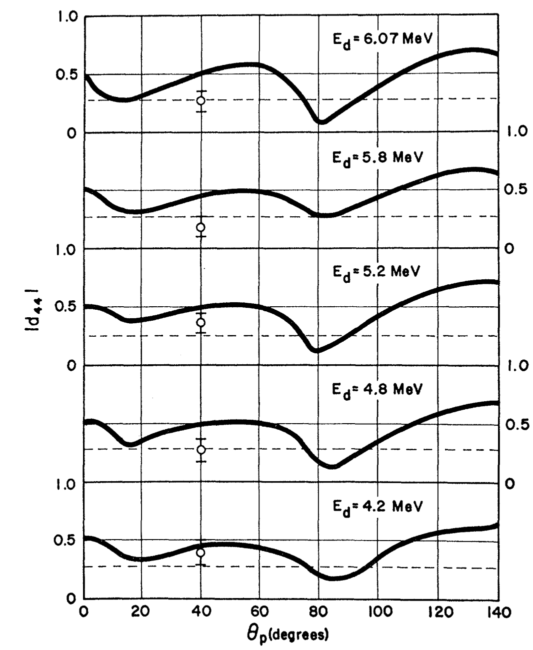

FIG. 12. The solid curves show the calculated values of the magnitude of the normalized statistical tensors d_{44} as a function of proton detector angle. The solid curves are *not* fits to the experimental data shown. (See caption of Fig. 10.) Plane-wave predictions are shown as dashed curves.

Table VI contains the values of B_2/B_0 calculated from the complete set of statistical tensors obtained from the 2.03-MeV correlation as well as the values of B_2/B_0 measured by the 1.28-MeV angular correlations. Within the relatively large statistical uncertainties of the values, there appears to be reasonable agreement between the values.

TABLE VI. Comparison of the 1.28-MeV reaction-plane coefficients with those calculated by the statistical tensors from the 2.03-MeV reaction-plane data.

E_d (MeV)	Measured (B_2/B_0)	Calculated (B_2/B_0)
4.2	-0.12 ± 0.02	-0.06 ± 0.03
4.8	-0.11 ± 0.01	-0.09 ± 0.03
5.2	-0.13 ± 0.02	-0.06 ± 0.03
5.8	-0.12 ± 0.02	-0.03 ± 0.02
6.07	-0.11 ± 0.03	-0.05 ± 0.03

In their analysis of their experimental data, the Chalk River group compared the predicted values of (B_2/B_0) for their different statistical tensor solutions with the experimentally determined values. Both the energy variation and magnitude of the experimentally determined (B_2/B_0) appeared to agree better with one particular solution (the triangular points in Figs. 7, 8, and 9). Our analysis of the reaction-plane data shows no such clear-cut distinction. If anything, using a similar analysis alone on our data, we favor solution 2 (see Tables IV and V) over that of solutions 1 or 3. (Our solution 3 would correspond to the Chalk River solution 1.) Indeed, it is our solution 2 which is most consistent with (a) the reaction-plane angular correlation, (b) the azimuthal-plane angular correlation, (c) the predictions of the coefficient (B_2/B_0) for the 1.28-MeV decay. Surprisingly enough, it is also the one solution of the three which is closest in magnitude to the plane-wave limit.

4. Polarization of Outgoing Protons

Although the spin independent DWBA approximation has proven inadequate in predicting the polarizations of the emitted protons, it may be instructive to calculate the magnitude of the polarization from the set of statistical tensors that we have measured. The polarization vector lies along the quantization axis, $\mathbf{k}_d\times\mathbf{k}_p$, and the polarization magnitude taken from HRS gives

$$P=\pm\frac{4}{(3\sqrt{5})(2j+1)}\left[\left\{\frac{\sqrt{5}}{2}+\frac{\sqrt{7}}{2\sqrt{2}}\left(\frac{\rho_{20}}{\rho_{00}}\right)\right\}^2-4\left|\frac{\rho_{44}}{\rho_{00}}\right|^2\right]^{1/2}, \tag{22}$$

where j = the spin of the state.

The polarization of the protons for the case $l_n=2$ is plotted in Fig. 13 as a function of energy for the "best" fit set of parameters. If one assumes, as one does in this analysis, that only one j contributes to the reaction involving the 2.03-MeV $j=\frac{5}{2}$ state, then

$$|P|\leq\tfrac{1}{3}l/(l+1) \tag{23}$$

or

$$|P|\leq 2/9 \quad \text{or} \quad 22\%.$$

The dotted curve in the figure is the polarization as calculated from the SALLY code with the same parameters as those used in Figs. 10, 11, and 12.

In addition to predicting the polarization of the protons leaving Si^{29} in its second excited state, the statistical tensors can be used to predict the polarization of the proton leaving Si^{29} in its first excited state. Again, that this be true is an assumption of the HRS spin-independent formalism. It would be of interest to compare measured polarizations in the vicinity of the stripping peak with those predicted by the statistical tensors derived from $(d,p\gamma)$ correlations. Only a small number of proton polarization measurements on excited state reactions[14,15] are found in the literature due to the difficulty of separating proton groups from closely spaced levels. However, Reber and Saladin,[15] using magnetic analysis of the emitted protons, have measured the polarization of protons from excited nuclear states including the $Si^{28}(d,p)Si^{29}$ 1.28-MeV polarization. Their incident deuteron energy was 15 MeV and the measured polarization in the vicinity of the stripping peak was ~-0.18. The value we calculate, using our measured statistical tensors at $E_d=6.0$ MeV, is $\pm(0.24\pm0.03)$.

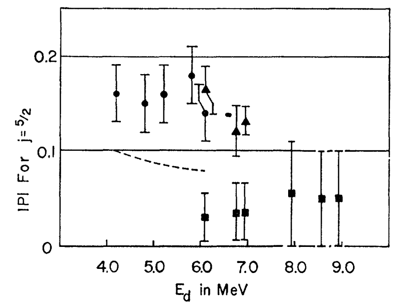

FIG. 13. Predicted polarization magnitudes for protons at the peak of the $l_n=2$ stripping angular distribution from the reaction $Si^{28}(d,p_2\gamma)Si^{29}$ (2.03 MeV). The circular points are from this experiment whereas the square and triangular points are taken from Ref. 1. The dashed curve is taken from the DWBA calculation referred to in the text. The plane-wave prediction for the magnitude of the polarization is zero.

VI. CONCLUSION

We have found it possible to fit the $Si^{28}(d,p\gamma)Si^{29}$ angular correlations assuming the HRS formalism. For the reactions studied, only four statistical tensors are required to predict the general (p,γ) angular correlations over the complete sphere. The reaction plane angular correlations involving the 2.03-MeV state alone yield these four statistical tensors. Hence we compared the azimuthal correlation coefficients, predicted from the reaction plane statistical tensors, with the measured azimuthal coefficients (see Tables IV and V). At the energies $E_d=5.2$ MeV and 5.8 MeV the agreement is within statistical accuracy. On the basis of the agreement of the HRS formalism and the measured correlations, we have joined the many experiments which show the general validity of the distorted-wave stripping theory. In addition, by extending the angular correlations to lower bombarding energies than used at Chalk River and by making azimuthal-plane correlations, we have removed the ambiguities of the multiple statistical-tensor solutions which occur when one analyzes only the reaction plane correlations for $l=2$ transfer. However, the analysis of our measurements, at these low bombarding energies, in terms of an extreme direct-reaction model could be considerably in error because of the unknown compound-nucleus contributions to the reaction. In addition, we have analyzed the data with a spin-independent formalism although strong spin dependence has been observed in a number of experiments.

First, we examine the possible compound nucleus contributions and their effect upon the analysis. There appears to be no doubt that compound-nucleus reactions are present over our energy range. Kuehner, Almqvist, and Bromley[26] observed distinct compound nucleus contributions in the study of the $Si^{28}(d,p)Si^{29}$ angular distributions in the energy range of 6 to 10 MeV. They observed prominent resonance structure in their excitation curve particularly at proton detector angles off the forward stripping peak. They estimate a compound nucleus contribution between 15% to 30%. Read, Calvert, and Schork[27] at lower bombarding energies and for a lighter target nucleus than ours, $Be^9(d,p\gamma)Be^{10}$ at $E_d=3.37$ MeV, also observe strong compound-nucleus contribution in their correlation data taken at proton angles off the stripping peak. Finally, the extensive investigation of Wildenthal, Krone, and Prosser[28] on the $Si^{28}(d,p)$ reaction below 3 MeV show strong compound nucleus contributions up to one-half of the observed cross section.

Since the amount of compound-nucleus contribution to our reaction under study was unknown and since in addition, interference effects between the direct and compound parts of the correlation are not understood, our analysis hinges upon minimizing the compound-nucleus contribution experimentally. Our attempt to minimize the effect of the expected compound-nucleus contribution to the reaction was done in two ways. First, the proton detector was placed at the angle corresponding to the maximum of the forward stripping peak. The assumption is that at this angle, where the stripping cross section is a maximum, the ratio of compound nucleus reaction to stripping reaction would be minimized. Second, the target was made thick so that hopefully we would average experimentally over a large number of compound resonance states. The similarity in the shape of the angular correlations as a function of bombarding energy probably indicates that we have averaged over sharp compound resonances. At the present time, however, there appears to be no way of extracting that part of the angular-correlation coefficients which are due solely to stripping reactions.

One of the purposes of this experiment was to check the general predictions of the HRS spin-independent formalism with the idea that deviations from such predictions would serve as indications of spin-dependent forces in the reaction. The presence of compound-nucleus events precludes such a simple interpretation. However, even should complicating compound-nucleus events be absent from the reaction, our particular choice of proton detector angle, the maximum of the forward stripping peak, appears to be an unfortunate one for observing such deviations. From an inspection of Figs. 10, 11, and 12 one can see that in the region of the forward stripping peak the plane-wave value and the DWBA calculated value are close in magnitude and are well within the statistical accuracy of the measured values. It appears that a more stringent test of the spin-independent formalism, and hence the ability to obtain more information on the spin-dependent forces, would be to measure angular correlations with the proton detector off the stripping peak and for backward angles. It is also at the backward angles that proton polarizations have been measured as being large compared to polarizations near the stripping peak. We are currently conducting $(d,p\gamma)$ angular correlations at a fixed bombarding energy but as a function of proton detector angle.

ACKNOWLEDGMENTS

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