Quadrupole Elastic Scattering of Alpha Particles by Polarized Nuclei

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The effect of the charge distribution of target nuclei on elastic scattering between nuclei at an incident energy well below the Coulomb barrier has been investigated in terms of the deviation from Rutherford scattering. In the present paper, we report the result of the calculation for the case in which the target nuclei are oriented during the collision. For the head-on collision of alpha particles with U^{238} and Ta^{181} at 24 and 16 MeV, we find that the deviations from Rutherford scattering should be about 18 and 20%, respectively, if the polarization of the target is complete. Therefore, we conclude that the deviation should be large enough for detection even for a reasonable degree of polarization of the target.

I. INTRODUCTION

The effect of the charge distribution in the nucleus on the scattering of charged particles has been very intensely investigated under various conditions. In the case of scattering at low energy,[1] one may expect in general that the interaction is predominantly electric if the incident energy is so low that the probability of the incident particle's penetration of the Coulomb barrier is negligible. In fact, however, other interactions may compete. For example, in case of elastic scattering of low-energy deuterons by nuclei, such interactions as the deuteron breakup in flight may play an important role.[2]

In investigating Coulomb scattering of the projectile by the nucleus, the first step is to take into account the extension of the target tacitly assuming that it remains in its ground state throughout the interaction. For a more satisfactory theory, however, we have to include the effect of charge polarization[3] which takes account of the contributions from all the possible excited states of the nucleus.

It has been pointed out that virtual Coulomb excitation may cause a significant modification on the elastic scattering cross section.[4–10] If the process is adiabatic, the polarization potential as obtained by a perturbation calculation may appear to show that in most cases the quadrupole interaction effect is larger than that of dipole interaction in view of the high excitation frequencies associated with the dipole oscillator strength. However, in most cases of interest, the adiabatic approach is not justified for the quadrupole polarization interaction because of the low excitation frequencies.[9,11] It would be therefore necessary in general to take account of the coupled motion of the colliding particles in detail to evaluate the effect of the quadrupole polarization.

Under ordinary conditions, the nuclear axis of the target is oriented at random. Therefore, if we orient the strongly deformed target nuclei, the quadrupole effect will be greatly enhanced. In fact, in this case, the interaction is proportional to r^{-3} if r is larger than the nuclear radius. Although perfect orientation of nuclei cannot be achieved at present, even limited degrees of orientation may produce a large amount of the quadrupole effect.

In the present paper, we calculated the quadrupole effect in the collision of alpha particles with U^{238} and Ta^{181} at the incident energy (c.m. system) of 24 and 16 MeV, respectively. These energies are roughly half of the Coulomb barrier in each case. We have used essentially the same method as employed in Ref. 9, introducing the quadrupole interaction as a perturbation. After presenting the general formalism, the particular case of head-on collision is discussed in detail.

II. CALCULATION

Considering only the quadrupole interaction, the Schrödinger equation becomes

$$(H_0+H')\psi=E\psi\,, \tag{1}$$

where

$$H_0=-\frac{\hbar^2}{2m}\nabla_r^2+\frac{Z_1Z_2e^2}{r}\,, \tag{2}$$

and

$$H'=\tfrac{1}{2}Z_1Q_0e^2f(r)P_2(\cos\angle(\alpha N))\,. \tag{3}$$

In Eqs. (2) and (3), $\mathbf{r}$ is the vector distance from the target to the projectile, $\angle(\alpha N)$ is the angle between $\mathbf{r}$ and the nuclear axis of the target, Z_1 and Z_2 are the charges of the projectile and the target, respectively, and $f(r)$ can be written as

$$\begin{aligned} f(r)&=r^{-3} && \text{if} \quad r>R \\ &=r^2R^{-5} && \text{if} \quad r<R\,, \end{aligned} \tag{4}$$

[1] For references see, e.g., K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Rev. Mod. Phys. **28**, 432 (1956); *Comptes Rendus du Congrès International de Physique Nucléaire* (Centre National de la Recherche Scientifique, Paris, 1964), Sec. 4d; L. C. Biedenharn and P. J. Brussard, *Coulomb Excitation* (Oxford University Press, London, 1965), Chap. 1.

[2] J. K. Dickens and F. G. Perey, Phys. Rev. **138**, B1083 (1965) and papers cited therein.

[3] In this paper, the term "charge polarization" is used to indicate the mode of Coulomb excitation of the nucleus while "polarization" refers to the mode of nuclear orientation.

[4] P. Debye and W. Hardmeier, Physik. Z. **27**, 196 (1926).

[5] W. Hardmeier, Physik. Z. **28**, 181 (1927).

[6] N. F. Ramsey, Phys. Rev. **83**, 659 (1951).

[7] B. J. Malenka, U. E. Kruse, and N. F. Ramsey, Phys. Rev. **91**, 1165 (1953).

[8] G. Breit, M. H. Hull, and R. L. Gluckstern, Phys. Rev. **87**, 74 (1952).

[9] Y. N. Kim, Nuovo Cimento **22**, 885 (1961).

[10] C. F. Clement, Phys. Rev. **128**, 2728 (1962).

[11] See, e.g., the first paper in Ref. 1, Sec. II D.4.

where R is the radius of the nucleus. Q_0 is the intrinsic quadrupole moment of the nucleus and is oriented along the nuclear axis, and P_2 is the Legendre polynomial.

In our case, because of the low incident energy, $r>R$, and, therefore, Eq. (1) becomes

$$\left\{\nabla_r^2+k^2-\frac{2m}{\hbar^2}\frac{Z_1Z_2e^2}{r}-\frac{mZ_1Q_0e^2}{\hbar^2r^3}P_2(\cos\angle(\alpha N))\right\}\psi=0. \quad (5)$$

Equation (5) may be rewritten as

$$\left\{\nabla_R^2+k^2-\frac{2m}{\hbar^2}\frac{Z_1Z_2e^2}{r}\right\}\psi$$
$$=\frac{4\pi}{5}\frac{mZ_1Q_0e^2}{\hbar^2r^3}\sum_{q=-2}^{2}(-1)^qY_{2q}(\theta_N,\phi_N)Y_{2-q}(\theta_\alpha,\phi_\alpha)\psi, \quad (6)$$

where $(r,\theta_\alpha,\phi_\alpha)$ refer to projectile and (θ_N,ϕ_N) determine the direction of the nuclear axis of the target.

The solution of Eq. (6) defined by the requirement of adding only outgoing waves to ψ_c at large r may be written in the following form:

$$\psi=\psi_c+\frac{4\pi}{5}\frac{mZ_1Q_0e^2}{\hbar^2}\int K(\mathbf{r},\mathbf{r}')\frac{1}{r'^3}\sum_{q=-2}^{2}(-1)^q$$
$$\times Y_{2q}(\theta_N,\phi_N)Y_{2-q}(\theta_\alpha,\phi_\alpha)\psi_c(\mathbf{k}_1,\mathbf{r}')d\mathbf{r}', \quad (7)$$

where, for large r,

$$K(\mathbf{r},\mathbf{r}')\sim\frac{1}{4\pi r}\exp i[(kr-\eta\ln 2kr)]\psi_c(-\mathbf{k}_2,\mathbf{r}') \quad (8)$$

and

$$\eta=Z_1Z_2e^2/\hbar v.$$

$\psi_c(\mathbf{k},\mathbf{r})$ is the Coulomb wave function and k_1 and k_2 are the wave vectors of the Coulomb-distorted plane waves of the projectile before and after the interaction with the target, respectively. Since we confine our investigation to elastic scattering only, $k_1=k_2(=k$, say) and $\psi_c(-k_2,r)$ is the Coulomb scattering wave function for the incident wave vector $-k_2=-kr/r$.

The scattering amplitude $f(\theta)$ may be expressed as the sum of $f_c(\theta)$, the Coulomb scattering amplitude, and $f_q(\theta)$, the amplitude due to the quadrupole interaction

$$f(\theta)=f_c(\theta)+f_q(\theta), \quad (9)$$

where

$$f_c(\theta)=\frac{\eta}{2k\sin^2(\theta/2)}\exp[-i\eta\ln\sin^2(\theta/2)+i\pi+2i\sigma_0], \quad (10)$$

and

$$f_q(\theta)=-mZ_1Q_0e^2/5\hbar^2$$
$$\times\int\psi_c(-\mathbf{k}_2,\mathbf{r})\frac{1}{r^3}\sum_{q=-2}^{2}(-1)^qY_{2q}(\theta_N,\phi_N)$$
$$\times Y_{2-q}(\theta_\alpha,\phi_\alpha)\psi_c(\mathbf{k}_1,\mathbf{r})d\mathbf{r}. \quad (11)$$

We expand[12]

$$\psi_c(\mathbf{k}_1,\mathbf{r})=\sum_l\psi_l(\mathbf{k}_1,\mathbf{r})$$
$$=\sum_{l,m}[4\pi(2l+1)]^{1/2}(k,r)^{-1}\exp\{i(\sigma_l-\sigma_0)\}$$
$$\times F_l(\eta,k,r)D_{m0}{}^l(\hat{k}_1)i^lY_{lm}(\theta_\alpha,\phi_\alpha), \quad (12)$$

where $F_l(\eta,k,r)$ is the regular solution to the radial wave equation for the orbital angular momentum l, and σ_l is the Coulomb phase shift

$$\sigma_l=\arg\Gamma(l+1+i\eta).$$

The function σ_l satisfies the recurrence relation

$$\sigma_{l+1}=\sigma_l+\tan^{-1}\left(\frac{\eta}{l+1}\right). \quad (13)$$

$D_{m0}{}^l$ are matrix elements of the rotation matrices and $\hat{k}_1$ is a unit vector in the $\mathbf{k}_1$ direction. The integral on the right-hand side of Eq. (11), which will be denoted by J, becomes

$$J=(4\pi)^2\sum_{ll'}i^{l+l'}\exp\{i(\sigma_l+\sigma_{l'}-2\sigma_0)\}M_{ll'}{}^{-3}$$
$$\times\sum_{q,m,m'}(-1)^{q-m-m'}Y_{2q}(\theta_N,\phi_N)Y_{l'm'}(\mathbf{k}_2)$$
$$\times Y_{l,-m}(\mathbf{k}_1)\left[\frac{5(2l+1)(2l'+1)}{4\pi}\right]^{1/2}$$
$$\times\begin{pmatrix}2 & l & l'\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}2 & l & l'\\ q & -m & m'\end{pmatrix}, \quad (14)$$

where

$$\begin{pmatrix}2 & l & l'\\ 0 & 0 & 0\end{pmatrix}\text{ etc.}$$

are Wigner $3j$ symbols, and[13]

$$M_{ll'}{}^{-3}=(k_1k_2)^{-1}\int_0^\infty F_{l'}(k_2r)r^{-3}F_l(k_1r)dr. \quad (15)$$

If the nuclear axis of the target is parallel to the direction of the relative motion before the collision, i.e., $(\theta_N,\phi_N)/\!\!/\mathbf{k}_1$, then

$$Y_{2q}(\theta_N,\phi_N)Y_{l,-m}(\mathbf{k}_1)$$
$$=\sum_{\lambda,\mu}\left[\frac{5(2l+1)(2\lambda+1)}{4\pi}\right]^{1/2}\begin{pmatrix}2 & l & \lambda\\ q & -m & \mu\end{pmatrix}$$
$$\times Y_{\lambda\mu}{}^*(\theta_N,\phi_N)\begin{pmatrix}2 & l & \lambda\\ 0 & 0 & 0\end{pmatrix}. \quad (16)$$

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[12] See, e.g., A. Sommerfeld, *Atombau und Spektrallinien*, (F. Vieweg und Sohn, Braunschweig, 1939), Vol. II, Chap. VII.
[13] L. C. Biedenharn, J. L. McHale, and R. M. Thaler, Phys. Rev. **100**, 376 (1955).

From Eqs. (14) and (16), by using the orthogonality relations of the $3j$ symbols, we obtain

$$J=5\times 4\pi \sum_{l,l'} i^{l+l'} \exp\{i(\sigma_l+\sigma_{l'}-2\sigma_0)\} \times M_{ll'}^{-3}\begin{pmatrix}2 & l & l'\\ 0 & 0 & 0\end{pmatrix}^2 (2l+1) \times\sum_{m'} Y_{l'm'}^*(\theta_N,\phi_N)Y_{l'm'}(\mathbf{k}_2) \quad (17)$$

$$=5\sum_{l,l'} i^{l+l'} \exp\{i(\sigma_l+\sigma_{l'}-2\sigma_0)\} \times M_{ll'}^{-3}\begin{pmatrix}2 & l & l'\\ 0 & 0 & 0\end{pmatrix}^2 (2l+1)(2l'+1)P_{l'}(\cos\theta).$$

From Eqs. (11) and (17), we find

$$f_q(\theta)=\frac{mZ_1Q_0e^2}{2\hbar^2}\sum_{l,l'} i^{l+l'} \times\exp\{i(\sigma_l+\sigma_{l'}-2\sigma_0)\}M_{ll'}^{-3}\begin{pmatrix}2 & l & l'\\ 0 & 0 & 0\end{pmatrix}^2 \times(2l+1)(2l'+1)P_{l'}(\cos\theta). \quad (18)$$

The $3j$ symbol on the right-hand side of Eq. (18) shows that $f_q(\theta)$ vanishes unless $l'=l+2$, l or $l-2$. Also it is clear that $l'=l=0$ gives no contribution to $f_q(\theta)$.

Much work has been done on the evaluation of integrals of the type appearing in Eq. (15).[14] Making use of this work, one can readily obtain the following results:

$$M_{ll}^{-3}=\frac{1}{2l(l+1)(2l+1)}\{(2l+1)-\pi\eta +i\eta\Psi(l+1-i\eta)-i\eta\Psi(l+1+i\eta)\}$$
$$=\frac{1}{2l(l+1)(2l+1)}\left\{(2l+1)-\pi\eta+2\eta^2 \times\lim_{n\to\infty}\sum_{n=1}\frac{1}{(l+n)^2+\eta^2}\right\}, \quad (19)$$

where $\Psi(z)$ is the logarithmic derivative of the Γ functions.

$$M_{l,l+2}^{-3}=\frac{(l+1)(l+2)}{4(2l+1)(2l+3)(2l+5)}\begin{pmatrix}2 & l & l+2\\ 0 & 0 & 0\end{pmatrix}^2 \times\frac{1}{[\{(l-1)-\eta^2\}^2+(2l-1)^2\eta^2]^{1/2}}. \quad (20)$$

Since the integral in Eq. (15) must converge at the origin, the inequality $2l+3+(l'-l)-3>0$ must hold.[15] Therefore, Eq. (19) is valid for $l\geqslant 1$ whereas Eq. (20) is valid for all values of l including $l=0$.

The deviation δ from the Rutherford scattering is given by

$$\delta=\frac{\sigma_c(\theta)-\sigma(\theta)}{\sigma_c(\theta)}=\frac{|f_c(\theta)|^2-|f(\theta)|^2}{|f_c(\theta)|^2} =-\frac{2\,\mathrm{Re}\{f_c^*(\theta)f_q(\theta)\}}{|f_c(\theta)|^2}, \quad (21)$$

where $\sigma_c(\theta)$ is the Coulomb cross section and $\sigma_q(\theta)$ is that part of the cross section which is caused by the quadrupole interaction. The squared term of $f_q(\theta)$ on the right-hand side of Eq. (21) is neglected. In the case of head-on collisions, we obtain from Eqs. (10), (18), and (21),

$$\delta=-\frac{2m}{\hbar^2}\frac{Q_0Z_1e^2k}{\eta}\sum_{l=1}\cos(2\sigma_l-4\sigma_0) \times\frac{1}{(2l-1)(2l+3)}\left\{2l-2\eta^2\sum_{m=1}\frac{1}{m^2+\eta^2}\right\}, \quad (22)$$

for $l'=l$, and

$$\delta=\frac{2m}{\hbar^2}\frac{Q_0Z_1e^2k}{\eta}\sum_{l=0}\cos(\sigma_l+\sigma_{l+2}-4\sigma_0) \times\frac{(l+1)(l+2)}{(2l+3)[\{l(l-1)-\eta^2\}^2+(2l-1)^2\eta^2]^{1/2}}, \quad (23)$$

for $l'=l+2$.

Obviously the following three cases, $l'=l$, $l'=l+2$, and $l'=l-2(l\geqslant 2)$ contribute to the total deviation. However, for $\theta=\pi$, $i^{l+(l+2)}P_{l+2}(\cos\theta)=i^{(l+2)+2}P_l(\cos\theta)$, and also from Eq. (15), $M_{l,l+2}^{-3}=M_{l+2,l}^{-3}$. The $3j$ symbol in Eq. (18) does not change value when l and l' are interchanged. Since $l=0$ and 1 give no contribution to the deviation in the case of $l'=l-2$, and each term, though not the sum, on the right-hand side of Eq. (23) for large l becomes negligible, one can readily see, from Eqs. (18) and (21), that the contributions to the total deviation from the two cases, $l'=l+2$ and $l'=l-2$, cancel each other. We will, therefore, confine our considerations to the case of $l'=l$ only.

In general, to carry out a calculation involving the Coulomb wave functions with reasonable accuracy, we have to take a very large number of partial waves. We need in our problem the evaluation of δ as given by Eq. (22). For large values of l, each term of the series in this equation decreases as $1/l$, oscillating between positive and negative values as l increases. The sum of the first l terms of this series, which will be denoted by $\sum_{l=1}^{l}\delta_l$, also oscillates, as a function of l, with decreasing amplitude. However, as Eq. (13) shows, σ_l changes its value more and more slowly as l becomes larger, so that the intervals of the values of l over which $\sum_{l=1}^{l}\delta_l$ undergoes oscillation will become larger, thus

[14] See, e.g., papers cited in Ref. 1.
[15] Reference 13, p. 381.

making it difficult to evaluate the sum of this series. This is, of course, due to the fact that we have employed a pure Coulomb field. In nature, however, such a field is never encountered; in our case the Coulomb field of the nucleus is screened by the electron cloud at distances larger than the radius of the atom. This effect may be taken into account by adopting a screened Coulomb potential such as

$$(Z_1Z_2e^2/r)e^{-\lambda r},$$

where λ^{-1} is of the order of the radius of the atom, instead of the pure Coulomb potential. A calculation along these lines appears to be very difficult however and has not been carried out.

Attempts to obtain an approximate value of the sum by cutting off the summation at a value of l corresponding to an average radius of electron orbits, such as is obtainable from the Thomas-Fermi statistical model, also did not prove successful. The difficulty lies in the indefiniteness of the value of the radius which when combined with the oscillatory character of the sum of the first l terms of the series introduces a large uncertainty in the answer.

We carried out the computation of the series in Eq. (22) up to $l=20\,000$ for the following two cases:

Case	Projectile	Target	Incident energy (c.m. system)	Coulomb barrier
I	alpha	U^{238}	24 MeV	28.6 MeV
II	alpha	Ta^{181}	16 MeV	24.8 MeV

As long as we consider the simple sum of this series, it showed little sign of convergence even at this large value of l. However, we can find the sum of this series by invoking the Césaro method of summation known as $(C,1)$.[16] We found it convenient to calculate the sums of the terms in each lump with the same sign in the series and apply the Césaro method to the series of these sums. The values thus obtained definitely showed convergence and by calculating terms for l's up to 10 000 in both cases, we found the following values to within the uncertainty of 3×10^{-4}:

$$\left(-\frac{2m}{\hbar^2}\frac{Q_0Z_1e^2k}{\eta}\right)^{-1}\sum_l \delta_l$$
$$\approx 0.19\times10^{-2} \text{ for case I}$$
$$\approx 0.35\times10^{-2} \text{ for case II.} \quad (24)$$

These procedures are justified because as long as the series is convergent (in the sense of the simple sum), the simple sum and $(C,1)$ should give the same answer in view of the condition of consistency. That the series in Eq. (22) indeed converges even in the sense of the simple sum, at least for our problems, can be seen in the following way: $\sum_{l=1}^{\infty}\delta_l$ is summable $(C,1)$, and as was mentioned shortly after Eq. (23), $\delta_l=O(1/l)$. Then according to Hardy's convergence theorem,[16] the series converges in the sense of the simple sum. Therefore the answers obtained above $(C,1)$ must be equal to the simple sum of the series $\sum_{l=1}^{\infty}\delta_l$. Only the simple sum is very difficult to get.

The value of the intrinsic quadrupole moment Q_0 of the nuclei cannot be determined very accurately. It varies considerably according to different methods of determination.[17] For example, the value of Q_0 for Ta^{181} obtained from the hyperfine structure intervals is much larger than the value used in this paper, a value estimated from the Coulomb excitation. For our purpose, however, a rough estimation will suffice, and if we take the following values,[18]

$$Q_0=11\times10^{-24}\text{ cm}^2 \quad \text{for} \quad U^{238}$$

and

$$Q_0=6.8\times10^{-24}\text{ cm}^2 \quad \text{for} \quad Ta^{181},$$

we obtain, from Eqs. (22) and (24),

$$\delta\approx18\% \text{ for case I, and } \delta\approx20\% \text{ for case II.}$$

III. CONCLUSION

The large deviation from the Coulomb scattering cross section obtained above corresponds, of course, to complete polarization of the target nucleus, a situation which cannot be achieved at present. However, even assuming the degree of polarization to be 0.5, we still get very large deviations of 9 and 10% for case I and case II, respectively.

These deviations should be compared with those arising from the charge polarization of the nucleus taking into account the virtual Coulomb excitations. Of these, the dipole and quadrupole polarization effects would be the most important. According to the evaluation of Malenka, Kruse, and Ramsey,[7] the deviation from Coulomb scattering due to the dipole polarization (polarizability) for the elastic scattering of 10-MeV deuterons by U is 4.2% in case of head-on collisions. Although we do not know the exact value of the polarizability of alpha particles, undoubtedly this must be very much smaller than that of the deuteron because of the great difference in the looseness of the binding of these two nuclei. Therefore, the deviations from the Coulomb scattering due to the dipole polarization should be negligible.

As was pointed out in Sec. I, the quadrupole interaction would be in most cases nonadiabatic. This can result in anomalously strong excitations if partial resonance is involved. The exact evaluation of this effect taking account of the detailed coupled motion of the projectile and target has not been attempted in this paper. However, we note that the cross sections for quadrupole excitations are, for the fixed energy of pro-

[16] E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis* (Cambridge University Press, New York, 1946), Chap. VIII.

[17] B. M. Spicer, H. H. Thies, J. E. Baglin, and F. R. Allum, Australian J. Phys. **11**, 298 (1958).

[18] See, e.g., the first paper in Ref. 1, Chap. VB.

jectiles, proportional to the mass of the projectiles, when the interaction is nonadiabatic because of the low excitation energies.[19] On the other hand, the dipole excitations are more easily produced by particles with small charges such as protons. Therefore the quadrupole excitation effect in our problem would be rather small because of the smallness of the charge of the alpha particle.

We have entirely neglected other interactions which might occur in the course of the collisions. Of these the most serious one would be the disintegration of the projectile in flight due to the electric interaction. However, the cross sections of the photodisintegration of the alpha particle at the energies considered in the present work would be of the order of millibarns,[20] whereas the Rutherford scattering cross sections of the head-on collisions are 0.13 b for case I and 0.14 b for case II.

The incident energy in case I is above the threshold of the photofission of U^{238}, which is about 5.8 MeV. Also photoneutron, and to less extent, photoproton and other disintegration processes of the target are possible. Although the effects of these interactions should be carefully analyzed, it does not seem likely that they could overshadow the large quadrupole moment effect on the elastic scattering by the polarized nuclei.

Lastly we examine whether inelastic scattering can be sufficiently well separated from the truly elastic scattering experimentally. It would be very difficult to make any accurate evaluation without detailed calculation of the inelastic scattering cross section.

[19] See, e.g., the first paper in Ref. 1, Chap. III.

[20] See, e.g., P. Goldhammer and H. S. Vaulk, Phys. Rev. **127**, 945 (1962).

Although the experimental evidence on the scattering of alpha particles by nuclei over a wide range of scattering angle well below the Coulomb barrier is scarce, there are numerous experiments at incident energies not very far above the Coulomb barrier. For example, Wilson and Sampson[21] measured the angular distributions for elastic scattering and for inelastic scattering to the lowest 2^+ and other states in Fe and Zn using 22.2-MeV alpha particles. Their measurements show that the ratio of the elastic and inelastic scattering is at least about 100 even for a large scattering angle, (167°).

The excitation energies ΔE we have to consider for U^{238} and Ta^{181} are $\sim$0.048 MeV and $\sim$0.136 MeV, respectively,[22] whereas the typical values of excitation energies in the work of Wilson and Sampson are 0.845 MeV for Fe^{56} and 0.99 MeV for Zn^{64}, respectively. These differences in ΔE are partially compensated by the differences in atomic numbers Z_2 of the targets. In general,[19] the excitation cross sections for a fixed projectile and energy depends on ΔE and Z_2 approximately as a function of $(Z_2\Delta E)^{-2/3}$. We also note that the ratio of the cross sections for elastic and $E2$ Coulomb excitation for a fixed projectile increases rapidly as the incident energy decreases.[19] From the above considerations, it would be safe to assume that the resolution of elastic and inelastic scatterings in our problem would not cause any difficulty because of the overwhelming dominance of the former over the latter.

[21] H. L. Wilson and M. B. Sampson, Phys. Rev. **137**, B305 (1965).

[22] *Nuclear Data Sheets*, compiled by K. Way *et al.* (National Academy of Sciences—National Research Council, Washington, D. C., 1958–65); W. Kunz and J. Schintlmeister, *Tabellen der Atomkerne* (Akademie-Verlag, Berlin, 1958).