involved the $7.34 \to 0$ $M2$ rate and the $6.59 \to 6.09$ $E1$ rate. We note that the calculations of Sebe[26] indicate substantial departures from $p_{1/2}(s,d)$ for both the 1^- and 2^- levels in question. This could well be the major cause of the noted discrepancies. It is also possible that the model used does not give a good description of the 0^+ state at 6.59 MeV. The situation may be similar to O^{16}; that is, the O^{16} 6.06-MeV 0^+ state does not appear to be amenable to a simple shell-model description.

It would appear to be worthwhile to repeat the present calculations using the wave functions of Sebe[26] for the odd-parity states. In this case it would appear that the best wave function to use for the C^{14} ground state would be that of True[23] with the $C_{1/2,1}{}^0\Psi(p_{1/2}{}^2)$ component replaced by $C_{1/2,1}{}^0[\alpha\Psi(p_{1/2}{}^2)+\beta\Psi(p_{3/2}{}^2)]$, where $\alpha^2+\beta^2=1$ and α and β are given by the shell-model calculations[24] for s^4p^{10}. This procedure seems valid since the s^4p^{10} calculations[24] give $\alpha^2 \gg \beta^2$. The decision as to what to use for the wave function of the 0^+ 6.59-MeV level is more difficult.

Only one of the five C^{14} states considered has a measured lifetime. The theoretical predictions are that three of the remaining four have lifetimes which could be determined by Doppler-shift techniques. A knowledge of these lifetimes would be valuable in future comparisons between theory and experiment.

ACKNOWLEDGMENTS

One of us (D. E. A.) would like to express his thanks to Professor S. Gorodetzky for the opportunity of carrying out part of this work as a guest at the Institut de Recherches Nucléaires during May, 1965. We are indebted to Hugh Williams for considerable assistance in taking and analyzing data in the work at Brookhaven. The Ge(Li) detector used in this work was constructed by C. Chasman and R. Ristinen.

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Two-Nucleon Distorted-Wave Born Approximation Analysis of $F^{19}(d,\alpha)O^{17}$ Data*

Jerome J. Wesolowski, Luisa F. Hansen, Jose G. Vidal, and Marion L. Stelts
Lawrence Radiation Laboratory, University of California, Livermore, California
(Received 6 April 1966)

The $F^{19}(d,\alpha)O^{17}$ absolute differential cross sections have been measured at bombarding energies of 5.5, 6.5, 7.5, 8.5, 9.5, and 11.5 MeV for the ground and first four excited states of O^{17}. Attempts were made to fit the ground- and first-excited-state relative differential cross sections, using the two-nucleon DWBA theory of Glendenning. Reasonable fits were obtained only for the ground-state distributions at higher bombarding energies.

INTRODUCTION

The interpretation of (d,p) and (d,n) reactions and their inverses in terms of direct interactions populating single-particle levels of the nucleus have proved highly successful.[1,2] This success has awakened interest in the two-nucleon transfer reactions, such as (d,α) and (α,d). Several theoretical treatments[3–11] have been developed in recent years which deal with these reactions in a similar way to the one-nucleon transfer reactions, i.e., as pickup or stripping. In these theories certain correlations are assumed between the transferred nucleons. Such correlations impose stringent conditions on the levels populated by the reaction. If this picture of the process is shown to provide an adequate description, then the reaction gives the means for obtaining spectroscopic information about the levels it populates.

A considerable amount of experimental work has been done in the investigation of the two-nucleon transfer reaction. However, because of the complexity of the problem of fitting such angular distributions with distorted-wave Born approximation (DWBA), most authors have used plane-wave Born approximation (PWBA) for an analysis of their data. Although plane-wave theories are of limited value in obtaining some spectroscopic information, the work done in single-nucleon transfer reactions has clearly shown the plane-wave theory to be only a crude approximation to the

* This work performed under the auspices of the U. S. Atomic Energy Commission.
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[11] N. K. Glendenning, *Annual Review of Nuclear Science* (Annual Reviews, Palo Alto, California, 1963), Vol. 13; and Phys. Rev. **137**, B102 (1965).

more sophisticated, albeit still inadequate, distorted-wave theory.[12] There is no reason for this not also to be the case for multiple-nucleon transfer reactions.

Hence, in an attempt to do a more complete study of a two-nucleon transfer reaction, angular distributions were obtained for the ground and first four excited states of the $F^{19}(d,\alpha)O^{17}$ reaction at a number of bombarding energies between 5.5 and 11.5 MeV, and an attempt was made to fit these data with Glendenning's[11] DWBA two-nucleon transfer reaction theory, fully taking into account the various anticipated configurations of the ground-state wave function[13] of F^{19}.

The $F^{19}(d,\alpha)O^{17}$ reaction has been investigated at lower energies (1–3 MeV) by a number of authors.[14–17] Their interpretations of the reaction were done on the basis of PWBA[3–10] and compound-nucleus theories.[18,19] Similar interpretations were used for the studies at 9.2 MeV,[20] 11.1 and 11.4 MeV,[21] 13 MeV,[22] 14.7 MeV,[23] and 27.5 MeV.[24]

EXPERIMENTAL PROCEDURE

The deuteron source was the Lawrence Radiation Laboratory 90-in. variable-energy cyclotron. Appropriate bending, focusing, and collimation allowed a $\frac{1}{8}$-in.-diam beam to enter a 40-in.-diam scattering chamber and impinge upon a centrally located Teflon (CF_2) target of 0.60 mg/cm^2 thickness. The target was automatically oriented at half the scattering angle. For absolute-cross-section measurements, a 1.24-mg/cm^2 thick target was also used. The target material was mounted on two concentric aluminum rings and the entire target assembly was rotated at a speed of approximately 1 cm/sec to eliminate excessive target deterioration due to beam heating effects. Under these conditions, target damage did not become noticeable for as long as 12 h of continuous bombardment with beam currents of the order of 0.2 μA.

Data collection was speeded by the simultaneous use of three silicon surface-barrier detectors. The three detectors were mounted in fixed positions on a curved brass arm at 10° intervals. The entire detector assembly was mounted on a rotating table located just above the floor of the scattering chamber. Each detector subtended a solid angle of 0.406×10^{-3} sr and had an angular spread of $\pm0.6°$.

The output of each detector, after suitable amplification, was routed to a 200-channel subgroup of an 800-channel pulse-height analyzer. The fourth 200-channel subgroup of the analyzer received pulses from a monitor detector, also a silicon surface barrier detector, permanently fixed at a forward angle in the scattering chamber. A sufficient thickness of aluminum absorber was placed in front of the monitor so that only protons from the (d,p) reaction on the target nucleus were observed. For relative cross-section measurements, the use of a monitor system eliminates the need for corrections resulting from changing target thickness, analyzer dead time, and cyclotron beam modulations.

A Faraday cup was used to monitor the beam current for absolute cross-section determinations, and also as an independent check of the monitor system during the relative measurements.

Alpha particles were observed in the presence of deuterons and protons by using the intrinsic particle discrimination characteristics of a solid-state counter, i.e., by setting the bias voltage at such a value to make the effective thickness of the counter just equal to the range of the maximum-energy α particle, hence not allowing the less ionizing protons and deuterons to lose any significant energy in the detector.

Angular distributions for the ground and first four excited states were measured for deuteron bombarding energies of 5.5, 6.5, 7.5, 8.5, 9.5, and 11.5 MeV. For most distributions, spectra were taken in 5° intervals from 7.5° to 163° in the laboratory.

EXPERIMENTAL RESULTS

A typical pulse-height spectrum is shown in Fig. 1. For almost all spectra taken, the ground and first four excited states were easily resolvable, thus making data reduction correspondingly simple.

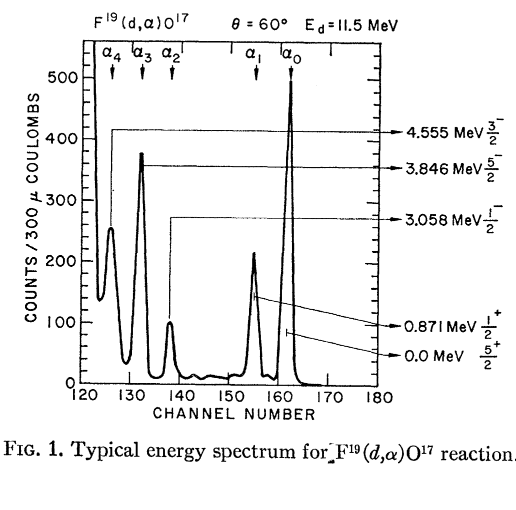

FIG. 1. Typical energy spectrum for $F^{19}(d,\alpha)O^{17}$ reaction.

[12] D. H. Wilkinson, Brookhaven National Laboratories, Report No. BNL 5013, 1960 (unpublished).
[13] M. G. Redlich, Phys. Rev. **99**, 1427 (1955); **110**, 468 (1958).
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[16] A. Z. El-Behay, A. M. Farouk, M. H. Nassef, and I. I. Zaloubovsky, Nucl. Phys. **61**, 282 (1965).
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[24] S. Mayo, J. Testoni, and O. M. Bilaniuk, Phys. Rev. **133**, B350 (1964).

Although the relative experimental errors were not identical at each angle, in general the relative error can be taken to be about 5%. The error associated with the absolute cross-section measurements is 12%.

In order to make presentation of the data easier, Legendre polynomial fits were made to each angular distribution. Since data were collected, in general, every 5° from 7.5° to 163° and the relative errors were small, the Legendre fits which extend from 0° to 180° can be considered to be accurate representations of the actual angular distributions. The data are presented in Figs. 2–6. It is to be noted that angular distributions obtained at another laboratory for the ground and first

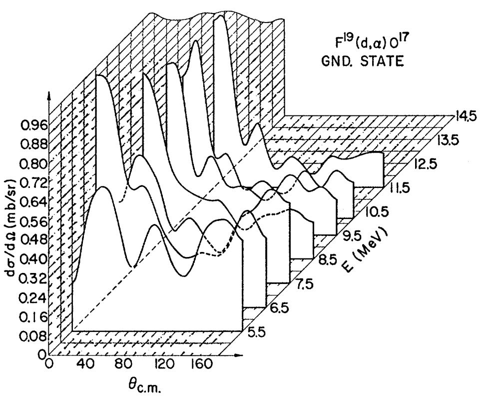

FIG. 2. Legendre fits to ground-state data.

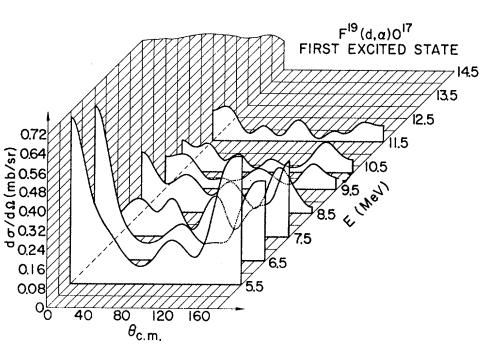

FIG. 3. Legendre fits to first-excited-state data.

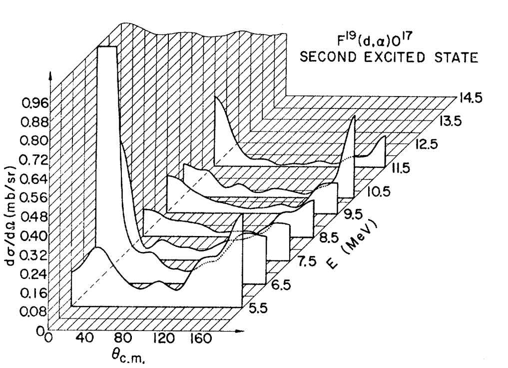

FIG. 4. Legendre fits to second-excited-state data.

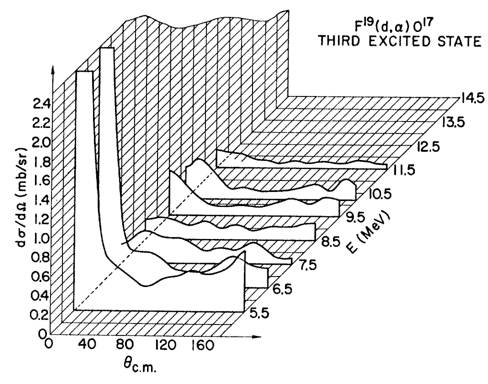

FIG. 5. Legendre fits to third-excited-state data.

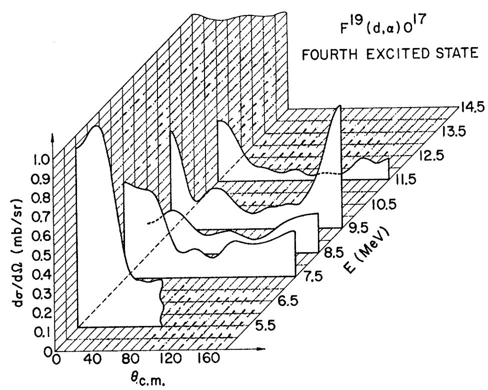

FIG. 6. Legendre fits to fourth-excited-state data. The cross section at 5.5 MeV was not measured past 90°.

three excited states at 10.2 MeV are also presented.[25] The Legendre fit to the fourth excited 5.5-MeV distribution is cut off at 90°, since for this particular case data were collected only in the first quadrant.

DISCUSSION OF DATA

Except for their oscillatory behavior, the angular distributions do not lend themselves to any sweeping generalities. In the bombarding range covered here, one anticipates some contributions to the reaction mechanism from compound-nucleus effects. These effects should diminish as the bombarding energy is increased. This effect is perhaps demonstrated by the predominantly forward peaking of the angular distribution for all states of the 11.5-MeV data, although it must be emphasized that the shape of a distribution is not sufficient to characterize definitively the reaction mechanism. Within the experimental errors, the 11.5-MeV data were in excellent agreement with the 11.4-MeVdata of Hu.[21]

Another salient feature of the data is the large backward peaks found in many of the distributions. Al-

[25] The authors are indebted to R. J. Wilson of Washington University, St. Louis, Missouri, for allowing them the use of these 10.2-MeV data prior to their publication.

though such effects have often been attributed to the so-called heavy particle stripping process, there is no justification for completely dismissing distortion effects as a possible cause.[12] Furthermore, as has been recently discussed for the (p,α) and (α,p) reactions, a compound-nucleus model can also be evoked to produce such backward peaks.[26]

Of the various possible direct interactions, a pickup of two nucleons from the target appears the most attractive, not only from a calculational point of view but also on purely physical grounds; this latter assumption deriving some justification from work done in heavier elements.[27]

In order to make some assertions regarding the reaction mechanism, the integrated cross section was plotted as a function of $(2I+1)$, where I is the spin of the final state, for each bombarding energy. The statistical compound-nucleus theory predicts under certain conditions[18,19] a proportionality to $(2I+1)$, whereas a direct reaction pickup mechanism contains no such dependence.[28,29] The integrated cross sections were not proportional to $(2I+1)$ even at the lowest bombarding energies where one anticipates compound effects to be most prevalent. Since one could argue that large-angle particles might originate from a compound-nucleus process, while the forward angles might be complicated by both direct and compound processes, cross sections integrated from 90° to 180° were also plotted as a function of $(2I+1)$. Here, also, no $(2I+1)$ dependence was found. All these results are consistent with the data of Cosper *et al.*[20] at 9.5 MeV and those of Jahns *et al.*[15] at 2–3 MeV. The Legendre fits to the cross sections were integrated from 0° to 180° for the various states and plotted as a function of bombarding energy. These results are presented in Fig. 7. The integrated cross sections for the ground, first, and third excited states decrease more or less smoothly with increasing energy. This behavior is certainly not in contradiction to a direct reaction model.

In view of the above discussion, it would not be amiss to state that the direct reaction mechanism plays a significant role in this reaction. Thus an attempt was made to fit the relative distributions and to obtain ratios of the cross sections to the various states using direct pickup theory.

THEORY

The DWBA two-nucleon exchange theory of Glendenning was used to fit the data.[11] This theory assumes that the (d,α) reaction proceeds by the pickup of a neutron-proton pair from the target nucleus by the incoming deuteron. It further assumes that the n-p pair is in a relative $l=0$, $S=1$, $T=0$ state. Thus the reaction

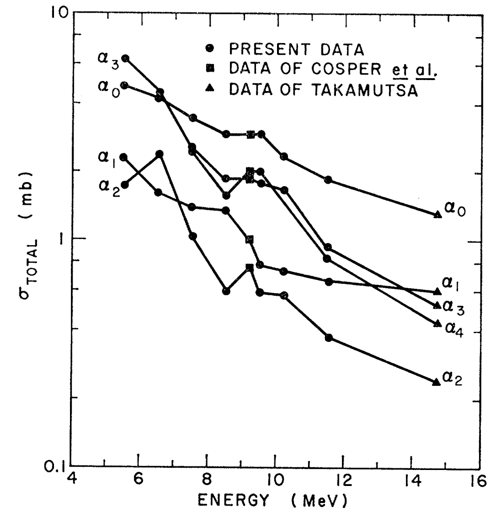

FIG. 7. Total cross section as obtained from Legendre fits to data.

will proceed predominantly from the components of the target-nucleus shell-model wave function where that relative configuration is strong, to states of the final nucleus with the same isotopic spin as the target nucleus and where the core is left unexcited.

Under these assumptions the cross section for the (d,α) reaction is given by

$$d\sigma/d\Omega \propto \sum_{LJ}\sum_{M} \left|\sum_{N} G_{NLJ}B_{NL}{}^{M}\right|^2, \qquad (1)$$

where L and J are the orbital and total angular momentum of the transferred deuteron. The kinematic factor $B_{NL}{}^{M}$ represents the probability amplitude for transferring a neutron-proton pair into the orbital state (N,L) in a structureless nucleus (the structure is carried in the factor G, discussed below). It can be represented by

$$B_{NL}{}^{M} \propto \int \Psi_\alpha{}^{(-)*}\mathfrak{U}_{NL}{}^{*}Y_L{}^{M*}V\Psi_d{}^{(+)}\phi_\alpha d\tau.$$

Here $\Psi_d{}^{(+)}$ and $\Psi_\alpha{}^{(-)}$ are the distorted waves for the entrance and exit channels, respectively. They are generated from optical-model potentials which describe the observed elastic scattering. $\mathfrak{U}_{NL}$ represents the center-of-mass motion of the n-p pair with respect to the core, i.e., the bound-state wave function. In all calculations made these were taken to be harmonic oscillator functions in the interior region, matched to a tail corresponding to the appropriate binding energy. The function ϕ_α is that part of the α-particle internal wave function which depends on the distance between the center of mass of the incoming deuteron and the proton-neutron pair which is picked up. The fact that the α-particle internal wave function can be separated in this manner is a result of using a Gaussian wave function for the α

[26] H. R. Blieden, Phys. Letters **3**, 257 (1963).
[27] J. B. Mead and B. L. Cohen, Phys. Rev. Letters **5**, 105 (1960); and Phys. Rev. **125**, 947 (1962).
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[29] H. A. Enge, Phys. Rev. **94**, 730 (1954).

particle, i.e.,

$$\psi_\alpha = N \exp(-\eta^2 \sum r_{ij}^2).$$

In order to evaluate the above integral the interaction operator V, which is a function of the distance between the centers of mass of the deuteron and the n-p pair, is effectively replaced by a delta function in this coordinate. This "zero-range" approximation is a limitation of the theory. Austern *et al.*[30] have shown that finite-range corrections for deuteron stripping are not expected to be drastic, especially in the case where a cutoff close to the nuclear surface is employed. On the other hand, the plane-wave estimate of Rodberg[31] indicates that other reaction processes may be much more sensitive to finite-range effects. Thus, the extent to which the present calculation is hampered by the zero-range approximation is a matter for conjecture.

Detailed information concerning the nuclear structure is carried by the factor G_{NLJ}. Equation (1) shows that the relative weights with which the various radial states, characterized by N, contribute coherently to the cross section, are determined by these structure factors G_{NLJ}. It is necessary to discuss this factor in some detail. It is given by

$$G_{NLJ} = \sum_\gamma g\beta_{\gamma LJ}\Omega_n \langle n0,NL; L | n_1l_1,n_2l_2; L\rangle, \quad (2)$$

where the sum over γ is a sum over all the possible states $(n_1,l_1,j_1; n_2,l_2,j_2)$ from which the neutron and proton that form the deuteron can be picked up. The sum on configurations, γ, is coherent which expresses the fact that the two-nucleon transfer reaction is sensitive to certain correlations between the nucleons in the nuclear state. If the nuclear wave functions have definite symmetry under exchange of the two particles, then

$$\begin{aligned} g &= 1 \quad \text{if} \quad n_1l_1j_1 \equiv n_2l_2j_2 \\ &= \sqrt{2} \quad \text{otherwise.} \end{aligned}$$

The bracket in Eq. (2) is a Talmi transformation coefficient.[32] It arises when we describe the spatial part of the wave function for the two nucleons in the state γ in terms of wave functions of relative motion and center-of-mass motion, i.e.,

$$[\Phi_{n_1l_1}(\mathbf{r}_1)\Phi_{n_2l_2}(\mathbf{r}_2)]_L = \sum_{n\lambda N\Omega} \langle n\lambda,N\Omega; L | n_1l_1,n_2l_2; L\rangle \times [\Phi_{n\lambda}(\mathbf{r})\Phi_{n\Lambda}(\mathbf{R})]_L, \quad (3)$$

where the square bracket denotes vector coupling. Here n and N are the principal quantum numbers of the relative and center-of-mass motions and λ and Λ are the orbital angular momenta of these motions. The factor Ω_n in Eq. (2) denotes the overlap between the relative motion of the neutron and proton in the nucleus, i.e., $\Phi_{n\lambda}(r)$, and the motion of the pair in the α particle. Now it is assumed that the relative motion in the α particle is pure s state. As a result, the only part of $\Phi_{n\lambda}(r)$ which can contribute to the reaction is the $\lambda=0$ component. Hence the zero in the bracket in Eq. (2). The wave functions in Eq. (3) were taken to be harmonic-oscillator functions. In this case the transformation coefficients are obtained in closed form, and numerical tables are found in Ref. 32. Also, a simple expression is given for Ω_n in Ref. 11 for the case of a Gaussian wave function for the α particle and an harmonic-oscillator function for the bound deuteron in the nucleus.

The quantity $\beta_{\gamma LJ}$ in Eq. (2) measures the parentage of the ground state of the nucleus $(A+2)$ based on the nucleus (A) that is formed in the reaction, plus a neutron and a proton in the state γ,L,J, with $\lambda=0$, $T=0$, and $S=1$. This overlap integral is of the same form as appears in (d,p) stripping theory, and its square is proportional to the spectroscopic factor. It contains the information about the nuclear coupling scheme and is given by

$$\beta_{\gamma LJ} = \sqrt{3}\int [\psi_{J_FT_F}{}^*(A)\phi_{\gamma LJ}{}^*(\mathbf{r}_1,\mathbf{r}_2)]_{J_iT_i}\psi_{J_iT_i} \times (A,\mathbf{r}_1,\mathbf{r}_2)dA d\mathbf{r}_1 d\mathbf{r}_2.$$

COMPARISON WITH THEORY

The calculations were carried out, using the IBM 7094 in the computer complex at this laboratory. The code essentially performed the calculation of Eq. (2). Although absolute cross sections were not computed, the program was capable of computing ratios of cross sections to the various excited states, as well as relative angular distributions.

The numerical values of the spectroscopic amplitudes, $\beta_{\gamma LJ}$'s, which embody the nuclear structure information, had to be calculated by hand and fed into the program as parameters. The ground state of F^{19} was assumed to be composed of an O^{16} core plus three nucleons in the $1d$, $2s$ shell. The amplitudes of the various possible configurations were taken from the shell-model calculations of Redlich.[13] These configurations and their amplitudes are listed in Table I. The ground and first excited states of O^{17} were taken to be pure single-particle levels; an O^{16} core plus a $1d_{5/2}$ neutron for the ground state, a $2s_{1/2}$ neutron for the first excited state.[33] Under such a model the spectroscopic factor amplitude can be expressed as

$$\beta_{j_2j_3LJ} = \sqrt{3}\begin{Bmatrix} l_2 & \frac{1}{2} & j_2 \\ l_3 & \frac{1}{2} & j_3 \\ L & 1 & J \end{Bmatrix} \sum_i A_i \langle (j_2j_3)[0,J]j_1; \tfrac{1}{2}\tfrac{1}{2}] \times (j_1j_2)[T',J']j_3; \tfrac{1}{2}\tfrac{1}{2}\rangle. \quad (4)$$

[30] N. Austern, R. M. Drisko, E. C. Halbert, and G. R. Satchler, Phys. Rev. **133**, B3 (1964).

[31] L. Rodberg, Nucl. Phys. **47**, 1 (1963).

[32] T. A. Brody and M. Moshinsky, *Tables of Transformation Coefficients* (Monografias del Instituto de Fisica, Mexico, 1960).

[33] I. Talmi and I. Unna, *Annual Review of Nuclear Science* (Annual Reviews, Palo Alto, California, 1960), Vol. 10.

TABLE I. Ground-state configurations of F^{19}.

Configuration O^{16} core plus[a]	Amplitude (A_i)
$(d_{5/2})^2[1,2]d_{5/2}$	0.30
$(d_{5/2})^2[0,1]2s_{1/2}$	−0.37
$(d_{5/2})^2[1,0]2s_{1/2}$	0.52
$(2s_{1/2})^2[1,0]2s_{1/2}$	0.55
$(d_{5/2})^2[1,2]d_{3/2}$	0.17
$(d_{5/2})^2[0,1]d_{3/2}$	−0.08
$(d_{5/2})(d_{3/2})[1,1]2s_{1/2}$	0.01
$(d_{5/2})(d_{3/2})[0,1]2s_{1/2}$	−0.33
$(d_{3/2})^2[1,2]d_{5/2}$	0.12
$(d_{3/2})^2[0,3]d_{5/2}$	0.01
$(d_{3/2})^2[1,0]2s_{1/2}$	0.22
$(d_{3/2})^2[0,1]2s_{1/2}$	0.09
$(d_{3/2})^2[0,1]d_{3/2}$	0.03

[a] The notation is explained in the text after Eq. (4).

The sum is over all configurations occupied by the two nucleons, the A_i being the amplitudes of the various configurations as listed in Table I. The bracket in the sum is a coefficient of fractional parentage. In order to understand the role of this coefficient in the $F^{19}(d,\alpha)$ reaction, consider a specific configuration from Table I, say $(d_{5/2})^2[0,1]2s_{1/2}$. This notation states that the two nucleons in the $(d_{5/2})$ shell couple to $T=0$ and $J=1$. Hence a neutron and a proton occupy the $d_{5/2}$ shell. Labeling these particles j_1 and j_2, respectively, we note the reaction can proceed from the ground state of $F^{19}(\frac{1}{2}+)$ to the ground state of $O^{17}(\frac{5}{2}+)$ only by the pickup of a neutron from the $2s_{1/2}$ shell (j_3) and a proton from the $1d_{5/2}$ shell (j_2). Now these two nucleons must be coupled to $T=0$ (a deuteron) and some value of J. Hence the coefficient of fractional parentage which is a coefficient of the unitary transformation connecting the two different representations of the three angular momenta would be written

$$\langle d_{5/2}2s_{1/2}[0,J]d_{5/2};\tfrac{1}{2}\tfrac{1}{2}]d_{5/2}d_{5/2}[0,1]2s_{1/2};\tfrac{1}{2}\tfrac{1}{2}\rangle.$$

Coefficients such as these can be expressed in terms of 6-j symbols.[13] The numerical values of the 6-j symbols were obtained from tables.[34]

TABLE II. Calculated values of the nuclear structure factors.

J	L	G_{NLJ} $N=1$	$N=2$	$N=3$
		Ground state		
2	2	−0.0110	+0.0394	···
3	2	+0.0290	−0.402	···
	4	+0.0782	···	···
		First excited state		
1	0	−0.00899	+0.0263	−0.285
	2	−0.0313	+0.111	···

[34] M. Rotenberg, R. Bivens, N. Metropolis, and J. R. Wooten, Jr., *The 3-j and 6-j Symbols* (MIT Press, Cambridge, Massachusetts, 1959).

The matrix before the sum in Eq. (4) is just a transformation coefficient between LS and JJ coupling. Numerical values were obtained from tables.[35]

The selection rules allow $L=2$ and 4 for the ground-state transition and $L=0$ and 2 for the first excited state.

The nuclear structure factors G_{NLJ} were obtained as discussed above and are listed in Table II. This table indicates that the predominant momentum transfers for the ground and first excited states were $L=2$ and $L=0$, respectively. This was borne out by the calculations, the $L=2$ component of the cross section being about 85 times greater than the $L=4$ component for the ground state, whereas the first-excited-state cross section contained an $L=0$ component approximately nine times larger than the $L=2$ contribution. These ratios are greater than those used by Hu[21] and Takamatsu[23] who used PWBA theories and treated the spectroscopic factors as adjustable parameters.

No structure factors were calculated in the present work for the negative parity states of O^{17} since the configuration of these states is not known. Hence no theoretical calculations were made for these states.

Since the elastic scattering data were not available, optical-model parameters for the entrance channel were obtained by fitting the 11.6-MeV natural neon elastic data previously measured by R. Jahr.[36] The calculations were carried out using an optical-model search program called LOKI.[37] The nuclear potential used was given by

$$U=-[V+iW][1+\exp(r-R/a)]^{-1}, \quad (5)$$

where

$$R=R_0A^{1/3}.$$

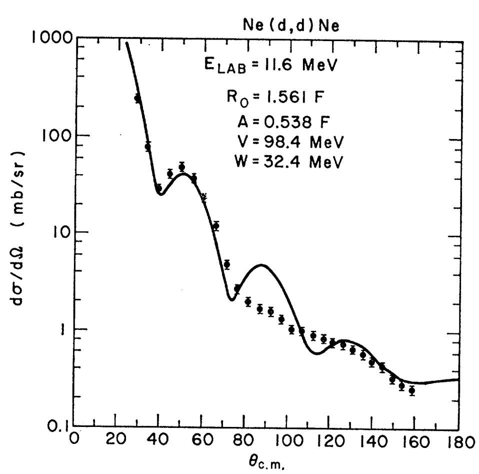

FIG. 8. Optical-model fit to Jahr's data.

[35] J. M. Kennedy and M. J. Cliff, Atomic Energy of Canada, Ltd., Chalk River Project, Report No. CRT-609, 1955 (unpublished).

[36] R. Jahr and G. Mairle, Nucl. Phys. 70, 383 (1965).

[37] The authors are indebted to E. Schwarz for the use of the code.

The Coulomb radius was taken to be $1.3A^{1/3}$. The best fit obtained after searching on R_0, a, V, and W is shown in Fig. 8. To ensure that the optical model displays no anomalies in this energy and mass region, the same parameters used above were also used to fit the 10.95-MeV natural neon data of Takeda[38] and the 12.1-MeV $Ne^{22}(d,d)$ data of Lutz.[39] The data and the theoretical curves were similar enough to those shown in Fig. 8 to indicate that no anomalies exist and that therefore the use of these parameters for the entrance channel are probably quite reasonable.

For the exit channel the 21.6-MeV $O^{18}(\alpha,\alpha)$ data of Lutz[40] was used. The form of the potential used is given by Eq. (5), and searches were made both in regions of "shallow" and "deep" well depths. The best fits obtained are shown in Fig. 9. Clearly the shallow potentials give better fits than the deep potentials. The form of the potential is identical to that used by McFadden and Satchler[41] in their optical-model analysis of the scattering of 24.7-MeV α particles from O^{16}. The actual values of the parameters used in this work are similar to those found by McFadden; her values for V, W, R_0, and a being 43.9 MeV, 3.85 MeV, 1.912 F and 0.451 F, respectively. As McFadden points out, the oxygen fits were not particularly good but then the data available were rather meager, only spanning the angular range from 45° to 125°.

Attempts were made to fit the (d,α) distributions with the neon parameters for the incident channel and both the O^{18} "shallow" and "deep" potentials for the exit channel. The fits for the "shallow" potential are shown in Figs. 10 and 11 for the ground and first excited state, respectively. The theoretical curves are

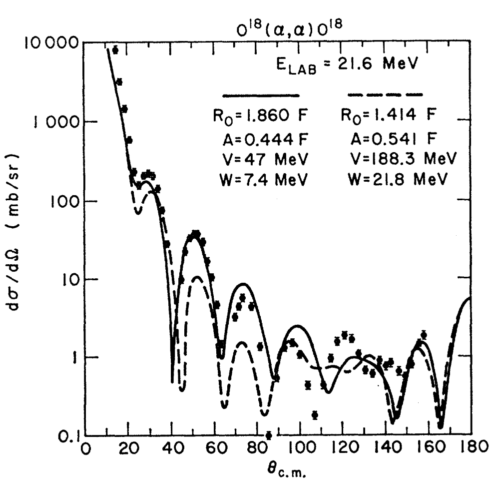

Fig. 9. Optical-model fit to Lutz's data for deep and shallow potentials.

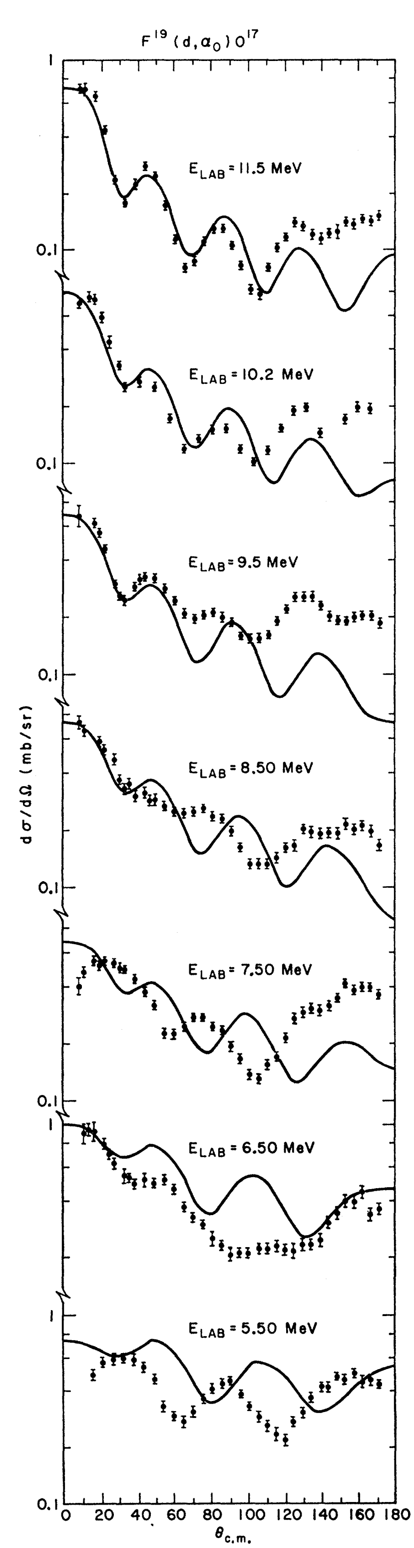

Fig. 10. "Shallow" potential DWBA fits to the ground-state data.

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38 M. Takeda, Phys. Soc. (Japan) **15**, 557 (1960).
39 H. F. Lutz (private communication).
40 H. F. Lutz and S. F. Eccles, Nucl. Phys. (to be published).
41 L. McFadden and G. R. Satchler (to be published).

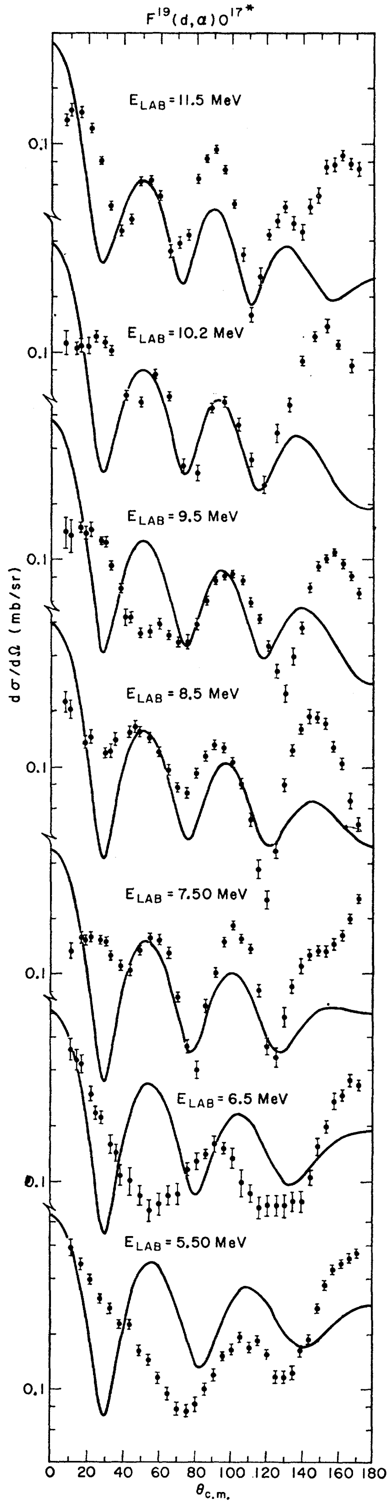

FIG. 11. "Shallow" potential DWBA fits to the first-excited-state data.

arbitrarily normalized to the experimental data. It should be emphasized that only one set of parameter values was used to generate all these curves, and that the values connected with the entrance and exit channels were identical to those used to obtain the fits shown in Figs. 8 and 9. The spectroscopic factors of Table II were used. In all cases a cutoff radius of 5.27 F was employed.

To give the proper slope to the theoretical angular distributions, it is customary to include in the DWBA analysis a damping factor $e^{-\kappa^2/8\eta^2}$, where κ is the momentum transferred to the incident deuteron by the picked-up pair of nucleons, and η is a constant parameter defining the Gaussian form of the α-particle wave function. The validity of introducing such an *ad hoc* factor, with the justification that it compensates for the zero-range approximation, is doubtful.[42] In all the calculations presented, no such damping factor was used.

DISCUSSION AND CONCLUSIONS

The theoretical fits to the 11.5-, 10.2-, and 9.5-MeV ground-state data are quite reasonable. Attempts to get better fits by changing the values of various parameters met with little success. Indeed it does appear as if the best agreement between theory and experiment is obtained when the parameter values used are the same ones used to fit the elastic data.

It is not surprising that the agreement between theory and experiment is poor at the lower bombarding energies. One anticipates that contributions from the compound-nucleus mechanism will increase with decreasing energy. Indeed some of the angular distributions at low energies are quite symmetric about 90°. Furthermore, the optical-model parameters for the entrance and exit channels were obtained only at energies corresponding to the higher energies of the (d,α) data.

What is perhaps surprising is the lack of agreement for the first excited state, even at the higher energies. Attempts to improve the fits by parameter-value changes were unsuccessful. From the calculations, one anticipates improved fits by allowing a larger contribution from the $L=2$ momentum terms. Such a change could be brought about by changes in the values of the nuclear structure factors, B_{NLJ}. Such indiscriminate changes, made simply in order to improve the fits, would be unwarranted. However, since the theory can be regarded as a test of whether the nuclear wave functions used properly describe the relevant correlations, the poor fits to the first-excited-state data are perhaps an indication that the O^{17} first-excited-state wave function used does not, in fact, properly describe the relevant correlations.

The experimental and theoretical values of the ratio of the ground- to the first-excited-state integrated cross sections were in sharp disagreement. The poor fits obtained for the first-excited-state relative distributions

[42] Richard H. Pehl, Ph.D. thesis; University of California, Radiation Laboratory Report No. UCRL-10993, 1962 (unpublished).

also made it unreasonable to try to improve the agreement of the ratios by parameter-value changes. The ratios are presented in Fig. 12. Also included are points taken from the work of Cosper *et al.*[20] and Takamatsu.[23] It is interesting to note that the discontinuity in the ratio values occurs at energies where there are deviations from the monotonic behavior of the excitation functions (see Fig. 7).

Calculations were also done for the higher energy (d,α) data using the "deep" potential parameters of Fig. 9. The fits were very poor, even those for the high-energy ground-state data. Little or no improvement could be made by small changes in the values of the parameters.

Attempts to fit the data were also made for parameter values far removed from those predicted by the elastic data. Occasionally reasonable fits could be obtained for either the ground or first excited state at a particular energy. No significance could be attached to such parameter values for two reasons. First, they did not give fits to the elastic data. Second, rather large parameter-value changes had to be made in order to obtain fits to the (d,α) data at neighboring bombarding energies.

It is important when comparing theory and experiment to use entrance and exit channel optical parameters which are in reasonable agreement with the optical-model predictions for these channels, and also to use spectroscopic factors which have been derived from the relevant initial- and final-state configurations. The latter is strongly recommended because the theory should be considered more as a tool for checking the validity of predicted configurations than as a means for determining such configurations, since the spectroscopic factors add coherently [see Eqs. (1) and (2)]. It is also wise to compare theory and experiment at energies high enough to ensure minimum interference from the compound-nucleus mechanism.

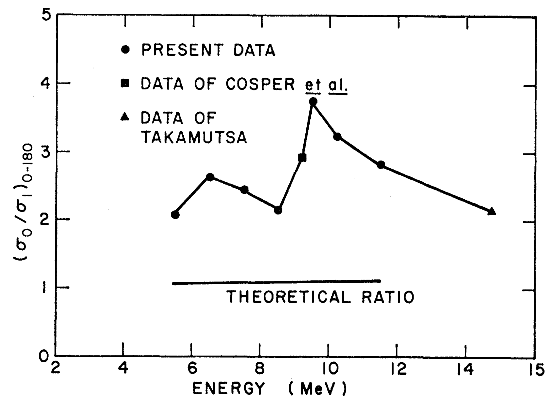

FIG. 12. Ratio of integrated ground- to first-excited-state cross sections as a function of energy.

Although the fits to the data leave much to be desired, the fact that theoretical distributions, calculated with the appropriate optical-model parameter values and spectroscopic factors, fit the higher energy ground-state data quite well gives encouragement to further study of the two-nucleon transfer reaction, both experimentally and theoretically.

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