Higher Symmetries and the 2^+ Mesons

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The two-body decay widths of the nine $J^P=2^+$ mesons are calculated in the framework of relativistic $SU(6)$. It is shown that one cannot distinguish from their decays whether the mesons should be accommodated in a **189** or **405** representation. An analysis of the masses of these mesons indicates that the **189** may be definitely ruled out. A mass formula pertinent to the **405** is derived which is fairly well satisfied and which indicates no inconsistency with placing the 2^+ mesons in this representation. Conclusions about the quark-quark and quark-antiquark interactions are drawn from this assignment.

I. INTRODUCTION

THE two-body decays of the recently completed nonet[1–3] of $J^{PC}=2^{++}$ mesons ($f^*=1.50$ BeV, $f^0=1.25$ BeV, $K^{**}=1.40$ BeV, $A_2=1.30$ BeV)[4] offer the tempting possibility of delineating in which bigger multiplet of some particular higher symmetry scheme they might be embedded. In our treatment of the subject we choose as the underlying theoretical basis the relativistic $SU(6)$ theory[5] in which the particles at rest are grouped according to the nonchiral $U(6)\times U(6)$ [6,7] and identified in $SU(6)_S$.[8] Their decays are calculated according to $SU(6)_W$.[6,9,10]

In such an approach the mesons may be accomodated in either the **189** or **405** representations of $SU(6)$.[11] We show below that, using only the two-body decay modes of the nonet of spin-2 mesons, one cannot in principle determine whether they belong in a **189** or **405** representation. This impasse may be broken by ordering states by their λ quark content in which case the **189** is eliminated from consideration. In the process, we find that the Schwinger mass formula,[12] which is not satisfied by the experimental masses, should not properly be used for the **405**, but should be replaced by a similar relation which is extremely well satisfied. That the Schwinger mass formula does not work may, in fact, be taken as an argument against assigning the 2^+ mesons to the **189**,[11] and also against the picture of the spin-2 mesons as being quark-antiquark P-wave composites.[13,14]

Two distinct cases have already been treated in the literature. In the paper of DRS,[11] who break $\tilde{U}(12)$ by the use of momentum spurions, the 2^+ mesons are regarded as S-wave composites of $qq\bar{q}\bar{q}$, are assigned to the **4212** representation of $\tilde{U}(12)$, the $(\mathbf{15},\bar{\mathbf{15}})$ of $U(6)\times U(6)$, and finally to the **189** of $SU(6)_S$. They calculate the decay amplitudes into the 0^-0^- and 0^-1^- meson modes; the branching ratios for the widths are in good agreement with experiment. In a more recent paper, Maiani and Preparata,[15] using the kinetic super-

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multiplet approach in which the quark and antiquark are in a **35** of $SU(6)$ and are in an $L=1$ state (also a **35**), have calculated the same decay widths and obtain the same branching ratios for the octet. In both papers the choice of the octet-singlet mixing angle for the f^* and f^0 was not dictated by any higher symmetry arguments. Glashow and Socolow[3] (and also Leitner *et al.*),[16] have carried out an $SU(3)$ analysis of the experimental decay widths and have also obtained from the masses an estimate of the amount of octet singlet mixing in both the f^* and f^0. The f^* has a mixing angle θ of approximately 30 deg; depending on the precise masses used this may vary by several degrees but still yields roughly

$$f^*=\tfrac{1}{2}\sqrt{3}f_8-\tfrac{1}{2}f_1,$$
$$f^0=\tfrac{1}{2}f_8+\tfrac{1}{2}\sqrt{3}f_1. \qquad (1)$$

The 2^+ mesons must lie in such an $SU(6)_S$ representation that their decays into 0^-0^- and 0^-1^- modes are allowed both by $SU(6)_W$ and charge conjugation. Only the **189** and **405** fulfil both of these requirements. The spin-2 $SU(3)$ representations included in the **189** are **1** and **8**, and those in the **405** are **1**, **8**, and **27**.

There is no *a priori* reason for the **189** to be the choice for the 2^+ mesons. In fact, *a priori*, the **405** might seem to be a more desirable choice. The **189** arises from the $(15,\overline{15})$ representation of $U(6)\times U(6)$ and is therefore antisymmetric in quarks and separately in the antiquarks, whereas the **405** stems from the $(21,\overline{21})$ and is correspondingly symmetric in both of these pairs. [In this discussion the symmetry refers to the $SU(6)$ part of the state.] Inasmuch as we know that the baryon **56** is totally symmetric in its three quarks, it must stem from symmetric quark-quark parents. We may therefore assert that the symmetric coupling of quarks is preferred and leads to lower mass states. This of course, favors the **405**. This choice agrees with the suggestion of $U(6,6)$ as a spectrum-generating algebra.[17]

Another way of choosing between the **189** and the **405** is given by the consideration of the relative amounts of octet and singlet for the f^* and the f^0. We take seriously the idea that the masses of states may be roughly ordered according to increasing λ quark content, n_λ.[18] Mathematically, we evaluate n_λ by taking the matrix elements of $S_{\lambda Z}$, for the highest S_Z state. S_λ is the lambda quark spin or strange spin. This assumption of ordering has, in the past, given as the lower state of the 1^- system, the $\omega(782)$ which has $n_\lambda=0$. The upper state is the $\varphi(1020)$ which has $n_\lambda=1$.

II. DECAY WIDTHS

We have calculated all of the two-body decay amplitudes of the 2^+ mesons for the case of both the **189** and **405** using the $SU(6)_W$ formalism, taking full cognizance of the technically complicated problem of $SU(6)_S-SU(6)_W$ mixing.[19] The $(\mathbf{15},\overline{\mathbf{15}})$ of $U(6)\times U(6)$ gives rise to a **189**, **35**, and **1** of $SU(6)_S$. (See Table I in Ref. 7.) The $(21,\overline{21})$ yields a **405**, **35**, and **1**. Each helicity component of the 2^+ octets and singlets of the **189** and **405** transform into different linear combinations of $SU(6)_W$ states stemming from their $U(6)\times U(6)$ rest multiplet[7] and are listed in Table I. The mixing implied by these linear combinations immediately produces the trivial[20] selection rule that the decay of a spin-2 meson into one vector meson and one pseudoscalar meson can only take place from the helicity ±1 state, not from the helicity zero state.

Using the tables of Clebsch-Gordan coefficients for the product 35×35 [21,22] together with the mixings required by S–W mixing, all possible two-body decay amplitudes from each helicity state in the spin-2 $SU(3)$ multiplets of the **189** and **405** have been evaluated and

TABLE I. $SU(6)_S$ to $SU(6)_W$ transformations. The various spin-2 states of the **189** and the **405** with helicity 2, 1, and 0 are given as linear combinations of $SU(6)_W$ states. Each entry in a row is to be divided by the normalization factor N listed in the last column.

Helicity 2

$S \backslash W$	$189,8^5$	$S \backslash W$	$189,1^5$	$S \backslash W$	$405,8^5$	$S \backslash W$	$405,1^5$
$189,8^5$	1	$189,1^5$	1	$405,8^5$	1	$405,1^5$	1

Helicity 1

$S \backslash W$			
	$189,8_{(2)}{}^3$	$35,8^3$	N
$189,8^5$	$-\sqrt{3}$	1	2
	$35,1^3$		
$189,1^5$	1		
	$405,8_{(2)}{}^3$	$35,8^3$	N
$405,8^5$	$-\sqrt{3}$	$\sqrt{5}$	$2\sqrt{2}$
	$35,1^3$		
$405,1^5$	1		

Helicity 0

$S \backslash W$				
	$189,8^5$	$189,8^1$	$35,8^1$	N
$189,8^5$	-1	$-\sqrt{5}$	$-\sqrt{3}$	3
	$189,1^5$	$189,1^1$	$1,1^1$	N
$189,1^5$	$-\sqrt{5}$	4	$-2\sqrt{6}$	$3\sqrt{5}$
	$405,8^5$	$405,8^1$	$35,8^1$	N
$405,8^5$	$-\sqrt{2}$	1	$-\sqrt{15}$	$3\sqrt{2}$
	$405,1^5$	$405,1^1$	$1,1^1$	N
$405,1^5$	$-\sqrt{7}$	$2\sqrt{2}$	$4\sqrt{3}$	$3\sqrt{7}$

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TABLE II. Two-body decay amplitudes for spin-2 states. The decay amplitudes for the spin-2 states into vector-pseudoscalar and pseudoscalar-pseudoscalar mesons are given for the **189** and the **405**. α and β are linear combinations of decay amplitudes from **189** to **189**, **35**, and **1** as required by the transformations listed in Table I. α' and β' are the corresponding amplitudes from the **405** to **405**, **35**, and **1**.

		189	**405**
Helicity 1	$8_1^5 \to 8_1^3 8^1$	$\frac{(\sqrt{6})}{8} SU(3)_a \alpha$	$\frac{(\sqrt{30})}{16} SU(3)_a \alpha'$
	$1_1^5 \to 8_1^3 8^1$	0	0
Helicity 0	$8_0^5 \to 8^1 8^1$	$-\frac{(\sqrt{5})}{12} SU(3)_s \alpha$	$-\frac{5}{24} SU(3)_s \alpha'$
	$1_0^5 \to 8^1 8^1$	$\frac{4}{15} SU(3) \beta$	$\frac{8}{21} SU(3) \beta'$
Ratio	$\frac{8_1^5 \to 8_1^3 8^1}{8_0^5 \to 8^1 8^1}$	$-\frac{3(\sqrt{30}) SU(3)_a}{10 SU(3)_s}$	$-\frac{3(\sqrt{30}) SU(3)_a}{10 SU(3)_s}$

are listed in Table II. An immediate result of the calculation is that for decays from the helicity 0 and ± 1 states of the octets the ratios of any two decay widths are the same, whether the decaying object is in a **189** or a **405** representation. This means that examining the decays of the A_2 or K^{**} mesons cannot be used to distinguish their source.[23] Unfortunately, neither the decays of the f^0 nor those of the f^* offer any help in this matter, because (a) the $SU(3)$ singlet has no decay into the $0^- 1^-$ mode, and (b) in the $0^- 0^-$ mode an additional amplitude must arise (the parameter β of Table II).

III. MASS FORMULAS

The impasse in deciding between the **189** and **405** may, however, be broken by invoking an additional criterion for the ordering of the f^* and f^0 which will allow us to decide which of the two $I=0$, $Y=0$ states of the **189** and which of the three $I=0$, $Y=0$ states of the **405** should correspond to the physical particles. We assume that the λ quark is more massive than the p or n quarks, so that states with the smallest number of λ quarks should lie lowest, as mentioned above.

From the properties of $SU(6)$, each of the states characterized by a particular S_λ corresponds to a definite linear combination of $SU(3)$ multiplets. We list in Table III the linear combinations of $SU(3)$ multiplets for the $Y=0$, $I=0$, 1, and 2 and the $Y=1$, $I=\frac{1}{2}$ and $\frac{3}{2}$ wave functions of the **189** and **405** representations of $SU(6)$ arranged according to their strange spin.

In we consider the **405** representation, then an immediate complication arises in that in addition to the $\mathbf{1}^5$ and $\mathbf{8}^5$ representations, the **405** also contains a $\mathbf{27}^5$; yet only 9 spin-2^+ mesons have been observed. This difficulty can be overcome by invoking another property that is known from the meson and baryon systems, namely that the smaller dimensional $SU(3)$ representations have the lower masses. Therefore the $\mathbf{27}^5$ is assumed to be very much higher in mass than any of the observed states.

Our mass formula is

$$m = a(SU(3)) + b n_\lambda . \tag{2}$$

$b n_\lambda$ may be viewed as a perturbation of the $a(SU(3))$ term. For example, for the $I=0$, $Y=0$ terms, the mass matrix becomes

$$\begin{array}{c} \\ f_{27} \\ f_8 \\ f_1 \end{array} \begin{array}{c} \begin{array}{ccc} f_{27} & f_8 & f_1 \end{array} \\ \begin{bmatrix} M_{27}+(6/5)b & \frac{3}{5}b & 0 \\ \frac{3}{5}b & M_8+(17/15)b & -\frac{1}{3}(\sqrt{5})b \\ 0 & -\frac{1}{3}(\sqrt{5})b & M_1+\frac{2}{3}b \end{bmatrix} \end{array} . \tag{3}$$

M_1, M_8, and M_{27} are $a(1)$, $a(8)$, and $a(27)$ and shall be treated as arbitrary parameters. The physical states are obtained by diagonalizing this mass matrix. We assume that $M_{27} \gg M_1, M_8$, so that the $\mathbf{8}^5$, $\mathbf{1}^5$ submatrix is effectively decoupled from the $\mathbf{27}^5$. The $I=1$, $Y=0$, and $I=\frac{1}{2}$, $Y=1$ matrices are similarly decoupled. For our 2^+ meson states we therefore obtain the following mass matrix, M_{405}:

$$M_{405} = \begin{array}{c} \\ K^{**} \\ A_2 \\ f_8 \\ f_1 \end{array} \begin{array}{c} \begin{array}{cccc} K^{**} & A_2 & f_8 & f_1 \end{array} \\ \begin{bmatrix} M_8+(9/10)b & & & \\ & M_8+(2/10)b & & \\ & & M_8+(17/15)b & -\frac{1}{3}(\sqrt{5})b \\ & & -\frac{1}{3}(\sqrt{5})b & M_1+\frac{2}{3}b \end{bmatrix} \end{array} . \tag{4}$$

The corresponding matrix for the **189** is

$$M_{189} = \begin{array}{c} \\ K^{**} \\ A_2 \\ f_8 \\ f_1 \end{array} \begin{array}{c} \begin{array}{cccc} K^{**} & A_2 & f_8 & f_1 \end{array} \\ \begin{bmatrix} M_8+\frac{1}{2}b & & & \\ & M_8+b & & \\ & & M_8+\frac{1}{3}b & \frac{1}{3}\sqrt{2}b \\ & & \frac{1}{3}\sqrt{2}b & M_1+\frac{2}{3}b \end{bmatrix} \end{array} . \tag{5}$$

[23] The decay of these mesons into the two-vector-meson mode is not useful because they are forbidden kinematically.

Invoking our assumption that the state with the smallest $\langle n_\lambda \rangle$ lies lowest, we see that in M_{189} the K^{**} and A_2 are in the wrong order compared to experiment. Further, assuming that $M_8 = M_1$, as has been done in the $SU(3)$ analysis of Glashow and Socolow, one obtains the f^0 and f^* in the wrong order. Another way of presenting this result is to say that in the f^* the amount of octet to singlet mixing is predicted to be $\frac{1}{2}$, whereas the ratio suggested by Glashow and Socolow is 3/1. This is a strong argument against the **189** representation.

One may derive, by diagonalizing the matrix M_{189}, a consistency relation which must be satisfied by the masses of the observed particles in order to fit into this scheme. This is the well-known Schwinger relation[12]:

$$(f^0 - A_2)(f^* - A_2) = \tfrac{4}{3}(K^{**} - A_2)(f^0 + f^* - 2K^{**}). \quad (6)$$

The same relation holds for the **35** with arbitrary L,[24] because the matrix for a **35** with arbitrary L is related to the matrix for the **189** by the equation

$$(n_\lambda)_{35} = \mathbf{1} - (n_\lambda)_{189}. \quad (7)$$

The Schwinger relation is well satisfied for the nonet of vector mesons.

The analogous relation derived from M_{405} is different, namely,

$$(f^0 - A_2)(f^* - A_2) = \tfrac{4}{3}(K^{**} - A_2) \times (f^0 + f^* - 2K^{**} - (9/49)(K^{**} - A_2)). \quad (8)$$

A critical test to decide what is the appropriate theory for the spin-2 mesons would be to see which formula they satisfy. For Eq. (6) the values of $R =$ (right-hand side)/(left-hand side) are 0.67 and 0.54, using linear masses and squared masses, whereas for Eq. (8) the corresponding values of R are 0.91 and 0.79. Equation (8) seems to be better satisfied.[25] An unequivocal choice can be made when the masses are determined more precisely. Using M_{405} and scaling our masses by the K^{**}, A_2, and f^0 masses, we obtain $b = 142.8$ MeV. The splitting $M_8 - M_1 = 0.38\text{b} = 54.8$ MeV, and we predict the f^* mass to lie at 1495 MeV. The eigenstates for f^* and f^0 are

$$\begin{aligned} f^* &= 0.87\,|8^5\rangle - 0.49\,|1^5\rangle, \\ f^0 &= 0.49\,|8^5\rangle + 0.87\,|1^5\rangle. \end{aligned} \quad (9)$$

The ratio of octet to singlet in the f^* is 3.0. The **405** thus strongly presents itself as the location of the nine 2^+ mesons.

TABLE III. Wave functions for states of definite S_λ. The various spin-2 states of definite lambda spin are expressed as linear combinations of $SU(3)$ multiplets. Each entry in a row is to be divided by the normalization factor N listed in the last column.

Rep.	Y	S_λ	I	27^5	8^5	1^5	N
189	0	1	0		1	$-\sqrt{2}$	$\sqrt{3}$
	0	1	1		1		1
	0	0	0		$\sqrt{2}$	1	$\sqrt{3}$
	1	$\frac{1}{2}$	$\frac{1}{2}$		1		1
405	0	2	0	3	4	$-\sqrt{5}$	$\sqrt{30}$
	0	1	0	3	-1	$\sqrt{5}$	$\sqrt{15}$
	0	1	1	2	-1		$\sqrt{5}$
	0	0	0	1	-2	$-\sqrt{5}$	$\sqrt{10}$
	0	0	1	1	2		$\sqrt{5}$
	0	0	2	1			1
	1	$\frac{3}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\sqrt{2}$		$\sqrt{5}$
	1	$\frac{1}{2}$	$\frac{1}{2}$	$\sqrt{2}$	$-\sqrt{3}$		$\sqrt{5}$
	1	$\frac{1}{2}$	$\frac{3}{2}$	1			1

IV. QUARK-QUARK AND QUARK-ANTIQUARK INTERACTIONS

The assignment of the 2^+ mesons to the **405** means, as assumed originally, that there is consistency in the nature of the quark-quark interaction as obtained from the $SU(6)$ symmetric baryons and as obtained from the 2^+ mesons. States in which two $SU(6)$ quarks are coupled symmetrically lie lowest.

We next consider the $SU(6)$ quark-antiquark interaction. Formerly the only sources of information about this were the mesons which arose from the $(6,\bar{6})$ representation. The $(6,\bar{6})$ gives rise to an $SU(6)$ **35** and an $SU(6)$ **1**. The **35** contains a $Y=0$, $I=0$, $J=0$ state, the η meson (548 MeV); this meson lies much lower than the $SU(6)$ **1**, which is also a $Y=0$, $I=0$, $J=0$ state, the X^0 (960 MeV). From this we conclude that the $SU(6)$ quark-antiquark interaction is such that the $SU(6)_S$ states which come from a given representation of $U(6) \times U(6)$ are ordered according to the dimensionality of the product representation, with the lowest states being those of the greatest dimensionality. This conclusion is consistent with the observation that two $SU(6)$ quarks like to couple as symmetrically as possible. If we consider an antiquark to be equivalent to five antisymmetrically coupled quarks, then the $SU(6)$ **1** is antisymmetric in all quarks, and should lie higher than the **35** which has less antisymmetry. Loosely speaking, the quark and antiquark seem to prefer coupling in a symmetrical way. Applying these observations to the structure of the $qqqq$ system we have to order the **405**, **35**, and **1** which come from the **(21,21)** such that the **405** lies lowest. Similarly in meson-baryon resonant systems, we would expect the $(\mathbf{126},\bar{\mathbf{6}})$, which gives rise to the $\mathbf{700}^-$ and $\mathbf{56}^-$, to lie lower than the $(\mathbf{210},\bar{\mathbf{6}})$ which yields the $\mathbf{1134}^-$, $\mathbf{70}^-$, and $\mathbf{56}^-$.[26] Further, we expect the $\mathbf{700}^-$ to lie lower than the $\mathbf{56}^-$.

[24] L. Maiani and G. Preparata have included additional spin-dependent terms in their Hamiltonian, which contribute an additional parameter to their mass matrix. We thank these authors for a discussion of this point.

[25] Including a spatial overlap term in the nondiagonal matrix element can only lead to worse agreement with experiment.

[26] This is consistent with the $U(6,6)$ work of Dothan, Gell-Mann, and Ne'eman because they obtain just those representations that are totally symmetric in the quarks and separately in the antiquarks.

V. CONCLUSIONS

In summary, we have found the following:

(1) Using the two-body decay modes of the 2^+ octet and singlet one cannot determine whether they belong in a **189** or **405** representation of $SU(6)$.

(2) Ordering states by their n_λ eliminates the **189** because it gives the wrong order for the K^{**} and A_2 and has a combination of octet and singlet which disagrees with experiment.

(3) Using n_λ and invoking an interaction which moves the **27** representation high, allows the 2^+ mesons to be accommodated in the **405**. The ratio of octet to singlet for the f^* and f^0 is in good agreement with experiment.

(4) The Schwinger formula should apply to any $q\bar{q}$ system with a relative orbital angular momentum[24] and also to the **189** representation model. It should not apply to the **405** representation model, but Eq. (8) should be used instead. The present experimental data on the masses of the 2^+ mesons seem to favor Eq. (8) rather than the Schwinger formula.

(5) From the systematics of the meson and baryon masses it seems that several interesting features of the quark-quark and quark-antiquark interactions emerge. Their nature may be further tested in the classification of higher baryon and meson resonances.

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Exact Solution of the One-Photon-Exchange N/D Equations*

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The exact solution of the single-Yukawa-meson-exchange N/D equations is obtained in the zero-meson-mass (Coulomb) limit. The solution sums an infinite series of infrared-divergent ladder-graph fragments into finite, unitary partial-wave amplitudes. As with the Schrödinger equation for the hydrogen atom, one finds an infinite number of bound states (zeros of the D function) with an accumulation point at threshold. These bound-state poles of the scattering amplitude arise from the long-range force in much the same way as dynamical bound states arise generally in dispersion theory, thus allowing a discussion of the long-range force rather naturally in the usual dispersion-theoretic terms. The bound-state poles are neither so deeply bound nor so dense as those of the hydrogen atom, thus providing some understanding of the role of the one-photon-exchange force relative to the (long-range) multiple-photon-exchange forces. The possibilities for extending the technique to the relativistic one-photon case and the question of electromagnetic corrections to the strong interactions are briefly discussed. Finally, some possible approaches to including higher order photon exchanges are considered.

I. INTRODUCTION

ACCORDING to Chew,[1] there are several fundamental reasons why one should not expect to be able to formulate a self-consistent S-matrix theory for electrodynamics in which, for example, it would be possible to "bootstrap" the photon. On the other hand, there is the lesser goal of calculating electrodynamics and electrodynamic corrections to the strong interactions by using dispersion techniques that do not employ self-consistency as a calculational device, e.g., the N/D equations. Since the simplest electrodynamic problem is nonrelativistic Coulomb scattering, we shall concentrate our attention on that.

Several dispersive investigations of Coulomb scattering in the presence of the strong interactions have been carried out.[2–4] However, these studies have all treated the Coulomb part, philosophically, as "known". To date, the pure Coulomb problem has been investi-

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