the same manner as the ordinary Ward identity for the conserved electromagnetic current yields the Kroll-Ruderman theorem. We can then obtain from field theory alone many interesting relations which have recently been obtained from the algebra of currents. Such an investigation will be reported elsewhere.

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$SU(6)$ Algebra of Gell-Mann: Mass Splittings and Nonleptonic Decays of Hadrons*

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We apply the $SU(6)$ algebra of Gell-Mann to nonleptonic decays of hadrons. We assume that the weak-interaction Hamiltonian for s-wave decays transforms like S_7^5 (the seventh component of the space integral of the pseudoscalar quark density). Using the Fubini-Furlan-Adler-Weisberger technique and the partially conserved axial-vector current, we express the s-wave decay amplitudes in terms of the reduced matrix elements, which are the same as the ones that appear in the expression for mass splittings if one assumes that the symmetry-breaking part of the mass operator transforms like S_8 (the eighth component of the space integral of the scalar quark density). Making use of the fact that the mass-splitting parameter is universal, and assuming that the *s-wave coupling constant G_s is also universal*, we predict a ratio $(K_1^0 \to 2\pi)/A(\Sigma_-^-)$ which is in excellent agreement with experiment. Knowing any one of the hyperon s-way decay amplitudes we can predict the others. Also we get $A(\Sigma_+^+)=0$. The implications of these results are discussed.

I. INTRODUCTION

It has been propounded by Gell-Mann[1] that one can extract physical information concerning hadrons out of a set of equal-time commutation relations of various currents which form an algebra. In particular the algebra of chiral $SU(3)\times SU(3)$,[1,2] which consists of eight space integrals of the fourth components of vector current densities and eight space integrals of the fourth component of axial-vector current densities of quarks, has been used with remarkable success to get the ratio G_A/G_V,[3,4] to relate various leptonic decays of hadrons,[5] to get sum rules of nonleptonic decays of hadrons,[6,7] to study the decays of vector mesons into pseudoscalar mesons,[8] and to obtain sum rules for the magnetic moment[9] and axial-vector form factors.[10] Gell-Mann[1] has observed that if one includes the space integrals of pseudoscalar and scalar densities of quarks, the algebra thus generated leads to $SU(6)$. More explicitly the fourth components of the vector and axial-quark current densities are

$$\mathfrak{F}_{i4}=i\bar{q}(\lambda_i/2)\gamma_4 q\,,\quad (i=1,\cdots,8);\tag{1a}$$

$$\mathfrak{F}_{i4}{}^5=i\bar{q}(\lambda_i/2)\gamma_4\gamma_5 q\,,\quad (i=0,1,\cdots,8);\tag{1b}$$

and the scalar and pseudoscalar quark densities are

$$\mathcal{S}_i=\bar{q}(\lambda_i/2)q\,,\quad (i=0,1,\cdots,8);\tag{1c}$$

$$\mathcal{S}_i{}^5=i\bar{q}(\lambda_i/2)\gamma_5 q\,,\quad (i=0,1,\cdots,8).\tag{1d}$$

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[7] L. M. Brown and C. Sommerfield (private communication); Y. Nambu, Proceeding of the Coral Gables Conference on Symmetries in High Energy Physics, 1966 (to be published). The way of relating $K_1^0\to 2\pi$ to hyperon decays was also suggested by Nambu, but the underlying philosophy is different. We like to point out that in the current-current picture there is no reason for the spurion which leads to nonleptonic decays to belong to the same octet as the mass splitting spurion.

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[9] V. S. Mathur and L. K. Pandit, Phys. Letters 20, 308 (1966); S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento 43A, 161 (1966).

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The corresponding quantities integrated over space are

$$F_i=-i\int \mathfrak{F}_{i4}d^3x\,, \tag{2a}$$

$$F_i^5=-i\int \mathfrak{F}_{i4}^5d^3x\,, \tag{2b}$$

$$S_i=\int \mathcal{S}_id^3x\,, \tag{2c}$$

$$S_i^5=\int \mathcal{S}_i^5d^3x\,. \tag{2d}$$

The commutation relations are given by

$$[F_i(t),F_j(t)]=if_{ijk}F_k(t)\,, \tag{3a}$$

$$[F_i(t),F_j^5(t)]=if_{ijk}F_k^5(t)\,, \tag{3b}$$

$$[F_i^5(t),F_j^5(t)]=if_{ijk}F_k(t)\,, \tag{3c}$$

$$[S_i(t),S_j(t)]=if_{ijk}F_k(t)\,, \tag{3d}$$

$$[F_i(t),S_j(t)]=if_{ijk}S_k(t)\,, \tag{3e}$$

$$[S_i^5(t),S_j^5(t)]=if_{ijk}F_k(t)\,, \tag{3f}$$

$$[F_i(t),S_j^5(t)]=if_{ijk}S_k^5(t)\,, \tag{3g}$$

$$[F_i^5(t),S_j^5(t)]=-id_{ijk}S_k(t)\,, \tag{3h}$$

$$[F_i^5(t),S_j(t)]=id_{ijk}S_k^5(t)\,, \tag{3i}$$

$$[S_i(t),S_j^5(t)]=id_{ijk}F_k^5(t)\,. \tag{3j}$$

The subalgebra given by the commutation relations (3a,3b,3c) which form the chiral $SU(3)\times SU(3)$ has been used with remarkable success as remarked earlier. The subalgebra given by (3a,3d,3e) which form a nonchiral $SU(3)\times SU(3)$ has recently been exploited with encouraging results to give a uniform picture of mass splitting within a multiplet.[11] The subalgebra (3a,3f,3g) has not yet been used. The purpose of this paper is to exploit commutators (3h,3i,3j) in the nonleptonic decays of hadrons. In the case of semileptonic decays of hadrons we assume that the Hamiltonian transforms like F_1+iF_2 or F_4+iF_5. It turns out, using Cabibbo picture[12] and the commutation relations (3a,3b,3c) of the chiral $SU(3)\times SU(3)$, that all semileptonic decays of hadrons can be expressed in terms of a single universal Fermi constant G and the Cabibbo angle θ. If one looks for how the Hamiltonian should transform in the case of nonleptonic decays, it turns out it should transform like λ_6 or λ_7 and also CP of the decay amplitude must be $+1$. Because $C=+1$ for quark densities, the Hamiltonian which gives rise to s-wave decays of hadrons must transform like S_7^5 and that which gives rise to p-wave decays must transform like S_6. Thus in this picture $K_1^0\to 2\pi$ is *not* forbidden in the limit of exact symmetry. It should be noted that the transformation property for the Hamiltonian for s-wave decays is different from what one would get from current-current picture if the currents are of Cabibbo type (or belong to first class). We do not think that there is any compelling reason for using the current-current picture for nonleptonic decays of hadrons. On the other hand, the following attitude is more appealing to us. The $SU(6)$ algebra is a good and useful algebra in the same sense as the chiral $SU(3)\times SU(3)$ algebra. Just as semileptonic decays are described by vector and axial-vector currents, nonleptonic decays are described by scalar and pseudoscalar densities. In this way we give scalar and pseudoscalar densities also a character of observables, measurable in Gell-Mann–Okubo mass splittings and nonleptonic decays. In our approach we show that for the case of s-wave nonleptonic hyperon decays, the triangular relation[13] and $\Sigma_+^+=0$[14] follow from the commutation relation (3h) and the partially conserved axial-vector current (PCAC)[15] hypothesis. Also just as semileptonic hadron decays are described in terms of a single Fermi coupling constant by using the chiral $SU(3)\times SU(3)$ subalgebra of the $SU(6)$ and mass splitting within every multiplet in terms of a single parameter δm by using the nonchiral $SU(3)\times SU(3)$ subalgebra (3a,3d,3e) of the $SU(6)$, we show that all hadron (hyperon and K_1^0) nonleptonic s-wave decays are correlated and described by a single coupling constant G_s. It would also be interesting to see that just as $G_a\approx G_v$ within 20% whether G_p (the coupling constant for p-wave nonleptonic decays) is equal to G_s within a few percent. This also seems to be true.

We use the techniques adopted by Fubini and Furlan,[16] Adler and Weisberger,[3] or the improved form thereof[17] and express all the s-wave baryon nonleptonic decays in terms of two reduced matrix elements s_d and s_f and the s-wave $K_1^0\to 2\pi$ decay in terms of s_d'. If we assume that the symmetry-breaking part of the mass operator transforms like S_8, the mass differences are related to s_f and s_d and hence we extract the ratio s_d/s_f. It has been shown that the mass splitting parameter is universal[11] for all $SU(3)$ multiplets. If we make use of this fact we can extract s_d'/s_f from the known

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mass differences of mesons and baryons. With our assumption that s-wave hadron nonleptonic decays are described by a single coupling constant G_s, this ratio determines the ratio of $K_1^0 \to 2\pi$ to $\Sigma^- \to n+\pi^-$ (or s-wave part of any hyperon decay); the latter ratio turns out to be in remarkable agreement with experiment, thus verifying our assumption.

In the next section we write down the various decay matrix elements of s-wave hadron decays in terms of the corresponding reduced matrix elements and get the usual sum rules for the decay amplitudes. In Sec. III we derive the relation between the mass differences and the reduced matrix elements and derive the final result. In the course of doing so we have made some improvements over the previous work on mass splitting which are indicated in the Appendix. In the last section we discuss the consequences of a similar treatment in the case of p-wave decays. Also we discuss the further scope of the $SU(6)$ algebra.

II. S-WAVE DECAY AMPLITUDES OF HADRONS

As mentioned in the Introduction, for s-wave decays the weak-interaction Hamiltonian H_w transforms like S_7^5

$$H_w = G_s S_7^5 .$$

Now consider the decay process

$$\beta \to \alpha + \Pi_i ,$$

where Π_i is the ith member of the pseudoscalar-meson octet. Consider the amplitude

$$M_{\mu,ij} = i\int dx\, e^{-iq\cdot x}\theta(x_0)\langle\beta|[\mathfrak{F}_{\mu,i}^5(x),S_j^5(0)]|\alpha\rangle . \quad (4)$$

Then we have the following identity:

$$iq_\mu M_{\mu,ij} = i\int dx\, e^{-iq\cdot x}\theta(x_0)\langle\beta|[\partial_\mu\mathfrak{F}_{\mu,i}^5(x),S_j^5(0)]|\alpha\rangle$$
$$+i\int dx\, e^{-iq\cdot x}\delta(x_0)\langle\beta|[\mathfrak{F}_{0,i}^5(x),S_j^5(0)]|\alpha\rangle . \quad (5)$$

Making use of PCAC and the Goldberger-Treiman relation and letting $q\to 0$, we get

$$\langle\beta|S_j^5|\alpha\Pi_i\rangle = ic\mu^2\langle\beta|[S_j^5,F_i^5]|\alpha\rangle . \quad (6)$$

If Π_i is a pion, $c = g_r/G_A m\mu^2$, where g_r is the pion-nucleon coupling constant, m is the mass of the nucleon, and μ the mass of the pion. We rewrite (6) for s-wave decay as

$$\langle\beta|S_7^5|\alpha\Pi_i\rangle = ia\langle\beta|[S_7^5,F_i^5]|\alpha\rangle , \quad (7)$$

where $a = (g_r/G_A m)$. The use of the commutator (3h) gives

$$\langle\beta|S_7^5|\alpha\Pi_i\rangle = -id_{i7k}a\langle\beta|S_k|\alpha\rangle . \quad (8)$$

Now consider $\beta = K_1^0$, $\alpha=\pi^0$, and $\Pi_i = \pi^0$; then Eq. (8) gives

$$A(K_1^0\to 2\pi^0) = \tfrac{1}{4}iaG_s s_d' = (1/\sqrt{2})A(K_1^0\to\pi^+\pi^-) , \quad (9a)$$

where s_d' is the reduced matrix element of S_k that is associated with d-type coupling. Similarly in the case of baryons we get

$$\begin{aligned} A(\Sigma_0^+) &= -i(\tfrac{1}{4}a)G_s(s_d - s_f) ,\\ A(\Sigma_-^-) &= i(a/2\sqrt{2})G_s(s_d - s_f) ,\\ A(\Lambda_-^0) &= -i(a/4\sqrt{3})G_s(s_d + 3s_f) ,\\ A(\Xi_-^-) &= -i(a/4\sqrt{3})G_s(s_d - 3s_f) , \end{aligned} \quad (9b)$$

where s_d and s_f are the reduced matrix elements of S_k associated with d- and f-type couplings. Also it follows that

$$A(\Sigma_+^+) = 0 , \quad (9c)$$

so that Σ_+^+ decay must be pure p wave and therefore we take Σ_-^- decay to be pure s wave. Equation (9b) gives the Lee-Sugawara-Gell-Mann-Okubo[13] triangle

$$A(\Lambda_-^0) + 2A(\Xi_-^-) = \sqrt{3}A(\Sigma_0^+) . \quad (10)$$

From (9a) and (9b) we obtain

$$\left|\frac{A(K_1^0\to\pi^+\pi^-)}{A(\Sigma^-\to n\pi^-)}\right| = \left|\frac{s_d'/s_f}{(s_d/s_f)-1}\right| . \quad (11)$$

Also note that the ratio of any two amplitudes in (9b) depends only on (s_d/s_f).

III. REDUCED MATRIX ELEMENTS AND MASSES OF HADRONS

If we assume that the symmetry breaking part of the mass operator transforms like S_8, the same reduced matrix elements as those occurred in Sec. II appear in the expression of masses of particles. Thus we take the mass operator to be

$$M = M_0 + \delta m S_8 .$$

For pseudoscalar mesons, we have (indicating masses by letters)

$$\begin{aligned} K^2 &= m_0'^2 - (\delta m'/2\sqrt{3})s_d' ,\\ \pi^2 &= m_0'^2 + (\delta m'/\sqrt{3})s_d' ,\\ \eta^2 &= m_0^2 - (\delta m'/\sqrt{3})s_d' , \end{aligned} \quad (12)$$

where s_d' has dimension of mass (see below and Appendix), and this fact compels us to use the quadratic mass formula. Similarly, for baryons

$$\begin{aligned} \Sigma &= m_0 + (\delta m/\sqrt{3})s_d ,\\ \Lambda &= m_0 - (\delta m/\sqrt{3})s_d ,\\ N &= m_0 - (\delta m/2\sqrt{3})(s_d - 3s_f) ,\\ \Xi &= m_0 - (\delta m/2\sqrt{3})(s_d + 3s_f) . \end{aligned} \quad (13)$$

From (13) and using experimental values of masses we get

$$s_d/s_f=-\tfrac{3}{2}(\Sigma-\Lambda)/(\Xi-N)=-0.31\pm.02. \quad (14)$$

We note at this point that because the ratio of any two hyperon decay amplitudes depends only on the ratio (s_d/s_f), as indicated at the end of Sec. II, knowing any one of them we can predict the others using the value given in (14).[18]

If we take $\delta m=\delta m'$ we see that (s_d'/s_f) is also determined from (12) and (13) and is given by

$$s_d'/s_f=2(K^2-\pi^2)/(\Xi-N). \quad (15)$$

In order to justify that $\delta m\approx\delta m'$, we have to disentangle δm from s_f and s_d in (12) and $\delta m'$ from s_d' in (13). For this purpose we make use of the following commutation relation:

$$[S_i,S_j]=if_{ijk}F_k. \quad (3d)$$

Taking the matrix elements of this between the meson states α and β at rest and keeping only a nonet of single-meson intermediate states, we obtain (see Appendix)

$$|s_d'|=2m_0',$$
$$|s_0'/s_d|=1, \quad (16)$$

where s_0' is the reduced transition matrix element between a unitary singlet and unitary octet and m_0' is the mean mass of the meson multiplet. This situation is independent of the spin of the meson multiplet. This solution for each nonet gives a Schwinger formula[19]; for the 1^- nonet we have

$$(\varphi^2-\rho^2)(\omega^2-\rho^2)=\tfrac{4}{3}(K^{*2}-\rho^2)(\varphi^2+\omega^2-2K^{*2}), \quad (17)$$

independent of the value of $\delta m'$ for each multiplet and we have the following relation if the same $\delta m'$ is used for each nonet:

$$\frac{K^2-\pi^2}{2(m_0')_{0^+}}=\frac{K^{*2}-\rho^2}{2(m_0')_{1^-}}=\frac{K^{*\prime 2}-A_2{}^2}{2(m_0')_{2^+}}=(\tfrac{1}{2}\sqrt{3})\delta m', \quad (18)$$

where $K^{*\prime}(1430)$ and $A_2(1310)$ belong to a 2^+ nonet together with $f(1250)$ and $f'(1525)$. The Schwinger formula is a consequence of the relation (16) because s_0' fixes the mixing ratio between the $I=0=Y$ member of the unitary octet and the unitary singlet. From (18) one obtains $\delta m'$ to be nearly the same and equal to about 145 MeV for both 1^- and 2^+ mesons; the Schwinger formula is also very well satisfied for these nonets. However, for 0^- mesons, the relation (18) gives $\delta m'\approx 290$ MeV; and also we know that the Schwinger formula is not satisfied by a 0^- nonet. We therefore take the attitude that $\delta m'$ in this case should be the same as for other nonets and the approximation which leads to (16) is bad for this particular case whereas the same approximation is good for 1^- and 2^+ nonets. We believe that if one takes scattering states in addition to single-particle states for solving the commutation relation (3d), one will get a different structure for Eq. (16) in the case of 0^- which will give $\delta m'\approx 145$ MeV. This point will be discussed in detail elsewhere.

Let us solve Eq. (3d) for baryons, again taking the octet of single-baryon intermediate states. Then we get

$$s_f=1 \quad \text{and} \quad s_d=0, \quad (19)$$

which is approximately the case, since empirically from (14) we have $(s_d/s_f)=-\frac{1}{3}$. This makes the Λ and Σ masses degenerate and one obtains in this case $\delta m=160$ MeV, a value close to that obtained from meson multiplets. The scattering states in this case also may change (19) to a more realistic one which will give $s_d/s_f=-\frac{1}{3}$ and at the same time make δm come closer to $\delta m'$.

Thus it is reasonable to assert that $\delta m'\approx\delta m\approx 150$ MeV is universal for all $SU(3)$ multiplets. This gives rise to Eq. (15). Using Eq. (15), Eq. (11) yields

$$\frac{A(K_1{}^0\to\pi^+\pi^-)}{A(\Sigma^-\to n\pi^-)}\approx 6.5\mu, \quad (20)$$

which is to be compared with the experimental value[20] of 6.4 μ (μ being the pion mass).

IV. DISCUSSION AND CONCLUSION

We have demonstrated above that if we assume that the weak interaction Hamiltonian for s-wave nonleptonic decays transforms like $S_7{}^5$ [the only possibility in $SU(6)$ algebra] and the symmetry-breaking part of the mass operator transforms like S_8, and make use of the already developed techniques of computation, there exists a universal s-wave coupling constant for hadrons. A similar procedure can be adopted to study the p-wave decays of baryons. Then we have to use the formula similar to (7) with S_6 instead of $S_7{}^5$; the matrix element on the right-hand side of (8) will be $\langle\beta|S_6{}^5|\alpha\rangle$, where both β and α are $\frac{1}{2}^+$ baryons, and hence it should vanish as in the work of Suzuki.[6] But taking properly the surface terms and the singular terms that appear by introducing degenerate single-particle intermediate states, we can remedy the situation in the same fashion as Brown and Sommerfield and Nambu.[7] The former authors find that if one takes the current-current picture with the same coupling constant for both s and p waves, the calculated asymmetry parameters in various decays agree with experimental values within 20%. This means

[18] After the completion of this work, we came across an unpublished report by M. K. Gaillard (Orsay) who has calculated the hyperon decay amplitudes using $A(\Lambda_-{}^0)$ as input; the results agree with experiment very well. See also, R. Gatto, L. Maiani, and G. Preparata, Nuovo Cimento **41**, 622 (1966).

[19] J. Schwinger, Phys. Rev. Letters **12**, 237 (1964).

[20] For $A(\Sigma_-{}^-)$ we have taken the value given by N. P. Samios, Proceedings of the International Conference on Weak Interactions, Argonne National Laboratory, 1965 (to be published); for $A(K_1{}^0\to\pi^+\pi^-)$ we have taken the value from A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. **37**, 633 (1965).

that in our case G_p (the p-wave coupling constant) is equal to G_s within 20%. Thus the analogy between the semileptonic decays and the nonleptonic decays of hadrons is complete. Now one wonders whether there is a basic reason for the universal s-wave coupling as in the case of vector coupling of semileptonic decays (namely the vector current is conserved). There does not seem to be any obvious physical reason. But one can think in terms of $SU(6)$ algebra as being a good algebra in the sense that every interaction in nature transforms like one of its generators and the strength of the coupling for each type is universal. This way of thinking is already corroborated by the existence of universal values of G_V,[21] α (the coupling strength of electromagnetic interactions which transform like $F_3+(F_8/\sqrt{3})$, δm, G_s, and G_p. It would be interesting to investigate this point further.

Lastly we would like to remark that the commutation relations (3) could be used in the domain of strong interactions to get information concerning them.

Note added in proof. (A) If one assumes that the p-wave nonleptonic weak Hamiltonian transforms like S_6 and the mass breaking Hamiltonian transforms like S_8 of the same algebra, in general p-wave nonleptonic decay Hamiltonian can be eliminated by a unitary transformation thus leading to no p-wave decays [see S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964)]. But this is not true if one of the following happens: (i) The p-wave nonleptonic Hamiltonian does *not* transform like the member of the *same* octet to which the mass breaking Hamiltonian belongs. But in this case we cannot relate the ratios of p-wave amplitudes to those of s wave. For this reason this possibility is not attractive. (ii) As suggested by J. Schwinger [Phys. Rev. Letters **13**, 355 (1964); **13**, 500 (1964)], in pole approximation, the p-wave decays can occur if the values of the strong interaction coupling constants $g_{B\bar{B}\pi}$ and $g_{B\bar{B}K}$ are slightly different from those given by $SU(3)$. (iii) As suggested by B. W. Lee [Phys. Rev. **140**, B152 (1965)], again in the pole approximation, p-wave decays can occur if the meson poles are completely neglected. This choice is very attractive to us because this is precisely what happens in the current algebra approach (see Sec. IV and Ref. 7).

(B) Since this paper was written the following have been done using the $SU(6)$ algebra: (i) Making use of the commutator (3h) we have been able to calculate the four-pion coupling constant [K. T. Mahanthappa and Riazuddin, University Pennsylvania Report (unpublished)]. (ii) Making use of the nonchiral $SU(3)\times SU(3)$ subalgebra formed by the commutators (3a, 3f, 3g) given in Sec. I and using the fact that S_i^5 is proportional to the source density J_i of pseudoscalar meson octet [K. T. Mahanthappa and Riazuddin and J. W. Moffat, University of Toronto Report (unpublished)] have gotten the sum rules between the integrals over cross sections of $N\pi$, NK, $\pi\pi$ and πK scatterings.

APPENDIX

In this Appendix we derive relations (16) from the commutation relation (3d) by putting only a nonet of mesons as intermediate states. We also show how relations (16) lead to Schwinger's formula for a nonet of mesons. We illustrate the procedure for spin-zero mesons. The treatment for spin-1 and spin-2 mesons is similar. It is convenient to rewrite the commutation relation in spherical base having the phase convention of de Swart[22]:

$$[S^\lambda,S^\mu]=-\sqrt{3}\begin{pmatrix}8 & 8 & 8_a\\ \lambda & \mu & \nu\end{pmatrix}F^\nu. \qquad \text{(A.1)}$$

Taking matrix elements between states α and β of spin zero, we obtain

$$\langle\alpha|[S^\lambda,S^\mu]|\beta\rangle=-\sqrt{3}\begin{pmatrix}8 & 8 & 8_a\\ \lambda & \mu & \nu\end{pmatrix}\langle\alpha|F^\nu|\beta\rangle. \qquad \text{(A.2)}$$

We define the following matrix elements

$$\langle\alpha(k')|F^\nu|\beta(k)\rangle = (4k_0k_0')^{-1/2}(k_0+k_0')\begin{pmatrix}8 & 8 & 8_a\\ \beta & \nu & \alpha\end{pmatrix}\sqrt{3}, \qquad \text{(A.3)}$$

$$\langle\gamma_8(p)|S^\mu|\beta(k)\rangle=(4p_0k_0)^{-1/2}\begin{pmatrix}8 & 8 & 8_s\\ \beta & \mu & \gamma\end{pmatrix}H_s, \qquad \text{(A.4)}$$

$$\langle\gamma_0(p)|S^\mu|\beta(k)\rangle=(4p_0k_0)^{-1/2}\begin{pmatrix}8 & 8 & 1\\ \beta & \mu & 0\end{pmatrix}H_0. \qquad \text{(A.5)}$$

The factor (k_0+k_0') appears in (A.3) since $F^\nu=-i\int F_4{}^\nu(x,t)d^3x$ and $F_4{}^\nu$ is the fourth component of a vector. Since we deal with states at rest and since we are interested to solve Eq. (A.2) in the U_3 limit, we have $k_0=m_0'=k_0'=p_0$, where m_0' is the mean mass of the multiplet. The subscripts 8 and 0 on γ in (A.4) and (A.5) denote that the corresponding states belong to a unitary octet or a unitary singlet. Only D-type or symmetric coupling is possible in (A.4) or (A.5) because of charge-conjugation invariance. H_s and H_0 are related to s_d' and s_0' used in the text by the following relations:

$$H_s=-(5^{1/2}/\sqrt{3})s_d',\quad H_0=(4/\sqrt{3})s_0', \qquad \text{(A.6)}$$

where s_d' and s_0' are defined by

$$\langle\alpha(p)|S_i|\beta(k)\rangle=(4p_0k_0)^{-1/2}d_{i\alpha\beta}s_d',\quad \langle 0(p)|S_i|\beta(k)\rangle=(4p_0k_0)^{-1/2}d_{0i\beta}s_0'. \qquad \text{(A.7)}$$

[21] In this approach, G_A/G_V is calculable.

[22] J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963).

Putting in a nonet of spin-zero mesons as intermediate states, Eq. (A.2) becomes

$$\sum_\gamma [\langle\alpha|S^\mu|\gamma_8\rangle\langle\gamma_8|S^\lambda|\beta\rangle+\langle\alpha|S^\lambda|\gamma_0\rangle\langle\gamma_0|S^\mu|\beta\rangle-\{\lambda\leftrightarrow\mu\}]$$
$$=-3\begin{pmatrix}8 & 8 & 8_a\\ \lambda & \mu & \nu\end{pmatrix}\begin{pmatrix}8 & 8 & 8_a\\ \beta & \nu & \alpha\end{pmatrix}. \quad \text{(A.8)}$$

Using now the method of Lee,[23] we obtain from Eq. (A.8) in a straightforward way, the following two independent equations:

$$8H_s^2+2H_0^2=96m_0'^2, \quad \text{(A.9)}$$
$$-4H_s^2+(5/4)H_0^2=0.$$

These give the following solution:

$$H_s^2=(20/3)m_0'^2;\quad H_0^2=(16/5)H_s^2; \quad \text{(A.10)}$$

or in terms of s_d' and s_0'

$$s_d'^2=4m_0'^2;\quad s_0'^2/s_d'^2=1; \quad \text{(A.11)}$$

giving the relations (16) of the text. The fact that s_d' and s_0' have dimensions of a mass necessitates the use of quadratic mass relation for the mesons as used in the text.

We now show that the relations (A.11) lead to Schwinger's mass formula for a meson nonet. This can be seen as follows: The Hamiltonian consists of an invariant part H_0 plus S_8:

$$M=M_0+\delta m' S_8. \quad \text{(A.12)}$$

Now if the $I=Y=0$ members of the unitary octet by η_8 and the unitary singlet by η_1, then the physical particles η and X are given by

$$\eta=p\eta_8+q\eta_1,$$
$$X=q\eta_8-p\eta_1, \quad \text{(A.13)}$$

[23] B. W. Lee, Phys. Rev. Letters 14, 676 (1965); Lectures given at Brandeis Summer Institute of Physics, 1965 (to be published).

where the mixing angle θ is given by $\tan\theta=p/q$. Also in the presence of the Hamiltonian (A.12)

$$-i(dF_i/dt)=[M,F_i]$$
$$=\delta m'[S_8,F_i]$$
$$=-i\delta m' f_{i8l}S_l. \quad \text{(A.14)}$$

Taking the matrix elements of (A.14) between the states j and k at rest, we obtain

$$i(m_k-m_j)\langle k|F_i|j\rangle=\delta m' f_{i8l}\langle k|S_l|j\rangle. \quad \text{(A.15)}$$

Since we are calculating the mass difference to order δm, $\langle k|S_l|j\rangle$ is to be evaluated in the U_3 limit in which matrix elements are given by (A.7) and (A.11). On the other hand, m_k and m_j on the left-hand side of Eq. (A.15) are the physical masses of the states k and j. The matrix element $\langle k|F_i|j\rangle$ for states at rest is given by

$$\langle k|F_i|j\rangle=(4p_0k_0)^{-1/2}if_{ijk}(m_i+m_j). \quad \text{(A.16)}$$

Using (A.7), (A.16), and (A.13), we obtain from (A.15) by using appropriate values of k and j the following equations:

$$K^2-\pi^2=-(\sqrt{3}/2)\delta m' s_d',$$
$$X^2-\eta_8^2=-(\sqrt{2}/\sqrt{3})\delta m'(p/q)s_0', \quad \text{(A.17)}$$
$$\eta^2-\eta_8^2=(\sqrt{2}/\sqrt{3})\delta m'(q/p)s_0'.$$

Therefore

$$(X^2-\eta_8^2)(\eta^2-\eta_8^2)=-\tfrac{2}{3}(\delta m')^2 s_0'^2,$$
$$(K^2-\pi^2)^2=\tfrac{3}{4}(\delta m')^2 s_d'^2.$$

Using now the second of relations (A.11), we obtain Schwinger's formula:

$$(X^2-\eta_8^2)(\eta^2-\eta_8^2)=-(8/9)(K^2-\pi^2)^2,$$

or

$$(X^2-\pi^2)(\eta^2-\pi^2)=\tfrac{4}{3}(K^2-\pi^2)(X^2+\eta^2-2K^2). \quad \text{(A.18)}$$

We emphasize that this formula is independent of the mass-splitting parameter $\delta m'$ and is solely a consequence of relation (A.11).