Magnetic Moments of the Strange Baryons and the Algebra of Currents*

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Using equal-time commutation relations of the axial-vector "charges" with the isoscalar and isovector electromagnetic current densities, the magnetic moments of the strange members of the baryon octet and the Σ^0-Λ transition moment have been calculated in terms of the pseudoscalar-meson photoproduction amplitudes, which are evaluated using low-lying baryon resonant states. The results agree very well with the experimental values available for the Σ^+ and the Λ magnetic moments. A comparison is also made with the results of the $SU(3)$ symmetry.

INTRODUCTION

IT has been emphasized by Gell-Mann[1] that the algebra generated by equal-time commutation of the hadronic weak and electromagnetic currents may have much useful dynamical information. A powerful method of exploiting this information has recently been developed by Fubini, Furlan, and collaborators.[2] In this technique algebraic methods appear in conjunction with those of dispersion theory. The early application of these techniques, along with partially conserved axial-vector current (PCAC),[3] leading to the remarkable evaluation of the renormalization of the axial-vector β-decay coupling constant by Adler and Weisberger,[4] has brought this subject to the forefront. Since then many interesting results have been obtained by various authors,[5] thereby strengthening further one's confidence in this approach in particle physics.

The present paper is devoted to an evaluation of the magnetic moments of the strange baryons, Σ^+, Σ^-, Σ^0, Λ, Ξ^0, Ξ^- and the $\Sigma^0-\Lambda$ transition magnetic moment. The magnetic moments of the nucleons have been calculated earlier by Fubini, Furlan, and Rossetti,[6] and the present work is thus a completion of the program of calculating the magnetic moments of the baryon octet. The results are in very good agreement with the experimental results available in the case of the p, n, Λ, and the Σ^+. Further measurements of the magnetic moments will be of great interest.

Fubini, Furlan, and Rossetti[6] have obtained the isoscalar and the isovector nucleon magnetic moments in terms of the pion photoproduction amplitudes. In a related approach several authors[7] have also given a sum rule relating the nucleon isovector magnetic moment and the nucleon charge radius with the total photoproduction cross sections. An evaluation of the Λ and the Σ^+ anomalous magnetic moments in terms of the kaon photoproduction amplitudes has recently been reported by us.[8] An alternative method is to relate the hyperon moments also to the pion photoproduction processes of the type $\gamma+Y\rightarrow\pi+Y$. In the present paper both these approaches will be discussed in some detail and applied to the strange members of the baryon octet. The relevant photoproduction amplitudes will be assumed to satisfy unsubtracted fixed momentum-

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[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).

[2] S. Fubini and G. Furlan, Physics **1**, 229 (1965); G. Furlan, F. Lannoy, C. Rossetti, and G. Segré, Nuovo Cimento **38**, 1747 (1965); S. Fubini, G. Furlan, and C. Rossetti, *ibid.* **40**, 1171 (1965). For further references see S. Fubini, G. Segré, and J. D. Walecka, Stanford Report No. ITP-199, 1966 (to be published).

[3] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960); Y. Nambu, Phys. Rev. Letters **4**, 380 (1960); J. Bernstein, M. Gell-Mann, and L. Michel, Nuovo Cimento **16**, 560 (1960); J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, *ibid.* **17**, 757 (1960).

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[5] (a) For application to strangeness-changing semileptonic decays, see L. K. Pandit and J. Schechter, Phys. Letters **19**, 56 (1965); D. Amati, C. Bouchiat, and J. Nuyts, *ibid.* **19**, 59 (1965); A. Sato and S. Sasaki, Osaka, 1965 (unpublished); C. A. Levinson and I. J. Muzinich, Phys. Rev. Letters **15**, 715 (1965); V. S. Mathur, S. Okubo, and L. K. Pandit, Phys. Rev. Letters **16**, 371 (1966); C. G. Callan and S. B. Treiman, *ibid.* **16**, 153 (1966); M.Suzuki, *ibid.* **16**, 212 (1966).

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(c) For applications to vector meson decays, see V. S. Mathur and L. K. Pandit, Phys. Letters **19**, 523 (1965); **20**, 320 (1966); K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966).

(d) For application to the $\pi\pi$ and $K\pi$ systems, see S. L. Adler, Phys. Rev. **140**, B736 (1965); I. J. Muzinich and S. Nussinov, Phys. Letters **19**, 248 (1965); K. Kawarabayashi, W. D. McGlinn, and W. W. Wada, Phys. Rev. Letters **15**, 897 (1965); V. S. Mathur and L. K. Pandit, Phys. Rev. **143**, 1216 (1966); R. Oehme, **143**, 1138 (1966).

(e) For other applications, see S. L. Adler, Phys. Rev. **143**, 1144 (1966); G. S. Guralnik, V. S. Mathur, and L. K. Pandit, Phys. Letters **20**, 64 (1966); S. Okubo, Nuovo Cimento **41**, 586 (1966); I. S. Gerstein, Phys. Rev. Letters **16**, 114 (1966); S. Bergia and F. Lannoy, 1965 (unpublished); R. Gatto, L. Maiani, and G. Preparata, Phys. Rev. Letters **16**, 377 (1966); R. Oehme, *ibid.* **16** 215 (1966).

[6] S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento (to be published).

[7] N. Cabibbo and L. A. Radicati, Phys. Letters **19**, 697 (1966); and see M. Gell-Mann, in *Proceedings of the Coral Gables Conference on Symmetry Principles at High Energy, 1966* (W. H. Freeman and Company, San Francisco, 1966).

[8] V. S. Mathur and L. K. Pandit, Phys. Letters **20**, 308 (1966).

transfer dispersion relations, which we shall approximate by the low-lying baryonic resonances. We find, in agreement with the conclusion of Fubini, Furlan, and Rossetti,[6] that the major contribution is given by the so-called decuplet resonant states, whenever they can occur. The contributions of the $J=\frac{3}{2}^-$ baryon resonances have been omitted here for this reason. We find, however, that the contribution of the $Y_0^*(1405)$ is important and must be included.

In Sec. I we summarize our definitions for the current densities and charges, and quote the required equal-time commutation relations. Sections II–V are devoted, respectively, to the calculations of the magnetic moments of the Σ, the Λ, the $\Sigma^0-\Lambda$ transition, and the Ξ. In Sec. VI, we summarize our results and discuss their implications.

I. THE ALGEBRA OF CURRENTS AND PCAC

We specify in this section the equal-time commutation relations, based on the quark model,[9] which form the basis of the present work.

Denoting the vector and the axial-vector current densities by $(V_\mu(x))_b{}^a$ and $(P_\mu(x))_b{}^a$, respectively, (here a and b are SU_3 tensor indices), we define the corresponding "charges" by

$$A_b{}^a(t)=i\int_{x_0=t} d^3x(V_4(x))_b{}^a, \tag{1a}$$

$$B_b{}^a(t)=i\int_{x_0=t} d^3x(P_4(x))_b{}^a, \tag{1b}$$

and the isovector and the isoscalar electromagnetic current densities by

$$J_\mu{}^V(x)=\tfrac{1}{2}[(V_\mu(x))_2{}^2-(V_\mu(x))_1{}^1], \tag{2a}$$

$$J_\mu{}^S(x)=\tfrac{1}{2}(V_\mu(x))_3{}^3. \tag{2b}$$

The quark model [in which $(V_\mu)_b{}^a=i\bar{\psi}_a\gamma_\mu\psi_b$, and $(P_\mu)_b{}^a=i\bar{\psi}_a\gamma_\mu\gamma_5\psi_b$] then suggests the following equal-time commutation relations:

$$\begin{aligned}[][B_b{}^a(t),J_\mu{}^V(x)]_{t=x_0} &=\tfrac{1}{2}[\delta_b{}^2(P_\mu(x))_2{}^a-\delta_2{}^a(P_\mu(x))_b{}^2\\ &\quad-\delta_b{}^1(P_\mu(x))_1{}^a+\delta_1{}^a(P_\mu(x))_b{}^1],\end{aligned} \tag{3a}$$

$$[B_b{}^a(t),J_\mu{}^S(x)]_{t=x_0}=\tfrac{1}{2}[\delta_b{}^3(P_\mu(x))_3{}^a-\delta_3{}^a(P_\mu(x))_b{}^3]. \tag{3b}$$

We shall use Eq. (1b) in the form[10]

$$B_b{}^a(t)=\int d^4x\,\theta(x_0-t)\partial_\lambda(P_\lambda(x))_b{}^a, \tag{3c}$$

where we have dropped the surface term $B_b{}^a(t=-\infty)$, which has a nonzero matrix element only between degenerate states. In case of a degeneracy we have to be careful, and will follow the equivalent procedure of calculating such a matrix element as a properly defined limit of the corresponding matrix element between nondegenerate states.[11]

Using PCAC for the axial-vector current densities

$$\partial_\mu(P_\mu(x))_b{}^a=C_\phi\phi_b{}^a(x), \tag{4}$$

where the proportionality constant C_ϕ depends, in general, on the particular pseudoscalar field $\phi_b{}^a(x)$, we may write

$$B_b{}^a(t)=C_\phi\int d^4x\,\theta(x_0-t)\phi_b{}^a(x). \tag{5}$$

The values of the C_ϕ may be obtained, if desired, by Goldberger-Treiman relations[12]; however, we do not need to do this as in the present work the results do not depend on these values. This is very gratifying since the use of the four-divergence of the axial-vector currents as suitable definitions of the pseudoscalar fields is perfectly acceptable,[13] the only approximation in the PCAC Eq. (4) arising in the determination of the proportionality constants C_ϕ.

II. THE MAGNETIC MOMENT OF THE Σ

We take the matrix element of the commutation relations

$$[B_1{}^1(0)-B_2{}^2(0),J_\mu{}^{V,S}]=0, \tag{6}$$

between Σ^+ states. Using Eq. (5) we then obtain

$$\int d^4x\,\theta(x_0)\langle\Sigma^+(p_2)|[\pi^0(x),J_\mu{}^{V,S}(0)]|\Sigma^+(p_1)\rangle=0. \tag{7}$$

Define the functions

$$T_\Sigma{}^{V,S}(q)=\int d^4x\,e^{-iq\cdot x}\theta(x_0)\times\langle\Sigma^+(p_2)|[\pi^0(x),\epsilon_\mu J_\mu{}^{V,S}(0)]|\Sigma^+(p_1)\rangle, \tag{8}$$

where ϵ_μ is a space-like unit vector. The functions $T_\Sigma{}^{V,S}$ are essentially related to the isovector and the isoscalar amplitudes for the photoproduction process

$$\gamma(k,\epsilon)+\Sigma^+(p_1)\rightarrow\pi^0(q)+\Sigma^+(p_2), \tag{9}$$

where ϵ_μ and $k=p_2+q-p_1$ are the polarization and the

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[9] M. Gell-Mann, Phys. Letters 8, 214 (1964); G. Zweig, CERN (unpublished). Note, however, that the commutation relations used in the present work can also be obtained from other field-theoretic models.

[10] S. Fubini, G. Furlan, and C. Rossetti (see Ref. 2).

[11] S. Okubo, Nuovo Cimento 41, 586 (1966); V. A. Alessandrini, M. A. B. Bég, and L. S. Brown, Phys. Rev. 144, 1137 (1966); W. I. Weisberger, Proceedings of the International Conference on Weak Interactions, Argonne, 1965 (to be published); S. Fubini, 1965 (to be published).

[12] M. L. Goldberger and S. B. Treiman, Phys. Rev. 109, 193 (1958); M. Gell-Mann and M. Lévy, see Ref. 3. See also: M. Ida, Phys. Rev. 132, 401 (1963); K. Nishijima, *ibid.* 133, B1092 (1964).

[13] K. Nishijima, discussion at the International Conference on Weak Interactions, Argonne, (1965) (unpublished).

momentum vectors of the photon and q is the momentum of the pion. We, of course, choose ϵ_μ entering in the definition of Eq. (8) to satisfy the condition $\epsilon_\mu k_\mu=0$. From Eqs. (7) and (8) we have

$$\lim_{q\to 0}[T_\Sigma{}^{V,S}]=0. \tag{10}$$

Suppressing isospin indices for the moment, we may write the amplitude T for the general process involving the baryons B_1 and B_2 and the pseudoscalar meson M

$$\gamma(k)+B_1(p_1)\to M(q)+B_2(p_2), \tag{11}$$

in terms of the standard gauge-invariant amplitudes[14] A_i ($i=1,\cdots,4$) by

$$T=\left(\frac{M_{B_1}M_{B_2}}{p_{20}p_{10}}\right)^{1/2}\frac{1}{V}\bar{u}_{B_2}\sum_{i=1}^{4}A_iO_iu_{B_1}. \tag{12}$$

Here we have the following notations:

$$P=\tfrac{1}{2}(p_1+p_2); \tag{13}$$

$$O_1=i\gamma_5(\gamma\cdot\epsilon)(\gamma\cdot k), \tag{14a}$$

$$O_2=2i\gamma_5\{(P\cdot\epsilon)(q\cdot k)-(q\cdot\epsilon)(P\cdot k)\}, \tag{14b}$$

$$O_3=\gamma_5\{(\gamma\cdot\epsilon)(q\cdot k)-(\gamma\cdot k)(q\cdot\epsilon) -i(M_{B_1}-M_{B_2})(\gamma\cdot\epsilon)(\gamma\cdot k)\}, \tag{14c}$$

$$O_4=2\gamma_5\{(\gamma\cdot\epsilon)(P\cdot k)-(\gamma\cdot k)(P\cdot\epsilon) -\tfrac{1}{2}i(M_{B_1}+M_{B_2})(\gamma\cdot\epsilon)(\gamma\cdot k)\}; \tag{14d}$$

M_B stands for the baryon mass and V is the volume of normalization.

The amplitudes A_i will be considered as functions of the invariant variables:

$$\nu=-(P\cdot q)/M_{B_1}, \tag{15a}$$

$$\Delta^2=(p_2-p_1)^2. \tag{15b}$$

It is easily seen from the structure of the Dirac covariants (14) that the only invariant amplitude that contributes in the limit $q\to 0$ is the combination

$$\alpha(\nu,\Delta^2)=A_1(\nu,\Delta^2)+(M_{B_2}-M_{B_1})A_3(\nu,\Delta^2). \tag{16}$$

Thus Eq. (10) leads to the results that

$$\lim_{\nu\to 0}\alpha_\Sigma{}^{V,S}(\nu,\Delta^2)=0. \tag{17}$$

We shall assume that the amplitudes $\alpha(\nu,\Delta^2)$ satisfy unsubtracted dispersion relations at fixed momentum transfer:

$$\alpha(\nu,\Delta^2)=\frac{1}{\pi}\int\frac{\mathrm{Abs}\alpha(\nu',\Delta^2)}{\nu'-\nu-i\epsilon}d\nu'. \tag{18}$$

In anticipation of the limit $q\to 0$ in the sum rules of the type (17), we shall take $q^2=0$ and also $\Delta^2=0$. To calculate the absorptive parts of the $\alpha_\Sigma{}^{V,S}$ we note that

$$\mathrm{Abs}T_\Sigma{}^{V,S}=\frac{(2\pi)^4}{2i}\{\sum_l\delta^4(q+p_2-l)\langle\Sigma^+(p_2)|\pi^0(0)|l\rangle \times\langle l|\epsilon\cdot J^{V,S}(0)|\Sigma^+(p_1)\rangle-\sum_j\delta^4(q-p_1+j) \times\langle\Sigma^+(p_2)|\epsilon\cdot J^{V,S}(0)|j\rangle\langle j|\pi^0(0)|\Sigma^+(p_1)\rangle\}. \tag{19}$$

We shall approximate the sum over the intermediate states $|l\rangle$ and $|j\rangle$ by the single-particle states Σ^+ and $Y_1{}^{*+}(1385)$. For this purpose we define the relevant matrix elements as follows:

$$\langle\Sigma^+(p_2)|\pi^0(0)|\Sigma^+(l)\rangle =\left(\frac{M_\Sigma{}^2}{p_{20}l_0}\right)^{1/2}\frac{1}{V}\frac{iG(\Sigma^+\to\Sigma^+\pi^0)}{(p_2-l)^2+M_\pi{}^2}\bar{u}_\Sigma(p_2)\gamma_5u_\Sigma(l), \tag{20a}$$

$$\langle\Sigma^+(l)|\epsilon\cdot J^{V,S}(0)|\Sigma^+(p_1)\rangle =-\left(\frac{M_\Sigma{}^2}{p_{20}l_0}\right)^{1/2}\frac{1}{V}\bar{u}_\Sigma(l)\{iF_{\Sigma1}{}^{V,S}([l-p_1]^2)\gamma\cdot\epsilon +iF_{\Sigma2}{}^{V,S}([l-p_1]^2)\sigma_{\mu\nu}(l-p_1)_\nu\epsilon_\mu\}u_\Sigma(p_1), \tag{20b}$$

$$\langle\Sigma^+(p_2)|\pi^0(0)|Y_1{}^{*+}(l)\rangle =\left(\frac{M_{Y_1*}M_\Sigma}{p_{20}l_0}\right)^{1/2}\frac{1}{V}\frac{iG(Y_1{}^{*+}\to\Sigma^+\pi^0)}{(p_2-l)^2+M_\pi{}^2} \times(p_2-l)_\mu\bar{u}_\Sigma(p_2)u_\mu(l), \tag{20c}$$

$$\langle Y_1{}^{*+}(l)|\epsilon\cdot J^{V,S}(0)|\Sigma^+(p_1)\rangle =-\left(\frac{M_{Y_1*}M_\Sigma}{l_0p_{10}}\right)^{1/2}\frac{1}{V}if(Y_1{}^{*+}\to\Sigma^+\gamma^{V,S}) \times\bar{u}_\mu(l)\gamma_\nu\gamma_5u_\Sigma(p_1)[(l-p_1)_\mu\epsilon_\nu-(l-p_1)_\nu\epsilon_\mu]. \tag{20d}$$

In the above equations, the G and the f stand for the coupling constants explained by their parentheses. The form factors in Eq. (20b) are normalized by

$$F_{\Sigma1}{}^{V,S}(0)=1, \tag{21a}$$

$$F_{\Sigma2}{}^{V,S}(0)=\mu_\Sigma{}^{V,S}/2M_p, \tag{21b}$$

where $\mu_\Sigma{}^{V,S}$ denote the isovector and isoscalar anomalous magnetic moments of the Σ in Bohr nuclear magnetons ($e\hbar/2M_pc$). We may substitute the Eqs. (20) in the Eq. (19) and extract therefrom the absorptive part of $\alpha_\Sigma{}^{V,S}(\nu)$ by standard use of the Dirac algebra. However, we have to observe some care at this stage, since we encounter for the Σ^+ intermediate state precisely the degeneracy referred to after Eq. (3c). For a proper treatment we take the mass of the final Σ^+ different from that of the initial Σ^+ and take the degeneracy limit at the very end of the calculation of the left-hand side of the sum rule given in Eq. (17).

[14] G. F. Chew, M. L. Goldberger, F. Low, Y. Nambu, Phys. Rev. **106**, 1345 (1957). We have used Hermitian γ_μ matrices and $\sigma_{\mu\nu}=\frac{1}{2}i[\gamma_\mu,\gamma_\nu]$. We work in the metric such that $p\cdot q=\mathbf{p}\cdot\mathbf{q}-p_0q_0$. We also take $e=\hbar=c=1$.

Indeed, we find that it is the term containing A_3 in Eq. (16) that receives a nonvanishing contribution from the Σ^+ pole. Finally the sum rule of Eq. (17) becomes, in the stated approximation

$$F_{\Sigma 2}{}^V(0)=\frac{M_\Sigma}{3}\frac{f(Y_1{}^{*+}\to\Sigma^+\gamma^V)G(Y_1{}^{*+}\to\Sigma^+\pi^0)}{G(\Sigma^+\to\Sigma^+\pi^0)}\times\frac{M_\Sigma}{M_{Y_1^*}}\left(1+\frac{M_\Sigma}{M_{Y_1^*}}\right),\qquad(22a)$$

$$F_{\Sigma 2}{}^S(0)=\frac{M_\Sigma}{3}\frac{f(Y_1{}^{*+}\to\Sigma^+\gamma^S)G(Y_1{}^{*+}\to\Sigma^+\pi^0)}{G(\Sigma^+\to\Sigma^+\pi^0)}\times\frac{M_\Sigma}{M_{Y_1^*}}\left(1+\frac{M_\Sigma}{M_{Y_1^*}}\right).\qquad(22b)$$

The coupling constants appearing in the sum rules (22a) and (22b) are not all known experimentally. In this situation we estimate them by using the $SU(3)$ symmetry. The magnitude of the coupling constant $G(Y_1{}^{*+}\to\Sigma^+\pi^0)$ can be determined from the $Y_1{}^*$ width,[15] and is also found to be well given by the $SU(3)$ symmetry in terms of the coupling constant $G(N_{33}{}^{*+}\to p\pi^0)\equiv\lambda/M_\pi$ fixed by the $N_{33}{}^*$ width ($\lambda=1.81$). This also fixes the relative sign of $G(Y_1{}^{*+}\to\Sigma^+\pi^0)$. Similarly the coupling constants for the radiative decays, $f(Y_1{}^{*+}\to\Sigma^+\gamma^{V,S})$, are expressed in terms of the constant $C/M_\pi\equiv f(N_{33}{}^{*+}\to p\gamma^V)$. The value $C=0.345$ has been obtained from an analysis of the pion photoproduction by Gourdin and Salin.[16] The coupling constant $G(\Sigma^+\to\Sigma^+\pi^0)$ is similarly given by

$$G(\Sigma^+\to\Sigma^+\pi^0)=2fG_{NN\pi},\qquad(23)$$

where f is the usual parameter specifying that the F and the D couplings of octet appear with coefficients f and $(1-f)$, respectively. We have $G_{NN\pi}=13.5$, and shall choose[17] $f=0.25$. We find that within the present approximations the isoscalar and the isovector moments in Eqs. (22a) and (22b) are equal. Hence,[18]

$$\mu_{\Sigma^-}=0,\qquad(24a)$$

$$\mu_{\Sigma^0}=\tfrac{1}{2}\mu_{\Sigma^+},\qquad(24b)$$

$$\mu_{\Sigma^+}=\frac{\lambda c}{6}\frac{M_\Sigma M_p}{M_\pi{}^2}\frac{1}{fG_{NN\pi}}\frac{M_\Sigma}{M_{Y_1^*}}\left(1+\frac{M_\Sigma}{M_{Y_1^*}}\right).\qquad(24c)$$

[15] A recent measurement [see A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. **37**, 633 (1965)] of the $Y_1{}^*$ decay branching ratio into $\Sigma\pi$ and $\Lambda\pi$ is in good agreement with the $SU(3)$ prediction, thus removing earlier discrepancies.

[16] M. Gourdin and P. Salin, Nuovo Cimento **27**, 193 (1963). The value of C being used in the present work has been obtained on the basis of this paper by Fubini *et al.* (cf. Ref. 6).

[17] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963); R. Cutkosky, Ann. Phys. (N. Y.) **23**, 415 (1963); J. J. de Swart and C. K. Iddings, Phys. Rev. **130**, 319 (1963).

[18] R. E. Marshak, S. Okubo, and E. C. G. Sudarshan, Phys. Rev. **106**, 599 (1957).

Using the values for λ, C, f, and $G_{NN\pi}$ quoted above, we then find in nuclear Bohr magnetons the value

$$\mu_{\Sigma^+}=2.7.\qquad(25)$$

Adding the Dirac moment, also in the same units, we find for the total magnetic moments[19]

$$(\mu_{\Sigma^+})_{\text{total}}=3.5,\qquad(26a)$$

$$(\mu_{\Sigma^-})_{\text{total}}=-0.79,\qquad(26b)$$

$$(\mu_{\Sigma^0})_{\text{total}}=1.4.\qquad(26c)$$

Our value for $(\mu_{\Sigma^+})_{\text{total}}$ is quite consistent with the experimental value[20] 4.3 ± 1.5, and is a little larger than the $SU(3)$ value[21] 2.79. The $SU(3)$ values[21] for μ_{Σ^-} and μ_{Σ^0} are 0.12 and 0.96, respectively, and may be compared with our values given by Eqs. (24a) and (24b).

Alternatively we may relate the Σ magnetic moments to the amplitudes for the kaon photoproduction process[8]

$$\gamma^{V,S}+p\to K^0+\Sigma^+.\qquad(27)$$

For this purpose we start by taking the matrix element between Σ^+ and p states of the commutation relations

$$[B_2{}^3(0),J_\mu{}^{V,S}(0)]=\pm\tfrac{1}{2}(P_\mu(0))_2{}^3.\qquad(28)$$

Proceeding exactly as before we obtain for the Σ^+ anomalous magnetic moment the expression

$$\mu_{\Sigma^+}=-\mu_p+\frac{\lambda C}{6}\frac{(M_\Sigma+M_N)M_N}{M_\pi{}^2}\frac{1}{G_{KN\Sigma}}\times\left[\frac{M_\Sigma}{M_{Y_1^*}}\left(1+\frac{M_N}{M_{Y_1^*}}\right)+\frac{M_N}{M_{N^*}}\left(1+\frac{M_\Sigma}{M_{N^*}}\right)\right].\qquad(29)$$

It should be noted that, since K^0, $B_2{}^3$, and $(P_\mu)_2{}^3$ are related by PCAC according to Eqs. (4) and (5), the proportionality constant C_K disappears from both sides of the Eq. (28) on taking the matrix element. The intermediate states taken in Eq. (29) are the single-particle states Σ^+, p, $Y_1{}^{*+}$, $N_{33}{}^{*+}$. The coupling constant $G_{KN\Sigma}$ is given by the $SU(3)$ symmetry as

$$G_{KN\Sigma}=(1-2f)G_{NN\pi}.\qquad(30)$$

Using the numerical values of coupling parameters already quoted, we find

$$(\mu_{\Sigma^+})_{\text{total}}=3.6.\qquad(31)$$

This is in remarkable agreement with the earlier value given in Eq. (26a). In view of the different commu-

[19] The results quoted in Ref. 8 are actually in units of $e\hbar/2M_Yc$ and *not* in nuclear Bohr Magnetons $e\hbar/2M_pc$.

[20] A. D. McInturff and C. E. Roos, Phys. Rev. Letters **13**, 246 (1964).

[21] S. Coleman and S. L. Glashow, Phys. Rev. Letters **6**, 423 (1961). We have used the values $\mu_p=1.79$ and $\mu_n=-1.91$, for computing the $SU(3)$ values of the magnetic moments of the other baryons. See also, K. Kawarabayashi and W. W. Wada, Phys. Rev. **141**, 1323 (1966).

tations relations employed, and the appearance of different coupling parameters entering the two evaluations, we find the agreement as indicative of the inner consistency of the approach and the approximation scheme used.

III. THE MAGNETIC MOMENT OF THE Λ

The analysis of the Λ magnetic moment requires a somewhat different treatment from that in the Σ case discussed in Sec. II. This is because the Λ has isospin zero, so that the decuplet states do not contribute to the relevant sum over intermediate states. Hence, it is clear that the $J=\frac{1}{2}^-$ resonant state $Y_0^*(1405)$ will play an important role in the calculation. Since the coupling parameters entering with Y_0^*, especially the one corresponding to its radiative decay, are neither known experimentally nor relatable to known radiative decays of other baryon resonances by the $SU(3)$ symmetry, we shall set up two independent sum rules both involving the same Y_0^* parameters, from which these unknown quantities may be eliminated.

We first use the commutation relation

$$[B_3{}^3(0),J_\mu{}^S(0)]=0\,, \tag{32}$$

and take its matrix element between Λ states. Using PCAC to introduce the η-meson field by Eqs. (4) and (5), we then obtain

$$\int d^4x\,\theta(x_0)\langle\Lambda(p_2)|[\eta(x),J_\mu{}^S(0)]|\Lambda(p_1)\rangle=0\,. \tag{33}$$

The left-hand side of Eq. (33) is now related to the photoproduction process

$$\gamma+\Lambda\to\eta+\Lambda\,, \tag{34}$$

in the limit of zero four-momentum of the η. Now proceeding exactly according to the steps outlined in Sec. II, and taking the contributions of the Λ and the $Y_0^*(1405)$ to obtain the absorptive part of the photoproduction amplitude, we obtain the sum rule

$$\mu_\Lambda=-\frac{4M_\Lambda M_p}{M_{Y_0^*}-M_\Lambda}\,\frac{G(Y_0^*\to\Lambda\eta)f(Y_0^*\to\Lambda\gamma)}{G(\Lambda\to\Lambda\eta)}\,. \tag{35}$$

The Y_0^* parameters appearing in Eq. (35) are defined by the following matrix elements:

$$\langle\Lambda(p_2)|\eta(0)|Y_0^*(l)\rangle =\left(\frac{M_\Lambda M_{Y_0^*}}{p_{20}l_0}\right)^{1/2}\frac{1}{V}\,\frac{iG(Y_0^*\to\Lambda\eta)}{(p_2-l)^2+M_\eta{}^2}\bar{u}_\Lambda(p_2)u_{Y_0^*}(l)\,, \tag{36a}$$

$$\langle Y_0^*(l)|\epsilon\cdot J^S(0)|\Lambda(p_1)\rangle =-\left(\frac{M_\Lambda M_{Y_0^*}}{l_0p_{10}}\right)^{1/2}\frac{1}{V}if(Y_0^*\to\Lambda\gamma) \times\bar{u}_{Y_0^*}(l)\gamma_5\sigma_{\mu\nu}(l-p_1)_\nu\epsilon_\mu u_\Lambda(p_1)\,. \tag{36b}$$

We shall now set up a second sum rule involving the same Y_0^* parameters that enter Eq. (35). For this purpose we take the matrix element, between a Σ^+ and a Λ state, of the equal-time commutator

$$[B_1{}^2(0),J_\mu{}^S(0)]=0\,. \tag{37}$$

Using once again Eqs. (4) and (5) to introduce the π^+-meson field by PCAC, we obtain

$$\int d^4x\,\theta(x_0)\langle\Lambda(p_2)|[\pi^+(x),J_\mu{}^S(0)]|\Sigma^+(p_1)\rangle=0\,. \tag{38}$$

The expression in Eq. (38) is now related to the process

$$\gamma^S+\Sigma^+\to\pi^++\Lambda\,. \tag{39}$$

The intermediate states' contribution to the absorptive part of the amplitude for this process is now approximated by the states Σ^+, $Y_1{}^{*+}$, Λ, and also Y_0^*. Finally, the sum rule obtained is

$$\mu_\Lambda+\mu_\Sigma{}^S=\frac{2M_p(M_\Lambda+M_\Sigma)}{G(\Sigma^+\to\Lambda\pi^+)}\Bigg\{\tfrac{1}{6}f(Y_1{}^{*+}\to\Sigma^+\gamma^S) \times G(Y_1{}^{*+}\to\Lambda\pi^+)\frac{M_\Sigma}{M_{Y_1^*}}\left(1+\frac{M_\Lambda}{M_{Y_1^*}}\right)+f(Y_0^*\to\Lambda\gamma) \times G(Y_0^*\to\Sigma^+\pi^-)\frac{1}{M_{Y_0^*}-M_\Sigma}\Bigg\}\,. \tag{40}$$

Taking Y_0^* as an $SU(3)$ singlet, we can relate the Y_0^* coupling parameters entering Eqs. (35) and (40). Now eliminating these unknown parameters from the two equations we obtain finally

$$\mu_\Lambda\left[1+\frac{M_\Lambda+M_\Sigma}{M_\Lambda}\,\frac{M_{Y_0^*}-M_\Lambda}{M_{Y_0^*}-M_\Sigma}\right] =-\mu_\Sigma{}^S-\frac{\lambda C}{8}\,\frac{(M_\Lambda+M_\Sigma)M_p}{M_\pi{}^2}\,\frac{1}{(1-f)G_{NN\pi}} \times\frac{M_\Sigma}{M_{Y_1^*}}\left(1+\frac{M_\Lambda}{M_{Y_1^*}}\right). \tag{41}$$

From Eqs. (24), $\mu_\Sigma{}^S=\frac{1}{2}\mu_{\Sigma^+}=1.4$, so that we obtain in nuclear Bohr magnetons

$$\mu_\Lambda=-0.74\,. \tag{42}$$

This is in excellent agreement with the recent experimental value[22] -0.77 ± 0.27, and is smaller than the $SU(3)$ value[21] of -0.96.

It is interesting to note that we may also estimate the Y_0^* radiative decay coupling constant $f(Y_0^*\to\Lambda\gamma)$ from Eqs. (35) and (40) using the experimentally

[22] D. A. Hill, K. K. Li, E. W. Jenkins, T. F. Kycia, and H. Ruderman, Phys. Rev. Letters 15, 85 (1965).

measured width of Y_0^*. This parameter is useful in many problems, and will be discussed elsewhere.

An alternative evaluation[8] of the Λ magnetic moment can be made following the above method in terms of the experimentally studied kaon photoproduction process

$$\gamma^S+p\to K^++\Lambda\,. \tag{43}$$

For this purpose we use the parametrization given by Hatsukade, Pandit, and Zimerman.[23] We start here with the matrix element between a Λ and a p state of the commutator:

$$[B_1^3(0),J_\mu^S(0)]=-\tfrac{1}{2}(P_\mu(0))_1^3, \tag{44}$$

and obtain, as shown in Ref. (8), the result

$$\mu_\Lambda=-0.75\,, \tag{45}$$

in nuclear Bohr magnetons. Once again the remarkable agreement of the two independently obtained results of Eqs. (45) and (42) emphasizes the internal consistency of the techniques involved.

IV. THE Σ^0-Λ TRANSITION MOMENT

Taking the matrix element between Λ states of the equal time commutator in Eq. (6) for the isovector electromagnetic current density, and using Eqs. (4) and (5) we get

$$\int d^4x\,\theta(x_0)\langle\Lambda(p_2)|[\pi^0(x),J_\mu^V(0)]|\Lambda(p_1)\rangle=0. \tag{46}$$

The left-hand side of the Eq. (46) is obtained in the limit $q\to 0$ from the function

$$T_\Lambda=\int d^4x\,e^{-iq\cdot x}\theta(x_0)\langle\Lambda(p_2)|[\pi^0(x),\epsilon\cdot J^V(0)]|\Lambda(p_1)\rangle\,, \tag{47}$$

which is related to the pion photoproduction process

$$\gamma+\Lambda\to\pi^0+\Lambda\,. \tag{48}$$

The absorptive part of the amplitude T_Λ is now calculated in our approximation, whereby the intermediate states occurring are the Σ^0 and the Y_1^{*0}. The $\Sigma^0-\Lambda$ transition moment enters through the matrix element:

$$\begin{aligned}\langle\Sigma^0(l)|\epsilon\cdot J^V(0)|\Lambda(p_1)\rangle=&-(M_\Sigma M_\Lambda/l_0p_{10})^{1/2}(1/V)\bar{u}_\Sigma(l)\\&\times[iF_2^{\Sigma\Lambda}([l-p_1]^2)\sigma_{\mu\nu}(l-p_1)_\nu\epsilon_\mu]u_\Lambda(p_1)\,,\end{aligned} \tag{49}$$

where

$$F_2^{\Sigma\Lambda}(0)=\mu_T/2M_p\,, \tag{50}$$

μ_T being the $\Sigma^0-\Lambda$ transition moment in nuclear Bohr magnetons. The problem of degeneracy of states referred to after Eq. (3c) is again met with here and must be treated in the manner already discussed in the case of the Σ^+ magnetic moment calculation in Sec. II. Conservation of the isovector current also enforces[24] on us a degeneracy of the Σ^0, entering as an intermediate state, and the Λ; hence some extra care is required this time in the limiting procedure. Our sum rule finally reduces to

$$\mu_T=\frac{\sqrt{3}\lambda C}{4}\frac{M_pM_\Lambda}{M_\pi^2}\frac{1}{(1-f)G_{NN\pi}}\frac{M_\Lambda}{M_{Y_1*}}\left(1+\frac{M_\Lambda}{M_{Y_1*}}\right). \tag{51}$$

Using as before $\lambda=1.81$, $C=0.345$, $f=0.25$, and $G_{NN\pi}=13.5$, we obtain

$$\mu_T=2.0\,. \tag{52}$$

Again, an alternative way of evaluating μ_T is at hand, whereby we take the matrix element between Λ and p states of the commutator:

$$[B_1^3(0),J_\mu^V(0)]=-\tfrac{1}{2}(P_\mu(0))_1^3, \tag{53}$$

and relate μ_T to the process

$$\gamma^V+p\to K^++\Lambda\,. \tag{54}$$

The result obtained on using the intermediate states p, Σ^0, and Y_1^{*0} is

$$\mu_T=1.5\,. \tag{55}$$

The difference between the two results given in Eqs. (52) and (55) may be due to the probable importance of an isospin $\frac{1}{2}$ πN-resonance state contribution to the latter result. This may be felt in the absence of N_{33}^* contribution to Eq. (55). The agreement is, however, still quite fair. No experimental value for μ_T is available; for purpose of comparison we may quote the $SU(3)$ value[21] of 1.65.

V. THE MAGNETIC MOMENTS OF THE Ξ

If we take the matrix elements of the commutation relations in Eq. (6) between Ξ^0 states, we shall obtain sum rules for the isovector and isoscalar anomalous magnetic moments of the Ξ, $\mu_\Xi{}^V$, and $\mu_\Xi{}^S$, in terms of the amplitudes for the processes:

$$\gamma^{V,S}+\Xi^0\to\pi^0+\Xi^0\,, \tag{56}$$

in the limit of zero four-momentum of the π^0. The analysis is completely similar to that given in detail for the Σ case discussed in Sec. II. The intermediate octet and decuplet states Ξ and $\Xi^*(1530)$ are used for the evaluation of the absorptive parts of the amplitudes for the processes (56). We obtain the sum rules

$$\mu_\Xi{}^{V,S}=\frac{2M_pM_\Xi}{3}\frac{f(\Xi^{*0}\to\Xi^0\gamma^{V,S})G(\Xi^{*0}\to\Xi^0\pi^0)}{G(\Xi^0\to\Xi^0\pi^0)}\times\frac{M_\Xi}{M_{\Xi*}}\left(1+\frac{M_\Xi}{M_{\Xi*}}\right). \tag{57}$$

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[23] S. Hatsukade, L. K. Pandit, and A. H. Zimerman, Nuovo Cimento 34, 819 (1964).

[24] According to the conserved-vector-current hypothesis, in order that the charge $Q(t)=i\int J_4^V(\mathbf{x},t)d^3x$ be time-independent, it is necessary that M_{Σ^0} and M_Λ be taken equal.

Using, as before, the $SU(3)$ symmetry to estimate the coupling constants entering Eqs. (57), we obtain

$$\mu_{\Xi}{}^{V}=\mu_{\Xi}{}^{S}=\frac{\lambda C}{6}\frac{M_{\Xi}M_{p}}{M_{\pi}{}^{2}}\frac{1}{(2f-1)G_{NN\pi}}\times\frac{M_{\Xi}}{M_{\Xi^*}}\left(1+\frac{M_{\Xi}}{M_{\Xi^*}}\right); \tag{58}$$

so that we have

$$\mu_{\Xi^-}=0\,, \tag{59a}$$

$$\mu_{\Xi^0}=-3.1\,, \tag{59b}$$

for the anomalous magnetic moments of the Ξ in nuclear Bohr magnetons. The results of Eqs. (59a) and (24a) are also consistent with the $SU(3)$ prediction[21] $\mu_{\Sigma^-}=\mu_{\Xi^-}$. The total Ξ^- magnetic moment (including the Dirac moment) in nuclear Bohr magnetons is then

$$(\mu_{\Xi^-})_{\rm total}=-0.71\,. \tag{60}$$

Since no experimental information is available yet, we may compare our results (59a) and (59b) with the $SU(3)$ predictions[21] $\mu_{\Xi^-}=0.12$ and $\mu_{\Xi^0}=-1.91$.

VI. DISCUSSION

We summarize in Table I our results for the total magnetic moments of the strange members of the baryon octet together with the available experimental values and the values obtained from the $SU(3)$ symmetry.

At this point a few remarks are in order. The $SU(3)$ results are reasonably close to our results, excepting perhaps the cases of the Σ^0 and the Ξ^0. This is further evidence in support of $SU(3)$ as an approximate symmetry. We may also compare, from this point of view, the results implied by the model of Maki and Hara,[25] where, in contrast with the case of the quark model[9] the tensor, of which the electromagnetic current density is a component, is no longer traceless. The anomalous moments in the Maki-Hara model are now given in terms of the three independent parameters which may be taken to be μ_p, μ_n, and μ_Λ. Then using the experimental values $\mu_p=1.79$, $\mu_n=-1.91$, $\mu_\Lambda=-0.77$, we find $\mu_{\Sigma^-}=\mu_{\Xi^-}=1.23$ in contrast with the SU_3 values (or the quark-model values) $\mu_{\Sigma^-}=\mu_{\Xi^-}=0.12$, and the values obtained in the present work, $\mu_{\Sigma^-}=\mu_{\Xi^-}=0$. This comparison is interesting in view of the recent work of Okubo[26] devoted to testing various models for elementary particles.

The rather good agreement of our results with the experimental values quoted for the Λ and the Σ^+ magnetic moments, and the internal consistency of our calculations, indicate that the approximations of replacing the sum over the intermediate states by the decuplet resonances, besides the octet baryon states, is quite reasonable. This is in agreement with the conclusions of Fubini, Furlan, and Rossetti,[6] who obtained excellent values for the nucleon magnetic moments in the same approximation. However, in the case of the Λ, we find that the contribution of the $Y_0{}^*(1405)$ is quite large and must be taken into account. We should also like to emphasize that the use of the $SU(3)$ symmetry in our work was made only for estimating the presently unknown coupling parameters, and is, in principle, not necessary. In any case, the correct values of the coupling constants are not expected to be much different from the $SU(3)$ symmetric values, in terms of which the decay widths of resonant states are rather well described.[27]

Finally, we wish to point out that our calculations of all the magnetic moments depend directly on statements like Eq. (10) about the behavior of pseudoscalar-meson photoproduction amplitudes in the limit of vanishing four-momentum of the meson. These relations, which are obtained here from the algebra of currents,[28] are, in fact, generalizations of the well-known Kroll-Ruderman theorem[29] obtained originally as an exact result in the renormalized field theory. It is interesting to remark that such low-energy theorems, which have been known for so long, were not exploited before for calculations of the present kind. However, Nambu and Lurié[30] and Nambu and Schrauner[31] have, some time ago, obtained interesting relations for "soft" pion processes with a view to testing PCAC. It appears that the generalized Takahashi-Ward identity[32] for partially conserved axial-vector currents[33] can be exploited to derive relations between general processes differing only by a "soft" pseudoscalar meson, in much

TABLE I. The total magnetic moments of the strange baryons in nuclear Bohr magnetons ($e\hbar/2M_pc$).

Particle symbol	Results of the present calculation[a]	The $SU(3)$ values[b]	Experimental values
Λ	-0.75	-0.96	-0.77 ± 0.27[c]
Σ^+	3.6	2.79	$4.3\ \pm1.5$[d]
Σ^0	1.4	0.96	
Σ^-	-0.79	-0.88	
$(\Sigma^0-\Lambda)_T$	2.0(1.5)	1.65	
Ξ^0	-3.1	-1.91	
Ξ^-	-0.71	-0.88	

[a] Here we have quoted the mean of the values obtained by us up to the second significant figure. The exception is the value of μ_T, for which see the discussion following Eq. (55).
[b] Reference 21. [c] Reference 22. [d] Reference 20.

[25] Z. Maki, Progr. Theoret. Phys. (Kyoto) 31, 331 (1964); Y. Hara, Phys. Rev. 134, B701 (1964).
[26] S. Okubo, Phys. Letters 20, 195 (1966); and Ref. 5(e).
[27] See also K. C. Wali, and R. L. Warnock, Phys. Rev. 135, B1358 (1964).
[28] The Kroll-Ruderman theorem for the pion photoproduction was obtained from the algebra of currents first by S. Okubo, see Ref. 5(e).
[29] N. M. Kroll and M. A. Ruderman; Phys. Rev. 93, 233 (1954); A. Klein, Phys. Rev. 99, 998 (1955).
[30] Y. Nambu and D. Lurié, Phys. Rev. 125, 1429 (1962).
[31] Y. Nambu and E. Schrauner, Phys. Rev. 128, 862 (1962).
[32] J. C. Ward, Phys. Rev. 78, 182 (1950); Y. Takahashi, Nuovo Cimento 6, 371 (1957).
[33] J. Bernstein, M. Gell-Mann, and L. Michel, see Ref. 3.

the same manner as the ordinary Ward identity for the conserved electromagnetic current yields the Kroll-Ruderman theorem. We can then obtain from field theory alone many interesting relations which have recently been obtained from the algebra of currents. Such an investigation will be reported elsewhere.

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$SU(6)$ Algebra of Gell-Mann: Mass Splittings and Nonleptonic Decays of Hadrons*

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We apply the $SU(6)$ algebra of Gell-Mann to nonleptonic decays of hadrons. We assume that the weak-interaction Hamiltonian for s-wave decays transforms like S_7^5 (the seventh component of the space integral of the pseudoscalar quark density). Using the Fubini-Furlan-Adler-Weisberger technique and the partially conserved axial-vector current, we express the s-wave decay amplitudes in terms of the reduced matrix elements, which are the same as the ones that appear in the expression for mass splittings if one assumes that the symmetry-breaking part of the mass operator transforms like S_8 (the eighth component of the space integral of the scalar quark density). Making use of the fact that the mass-splitting parameter is universal, and assuming that the *s-wave coupling constant G_s is also universal*, we predict a ratio $(K_1^0\rightarrow 2\pi)/A(\Sigma_-^-)$ which is in excellent agreement with experiment. Knowing any one of the hyperon s-way decay amplitudes we can predict the others. Also we get $A(\Sigma_+^+)=0$. The implications of these results are discussed.

I. INTRODUCTION

IT has been propounded by Gell-Mann[1] that one can extract physical information concerning hadrons out of a set of equal-time commutation relations of various currents which form an algebra. In particular the algebra of chiral $SU(3)\times SU(3)$,[1,2] which consists of eight space integrals of the fourth components of vector current densities and eight space integrals of the fourth component of axial-vector current densities of quarks, has been used with remarkable success to get the ratio G_A/G_V,[3,4] to relate various leptonic decays of hadrons,[5] to get sum rules of nonleptonic decays of hadrons,[6,7] to study the decays of vector mesons into pseudoscalar mesons,[8] and to obtain sum rules for the magnetic moment[9] and axial-vector form factors.[10] Gell-Mann[1] has observed that if one includes the space integrals of pseudoscalar and scalar densities of quarks, the algebra thus generated leads to $SU(6)$. More explicitly the fourth components of the vector and axial-quark current densities are

$$\mathfrak{F}_{i4}=i\bar{q}(\lambda_i/2)\gamma_4 q\,,\quad (i=1,\cdots,8)\,;\tag{1a}$$

$$\mathfrak{F}_{i4}^5=i\bar{q}(\lambda_i/2)\gamma_4\gamma_5 q\,,\quad (i=0,1,\cdots,8)\,;\tag{1b}$$

and the scalar and pseudoscalar quark densities are

$$\mathcal{S}_i=\bar{q}(\lambda_i/2)q\,,\quad (i=0,1,\cdots,8)\,;\tag{1c}$$

$$\mathcal{S}_i^5=i\bar{q}(\lambda_i/2)\gamma_5 q\,,\quad (i=0,1,\cdots,8)\,.\tag{1d}$$

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