$\pi^- + p$ Interactions at 646 MeV*

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(Received 28 February 1966)

A 14-in. liquid-hydrogen-filled bubble chamber in a 17.5-kG magnetic field was exposed to a beam of negative pions produced by the Cosmotron at Brookhaven National Laboratory. About 26 000 pictures were taken and examined for the following final states: (1) elastic scattering ($\pi^- p$); (2) π^+ production ($\pi^-\pi^+ n$); (3) π^0 production ($\pi^-\pi^0 p$); (4) neutrals. Values for the cross sections for these processes are σ(elastic) $=17.56\pm0.43$ mb, $\sigma(\pi^+)=7.14\pm0.23$ mb, $\sigma(\pi^0)=4.65\pm0.17$ mb. The elastic-scattering angular dependence in the c.m. system is fitted by a power-series expansion in $\cos\theta$ and gives the following coefficients: $a_0=0.27\pm0.02$, $a_1=1.48\pm0.11$, $a_2=3.86\pm0.22$, $a_3=-0.29\pm0.53$, $a_4=-0.65\pm0.28$, $a_5=1.69\pm0.52$ (units: mb/sr). Cross sections for multiple-pion production were also measured: $\sigma(\pi^-\pi^+\pi^0 n)=0.33\pm0.04$ mb, $\sigma(\pi^-\pi^+\pi^- p)=0.08\pm0.02$ mb. The total neutral cross section was σ(neutrals) $=11.78\pm0.43$ mb; the total charged events cross section was σ(charged) $=29.76\pm0.69$ mb; and the total cross section was σ(total) $=41.54\pm0.82$ mb. For single-pion production events, two-body mass distributions and angular distributions were compared with the predictions of the Olsson-Yodh isobar model.

I. INTRODUCTION

IN this paper, we report on a measurement of the interactions of negative pions (646-MeV kinetic energy in the laboratory) with protons. This experiment together with two others, one at 557 MeV[1] and the other at 604 MeV,[2] form a triplet, designed to make accurate measurements of the elastic and inelastic cross sestions in the neighborhood of the second peak in the $\pi^- p$ total cross section. They were all performed using the Adair 14-in. liquid-hydrogen bubble chamber at the Brookhaven Cosmotron.

The reactions studied in this experiment are:

$\pi^- + p \to \pi^- + p$	elastic scattering	(1)
$\to \pi^- + \pi^0 + p$	single π^0 production	(2)
$\to \pi^0 + \pi^+ + n$	single π^+ production	(3)
$\to$ neutrals	charge exchange and π^0 production	(4)
$\to$ nucleon + pions	multiple pion production	(5)

We have measured the total $\pi^- p$ cross section as well as the partial cross sections for the reactions (1) through (5). In addition, for the case of elastic scattering, the differential cross section has also been measured, and for reactions (2) and (3), measurements of the angular distribution of the final-state particles and the two-body effective-mass distributions are compared with predictions of the Olsson-Yodh isobar model.[3]

II. EXPERIMENTAL PROCEDURE

The π^- beam was designed with the help of a ray-tracing program[4] and checked with the wire orbit technique. The wire measurement gave a momentum of 775 ± 23 MeV/c which was found to agree well with measurements of incident tracks made on the photographs. Figure 1 shows the beam layout[5] and the chamber orientation.

The fraction of the beam which was composed of muons and electrons was measured, over a range of momenta, using a CO_2-filled gas Čerenkov counter. At the energy of this experiment, the contamination was $(14.5\pm0.5)\%$ of the beam.[6]

Approximately 26 000 pictures were taken with an average of 13 tracks per frame. A large scanning volume, Fig. 2, was used so that good statistics would be obtained. The basic topology of an event consisted of an incoming track and two outgoing tracks (the so-called 2-p configuration). All events which had an incident track lying within $\pm5°$ of the average beam direction were accepted and only frames with less than 40 tracks were scanned in order to insure good scanning efficiency. A rescan of half the film showed that the average efficiency for a single scan for events was 98%.

* Supported in part by the U. S. Atomic Energy Commission.

† A thesis, based upon the work reported here has been submitted to the Department of Physics, Carnegie Institute of Technology, in partial fulfillment of the requirements for the degree of Doctor of Philosophy (NYO Report No. CAR 882-8, Carnegie Institute of Technology, 1965).

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[4] J. Sanweiss and J. R. Sanford, Brookhaven National Laboratory Report, 1959 (unpublished).

[5] For beam details see C. N. Vittitoe, thesis, University of Kentucky, 1963 (unpublished); and Ref. 2.

[6] Details of the measurement may be found in J. Oliver, thesis, Carnegie Institute of Technology, 1965 (U. S. Atomic Energy Report No. NYO CAR 882-8) (unpublished).

It was more difficult to identify stops than events, and therefore, a different scanning volume, approximately 20% smaller, was used (see Fig. 2) and only frames containing less than 25 tracks were accepted. With these restrictions, it was found that the average efficiency for a single stop scan was 96.8%.

All measured events were put through the DATPRO[7] reconstruction program which translated the measuring machine coordinates into appropriate three-dimensional quantities. Rejections from DATPRO were remeasured once and resubmitted to the computer. This procedure resulted in the reconstruction of all but about 7.5% of the measured events, and further remeasurement had no significant effect on this latter group. This was due to a systematic difficulty in DATPRO for handling tracks with a large ($\gtrsim 65°$) dip. Rather than change the reconstruction program, it was decided to correct for this difficulty by using the fact that the outgoing tracks should be independent of azimuthal rotation around the incident track. If the incident track is chosen as the x axis of an orthogonal coordinate system and the angle α is defined by $\tan\alpha = P_y/P_z$ for a given outgoing track, then the numbers of events in a range $\Delta\alpha$ should not depend upon α. The depletion of events appeared as an asymmetry in the plots of number versus α and this information was used to make the correction.

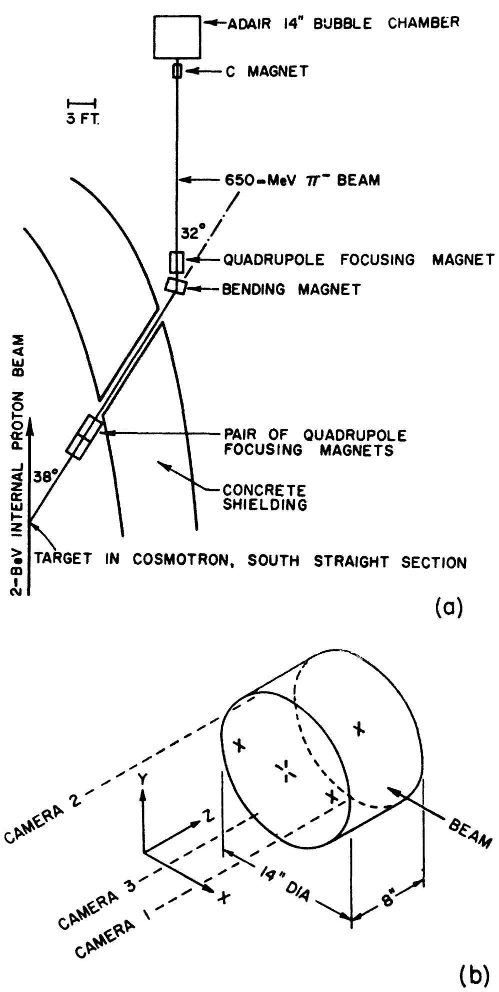

FIG. 1. (a) Plan view of apparatus. (b) Schematic sketch of chamber.

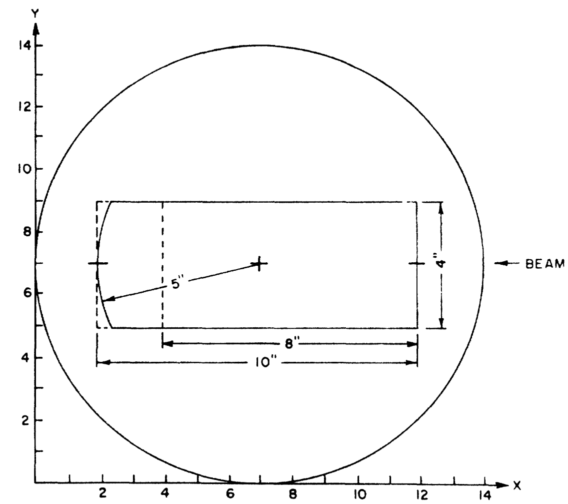

FIG. 2. Top view of chamber showing the fiducial volume. The dashed line at the extreme left is the limit for the scanning region. The solid line is the final fiducial volume for events. The dashed line at 4″ is the left-hand limit of the fiducial volume used for stops.

III. EXPERIMENTAL ANALYSIS

Events which passed the reconstruction program were put through the GUTS[8] kinematic fitting program where each event was tested for agreement with the hypotheses of elastic, single π^+ production, and single π^0 production. The criteria for deciding if a fit was successful were deliberately made broad so that if an event failed a given hypothesis, it was extremely unlikely that a mistake had been made. On the other hand, it was possible for an event to have a successful fit to all three hypotheses. Classification of events as to type will be described below. Successful fits were written out on magnetic tape for use with other programs. The record on this tape, for each event, contained the measured and fitted values of the input variables, a value of χ^2 for the fit, correlation and coplanarity test functions and stretch functions. The coplanarity function measured the degree to which the three tracks were not coplaner and the correlation function measured the deviations of an event from the known elastic kinematic curve. For elastic events, when measurement errors were taken into account, these functions would be normally distributed with a mean of 0 and a variance of 1.

[7] DATPRO was written by R. K. Adair, L. Leipuner, and B. Musgrave and modified for the experiment.

[8] J. P. Berge, F. T. Solmitz, and H. D. Taft, Rev. Sci. Instr. 32, 538 (1961).

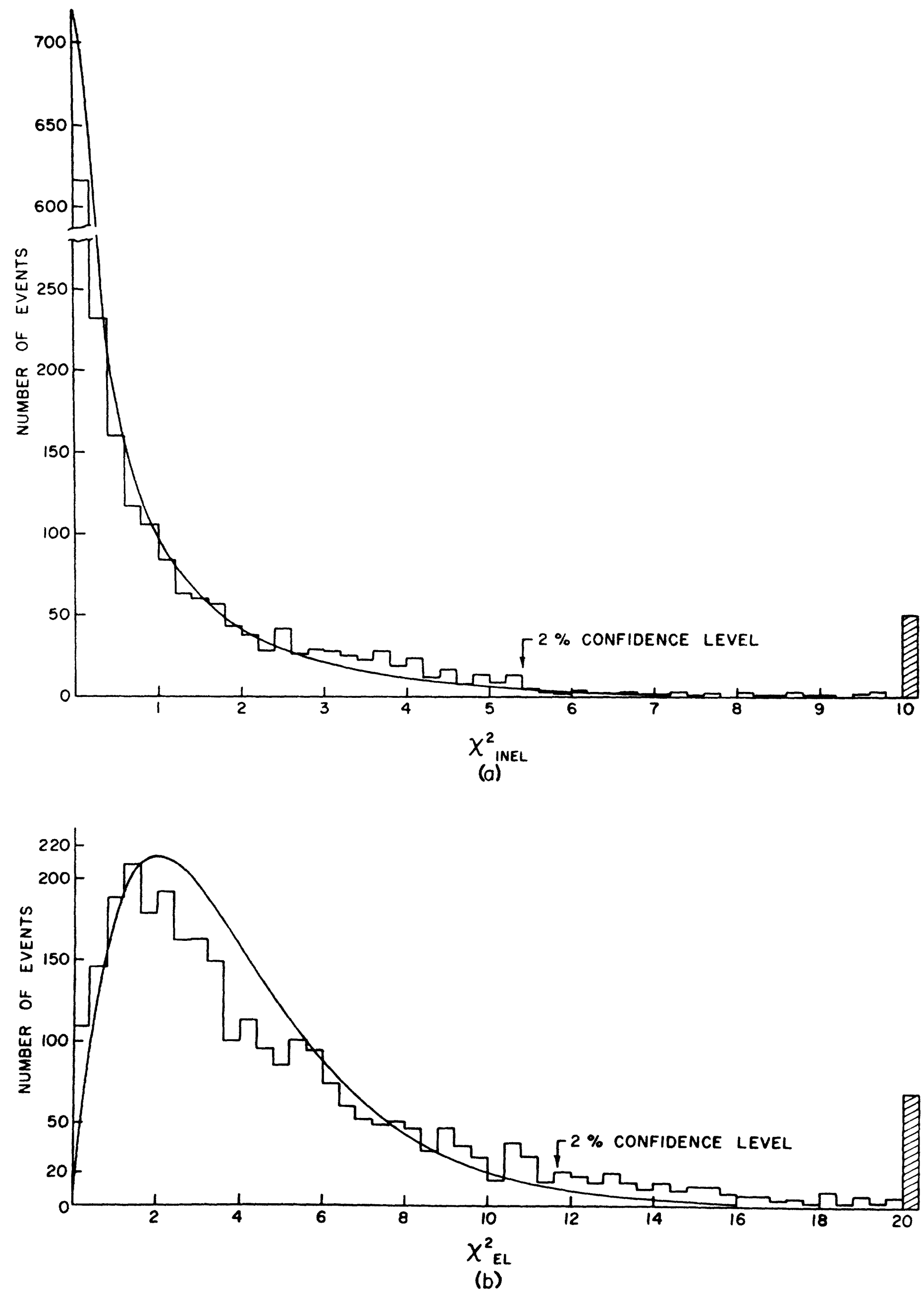

FIG. 3. (a) Combined χ^2 for π^+ and π^0 production. The cross-hatched events lie outside the limit of the plot. (b) χ^2 for elastic events. The solid lines are theoretical curves of the chi-square distribution function.

The stretch functions were defined as

$$S_i = (X_i^f - X_i^m)/\sigma_i,$$

where X_i^f is the fitted value, X_i^m the measured value, and σ_i is the error in the measured quantity. They were used to test the error assignments since properly assigned errors should give stretch functions with a normal distribution, having a mean of 0 and a variance of 1. The stretch function for the curvature of the incident track, $K=1/[P\cos(\text{dip})]$ was also used to check the measured value of the beam momentum. The average value computed by DATPRO was 812 MeV/c, in strong disagreement with the wire orbit measurement of 775 MeV/c. When 812 MeV/c was used in GUTS, the program shifted the momentum to 786 MeV/c and the stretch function had a mean of 0.5. It was found that the only incident momentum which would give a stretch function with a mean of 0 was 775 MeV/c. If any other value was used, GUTS shifted the momentum toward 775 MeV/c. It was assumed that the discrepancy between the GUTS and DATPRO momentum indicated an error in the value of the magnetic field

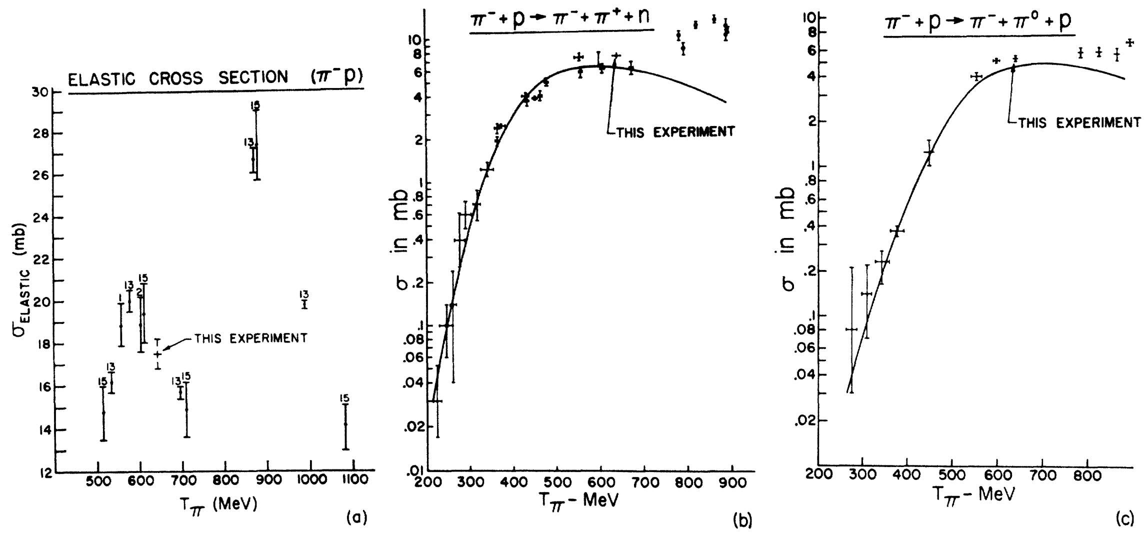

FIG. 4. (a) Elastic cross sections as a function of incident pion energy. The numbers correspond to the footnotes of this article. (b) Excitation function for the reaction $\pi^- + p \rightarrow \pi^- + \pi^+ + n$. The smooth curve is the Olsson-Yodh prediction. (c) Excitation function for the reaction $\pi^- + p \rightarrow \pi^- + \pi^0 + p$. The smooth curve is the Olsson-Yodh prediction.

used in DATPRO, and consequently the field was reduced by 5%. The stretch functions were found to be very close to the ideal, indicating that the assigned measurement errors were appropriate. These error assignments were also supported by the shapes of the χ^2 distributions as will be discussed below.

The output tape from GUTS was used as input to a program called EDITOR, which examined the fits made for each event and decided which, if any, were satisfactory. The test limits used for elastic fits were $\chi^2 < 11.67$, correlation < 2.3 and coplanarity < 2.3. For inelastic fits the limit was $\chi^2 < 5.4$. These limits correspond to 2% confidence levels. An event was considered classified by EDITOR only if it passed the tests for one fit and failed the tests for all other fits. Those events which passed more than one fit or failed all fits were examined by a physicist at a scanning table. Only 100 events were found which were impossible to classify. These events were thrown out and the total track length (see below) appropriately reduced. The χ^2 distributions obtained are shown in Fig. 3.

The classified events were then examined by a computer program called PLOTTER, which checked to determine that the event was inside the fiducial region and that the beam track had an acceptable entrance angle. These final requirements were more stringent than those used in scanning. This eliminated problems for scanner judgment since events near the border of the scanning criteria were not accepted by the computer. PLOTTER also imposed requirements on the length of an outgoing track and on the error in the measured momentum. Events rejected on the basis of these two tests are true events and were included when calculating the cross sections. The output of PLOTTER consisted of plots of the physically interesting variables for all classified events which passed the PLOTTER criteria.

To obtain absolute cross sections, the total amount of hydrogen traversed by incident pions was measured by making a track count of every tenth frame using criteria equivalent to the final acceptance criteria in PLOTTER. The average track length was measured by tracing the incident track for each acceptable event back to the entrance of the fiducial volume and forward to where it left the fiducial volume. The density of the hydrogen was found from the vapor pressure of a Ne vapor-pressure thermometer in contact with the chamber and, also, from the average range of muons from the reaction $\pi^+ \rightarrow \mu^+ + \nu$. These two methods gave results (the average of both methods was 0.0606 ± 0.0006 g/cm^3) which agreed very well with each other. It was assumed that 14.5% of the tracks were muons or electrons.

IV. RESULTS

A. Total Cross Sections

Our cross sections are given in Table I and in Fig. 4. The neutral cross section is independently normalized

TABLE I. Experimental results.

Type of event	No. of events	Absolute cross section (mb)
elastic	3960 ± 81	17.56 ± 0.43
π^0 production	1049 ± 34	4.65 ± 0.17
π^+ production	1609 ± 45	7.14 ± 0.23
$\pi^-\pi^+\pi^0 n$	74 ± 9	0.33 ± 0.04
$\pi^-\pi^+\pi^- p$	17 ± 4	0.08 ± 0.02
All charged events	6711 ± 118	$\sigma_{ch} = 29.76 \pm 0.69$
Neutrals	1336 ± 44	$\sigma_{neut} = 11.78 \pm 0.43$
Total π^- track length for events	$6.118 \pm 0.09 \times 10^6$ cm	
Total π^- track length for stops	$3.030 \pm 0.045 \times 10^6$ cm	
	Total cross section	$\sigma = 41.54 \pm 0.82$

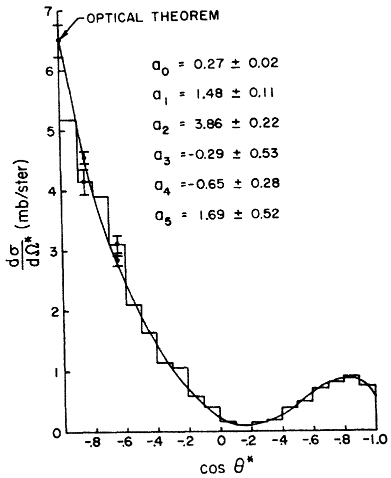

FIG. 5. The elastic differential cross section showing the fit to a fifth-order curve (solid line).

because the scanning criteria for neutrals were different from those for events. The errors assigned to these numbers are due to statistics and also to uncertainty in the various corrective procedures applied during the analysis. Of these two sources, the major contribution to the errors was statistical.

Our results for total and neutral cross sections are in good agreement with previous experiments.[9–12] Our elastic cross sections also agree well with previous experiments[13–15] but our π^+ cross section is somewhat higher than that of Kirz *et al.*[16] and our π^0 cross section is much lower than that of Detoeuf *et al.*[17]

B. Elastic Scattering

The differential cross section was fitted to a polynomial expansion in powers of $\cos\theta^*$ (θ^* is the scattering angle in the center-of-mass system). The optical theorem and dispersion relations were used to evaluate $d\sigma/d\Omega^*$ at zero degrees. All fits were made omitting those events with $\cos\theta^*>0.9$ because this bin was depleted due to the effects of scanning inefficiency for small-angle scattering. When the fitting was done without using the point at 0°, the first satisfactory fit was fourth order and the best fit was fifth order. However, when the forward-scattering point was included, the fourth-order fit was no longer satisfactory and the best fit remained fifth order. It was possible, but not necessary, to make a good fit to a sixth-order curve. We conclude that partial waves up to $F_{5/2}$ must be present. These results are in good agreement with other experiments.[13–15] Our differential cross section, with the forward-scattering point included, is given by

$$d\sigma/d\Omega^*=(0.27\pm0.02)+(1.48\pm0.11)\cos\theta^* +(3.86\pm0.22)\cos^2\theta^*-(0.29\pm0.53)\cos^3\theta^* -(0.65\pm0.28)\cos^4\theta^*+(1.69\pm0.52)\cos^5\theta^*.$$

The differential cross section and fitted curve are shown in Fig. 5.

C. Single Pion Production

Both π^0 and π^+ production strongly involve production of the $N^*(1238)$ isobar. This can be seen by examining the effective two-body mass plots in Fig. 6. Isobar production was first discussed by Sternheimer and Lindenbaum.[18] Their model has been elaborated by a number of authors,[19,20] the most recent being the model of Olsson and Yodh.[3] The latter consider S-wave production and P-wave decay of the isobar, and also include an S-wave π-nucleon rescattering, and production from both the $T=\frac{1}{2}$ and $T=\frac{3}{2}$ states. Their model is compared with the experimental results in Fig. 6. It accounts well for all the distributions except $M(\pi^+\pi^-)$. The enhancement of high-mass values for this plot has been reported previously at other energies.[16] It cannot be considered as a 2π resonance because it does not occur at a fixed-mass value. Several authors[21–23] have pointed out that triangle diagrams can cause this behavior. In any case, it appears that more than simple isobar production must be involved in this reaction. The peak in $M(\pi^+n)$ is probably a reflection of the $N^*(1238)$ in $M(\pi^-n)$ since other experiments indicate that the N^{*-} is produced more copiously than the N^{*+} in π^+ production.

The angular distributions are shown in Fig. 7. The agreement with the Olsson-Yodh model is not perfect. In particular, if we assume that the π^+ production proceeds via $\pi^-+p\rightarrow\pi^++N^{*-}$ followed by $N^{*-}\rightarrow\pi^-+n$, then the distribution of π^+ does not support the assumption of S-wave production. However, the angular distributions are subject to contributions from small amplitudes not included in the model.[24]

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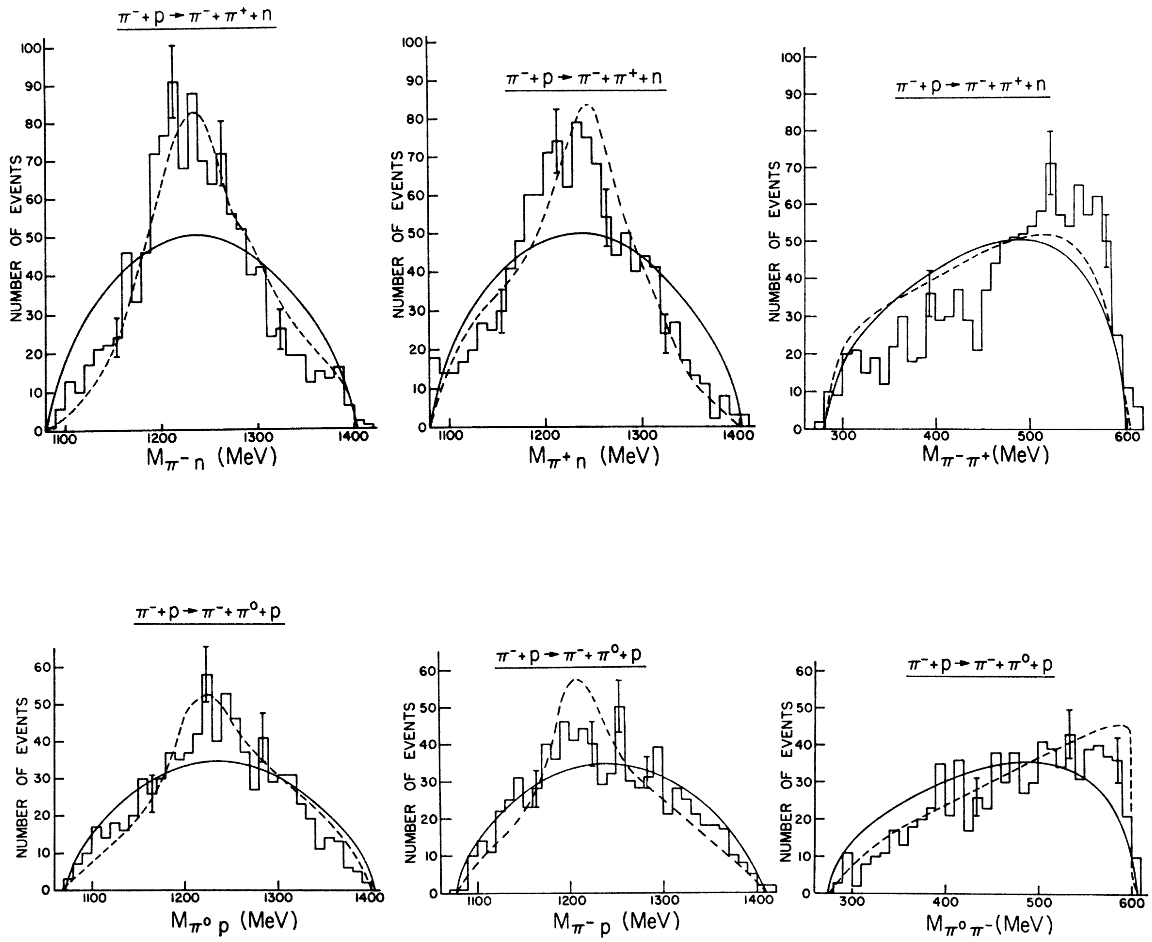

FIG. 6. The two-body invariant mass distributions. The solid lines are phase space and the broken lines are the Olsson-Yodh isobar model.

D. Multiple-Pion Production

The most striking fact about multiple-pion production is the preponderance of the reaction $\pi^- + p \to \pi^- + \pi^0 + \pi^+ + n$. The positive tracks of these events were identified as π^+ on the basis of bubble density but it was impossible to fit them to single-pion production. Moreover, the missing mass tended to be near 1100 MeV as would be expected if a π^0 were present. Our result of 0.33 ± 0.04 mb agrees with that of Kirz.[25] He reports a sharp rise in cross section near the threshold for η production and suggests that this may indicate that the reaction proceeds via η production. This suggestion is supported by a number of spark-chamber experiments which show that the cross section for η production rises rapidly near threshold.[26,27] However, our results for this cross section are significantly lower than would be expected on the basis of η production. The presently accepted branching ratios[28] indicate that the decay modes $\eta \to \pi^-\pi^+\pi^0$ and $\eta \to \pi^-\pi^+\gamma$ together should be about as common as the 2γ mode. On the basis of the results of Bulos *et al.*[26] for $\eta \to 2\gamma$, we would expect the reaction $\pi^- + p \to \eta + n$ followed by $\eta \to \pi^-\pi^+\pi^0(\gamma)$ to have a cross section of about 0.8 mb rather than 0.33 mb. The discrepancy between our results and those of Bulos *et al.*[26] points out the necessity of studying bubble-chamber reactions where all particles other than the π^0 are seen, for example, $\pi^+ + n \to \pi^+ + \pi^- + \pi^0 + p$.

The reaction $\pi^- + p \to \pi^- + \pi^+ + \pi^- + p$ is easy to see but can be confused with π^0 production followed by π^0 decay with a Dalitz pair. A total of 27 four-prong events was found. No detailed analysis was made, but the events were examined and 10 were eliminated as Dalitz pairs because of spiraling tracks. The remaining

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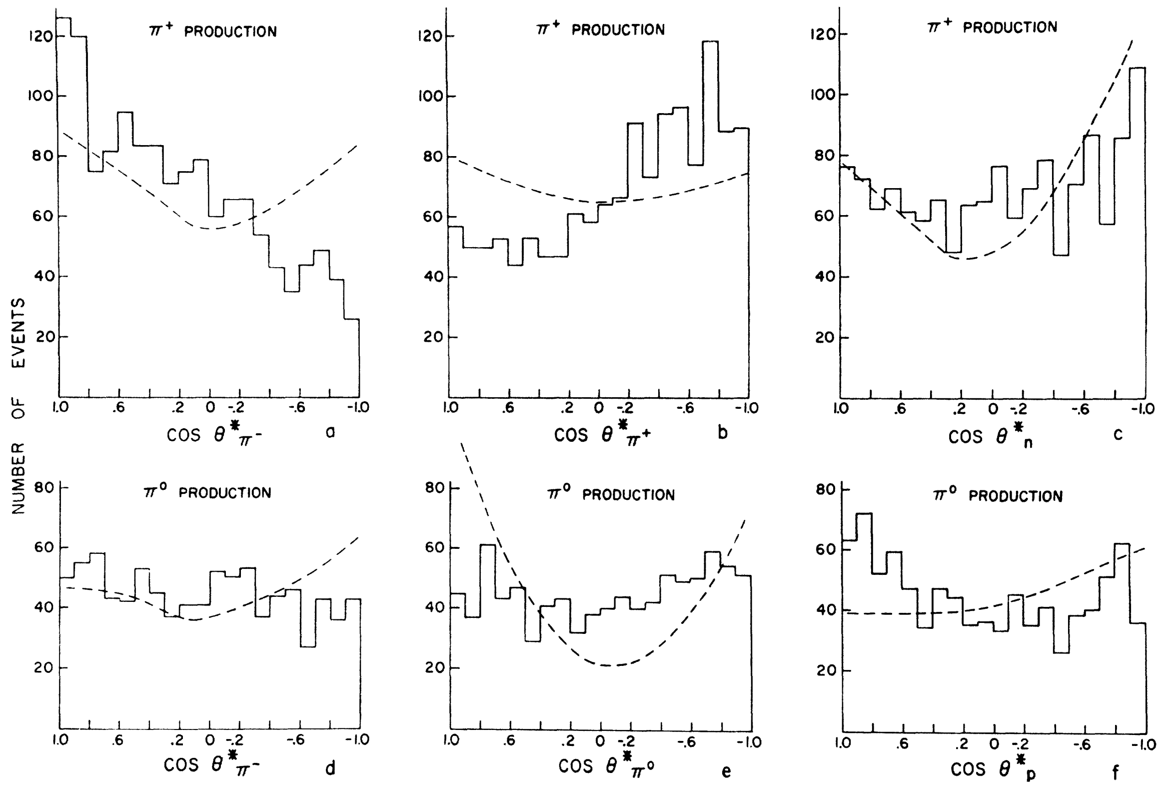

FIG. 7. The angular distribution of the final-state particles. θ_X* is the angle between particle X and the incident direction as measured in the center-of-mass system. The broken lines are the predictions of the Olsson-Yodh isobar model.

17 are an upper limit for the reaction because some may be high energy electrons.

No conclusive evidence was found for the reaction $\pi^-+p\rightarrow\pi^-+p+\pi^0+\pi^0$.

V. CONCLUSIONS

Our results for the total, neutral, and elastic cross sections support the results from numerous other experiments The detailed study of the inelastic reactions indicates that isobar production is the dominating mechanism at this energy. However, there is clear evidence that isobar production alone is not sufficient to explain the π^+ production reaction. Some evidence was found for η production but the cross sections are not consistent with the results of spark-chamber groups.

ACKNOWLEDGMENTS

We wish to acknowledge the cooperation of the Brookhaven National Laboratory Cosmotron staff and the 14-in. bubble-chamber crew who made the experiment possible.

The experiment was planned in collaboration with Dr. J. Ashkin. We gratefully acknowledge the help of the many people who played a part in this experiment. In particular, we mention Dr. R. K. Adair, Dr. L. Leipuner, Dr. B. Musgrave, Dr. T. Fields, Dr. E. G. Pewitt, Dr. V. P Kenney, and Dr. C. Vittitoe. We also thank J. Crump, Mrs. J. Fierst, and Miss M. E. Lipchak for help in writing and running computer programs. Finally, we thank the scanners: M. Godich, A Fine, D. Stasak, L. Stember, P. Bizub H. Scheller, S. Engles, M. Lentz, J. Griffith, and L. McGraw.