# Experimental Thermal Conductivity of Helium-3<sup>†</sup>

A. C. ANDERSON, J. I. CONNOLLY,\* O. E. VILCHES, AND J. C. WHEATLEY Department of Physics and Materials Research Laboratory, University of Illinois, Urbana, Illinois (Received 27 January 1966)

The thermal conductivity of liquid He<sup>3</sup> at pressures of 0.11, 6.78, and 27.0 atm has been measured down to 0.06'K. For each pressure the conductivity passes through a minimum, the temperature of the minimum decreasing with increasing pressure. At temperatures greater than  $\frac{1}{2}$ °K the thermal conductivity is nearly independent of pressure. The effect of pressure is to decrease the conductivity. The pressure effect increases monotonically down to the lowest temperature.

### I. INTRODUCTION

 $S$  EVERAL years ago Lee and Fairbank<sup>1</sup> measured the thermal conductivity of liquid  $\mathrm{He}^3$  at low pressur EVKRAI, years ago Lee and Fairbank' measured the and from 0.24 to 2.7°K. They found the conductivity to increase slowly with increasing temperature throughout the entire range investigated. More recently the low' pressure thermal conductivity was measured by Anderson, Salinger, and Wheatley<sup>2</sup> from 0.026 to 0.2°K and found to decrease as the temperature increased. Their results appear to blend with an extrapolation of those in Ref. 1.Below 0.04'K the data are in agreement with the  $T^{-1}$  temperature dependence predicted by the Fermi-liquid theory.<sup>3</sup> Together the two experiments indicated a minimum in the thermal conductivity between 0.<sup>2</sup> and 0.24'K. It was considered desirable to measure the thermal conductivity over a temperature range as wide as possible and to extend all measurements to higher pressures. The results would provide au experimental basis for theories of transport properties at temperatures above those for which the simplest Fermi-liquid picture may be valid, but still in a temperature region where degeneracy effects are important. The pressure data would also be useful in designing new experiments on He' under pressure. In addition, we had a particular interest in the pressure data so that previously published thermal-boundary resistance measurements<sup>4</sup> could be corrected for the effect of a series thermal resistance of the bulk He'.

In the present work the thermal conductivity has been measured at 0.11, 6.78, and 27.0 atm. The lowtemperature limit was that temperature beneath which the data became badly scattered due to irreproducibility in the resistance thermometers. At all pressures this was about  $0.055\textdegree K$ , a temperature too high for strongly degenerate Fermi-liquid behavior. In order to avoid convection currents while working with a single cell configuration, the high-temperature limit was chosen to be less than that temperature above which the thermal expansion coefficient is positive. This temperature ranges from about 0.52'K at saturated vapor pressure to about  $1.2\textdegree K$  at 27 atm.<sup>5</sup>

The experimental arrangement was a standard one of a column of He', two resistance thermometers, a heat source, and a heat sink. The He' was contained by a thin-walled nylon cylinder. Nylon was chosen for its very low thermal conductivity<sup>6</sup> and relative strength. Such an apparatus is limited eventually at high temperatures by the shunting effect of the cell wall and at low temperatures by the bulk resistance of the He<sup>3</sup> becoming comparable with or less than the series thermal-boundary resistance. Because of the shunting effect of the wall, no data were obtained above  $0.9^{\circ}$ K. To extend the measurements to temperatures considerably lower than those studied in this work would require other methods based on the high thermal conductivity of He' at low temperatures.

### II. EXPERIMENTAL ARRANGEMENT AND PROCEDURE

# A. General Arrangement

The cryostat and procedure for attaining and maintaining temperatures down to about 0.015'K has been I ne cryostat and procedure for attaining and mail<br>taining temperatures down to about  $0.015^{\circ}$ K has bee<br>described in detail in the literature.<sup>7,8</sup> The demagnetization apparatus used in the present work was that discussed in Ref. 8.

The thermal conductivity unit, which fitted onto a boss extending beneath the chrome-alum refrigerator, was assembled as indicated in Fig.  $1.9-11$  The spherical cerium magnesium nitrate (CMN) thermometer con-

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<sup>\*</sup>This work was submitted by J. I. Connolly in partial fulfillment of the requirements for the Ph.D. degree at the University of Illinois.<br><sup>1</sup> D. M. Lee and H. A. Fairbank, Phys. Rev. 116, 1359 (1959).

<sup>&</sup>lt;sup>1</sup> D. M. Lee and H. A. Fairbank, Phys. Rev. 116, 1359 (1959).<br><sup>2</sup> A. C. Anderson, G. L. Salinger, and J. C. Wheatley, Phys.<br>Rev. Letters 6, 443 (1961).<br><sup>3</sup> A. A. Abrikosov and I. M. Khalatnikov, Rept. Progr. Phys.

<sup>22,</sup> 329 (1959).

<sup>&</sup>lt;sup>4</sup> A. C. Anderson, J. I. Connolly, and J. C. Wheatley, Phys. Rev. 135, A910 {1964).

<sup>5</sup> D. M. Lee, H. A. Fairbank, and E.J. Walker, Phys. Rev. 121, 1258 (1961).

<sup>6</sup>A. C. Anderson, W. Reese, and J. C. Wheatley, Rev. Sci. Instr. 34, 1386 (1963).

<sup>&</sup>lt;sup>7</sup> A. C. Anderson, G. L. Salinger, and J. C. Wheatley, Rev. Sci. Instr. 32, 1110 (1961).

W. R. Abel, A. C. Anderson, W. C. Black, and J.C. Wheatley, Physics 1, 337 (1965). 'Furane Plastics, Inc., 4516 Brazil Street, Los Angeles, Cali-

fornia.<br>
<sup>10</sup> "Coil-foil" is a homemade insulated copper-wire sheet, with

the wires glued with a mixture of toluene, alcohol, and adhesive G.E. 7031.

G.E. 7031.<br>
<sup>11</sup> James G. Biddle Company, Township Lane, Plymout<br>Meeting, Pennsylvania.

sisted of finely powdered (0.006-in.-diam) CMN mixed, in a ratio of about 2:1 by weight, with Bow-Corning 200 Fluid" of 20 000 centistokes viscosity and packed into a network of fine copper wires. Such a thermometer has a time constant of about 100 sec at  $0.1\,{\rm K}$  and 1000 sec at 0.04'K. Hence, though it is easy to prepare because of the use of CMN powder, its performance is not particularly good at low temperatures. The magnetic susceptibility of the CMN was measured with a  $17$ -cps mutual-inductance bridge.<sup>13,14</sup>

The essentials of the pressure system have been described elsewhere.<sup>15</sup> In the present experiments, an aluminum "windmill" actuated by a small centrifugal blower was added to the stack of weights on the deadweight tester to keep them rotating constantly at about



FIG. 1. Thermal-conductivity unit. The thermal-conductivity cell (see Fig. 2) is inside an Epibond 100A shell (see Ref. 9). The CrK alum refrigerator is above the figure. The CMN thermometer and "coil-foil" (see Ref. 10) soldered to the copper-wire link to the and "coil-foil" (see Ref. 10) soldered to the copper-wire link to the<br>He<sup>3</sup> cell are thermally connected by several layers of "coil-foil,"<br>glued with Apiezon "N" grease (see Ref. 11).

<sup>12</sup> Electronics Products Division, Dow-Corning Corporation, Midland, Michigan.<br><sup>13</sup> Cryotronics, High Bridge, New Jersey.



 $\frac{1}{2}$  to 1 cps. In this way the dead-weight tester served as a manostat for the pressure on the He' sample.

#### B. Thermal-Conductivity Cell

The thermal-conductivity cell used in this experiment is diagrammed in Fig. 2. The body had to be sufficiently strong to withstand 27 atm, yet have a low thermal conductivity so as not to shunt thermally the liquid He<sup>3</sup>. Nylon was selected on the basis of its relatively high strength, low thermal conductivity, and ability to make strong, tight seals to Epibond 100A.<sup>6,16</sup> The nylon was machined from a  $\frac{1}{4}$ -in.-diam rod as shown in Fig. 2. Ridges of nylon were left on the wall to prevent the cell distorting under pressure and to increase its bursting point. It was later learned that for the pressure range of this experiment the ridges were probably not necessary. The cell i.d. was 0.152 in. with a wall thickness of 0.010 in. The ridges, separated by 0.152 in. , had an o.d. of 0.240 in. and were 0.020 in. thick, with a 0.010-in. radius on all internal and external contours. A test cell with a straight 0.011-in. wall did not explode at 2000 psi internal pressure while im-

<sup>&</sup>lt;sup>13</sup> Cryotronics, High Bridge, New Jersey.<br><sup>14</sup> W. R. Abel, A. C. Anderson, and J. C. Wheatley, Rev. Sci. Instr. 35, 444 (1964).<br> $1^{15}$  A. C. Anderson, W. Reese, and J. C. Wheatley, Phys. Rev.

<sup>127,</sup> 6?1 (1962).

<sup>&</sup>lt;sup>16</sup> J. C. Wheatley, Rev. Sci. Instr. 35, 765 (1964).

mersed in liquid nitrogen but then later exploded at 750 psi at room temperature.

The ends of the cell were feathered and grooved, and two small cylinders of Epibond 100A were molded over them, following a general technique described previthem, following a general technique described previously.<sup>16</sup> These cylinders were later machined and counterbored as shown in Fig. 2.

Thermal contact to the He<sup>3</sup> was made at each end by means of a "brush" of copper wires, constructed as follows. Six 1-in.-wide strips were formed, each of 500, 1.25-in.-long, 0.002-in.-diam copper wires. The heavy Formex insulation was removed for  $\frac{1}{4}$  in. at one end using a chemical stripper.<sup>17</sup> A fine line of Epibond  $121^9$ was laid across each strip about  $\frac{3}{8}$  in. from the end which was free of insulation. The 500 wires were then rolled together, and the bundle bound tightly with cotton thread over the Epibond 121 in such a way that the 121 formed an impervious barrier transverse to the wires. After the Epibond had hardened, the insulation-free portions of the wires were bound tightly around a 0.018-in. copper wire using a 0.004-in. copper wire. The bundle of wires was dipped through a pool of lowtemperature brazing flux into a bath of silver solder. (Silver solder is used instead of soft solder because, at the operating temperatures of this experiment, soft solder gives rise to bothersome magnetic interference plus more serious Joule heating problems. ) The tops of the small wires were held  $\frac{1}{16}$  in. below the solder surface for a few seconds until the solder was seen to wet the wires. The bundle was then quickly withdrawn and any excess solder ground off. To each of three such bundles a ring of Epibond 121 was added at the position of the 121 disk, and the three 0.018-in.-diam wires were hard soldered to a 1-in.-long, 0.040-in.-diam copper wire. Using a Teflon mold, a cylinder of Epibond 100A was cured over all solder joints such that 0.75 in. of the 0.040-in copper wire protruded from one end, and 0.30 in. of the 1500 wire brush was left, after trimming, to extend into the liquid He $3$ . The Epibond 121 barrier inhibited flow of the Epibond 100A, which otherwise would have filled the entire brush. Before applying the 121, it was allowed to become quite viscous (standing for 2 h at room temperature after the two components were mixed) so that it would not flow. A second brush was made, identical to the first, but with the inclusion of a  $\frac{1}{64}$ -in. o.d.  $\times$  0.003-in. wall cupro-nickel tube. This tube was laid in place after all silver-solder joints had been made. The final Epibond 121 application was postponed until after the soldering so that the tube could conveniently be placed within the brush but near one side. Machining the 100A cylinder, as shown in Fig. 2, and trimming the ends of the wires with a razor blade completed the brushes. For ease in assembling the cell, a very small strip of cellophane tape was fastened around the brush tips and held in place with General

Electric 7031 varnish. On examining the brushes under a binocular microscope, it seemed likely that this operation would have a negligible effect on the surface area of the brush. The resistance thermometers, supported by their leads, were fastened to the cylinder of Epibond 100A with Epibond 121. The brush and resistance thermometer assemblies were then sealed into the ends of the cell, using a coating of Epibond 121 at the Kpibond 100A interfaces. The average distance between the two thermometers was 0.275 in. At low pressure the cell contained approximately  $6.4 \times 10^{-3}$ mole of liquid He<sup>3</sup>.

Around the 0.040-in. copper wire connected to the bottom brush was wrapped a 0.002-in. bifilar Evanohm heater of electrical resistance  $138 \Omega$ . Manganin leads of 0.005-in. diam were hard soldered to the Evanohm and then covered with soft solder.

#### C. Resistance Thermometers

The resistance thermometers were prepared from a  $\frac{1}{2}$  W, grade 1002, 470- $\Omega$  Speer resistor.<sup>18</sup> This resistor was chosen for its temperature dependence, which was large enough for good sensitivity but not so large that the resistances greatly exceeded  $10k\Omega$ , the resistance at which our bridges are most sensitive. Using a diamond saw with a blade of 0.006-in. thickness, rectangular slabs 0.010 in.  $\times$  0.025 in.  $\times$  0.100 in. were cut from the resistor. The  $0.010$ -in.  $\times$  0.100-in. faces were coated the resistor. The 0.010-in. × 0.100-in. faces were coate<br>with silver paint.1º Manganin leads 0.002 in. in diamete were attached by stretching the wire across the slab, adding more silver paint, and then cutting off one of the ends of the wire with a razor blade. The resistors were finally covered with a protective coating of Epibond 121. This coating was quite thin (0.001—0.<sup>002</sup> in.) in order that the thermometers might remain in good thermal contact with the liquid He<sup>3</sup>. The 121 was cured for several hours at 100'F. All operations on the resistors were performed under a binocular microscope.

Previously, resistors as small as  $0.010 \times 0.015 \times$ 0.030 in. had been prepared as described above. In a preliminary run, these small resistors, thermally attached to a strip of coil-foil using General Electric 7031 varnish, exhibited serious heating problems at temperatures as high as 4.2'K. The size of the resistors used in this experiment were what we estimated would be the smallest resistors not to be troubled by selfheating when immersed in liquid He<sup>3</sup>. At the lowest temperature, about 0.05'K, it was nevertheless necessary to decrease the measuring power to approximately  $10^{-14}$  W.

The preparations of the two resistors were nominally identical. The resulting room-temperature resistance values were 321 and 260 $\Omega$ . This difference is probably due partly to the slabs' not being quite the same in size

<sup>&#</sup>x27;7 Strip-X No. 26-2, sold by G. C. Electronics Company, Los Angeles, California.

<sup>&</sup>lt;sup>18</sup> W. C. Black, W. R. Roach, and J. C. Wheatley, Rev. Sci. Instr. 35, 587 (1964). '9 G. C. Electronics Company, Los Angeles, California.

and partly to unequal silver-paint application. It is interesting to compare the resistance-versus-temperature characteristics of these resistors with that of an unmodified resistor nominally identical to the resistor from which our slabs were cut.<sup>18</sup> Neither resistor increased in resistance so rapidly as did the unmodified resistor. The 0.039 to 300'K resistance ratios for the 321 and the 269- $\Omega$  resistors were, respectively, 0.6 and 0.19 times the same ratio for the unmodified Speer resistor.

In the thermal conductivity cell of Ref. 2, the resistance thermometers were in the form of short, cylindrical tubes, 0.1 cm long by 0.35-cm o.d. by 0.25 mm thick, made from two layers of onionskin paper impregnated with Epibond 104. These were coated with RS-12 shielding Micropaint to which about 20% by volume of carbon black had been added.<sup>20</sup> The electrical leads, which served also as mechanical supports, were 0.003-in.-diam niobium wires attached to the resistors mechanically using Epibond 104' and electrically using a small amount of  $SC-13$  silver paint.<sup>20</sup> They had no protective coating. The temperature dependence for these film resistors was approximately  $T^{-1}$ , much less than for any of the above resistors. The 61m resistors were stable over short times to the lowest operating temperature of 0.022°K. However, below about 0.07°K there was a steady drift of order  $-0.1\%$  per day in the calibration curves.

It was because of the small temperature dependence and the long-term drift of the 61m resistors that the modified Speer resistors were used in the present work. However, in the present experiment the resistors gave erratic results below 0.05'K, even though untreated Speer resistors are not erratic down to at least 0.02'K. Whenever we have compared two resistors under similar conditions, the one with the smaller temperature dependence has been the more stable. In this experiment the  $321-\Omega$  resistor was more erratic, at all temperatures, than was the  $260-\Omega$  resistor. This erratic behavior probably was not caused by rf Joule heating because the cryostat and the measuring equipment were located within a double-thickness copper-screened  $room<sup>8</sup>$ 

The present calibration curves, in the region where the resistance readings were reliable, were more stable than those in Ref. 2, and there was no observable drift in the calibration if the cell was kept cold. On the other hand, there was frequently a shift in the calibration curves when the cell was warmed to room temperature and then recooled. Typically the shift would be about 1% at low temperatures and less at high temperatures. No clear pressure effect on resistance was observed at any temperature. The resistances were measured, in the present experiment as well as that of Ref. 2, with 33-cps bridges with phase-sensitive detection.

## D. Procedure

The CMN magnetic thermometer mas calibrated against the vapor pressure of the He' bath using He' exchange gas in the vacuum space. The chrome-alum heat sink was demagnetized from 0.3'K and <sup>15</sup> kG using a superconducting solenoid. After demagnetization the cell and heat sink were at slightly less than 0.02'K. No special care was taken during the demagnetizations, which were performed in 15 min, as the thermal-conductivity cell would not operate well below  $0.05$ °K.

After the heat sink, the thermal-conductivity cell, and the CMN thermometer were all nearly in thermal equilibrium with one another, the heater was turned on at approximately 1 erg/sec. After several hours the cell reached a dynamic equilibrium in which both resistance thermometers in the cell drifted warmer linearly in time. Under these conditions the mean temperature in the cell was usually about  $0.05{\text -}0.06^{\circ}\text{K}$ , the lowest temperature at which good quality data were obtained, and the temperature drop along the cell was about  $10\%$ of the temperature. Both resistances were plotted versus time, and five or six pairs of averaged readings mere selected to be converted into thermal conductivities. After 10000 sec of such data obtained under dynamic equilibrium, the heater power was increased. After another wait of several hours for dynamic equilibrium to obtain, another 10000 sec of data were taken but now at a slightly higher temperature. The input power was thus increased in steps, periods of approximately 10000 sec of dynamic-equilibrium data being taken after each step until finally a temperature of about 0.4'K was reached. Occasionally the heater power was turned off so that the resistance thermometers could be calibrated against the CMN thermometer. These calibration points were also dynamic equilibrium points since, with the heater turned off, the cell and chrome alum refrigerator warmed slowly under a residual heat leak of a few erg/min.

The high-temperature data were obtained by connecting thermally the He<sup>3</sup> refrigerator, iron-alum guard, chrome-alum heat sink, and the thermal conductivity cell. This was effected by soldering long strips of coilfoil to the  $He<sup>3</sup>$  refrigerator, smearing them with Apiezon " $N$ " grease, and then tightly binding them to the guard, heat sink, and thermal conductivity unit. With this configuration, data were taken to temperatures as low as possible (in order to overlap the demagnetization data) consistent with a moderately large temperature drop in the He'. For all three pressures this temperature was approximately 0.35°K. While taking these data, the temperature of the upper resistor was maintained constant during both power-on and power-off periods by regulating the pumping speed of the He' refrigerator. An electrical heater on the He<sup>3</sup> refrigerator provided a fine control. At 0.11, 6.78, and 27.0 atm, data were obtained up to 0.47, 0.75, and 0.87'K, respectively. At

<sup>&</sup>lt;sup>20</sup> Micro Circuits Company, New Buffalo, Michigan.



FIG. 3. Corrected thermal-conductivity data as a function of the absolute temperature, at three pressures. Lines have been drawn through the present data to facilitate reading.

0.11 atm the thermal conductivity was measured at 0.544'K, a temperature slightly above the density maximum, and found to be 50 times larger than expected. This result no doubt is to be attributed to the effect of convection.

# III. RESULTS OF THE MEASUREMENTS

Corrected data from this experiment for all three pressures and the data at low pressure from Refs. 1 and 2 are plotted in Fig. 3. The thermal conductivity decreases with increasing pressure at all temperatures. The pressure dependence increases with decreasing temperature and is relatively large at the lowest temperatures. The minimum in the conductivity occurs at lower temperatures as the pressure is increased. The temperature and conductivity at the three minima are given in Table I.

Two corrections had to be applied to the present data. One was to subtract from the measured heater power that amount shunted by the nylon wall of the cell. Since

TABLE I. The minima in the thermal conductivityversus-temperature curves.

$p(\text{atm})$	$T_{\rm min} ({}^{\circ} \rm K)$	$\kappa$ (erg/cm sec°K)
0.11	0.225	586
6.78	0.185	500
27.00	0.113	392

the thermal conductivity of nylon decreases rapidly with decreasing temperature, the shunting rapidly becomes unimportant at lower temperatures, being a  $0.1\%$  effect at  $0.1\,^{\circ}\text{K}$ . At the highest temperature,  $0.9\,^{\circ}\text{K}$ , 8% of the power was shunted. This effect was calculated, as it was not convenient to measure the proper shunting thermal conductance. The shunting effect of leads to the heater and to the resistance thermometers was negligible at all temperatures.

A second correction had to be applied when the cell temperature drifted due to the applied power. The correction was necessary because some of the measured heater power was used to increase the temperature of the He3. The heat current producing the temperature difference for a symmetrical cell is then less than the applied power  $Q_m$  (assumed corrected for any shunting effect of the nylon walls) by  $\frac{1}{2}Q_3$ , where  $Q_3$  is the power absorbed by the He<sup>3</sup> in the whole cell. The thermal conductivity may then be calculated from the formula

$$
\kappa = \frac{\hat{Q}_m - \left(\frac{1}{2}\right)\hat{Q}_3}{\Delta T(A/l)},\tag{1}
$$

where  $\Delta T$  is the temperature difference between the two thermometers located a distance  $l$  apart, and  $A$  is the cross-sectional area of He' perpendicular to the heat current.  $\dot{Q}_3$  may be calculated by estimating the volume of  $He<sup>3</sup>$  in the cell and then using the specific heat of  $He<sup>3</sup>$ 

as a function of temperature and pressure<sup>21</sup> and the measured rate of temperature drift of the resistance thermometer closer to the heater. That is, one has

$$
\dot{Q}_3 = C_3 \dot{T}.
$$
 (2)

One can also calculate  $\dot{Q}_3$  by recognizing that, if both the He' and the chrome-alum heat sink are increasing in temperature at the same rate under action of the measured heater power  $\dot{Q}_m$ , then

$$
\frac{\dot{Q}_3}{\dot{Q}_m} = \frac{C_3}{C_3 + C_{\text{CrK}}},\tag{3}
$$

where  $C_{\text{CrK}}$  is the heat capacity of the chrome alum. For the actual correction to the conductivity data, we measured  $C_3$  at  $0.35^{\circ}$ K by insisting that the low-pressure demagnetization data and the He<sup>3</sup> refrigerator data coincide there, the latter not requiring such a correction since  $\dot{Q}_3=0$ . The temperature dependence of  $C_3$  was since  $Q_3=0$ . The temperature dependence of  $C_3$  was<br>then assumed to be that for  $\mathrm{He^{3,21}}$  The set of values of  $\dot{Q}_3$  calculated using Eqs. (2) and (3) agreed within 10% with each other and with the above measurement. At low pressure the magnitude of the correction was  $3.6\%$ at 0.35'K. The correction decreased with decreasing temperature, being  $1\%$  at 0.1°K. The high-temperature thermal-conductivity data at high pressure did not define such a smooth curve as did the corresponding low-pressure data. For this reason the high-pressure corrections were scaled from the low-pressure correction using the known specific heat<sup>21</sup> and compressibility<sup>22</sup> data for He'. The magnitudes of the corrections at 6.78 and 27.0 atm, respectively, were 4.0 and  $4.2\%$  at  $0.35$ °K.

There was concern about the possibility of the cell distorting under pressure. After the experiment we measured, at room temperature and from 0 to 200 psi, the diameter of the four ridges and five valleys on the cell wall. The distortion was everywhere well under  $1\%$ . With the cell cold, the distortion would be expected to be even less. This effect was therefore neglected. The change in the cross-section-to-length ratio of the column of He' between the two resistance thermometers due to thermal contraction was calculated to be  $-0.1\%$  relative to room temperature. This was so much smaller than the scatter in the data that it was also neglected.

Although we spent several days taking data at low pressure between 0.029 and 0.055'K, these lowtemperature data were discarded because they were badly scattered, an effect probably coming from the mentioned erratic behavior of the resistors. The resistors did not appear to be in such good thermal contact with the helium as did the resistors in Ref. 2.

At high temperatures there is also some scatter which we do not understand and which indicates some uncontrolled feature of the experiment. Fluctuations in the He3 refrigerator temperature, as indicated by a carbon resistor in contact with it, were definitely less than  $0.001^{\circ}$ K. In addition, the period of the fluctuations was typically a few minutes, while the time constant of the cell was in excess of 1 h. In this temperature region the resistors themselves were well behaved, but the thermal conductivities calculated from these data were nevertheless scattered.

Our low-pressure data may be compared above 0.2'K with the data of Ref. 1 and below  $0.2^{\circ}$ K with those of Ref. 2. The conductivities of Ref. 1 lie at least  $10\%$ higher than the present data, somewhat outside of the combined scatter.

Between 0.08 and 0.3'K the scatter in our data is rather small, typically 1 or 2%. The data in Ref. 2, for which the scatter is also quite small, lie about  $10\%$ higher. During the present experiment we noticed that when we took data for several hours at the same power input, the later data yielded lower conductivities, even though all of the raw data appeared to be in dynamic equilibrium. In Ref. 2 it was known that equilibrium was a problem, and for this reason we believe that the Ref. 2 data may lie too high. They also were not corrected as in Eq. (1), this correction decreasing the conductivity. These may not account for the full  $10\%$ discrepancy, but systematic calibrational and geometrical errors are also present in both experiments. Below 0.08'K, however, the present data, although scattered, appear to be blending with the results of Ref. 2 as the temperature decreases. This fact would suggest that systematic calibrational and geometrical errors are not large in the two experiments.

Thermal conductivities were calculated from an expression of the form

$$
\kappa = (\dot{Q}/\Delta T)(l/A). \tag{4}
$$

Error arises from measurement of any of the above quantities.  $Q_m$  was measured with a Leeds and Northrup K-3 potentiometer using a four-lead connection to the heater. This measurement is undoubtedly less susceptible to error than any other measurement. The  $\dot{Q}$ appearing in (4), however, is the heater power corrected for the calculated shunting effect of the nylon and for the heat absorbed by the He<sup>3</sup> in the cell. We estimate  $\dot{Q}$ to be correct to better than  $1\%$  except possibly at the highest temperatures. Error in the primary temperature calibration (He' vapor pressure versus CMN), estimated to be accurate to about  $1\%$ , would lead to only a small error in  $\Delta T$  since both the hot and cold temperatures would be similarly shifted.  $\Delta T$  was determined by measuring the two resistances and finding the corresponding two temperatures independently from separate calibration curves. Hence, an error in the secondary temperature calibrations (resistance thermometers versus CMN) could cause a more serious error in  $\Delta T$ . The error in  $\kappa$ , resulting from all systematic errors in T and  $\Delta T$ , should not be greater than 5%.

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<sup>&</sup>lt;sup>21</sup> A. C. Anderson, W. Reese, and J. C. Wheatley, Phys. Rev.<br>**130**, 495 (1963).<br><sup>22</sup> R. H. Sherman and F. T. Edeskuty, Ann. Phys. (N.Y.) 9, 522

<sup>(1960).</sup>

The most serious error in  $\kappa$  may come from the measurement of  $l/A$ . We have assumed the isotherms to be planes perpendicular to the cell axis, though this assumption is probably not completely correct. It was also assumed that the temperature obtained from the resistance-versus-temperature-calibration curve corresponds to the temperature at the center of the resistor. Estimating an error from these sources and from measurements of the resistor separation and cell diameter, we estimate the total error in  $l/A$  to be not in excess of 5%. Hence, the total systematic error in  $\kappa$  from all sources is believed to be less than  $10\%$ .

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### IV. DISCUSSION

The majority of the data from this experiment lie at temperatures low enough for degeneracy effects to be important but too high for effects due to a strongly degenerate Fermi liquid to be observed. The data do, however, provide an experimental basis for future theories of transport properties in this intermediate degeneracy region.

It is possible' to derive, on the basis of the Fermiliquid theory, a theoretical estimate of the thermal conductivity on the basis of measurements of the heat capacity, sound velocity, and susceptibility. Such calculations have been carried out recently $^{23}$  for 0.28 and 27.0 atm, the only two pressures where definitive values of the heat capacity have been obtained. The result is that  $\kappa_{\text{theory}}=(80/T)$  (erg/cm sec) at 0.28 atm and  $\kappa_{\text{theory}} = (20/T)$  (erg/cm sec) at 27.0 atm. The

Fermi-liquid results might be expected to be valid at  $0.04\,{\rm ^oK}$  for 0.28 atm and  $0.02\,{\rm ^oK}$  for 27.0 atm, at which temperatures the conductivities would be respectively 2000 erg/cm sec  $\mathrm{K}$  and 1000 erg/cm sec  $\mathrm{K}$ . The agreement with extrapolations of the measurements of the present experiment is as good as can be expected.

A theory of thermal conductivity of Fermi systems has been worked out by Nishimura.<sup>24</sup> This theory does predict a minimum in the thermal conductivity of He'. Naive evaluation of the theory, using parameters valid at very low temperature, yields the qualitatively correct result that both the thermal conductivity at the minimum and the temperature of the minimum decrease as the pressure increases.

The effect of pressure on the thermal conductivity is given as a function of relative density in Fig. 4 with various temperatures as parameters. The smooth curves were obtained directly from Fig. 3.The effect of density becomes less as the temperature increases, approaching quasiclassical density independence at the higher temperatures. Presumably, if the Fermi-liquid theory were valid, at some sufficiently low temperature the effect of density mill become independent of temperature, but at 0.06'K the density dependence of the thermal conductivity is continuing to increase substantially as the temperature drops. Other evidence for quasiclassical behavior of the transport properties in liquid  $He<sup>3</sup>$  at higher temperatures is displayed in Fig. 5. Here, at low pressure, the ratio of the thermal conductivity (or an extrapolation) as obtained in the present work to the viscosity as determined by Betts, Osborne, Welber, and



FIG. 4. Relative thermal<br>
onductivity versus relative conductivity versus density at several temperatures.

<sup>24</sup> H. Nishimura, Progr. Theoret. Phys. (Kyoto) 27, 967 (1962).

J. C. Wheatley, Proceedings of Conference on Quantum Fluids, Sussex, England, 1965 (to be published).

Wilks<sup>25</sup> has been plotted versus temperature. Near 1<sup>o</sup>K this ratio approaches the classical<sup>26</sup> limit of  $\frac{5}{2}C$ , where C, the specific heat per unit mass, has been taken from<br>the work of Brewer, Daunt, and Sreedhar.<sup>27</sup> the work of Brewer, Daunt, and Sreedhar.<sup>27</sup>

We have used the thermal conductivity data at 27.0 atm to correct previously published data on thermal boundary resistance.<sup>4</sup> The resistance data actually



FIG. 5. Ratio of low-pressure thermal conductivity to viscosity versus temperature.

obtained in Ref. 4 represented the sum of the boundary resistances of two electropolished copper-liquid He'



FIG. 6. The Kapitza resistance between copper and He<sup>3</sup>, as measured by Anderson, Connolly, and Wheatley, corrected for the liquid-He' thermal resistance measured in this work.

surfaces plus the bulk resistance of a 0.0037-in. layer of liquid He'. At that time only the low-pressure data could be corrected for the bulk resistance of the He', as the high-pressure conductivity of liquid He' was not known. The corrected boundary resistances multiplied by the third power of the temperature  $(RT<sup>3</sup>)$  are shown as dashed lines in Fig. 6. The correction is negligible below  $0.3^{\circ}$ K. The pressure effect on the boundary resistance thus becomes quite small at higher temperatures, similar to the behavior of the thermalboundary resistance4 between copper and He4. The drop in  $RT^3$  with increasing temperature above  $0.1^{\circ}$ K is not understood.

<sup>~~</sup> D. C. Betts, D. W. Osborne, B. Welber, and J. Wilks, Phil. Nag. S, 977 (1963).

<sup>&</sup>lt;sup>26</sup> Sir James Jeans, *The Dynamical Theory of Gases* (Dover<br>Publications, 1961), 4th ed., p. 299.<br><sup>27</sup> D. F. Brewer, J. G. Daunt, and A. K. Sreedhar, Phys. Rev.<br>115, 836 (1959).