Dependence of the Angular Distribution of the (d,p) Reaction on the Total Angular-Momentum Transfer. II*

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The (d,p) reaction has been studied with targets of C12, O16, F19, Si28, S22.34, and Zr20,92.94. New evidence on the J dependence of the (d, p) angular distribution has been obtained in 1p and 2d transitions. Additional evidence for 1d transitions has also been obtained.

I. INTRODUCTION

N earlier publication¹ (hereafter referred to as I), A presented evidence that the proton angular distribution from the (d,p) reaction exhibits a marked dependence on the total angular-momentum transfer. The evidence was particularly strong for l=1 transitions in the 2p shell, and somewhat less strong but still quite unambiguous in the 1d and 1f shells. We have now studied a number of additional reactions, mainly l=1transitions in the 1p shell and l=2 transitions in both the 1d and 2d shells. The results indicate that Jdependent effects seem to be present, although these effects may well be obscured for weak transitions or at energies at which compound-nucleus processes are strong. Some of the results reported in this paper have been reported elsewhere.²

In addition to the measurements reported here we have surveyed the literature for any data relevant to the *J*-dependent effects we have studied. These are also reported. All the transitions we report are between states of known spin; in a few cases where spin assignments may be doubtful we have indicated this explicitly in the text or by parentheses. In the course of the measurements we also obtained angular distributions which are not relevant to the J-dependent effects. These are included in the present paper in case they may be of interest in future experimental or theoretical studies. Since the main emphasis of the paper is on J-dependent effects, we have not included a distorted-wave Born approximation (DWBA) analysis or extracted spectroscopic factors. We also have not obtained absolute cross sections except for the $Si^{28}(d, p)Si^{29}$ data.

II. EXPERIMENTAL RESULTS

The measurements reported in this paper were made in an 18-in.-diam scattering chamber³ using Si surface-

barrier detectors and deuteron beams from the Argonne tandem accelerator. Pulse-height spectra were recorded in multichannel analyzers and punched on IBM cards for further data processing.⁴ A monitor counter was used to correct for possible nonuniformities in the target.

 $C^{12}(d,p)C^{13}$. Self-supporting C foils (~30 $\mu g/cm^2$) were used as targets. Angular distributions were obtained at 11 and 13 MeV for the reactions leading to the ground state and the first three excited states. The results for the l=1, $J=\frac{1}{2}$ ground state and the l=1, $J = \frac{3}{2}$ 3.68-MeV state are shown in Fig. 1; the l = 2, 3.85-MeV state is shown in Fig. 2. Relative cross sections at the peak angles in the distributions are given in Table I.

 $O^{16}(d,p)O^{17}$. Angular distributions for the groundstate proton group were measured at 11- and 13-MeV incident energy. The results, obtained with SiO foil targets with a thickness of $\sim 200 \ \mu g/cm^2$ are shown in Fig. 3.

 $F^{19}(d,p)F^{20}$. Evaporated targets of PbF₂ were used. Data were obtained only for the 0.65-MeV state because





Conference on Bases for Nuclear Spin-Parity Assignments, Gatlinburg, Tennessee, 1965 (to be published). ^a J. T. Heinrich and T. H. Braid (to be published).

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¹L. L. Lee, Jr., and J. P. Schiffer, Phys. Rev. 136, B405 (1964), hereafter referred to as I.

² L. L. Lee, Jr., and J. P. Schiffer, Bull. Am. Phys. Soc. 9, 456 (1964); J. P. Schiffer, L. L. Lee, Jr., A. Marinov, and C. Mayer-Böricke, *ibid.* 10, 510 (1965); J. P. Schiffer, in Proceedings of the

⁴ R. J. Pecina and F. J. Spikas, Nucl. Instr. Methods 34, 245 (1964)



FIG. 2. Angular distributions for the l=2 transition in the $C^{12}(d,p)C^{13}$ reactions. The relative errors are less than the size of the points. The straight lines are drawn in to point out the difference between the 11- and 13-MeV data in developing a minimum at ~60°.



 $Si^{28}(d,p)Si^{29}$. The SiO foil targets had thicknesses of $\sim 200 \ \mu g/cm^2$. Angular distributions were obtained for the ground state and a number of excited states. Angular distributions are shown in Figs. 5, 6, and 7. Absolute cross sections were determined from comparison with α -particle scattering at 3 MeV; they are given in Table II. The angular distribution of the weak l=2 state at 3.07-MeV excitation indicates a probable spin of $\frac{3}{2}$ ⁺ from the break in the angular distribution at 55° and the relatively sharp decrease from the forward maximum. This, however, is a relatively weak state with a peak cross section only about 20% of that for the 1.28-MeV state. Compound-nucleus effects seem



FIG. 3. Angular distributions for the $O^{16}(d,p)O^{17}$ ground-state reaction. The relative errors are less than the size of the points. The 8-MeV data are those of Ref. b in Table IV. The straight lines are drawn in to emphasize the break developing at ~55°.



FIG. 4. Angular distributions for the $\Gamma^{19}(d, p)\Gamma^{20}$ reaction. The relative errors are less than the size of the points. The transition is probably $\Delta J = \frac{5}{2}$; note the resemblance of the angular distribution to those in Figs. 2 and 3 at the corresponding energies.

to be important at backward angles; the spin assignment should therefore be regarded as tentative. One should also note that the l=2, $\Delta J = \frac{5}{2}$ distribution of the 2.03-MeV state resembles the l=3 distribution of the 3.62-MeV state more than the l=2, $\Delta J = \frac{3}{2}$ distribution of the 1.28-MeV state. Only measurements at angles greater than 50° can resolve this difference unambiguously. The l=0 ground-state transition and the 2.42-MeV and 4.09-MeV excited-state transitions, which show no clear direct amplitudes, are also included.



FIG. 5. Angular distributions for the $Si^{28}(d, p)Si^{29}$ reaction. The relative errors are less than twice the size of the points. The lines are drawn in to smoothly connect the experimental points.





 $S^{32,34}(d,p)S^{33,35}$. Targets of PbS were bombarded with 9- and 12-MeV deuterons and angular distributions for the reactions to states of S³³ were measured. The results for the ground state and the 1.97- and 2.31-MeV excited states are shown in Fig. 8. The angular distribution of the weakly populated 1.97-MeV state shows forward peaking only at 12 MeV. We cannot assign an l value on this basis. The 2.31-MeV state appears to be spin $\frac{3}{2}$ +

TABLE I. Relative cross sections.

Excitation energy (MeV)	Relative cross	section (θ_{\max})	$ heta_{\max}^{a}$		
	$C^{12}(d,p)C^{13}$				
	At $E_d = 11 \text{ MeV}$	At $E_d = 13$ MeV			
g.s.	1.0 ^b	1.0 ^b	20°c		
3.09	0.55	0.65	40°d		
3.68	0.72	0.60	20°°		
3.85	4.3	4.3	25°°		
	$S^{32.34}(d,p)S^{33.35}$				
S32	At $E_d = 9$ MeV	At $E_d = 12$ MeV			
g.s.	1.0 ^b	1.0 ^b	30°		
0.84	0.21	0.45	30°ª		
1.97	0.03-0.06°	0.16°	20°		
2.31	0.19	0.27°	30°		
2.92	1.7	1.8	35°		
3.22	7.2	5.4	20°		
S ³⁴					
g.s.	1.0 ^b	1.0 ^b	30°		
1.99	2.4	2.1	30°		

* Angle at which the angular distributions showed maximum yield. ^b The cross sections are normalized to the ground-state (g.s.) yield at each energy. ^c This is the approximate angle for maximum yield; no clear decrease was observed in the yield for smaller angles. ^d The position of the secondary maximum in the l=0 angular distribution. ^e The angular distribution does not show stripping behavior. The range of cross sections is given.

although it is too weak to make this a definite assignment. This result is consistent with tentative assignments from gamma-ray angular correlations.⁵ Angular distributions at 12 MeV to states which are not l=2are shown in Fig. 9. The angular distributions for the $S^{34}(d,p)S^{35}$ reaction to the ground state and 1.99-MeV excited state were obtained with an isotopically enriched target. They are shown in Fig. 10. Relative cross sections are given in Table I.

 $Zr^{90,92,94}(d,p)Zr^{91,93,95}$. Angular distributions were obtained for 12-MeV deuterons bombarding self-sup-



⁵ Jean M. O'Dell, Atomic Energy Commission Report No. COO-1120-39 (unpublished).

FIG. 8. Angular distributions for the $S^{32}(d,p)S^{33}$ reaction. The relative errors are less than 5% for the ground state, 10% for the 2.31-MeV state and 20% for 1.97-MeV state. The lines are drawn in to smoothly connect the points.

ο 30 60 90 120 150

θ_{c.m.}

S³²(d,p)S³³

₽= 2, J= 3 ×E,=231 MeV \$= 2, J=(3)

> 120 150

E.= 12 MeV

porting isotopically enriched Zr metal foils. Angular distributions were extracted for only the two strongest l=2 transitions which are believed to have spins of $\frac{5}{2}$ + and $\frac{3}{2}^+$, on the basis of reasonable arguments regarding their spectroscopic factors.⁶ The results are shown in Fig. 11.

The l=0 angular distributions for C¹², Si²⁸, and S³² are compared in Fig. 12.

III. EVIDENCE FOR J-DEPENDENT EFFECTS

A. J Dependence for l=1 Transitions in the 1*p* Shell

There is only a limited number of possible (d,p)reactions in the 1p shell in which transitions with known total angular-momentum transfer proceed between bound states. These are listed in Table III together with (p,d) reactions which should show the same

TABLE II. Cross sections for the $Si^{28}(d,p)Si^{29}$ reaction.

Excitation energy	$\sigma(\theta)$	_{max})ª (mb/sr)		
(MeV)	At $E_d = 9$ MeV	At 11 MeV	At 13 MeV	θ_{\max}^{b}
0.0	1.7	2.0	2.9	~30°°
1.28	3.9	4.8	5.1	$\sim 30^{\circ}$
2.03	1.5	1.8	1.8	$\sim 35^{\circ}$
2.42	0.2 - 0.7	0.2 - 0.4	0.05 - 0.2	d
3.07	0.8	1.2	0.9	20°
3.62	4.8	4.4	4.4	30°
4.08	0.2-0.5	0.3	0.5	$\sim 15^{\circ}$

* The errors in the cross section are about 10%. ^b The angle at which the angular distribution showed the maximum yield. ^c At the secondary maximum of the l = 0 angular distribution. ^d The angular distribution does not show a stripping behavior. The range

of cross sections is given. • Slight forward peaking. The yield at the most forward angle is given.

effects. Some transitions for which ΔJ is inferred are also included as are some which proceed to narrow unbound states. References to experiments are included; many more experiments exist but most of them are over a very limited angular range or were performed at bombarding energies below 8 MeV where compoundnucleus effects may be important. The experimental evidence is summarized in Fig. 13.

The $\Delta J = \frac{1}{2}$ transitions all seem to show at least two minima, one at $\sim 40^{\circ}$ and another at 75° - 95° . The yield at the second minimum is not more than half of the yield below or above the angle of the minimum. The $\Delta J = \frac{3}{2}$ transitions exhibit a less pronounced minimum at $\sim 40^{\circ}$ -60° and virtually no second minimum in the vicinity of 80°. It should be noted that these data include results over a very wide region of Q values. Some results on the (p,d) reaction at $E_p = \sim 18$ MeV are shown in Fig. 14. These do not extend to large enough angles to cover the second minimum but the behavior of the first minimum is consistent with the (d, p) results.



FIG. 9. Angular distributions for the $S^{32}(d,p)S^{33}$ reaction. The rela-tive errors are less than 10%.

832

units)

E

SECTION (arbitrary

CROSS

0

E_+=9 Me∨

60

90

30

E_= 197 MeV

'(d,p)S³⁵ Ed= 12.0 MeV

3

FIG. 10. Angular distributions for the $S^{34}(d,p)S^{35}$ reaction. The relative errors are less than 10%.

FIG. 11. Angular distributions for the $Zr^{90,92,94}(d,p)Zr^{91,93,95}$ reactions. The spin values are probable, not definitely established. The relative errors are less than the size of the points.

FIG. 12. Angular distributions for various l=0transitions at $E_d = 12$ MeV for S³² (0.84-MeV state) and 13 MeV for C¹² (3.09-MeV state) and Si²⁸ (ground state). The lines are drawn to connect the points.



TABLE III. List of $l = 1$	transitions of known	ΔJ in the 1 <i>p</i> shell.
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	E_d	Q	. 7	D-f
Target	(MeV)	(MeV)	ΔJ	Ref.
		(d,p) reactio	ns	
Be ⁹	11.8	4.59	32	a
B_{10}	12.0	9.24	32	b
B^{10}	12.0	2.48	$\left(\frac{\overline{1}}{2}\right)$	b
C^{12}	11.0, 13.0	2.72	1/2	с
C^{12}	11.0, 13.0	-0.96	32	с
C^{13}	•	5.95	12	not available
N^{14}	13.6	8.62	$(\frac{1}{2})$	d
		(p,d) reaction	ons	
Be ⁹	11.5	0.6	32	e
B^{10}	7	-8.6	$\left(\frac{\overline{3}}{2}\right)$	e
C^{12}		-16.5	32	not available
N^{14}	8	-8.3	$\left(\frac{\overline{1}}{2}\right)$	f
O ¹⁶		-13.4	12	not available

* U. Schmidt-Rohr, R. Stock, and P. Turek, Nucl. Phys. 53, 77 (1964). b R. H. Siemssen and L. L. Lee, Jr. (to be published). The transition to the 6.76-MeV state proceeds with a large yield and should be predominantly $\Delta J = \frac{1}{2}$. [D. Kurath (private communication).] • Present work. • S. Morita, T. Ishimatsu, T. Cho, Y. Nakajima, N. Kawai, T. Murata, and Y. Hachiya, J. Phys. Soc. Japan 16, 1849 (1961). • J. B. Reynolds and K. G. Standing, Phys. Rev. 101, 158 (1956). * K. G. Standing, Phys. Rev. 101, 152 (1956).

B. J Dependence for l=2 Transitions in the 1d Shell

The results illustrating this effect are summarized in Fig. 15 from the experiments listed in Table IV. It is clear that the J dependence as formulated in I is very well confirmed by the additional data. The effect, however, seems to be most consistent at $E_d = 8-10$ MeV. At higher energies the $\frac{5}{2}$ angular distributions in some cases develop a break which may be confused with the minimum for the $\frac{3}{2}$ + transitions. This is illustrated by the $C^{12}(d,p)C^{13}$ and $O^{16}(d,p)O^{17}$ reactions shown in Figs. 2 and 3, where the $\frac{5}{2}^+$ angular distributions were plotted at various energies. It is also evident in Fig. 4 where the $F^{19}(d,p)F^{20}$ angular distri-



FIG. 13. Angular distributions for l=1 transitions in the 1 p shell with known ΔJ . The data for A and E are from Ref. b of Table III, B and F are from the present work, C is from Ref. d of Table III, and D is from Ref. a of Table III. The lines are drawn to smoothly connect the points.



FIG. 14. Angular distributions for some l=1 (p,d) reactions in 1p-shell nuclei with known or probable ΔJ . The data are those of Refs. e and f of Table III.

bution for $E_d = 13$ MeV develops a sharp break at 50° , where none was present at 9 MeV. The $Si^{28}(d, p)Si^{29}$ data, on the other hand, do not show such changes up to 13 MeV although, as discussed in I, similar difficulties develop at 15 MeV. It would seem, therefore, that this effect becomes unreliable at the higher bombarding energies.

C. J Dependence for l=2 Transitions in the 2d Shell

The evidence for such an l=2 effect in the 2d shell is rather meager and is restricted to the Zr isotopes. There seems to be a difference in the sharpness of maxima and minima, which is summarized in Table V. The effect appears to persist for the various isotopes, but it remains to be seen whether it is present for other 2d-shell nuclei. In other experiments on Zr and Cd,⁷ these effects are not evident. However, the statistical



FIG. 15. Summary of l=2 (d,p) angular distributions for nuclei in the 1*d* shell with ΔJ known. The sources of the data are given in Table IV. The lines are smoothed curves drawn through the

⁷ R. J. Silva and G. E. Gordon, Phys. Rev. 136, B618 (1964); F. G. Perey (private communication).

Designation in Fig. 15	Target	E _d (MeV)	Q (MeV)	ΔJ	Ref.
Δ	C12	11.0	-1 14	5	a
B	O^{16}	80	1 919	25	ĥ
č	018	7.0	1 731	25	č
ŏ	$M\sigma^{24}$	10.1	5.097	5	đ
Ē	Mo ²⁴	10.1	3.14	5	d
ĩ	Mo ²⁴	10.1	4.12	3	ā
Ť	Mo ²⁴	10.1	2.30	3	ā
ر ۲	Mg25	9.0	8.87	5	ē
Ĝ	Mg ²⁶	9.0	3.22	5	e
ĸ	Mg ²⁶	9.0	0.45	30	е
Ĥ	Si ²⁸	9.0	4.97	15	a
$\overline{\mathbf{L}}$	Si ²⁸	9.0	4.22	32	a
M	S ³²	9.0	6.63	32	a
Ñ	S ³⁴	9.0	4.76	3	a
0	Ca42	7.0	4.71	132	f
P	$Ca^{44}(p,d)$	8.1	-9.92	32	g

⁴ J. R. Holt and T. N. Marsham, Proc. Phys. Soc. (London) A66, 467 (1953).

D. Dehnhard and J. L. Yntema, Bull. Am. Phys. Soc. 9, 666 (1964)

⁴ T. A. Belote (private communication). ^e E. Kashy (private communication).

accuracy in these data was not sufficiently good to observe such small effects. In the work of Ref. 6, the curves given for the reaction $Zr^{90}(d,p)Zr^{91}$ at 15 MeV seem to show a qualitatively similar effect. As no crosssection scale is given in the relevant figure (Fig. 3), we cannot be certain whether these effects are statistically significant.

IV. CONCLUSIONS

Further evidence has been accumulated to show that the angular distribution of the (d,p) reaction depends systematically on the total angular-momentum transfer. It would almost appear that some measurable J dependence is present in every case in which sufficiently precise information has been obtained. Distorted-wave

TABLE V. List of l=2 transitions observed in reactions on Zr isotopes at $E_d = 12$ MeV. The ratios R_i are defined by

$$R_i = 2 [\sigma(\theta_{\max_i}) - \sigma(\theta_{\min_i})] / [\sigma(\theta_{\max_i}) + \sigma(\theta_{\min_i})],$$

where θ_{\max_i} and θ_{\min_i} are the *i*th observed maximum and minimum in the measured angular distributions.

Target	Q (MeV)	Spin	R_1	R_2	R_3
Zr ⁹⁰	4.98 2.92	5 2 3 2 3 2	0.35 0.15	0.36 0.27	0.29 0.13
Zr ⁹²	4.46 3.01	5232	0.43 0.05	$0.23 \\ -0.08$	$0.17 \\ -0.06$
Zr ⁹⁴	4.19 2.55	52 32 32	0.49 0.14	$0.14 \\ -0.18$	$0.13 \\ -0.14$
Maximum or minimum No.		-	1	2	3
$ heta_{\min_{i}}$ $ heta_{\max_{i}}$			40° 50°	75° 90°	120° 135°

TABLE IV. List of l=2 transitions of known ΔJ in the 1d shell.

FIG. 16. Angular distributions for the reaction. $\operatorname{Zr}(d,p)$ The curves labeled "Experiment" represent a smoothed average of the data in Fig. 11; the calculations are those of Ref. 18.



Born approximation calculations with spin-orbit parameters derived from elastic scattering and polarization data have not, so far been successful in reproducing the observed J dependences.⁸ The one apparent exception to this is for Zr, for which the calculations are shown in Fig. 16.9 This then raises the question of the usefulness of an empirical rule which is not yet understood theoretically. Is it justified to make spin assignments by use of these empirical rules or should one wait for a better understanding? It is our feeling that, at least in the well-documented cases for the 1d and 2p shell, one can assign spins in a favorable case with a probability of 95% or better that they are correct. In either case, such assignments should be made with care until such time as more supporting evidence has been accumulated. All the J-dependent effects observed in (d, p)reactions are schematically summarized in Fig. 17.

In recent experimental work on the (p,d) reaction, J-dependent effects similar to the ones reported in I have been observed for nuclei around $A \approx 60^{10}$ with deuteron energies in excess of 20 MeV.

J-dependent effects have also been found in other direct reactions such as (d,t),¹¹ (He³,d),¹² (He³, α),¹³ (α, p) ¹⁴ and (d, n).¹⁵ It would again seem that only a



FIG. 17. Summary of J-dependent effects for deuterons with $E_d = 10-12$ MeV. The cases shown are typical of the observed effects and are taken from either I or the present paper.

lack of sufficiently accurate or extensive experimental data prevents one from finding these effects in other cases. Because of the obvious usefulness of such effects in nuclear spectroscopy, it is to be hoped that more extensive systematic work will be done in the future in an effort to establish the general validity of J dependence in direct reactions.

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⁸ G. R. Satchler and R. H. Bassel (private communication).

⁹ We are indebted to R. H. Bassel for these DWBA calculations. ¹⁰ C. M. Glashausser, thesis, Princeton University, 1965

⁽unpublished). ¹¹ R. H. Fulmer and W. W. Daehnick, Phys. Rev. Letters 12, 455 (1964).

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¹³ C. Mayer-Böricke, R. H. Siemssen, and L. L. Lee, Jr., Bull. Am. Phys. Soc. 10, 26 (1956); and M. K. Brussel, D. E. Rundquist, and A. I. Yavin, Phys. Rev. 140, 838 (1965).
¹⁴ L. L. Lee, Jr., A. Marinov, C. Mayer-Böricke, J. P. Schiffer, R. H. Bassel, R. M. Drisko, and G. R. Satchler, Phys. Rev. Letters 14, 264 (1965).

 <sup>14, 261 (1965).
 &</sup>lt;sup>15</sup> S. G. Buccino, D. S. Gemmell, L. L. Lee, Jr., J. P. Schiffer, and A. B. Smith, Bull. Am. Phys. Soc. 10, 511 (1965).