Investigation of the Levels in Na^{23} below 3 MeV by the $Mg^{26}(p,\alpha\gamma)Na^{23}$ Reaction*

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The particle-gamma angular correlation method of Litherland and Ferguson was used to gain information on the decay modes and spin assignments of the levels in Na^{23} below 3 MeV as well as the mixing ratios of the transitions from these levels. The $Mg^{26}(p,\alpha)Na^{23}$ reaction was used at proton energies between 9.3 and 10.5 MeV. The spin of the first excited state was verified as $\frac{5}{2}$. Rigorous limitations are given for the spins of the other five levels, the most probable assignment being 2.08 MeV($\frac{7}{2}^{(+)}$), 2.39($\frac{1}{2}^{(+)}$), 2.64($\frac{1}{2}$), 2.71($\frac{9}{2}$), 2.98($\frac{3}{2}$). The results are discussed in terms of the Nilsson model for the N or $Z=11$ nuclei. In particular the phases of the mixing ratios for the transitions from the first two excited states of these nuclei are compared. The problem of the comparison of the signs of the experimentally determined mixing ratios quoted by different authors is also treated.

INTRODUCTION

IT has been known for a number of years that low-lying states of nuclei in the region $19\leqslant A\leqslant 25$ can be described in terms of the collective Nilsson[1] model for deformed nuclei, the deformation being prolate (cigar-shaped) with $\eta\sim 4$. For such a deformation, in the case of Na^{23} (for the ground state) the last odd proton would be on the $K^\pi=\frac{3}{2}^+$ Nilsson level 7. Excited configurations could be obtained by raising this proton to the Nilsson levels 5($K^\pi=\frac{5}{2}^+$) and 9($K^\pi=\frac{1}{2}^+$) or by raising a proton from the $K^\pi=\frac{1}{2}^+$ level 6 to level 7, leaving a hole in level 6. Paul and Montague[2] described the low-lying states of Na^{23} by considering the interaction between the levels of the rotational bands based on particle orbits 7, 9, and 5 with the same moment of inertia parameter for each band. Clegg and Foley[3] made a calculation in which a particle on level 7 and a hole in level 6 gave rise to two interacting rotational bands while Braben, Green, and Wilmott[4] used a similar picture to Paul and Montague but varied the moment of inertia for each band. In a recent paper, Glöckle[5] showed, however, by computing the total binding energy of Na^{23} as a function of deformation, that for $\eta=4$ the $K=\frac{3}{2}$ level 7 was indeed the lowest single-particle orbit but that particle excitations to levels 9 and 5 or a hole in level 6 were all at roughly the same energy of 3 MeV above the ground state. He therefore considered the interaction of states based on all four Nilsson levels 7, 9, 5, and 6 (hole). In this manner for the levels below 3 MeV the experimental energy spectrum was well reproduced, while predicted spins matched the most probable experimental spin assignments in each case except for the 2.98-MeV level.

The ground state and the first two excited states of Na^{23} have spins $\frac{3}{2}^+$, $\frac{5}{2}^+$, and $\frac{7}{2}^{(+)}$ (or $\frac{3}{2}^+$), respectively. If the 2.08-MeV level is indeed $\frac{7}{2}$, they are reasonably well described as unperturbed rotational states based on the $K^\pi=\frac{3}{2}^+$ ground state. Their dynamic properties (as pointed out by Howard, Allen, and Bromley[6]), should be quite well described by the simple Nilsson model. Elliott[7] showed that collective properties (in particular, rotational properties) of the nuclei in this region could be understood in terms of the usual shell model with only a small amount of configuration mixing. In view of the success of recent shell-model calculations in the p-shell in explaining energies of levels as well as dynamic properties such as $M1$ transition strengths and the application of similar methods in the s-d shell, further experimental results with which to confront the theoretical predictions are needed in the $19\leqslant A\leqslant 24$ region of the periodic table.

A careful examination of the literature showed that of the states below 3 MeV only one has so far received a rigorous spin assignment—the ground state. Even the assignment of $\frac{5}{2}$ to the first excited state ruled out $\frac{3}{2}$ by only two standard deviations as will be discussed below.

The purpose of the present work was then threefold:

(1) to make rigorous spin assignments where possible or to limit rigorously the possible level spins;

(2) to compare the experimentally determined spins with those predicted by the collective models;

(3) to compare the dynamic properties of the excited states (decay modes and mixing ratio) with the predictions of the simple Nilsson model as discussed above.

The results of the present work combined with previous

* Work supported in part by the U. S. Atomic Energy Commission.

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[2] E. B. Paul and J. H. Montague, Nucl. Phys. 8, 61 (1958).
[3] A. B. Clegg and K. J. Foley, Phil. Mag. 7, 247 (1962).
[4] D. W. Braben, L. L. Green, and J. C. Willmott, Nucl. Phys. 32, 584 (1962).
[5] W. Glöckle, Z. Physik 178, 53 (1964).
[6] A. J. Howard, J. P. Allen, and D. A. Bromley, Phys. Rev. 139, B1135 (1965).
[7] J. P. Elliott, Proc. Roy. Soc. (London) A245, 128 (1958).

spin and parity assignments are summarized in Fig. 1. We consider that the bracketed quantities have been neither rigorously assigned nor excluded as the case may be. In the next section the experimental method and results will be discussed.

EXPERIMENTAL METHOD AND RESULTS

Litherland and Ferguson[8] pointed out the simplification in the analysis of the gamma-ray angular distributions which can result when the reaction product in the reaction $X(a,b)Y^*$, $Y^* \rightarrow Y+\gamma$ is detected at 0° or 180° with respect to the beam. The maximum value of the magnetic quantum number of the gamma-emitting state Y^* is the sum of the spins a, b, X. Consequently the (p,α) reaction on a spin-zero target nucleus with the subsequent detection of the alpha particle at 180° populates only the $m=\pm\frac{1}{2}$ substates of the residual state and the alignment is complete. For this reason the (α,γ) angular correlations observed in the reaction $Mg^{26}(p,\alpha)Na^{23}$ with the alpha particles detected at 180° depended only on the spins of the levels involved and the multipole mixing ratio (x) of the emitted gamma ray.

The scattering chamber and correlation table which were used have been previously described.[9] Briefly, the alpha particles were detected in an annular surface-barrier detector (resistivity ~300 Ω-cm) placed symmetrically about the beam. The edges of the sensitive annulus subtended angles at the target of 171° and 175° with respect to the beam direction. The gamma rays were detected in either a 3×3-in. or a 5-in. (diam.) ×6-in. (long) NaI crystal whose front faces were 15 cm and 25 cm, respectively, from the target center. The 3×3-in. crystal was used to study all the levels which were investigated while the 5×6-in. crystal was used to study the 2.71-MeV level where maximum photopeak efficiency for the 2.27-MeV gamma radiation was needed. These detectors could be set at any angle from 20° to 90° with respect to the beam direction. The metallic targets were enriched to 95% in Mg^{26}. They were approximately 170 $\mu g/cm^2$ thick and were supported on carbon films approximately 20 $\mu g/cm^2$ thick.

Protons of energy between 9.3 and 10.5 MeV were provided either by the Tandem Electrostatic Generator, Atomic Energy Research Establishment, Harwell, or the Oxford Nuclear Physics Laboratory tandem Van de Graaff accelerator. As has been shown,[10] the cross sections for the formation of the various levels in Na^{23} by the reaction $Mg^{26}(p,\alpha)Na^{23}$ in this region of bombarding energy fluctuate markedly as the incident proton energy changes. For this reason it was possible to choose a bombarding energy for each level studied so that its excitation was a maximum, while the nearby levels were only weakly excited. Furthermore it was possible to clearly resolve all the α groups (except those leading to the 2.64- and 2.71-MeV levels) in the particle counter so that the windows set on the α groups were very clean. An example of the observed alpha spectra is given in Fig. 2. The angular-correlation results for each level

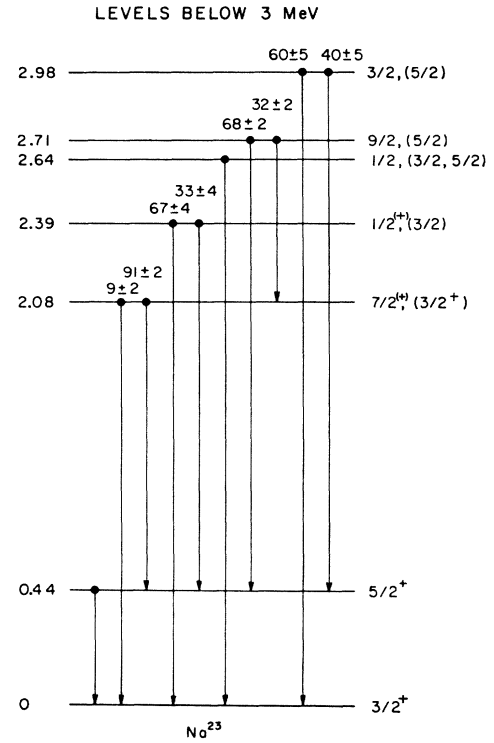

FIG. 1. A summary of the gamma-ray branchings measured in the present work for the excited states below 3 MeV in Na^{23}, together with spin and parity assignments from this and previous work.

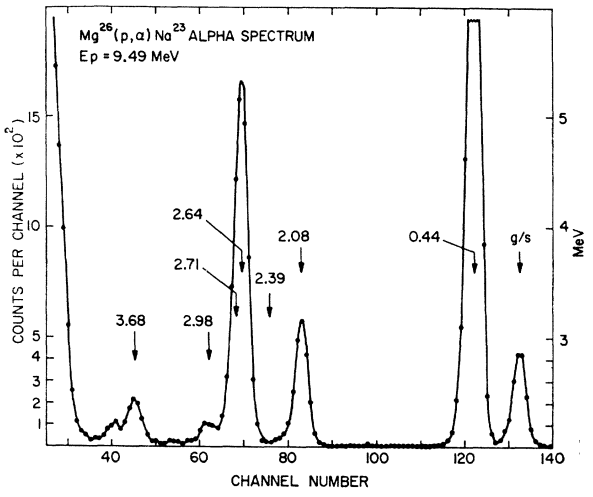

FIG. 2. Alpha-particle spectrum recorded in the annular detector at an incident energy of 9.49 MeV. At this energy the gamma-ray angular distributions in coincidence with the unreal solved α-groups leading to the 2.64- and 2.71 MeV levels were studied. Note the relatively small excitation of the nearby levels at this bombarding energy. The α groups are labeled by the energy of the state in Na^{23} to which they lead.

[8] A. E. Litherland and A. J. Ferguson, Can. J. Phys. **39**, 788 (1961).
[9] M. A. Grace and A. R. Poletti, Nucl. Phys. **78**, 273 (1966).
[10] B. Allardyce *et al.*, Nucl. Phys. (to be published).

will be discussed in the next section. The method of analysis which was used will be illustrated in the case of the 0.44-MeV level which will be discussed first.

The 0.44-MeV Level

The spin-parity assignment of $\frac{5}{2}^+$ to the 0.44-MeV state in Na^{23} has been made on the following evidence: Coulomb excitation[11,12] of the level, together with the observed anisotropy[13,14] of the 0.44-MeV gamma ray at the 1.29-MeV resonance in inelastic proton-scattering limited the spin to $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$ with positive parity. The $\frac{7}{2}$ assignment was eliminated by the measured partial[11,12] $E2$ and total[15–18] lifetimes which implied that the level de-excited mainly by $M1$ radiation. The assignment $\frac{5}{2}^+$ was made on the basis of Temmer and Heydenberg's[11] measurement of the angular distribution of the de-excitation gamma rays in Coulomb excitation. They observed an anisotropic distribution characterized by $a_2=0.054\pm0.027$, while in a similar experiment Stelson and McGowan[12] observed a distribution characterized by $a_2=(0.022\pm0.008)$. Since for a spin assignment of $\frac{3}{2}$, an (accidental) isotropy should be observed, these two experiments rule out $\frac{3}{2}$ to 2 and 2.7 standard deviations, respectively. For $J_1=\frac{5}{2}$, the angular distribution measured by Stelson and McGowan implied a mixing ratio $x=+0.137\pm0.022$. However, from the measured lifetimes[11,12,15–18] for a $\frac{5}{2}$ assignment, $|x|=(0.05\pm0.006)$, while from the independent work of Mizobuchi, Katoh, and Ruan,[14] which gives the sign as well, $x=+(0.045\pm0.015)$. Because these latter two determinations differ from the results of the very careful experiment of Stelson and McGowan,[12] we considered it worthwhile (as part of the experimental program) to investigate the spin of the level and the mixing ratio (x) of the de-exciting radiation in a completely independent way.

The absorption of the scattering chamber was too uncertain at angles further forward than 30° for the low energy of the gamma ray de-exciting the 0.44-MeV level. The correlation was therefore measured at four angles between 30° and 90° with respect to the beam. A least-squares computer fitting of the resulting distribution to the expansion $W(\theta)=1+a_2P_2(\cos\theta)+a_4P_4(\cos\theta)$ where $P_k(\cos\theta)$ is the kth-order Legendre polynomial, yielded $a_2=-(0.229\pm0.016)$ and $a_4=+(0.004\pm0.027)$. The experimental points were then further fitted to the theoretical distribution,

$$W(\theta)=\sum_k \rho_k(a)F_k(ab)Q_kP_k(\cos\theta),$$

where $\rho_k(a)=\sum_\alpha\rho_k(a,\alpha)P(\alpha)$ is the statistical tensor describing the alignment of the state a which decays to the state b. $F_k(ab)$ which describes the emission of the gamma radiation is defined as

$$F_k(ab)=\{F_k(LLba)+2xF_k(LL'ba)+x^2F_k(L'L'ba)\}/(1+x^2),$$

where x is the multipole mixing ratio (e.g., $E2$ to $M1$ amplitude ratio) defined by Eq. (3) of Ref. 19. The quantities $\rho_k(a,\alpha)$, $F_k(LL'ba)$, and the related coefficient $U_k(\text{lab})$ are defined and tabulated by Poletti and Warburton.[20] For the rest of this paper the spin of the initial level will be referred to as J, which is the usual notation. The results of the above least-squares fitting are shown in Fig. 3 in the form of χ^2 versus $\tan^{-1}x$ for values of the initial spin of $J=\frac{1}{2}$ to $\frac{7}{2}$. A description of the fitting method has been given previously.[20] It can be seen that $J=\frac{1}{2}$ and $\frac{7}{2}$ are immediately eliminated while for $J=\frac{5}{2}$ there is one solution: $x=+(0.08\pm0.02)$ (including any effect due to the finite size of the annular detector) and for $J=\frac{3}{2}$, two solutions: $x=-(0.50\pm0.04)$ or $x=-(5.0\pm0.80)$. In order to eliminate the solutions for $J=\frac{3}{2}$, it is necessary to consider the value of the mixing ratio determined by the lifetime measurements[11,12,15–18]: $x=0.05\pm0.006$, which is independent of

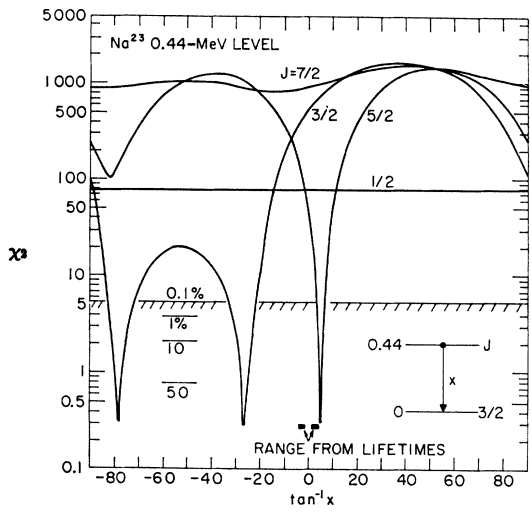

FIG. 3. χ^2 versus $\tan^{-1}x$ for the least-squares fit to the gamma-ray angular distribution observed in coincidence with α-particles leading to the 0.44-MeV level of Na^{23}. The assignment $J=\frac{3}{2}$ can be eliminated since the known total and partial $E2$ lifetimes limit the value of the mixing ratio to the two ranges indicated in the figure.

[11] G. M. Temmer and N. P. Heydenberg, Phys. Rev. **104**, 989 (1956).
[12] P. H. Stelson and F. K. McGowan, in *Proceedings of the Second Conference on Reactions between Complex Nuclei (1960)*, edited by A. Zucker, E. C. Halbert, and F. T. Howard (John Wiley & Sons, Inc., New York, 1960).
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[16] W. L. Mouton, J. P. F. Sellschop, and R. J. Keddy, Phys. Rev. **128**, 2745 (1962).
[17] V. K. Rasmussen, F. R. Metzger, and C. P. Swann, Nucl. Phys. **13**, 95 (1960).
[18] B. Ambrozy *et al.*, in *Proceedings of the Rutherford Jubilee Conference, Manchester, 1961*, edited by J. B. Birks (Heywood and Company, Ltd., London, 1961).
[19] M. Thomas, J. Lopes, R. Ollerhead, A. Poletti, and E. Warburton, Nucl. Phys. **78**, 298 (1966).
[20] A. R. Poletti and E. K. Warburton, Phys. Rev. **137**, B595 (1965).

TABLE I. Summary of spin assignments to levels and the observed angular distributions. The coefficients a_2 and a_4 of the expansion $W(\theta)=1+a_2P_2(\cos\theta)+a_2P_4(\cos\theta)$ have been corrected for the finite angle of the γ-ray detector.

Level (MeV)	Spin assigned	Bombarding energy (MeV)	Transition	a_1	a_4
0.44	$\frac{5}{2}$	9.23	0.44 to 0	-0.229 ± 0.015	0.004 ± 0.03
2.08	$\frac{7}{2}$, or $(\frac{3}{2})$	9.67	to 0.44	0.057 ± 0.036	0.031 ± 0.069
			0.44 to 0	-0.227 ± 0.023	$-0.07\ \pm0.05$
2.39	$\frac{1}{2}$, or $(\frac{3}{2})$	9.34	2.39 to 0	$0.08\ \pm0.06$	$-0.03\ \pm0.06$
			to 0.44	$-0.06\ \pm0.06$	$-0.04\ \pm0.09$
			0.44 to 0	$0.08\ \pm0.04$	$-0.04\ \pm0.06$
2.64	$\frac{1}{2}$, or $(\frac{3}{2}, \frac{5}{2})$	9.49 (Harwell)	2.64 to 0	$0.00\ \pm0.03$	$-0.02\ \pm0.05$
2.71	$\frac{9}{2}$, or $(\frac{5}{2})$	10.48 (Oxford)	2.71 to 0.44	$0.41\ \pm0.09$	$-0.34\ \pm0.13$
			2.71 to 2.08	$-0.09\ \pm0.09$	$-0.02\ \pm0.02$
			0.44 to 0	$-0.16\ \pm0.03$	$0.04\ \pm0.05$
2.98	$\frac{3}{2}$ or $(\frac{5}{2})$	9.98	2.98 to 0	$0.52\ \pm0.06$	$-0.01\ \pm0.10$
			2.98 to 0.44	$0.22\ \pm0.20$	$0.33\ \pm0.30$

the spin of the 0.44-MeV state. This solution and the smallest value from the angular-correlation work for $J=\frac{3}{2}$ are separated by a total of ten standard deviations—as can be seen from Fig. 3, $J=\frac{3}{2}$ is thus completely ruled out. To summarize: the spin of the first excited state of Na^{23} has been verified as $\frac{5}{2}$ and the mixing ratio for its de-excitation was measured as 0.08 ± 0.02 which is just outside the errors of the estimate from the lifetimes and that of Ref. 14.

The 2.08-MeV Level

At the bombarding energy used to study this level it was the most intense peak in the α-particle spectrum. Furthermore, the α group leading to the 2.39-MeV level had an intensity of $<3\%$ of that leading to the 2.08-MeV level. The coincident gamma-ray spectrum therefore represents accurately the decay mode of the 2.08-MeV level. From the summed γ-ray spectrum it was found to decay $(91\pm2)\%$ to the 0.44-MeV level and $(9\pm2)\%$ to the ground state. The angular distribution of both the 1.64-MeV and 0.44-MeV cascade radiations were obtained (see Table I). The ground-state branch was too weak for any meaningful extraction of an angular distribution. The results of a simultaneous fitting of the two cascade distributions is shown in Fig. 4. $J=\frac{9}{2}$ and $\frac{5}{2}$ are both ruled out, the former by a very large margin. The minimum value of χ^2 for $J=\frac{3}{2}$ lies at the 10% confidence limit and cannot be rigorously excluded. It is, however, roughly 5 times less likely than the solution $J=\frac{7}{2}$, $x=+(0.20\pm0.03)$.

The percentage branching measured by us agrees with the branching quoted by Endt and Van der Leun.[21] It agrees also with the recent result obtained by Lancman *et al.*,[22] and by Wernbom-Selin and Arnell[23] though not with the results of Howard *et al.*[24]

It has been assumed for a number of years that the spin parity of this level at 2.08 MeV is $J^\pi=\frac{7}{2}^+$. However, neither assignment has been at all rigorous, the spin being assigned as far as can be seen on the basis of the decay mode only (the angular-distribution work reported in Ref. 4 could not have distinguished between

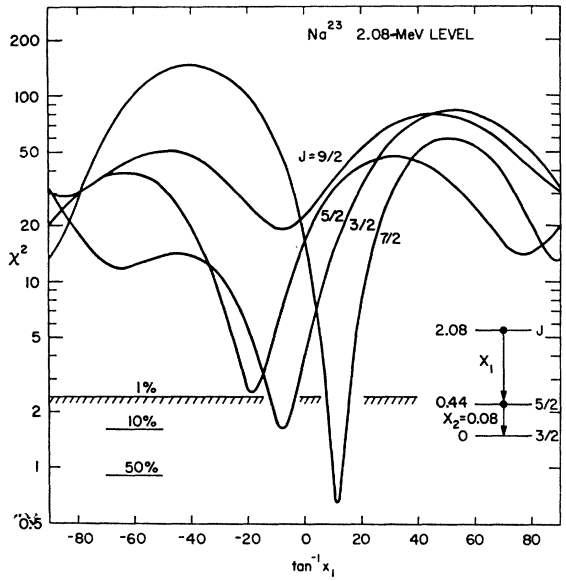

FIG. 4. χ^2 versus $\tan^{-1}x_1$ for the simultaneous least-squares fitting to the cascade gamma-ray angular distributions for the 2.08-MeV level of Na^{23}. The solution for $J=\frac{3}{2}$ is about 5 times less likely than that for $J=\frac{7}{2}$.

[21] P. M. Endt and C. Van der Leun, Nucl. Phys. **34**, 1 (1962).

[22] H. Lancman, A. Jasinski, J. Kownacki, and J. Ludziejewski, Nucl. Phys. **69**, 384 (1965).

[23] E. Wernbom-Selin and S. E. Arnell, (unpublished).

[24] A. J. Howard, J. P. Allen, D. A. Bromley, and J. W. Olness, Bull. Am. Phys. Soc. **9**, 68 (1964).

a number of spin assignments). The positive parity was assigned in an unpublished thesis on the basis of inelastic deuteron scattering (the logft value of 5.8 for the β decay from Ne^{23} is not conclusive evidence for an allowed transition). The present work favors the spin assignment $J=\frac{7}{2}$. For this spin, the deduced value of $x=+(0.20\pm0.03)$ can be compared with that of Howard *et al.*,[6] of $0.22\leqslant x\leqslant 0.60$ or $1.3\leqslant x\leqslant 2.5$ assigned on the basis of γ-γ correlation following the decay of Ne^{23}. The smaller of the two values of x obtained by Howard *et al.*,[6] just overlaps with the value determined by us. The phase is also the same, since Howard *et al.*,[6] have already converted their quoted phase to that corresponding to an emission type process according to the convention of Lloyd.[25]

The 2.39-MeV Level

At the bombarding energy used to study this level the intensity of the α-group leading to it was 4 times and 5 times those of the groups leading to the 2.08 and 2.64 and 2.71-MeV levels, respectively. From the summed γ-ray spectrum (Fig. 5) the decay modes $(67\pm4)\%$ to ground and $(33\pm4)\%$ to the first excited state were determined. This result was in good agreement with the mean values quoted by Endt and Van der Leun[21] as well as with the recent measurement by Wernbom-Selin and Arnell.[23] The angular correlations of the 2.39, 1.95, and 0.44-MeV gamma rays were analyzed. All three distributions were very nearly isotropic (see Table I). A simultaneous fitting of the 1.95 and 0.44-MeV γ-ray angular distributions (see Fig. 6) eliminated all spin assignments except $J=\frac{1}{2}$ and $J=\frac{3}{2}$ ($x=-4.6\pm1.6$ for the transition to the 0.44-MeV state). For the ground-state transition (Fig. 7) for $J=\frac{3}{2}$, two values of x were allowed: $x=-(0.20\pm0.05)$ or $x>10$. For both spin assignments, our observed angular distribution for the 2.39 MeV γ-ray is in sharp disagreement with the anisotropic distribution observed by Wernbom-Selin and Arnell.[23] We conclude on the basis of the present work that for this level at 2.39 MeV, $J=\frac{1}{2}$ or $\frac{3}{2}$ and for both spin assignments the transition to the 0.44 MeV state must be largely quadrupole. Other experiments, none of them conclusive, favor $\frac{1}{2}$ over $\frac{3}{2}$. Perhaps the best evidence available is the observation of a strong forward peak in the $Ne^{22}(d,n)Na^{23}$ reaction by Paul and Montague.[26] This could be interpreted as an $l=0$ stripping pattern which would imply spin-parity $\frac{1}{2}^+$ for the level.

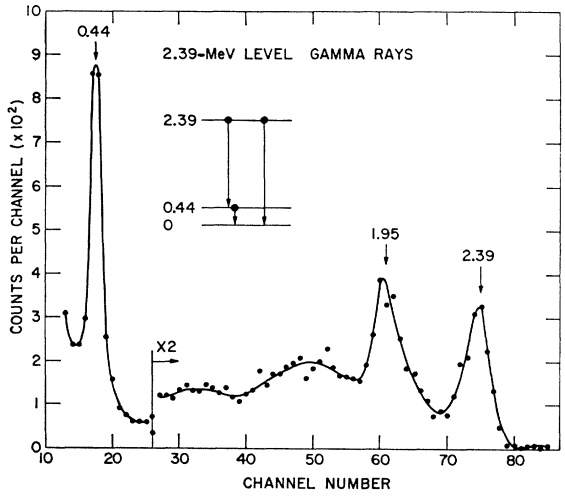

FIG. 5. Gamma rays in coincidence with alphas leading to the 2.39-MeV level in Na^{23}.

The 2.64- and 2.71-MeV Levels

It was not possible to resolve the α-particle groups leading to these two levels. The decay of both levels was therefore studied at the same time. The gamma ray spectrum in coincidence with alpha particles leading to these levels is shown in Fig. 8. Because of a deterioration of the α detector there is a low background beneath the peaks of interest due to some breakthrough from the higher lying 2.98-MeV level. The prominent peaks in the spectrum derive from the ground-state transition from the 2.64-MeV level and the cascades from the 2.71-MeV level through the 2.08 and 0.44-MeV levels. The decay modes of these levels at 2.64 and 2.71 MeV

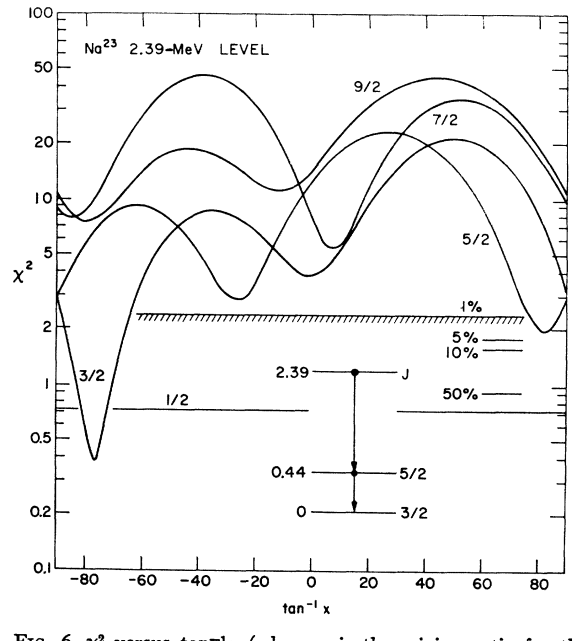

FIG. 6. χ^2 versus $\tan^{-1}x$ (where x is the mixing ratio for the transition to the 0.44 MeV level) for the simultaneous least-squares fitting to the cascade gamma-ray angular distributions for the 2.39-MeV level of Na^{23}. All assignments except $J=\frac{1}{2}$ and $\frac{3}{2}$ are rejected at the 5% confidence limit.

[25] S. P. Lloyd, Phys. Rev. 83, 716 (1951).

[26] E. B. Paul and J. H. Montague, Nucl. Phys. 54, 497 (1964).

are known to be very different[21] and the observed gamma-ray spectra were consistent with the hypothesis that the 2.64-MeV level decayed entirely to the ground state. The 2.71-MeV level was then found to decay 68±4% to the 0.44-MeV state and 32±4% to the 2.08-MeV state. This was assumed in the analysis of the angular distributions of the gamma rays de-exciting these levels. The measured branching for the 2.71-MeV level is in fair agreement with the results of Wernbom-Selin and Arnell[23] (55% and 45% to the 0.44- and 2.08-MeV state, respectively). Freeman and Montague[27] had previously concluded that the major decay mode of the 2.71-MeV state was to the 0.44-MeV level. The above three results all disagree with an earlier measurement of Kruse, Bent, and Lidofsky.[28]

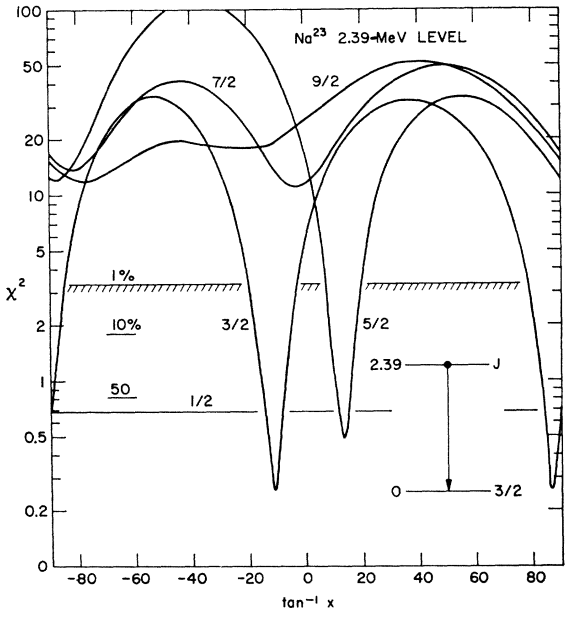

FIG. 7. χ^2 versus $\tan^{-1}x$ for the least-squares fitting to the angular distribution of the ground-state gamma ray from the 2.39-MeV level $J=\frac{1}{2}$, $\frac{3}{2}$, or $\frac{5}{2}$ are all allowed values, the latter two spins for particular values of x given in Table III.

The angular distribution of the full energy peak of the 2.64-MeV gamma ray was found to be isotropic within the statistical uncertainty. The allowed values of the spin for this level are then $J=\frac{1}{2}$, $\frac{3}{2}$, or $\frac{5}{2}$ (see Fig. 9), $\frac{3}{2}$ and $\frac{5}{2}$ being allowed only for certain values of x (see Table III). Because of the isotropy of the 2.64-MeV gamma ray and the availability of a standard gamma-ray source (Th^{228}) of practically the same energy we were able to extract the angular distribution of the 2.27-MeV gamma ray quite accurately. Nonzero values

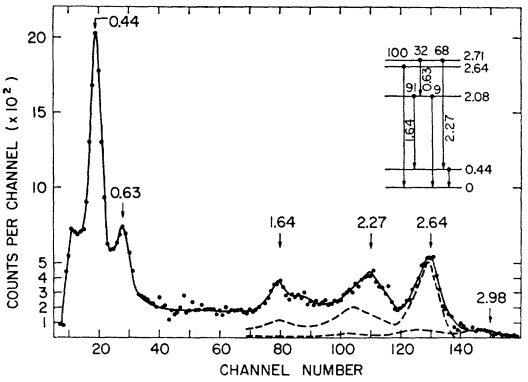

FIG. 8. Gamma rays in coincidence with alphas leading to the 2.64- and 2.71-MeV levels in Na^{23}. The small peak at 2.98 MeV is due to gamma rays from the 2.98 MeV level. The alpha counter had deteriorated and its resolution was somewhat worse than that shown in Fig. 2. The spectral shapes for the 2.98- and 2.64-MeV gamma rays have been sketched in, allowing the peaks due to the 1.64- and 2.27-MeV gamma rays to be clearly seen.

of a_2 and a_4 eliminated $J=\frac{1}{2}$ and $\frac{3}{2}$. When the distribution was fitted (see Fig. 10) to the theoretical angular distributions for spin assignments of $J=\frac{5}{2}$ through $\frac{11}{2}$ to the 2.71-MeV level, both $J=\frac{5}{2}$ ($x=+1.80\pm0.35$) and $J=\frac{9}{2}$ ($x=-0.05\pm0.07$) gave acceptable χ^2 values. $J=\frac{11}{2}$ gave an acceptable fit for a large value of x ($|x|>2.5$—i.e., mostly $E4$ or $M4$ radiation, but this possibility is eliminated by the lifetime limit of 10^{-7} sec established by the coincidence resolving time. We conclude that this state is either $J=\frac{9}{2}$ or $\frac{5}{2}$. We were unable to obtain any further substantial discrimination against the solution for $J=\frac{5}{2}$ by fitting the 2.27 and 0.44-MeV angular distributions simultaneously because

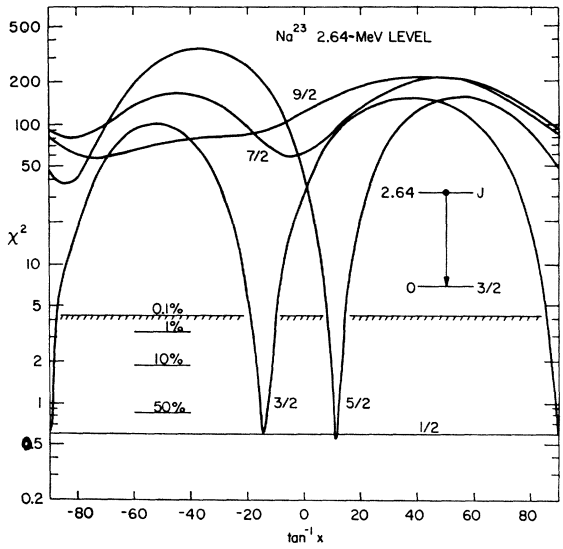

FIG. 9. χ^2 versus $\tan^{-1}x$ for the ground-state transition from the 2.64-MeV state. $J=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ are all allowed. Other assignments are eliminated at the 1.0% confidence limit.

[27] J. M. Freeman and J. H. Montague, Nucl. Phys. **9**, 181 (1958/59).
[28] T. H. Kruse, R. D. Bent, and L. Lidofsky, Phys. Rev. **119**, 289 (1960).

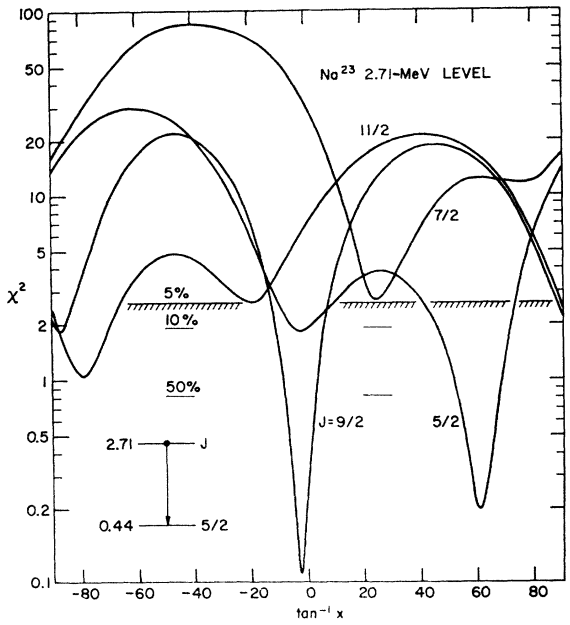

FIG. 10. χ^2 versus $\tan^{-1}x$ for the angular distribution of the 2.27-MeV transition between the 2.71- and 0.44-MeV states in Na^{23}. The solution for $J=\frac{1}{2}^1$ is rejected by lifetime considerations. Of the remaining possibilities both $J=\frac{9}{2}$ and $\frac{5}{2}$ give acceptable solutions.

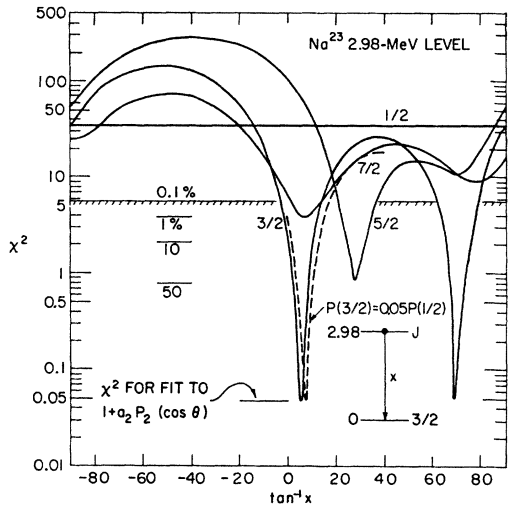

FIG. 12. χ^2 versus $\tan^{-1}x$ for the angular distribution to the ground-state transition from the 2.98-MeV level. Only $J=\frac{3}{2}$ and $\frac{5}{2}$ are not rejected at the 1% confidence limit. The dashed curve shows the effect on the mixing ratio for the $\frac{3}{2}$ assignment of a 5% population of the $m=\pm\frac{3}{2}$ substates.

of insufficient statistical accuracy on the angular distribution of the 0.44-MeV transition. In principle, however, this would provide the necessary discrimination. The almost isotropic distribution of the 0.63-MeV gamma ray leading from the 2.71-MeV level to the 2.08-MeV state could be fitted (for certain ranges of x) for the spin combinations $\frac{9}{2}\rightarrow\frac{7}{2}$, $\frac{5}{2}\rightarrow\frac{7}{2}$, and $\frac{5}{2}\rightarrow\frac{3}{2}$ and did not further limit the possible spins of these two levels.

The 2.98-MeV Level

The summed gamma-ray spectrum obtained in coincidence with α particles leading to this level is shown in Fig. 11. The measured decay modes [(60±5)% to ground and (40±5)% to the 0.44-MeV state] are in good agreement with the recent measurement by Wernbom-Selin and Arnell[23] (55 and 45%, respectively). The angular distribution of the ground-state transition was quite anisotropic (see Table I). $J=\frac{1}{2}$ was thus immediately eliminated. Figure 12 which is a plot of χ^2 versus $\tan^{-1}x$ allows $J=\frac{7}{2}$ to be eliminated at the 1% confidence limit. Solutions were, however, obtained for both $J=\frac{3}{2}$ ($x=0.11\pm0.05$ and 2.70 ± 0.40) and $J=\frac{5}{2}$ ($x=0.54\pm0.11$). The solution for $J=\frac{5}{2}$ is to some extent ruled against, since this would demand $a_4=0.13$ whereas the measured value is $a_4=-0.01\pm0.10$ (i.e., 1.4 standard deviations away). Figure 13 which shows the experimentally observed distribution together with the best fits for $J=\frac{3}{2}$ and $\frac{5}{2}$ also gives an idea of the extent to which $J=\frac{5}{2}$ is unfavored. Other experiments which

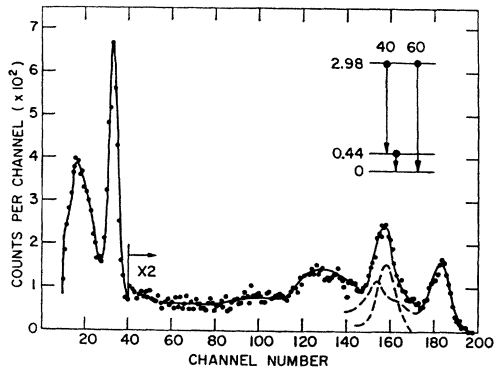

FIG. 11. Gamma rays in coincidence with alphas leading to the 2.98-MeV level in Na^{23}. The spectral shapes for a 2.98 and 2.54 gamma ray are also sketched in.

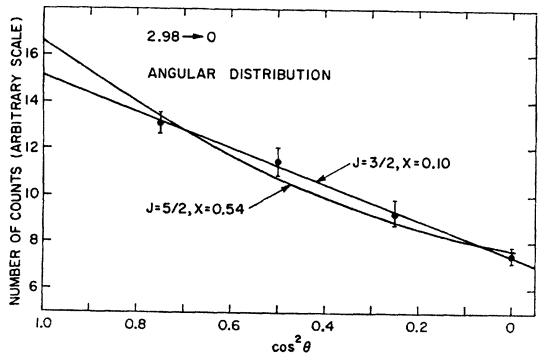

FIG. 13. The angular distribution of the ground-state transition from the 2.98-MeV level. The straight line through the points is the best fit for $J=\frac{3}{2}$, while the curve is the best fit for $J=\frac{5}{2}$ ($a_2=0.56\pm0.01$) and $a_4=0.13\pm0.003$).

give information about the spin of this level have been performed by Braben, Green, and Willmott:[4] $J=\frac{5}{2}$ ($-0.15\leqslant x\leqslant 0.10$), or $J=\frac{3}{2}$ (all values of x allowed), Wernbom-Selin and Arnell:[23] $J=\frac{5}{2}$ ($x=0.36$ and 11.4) and $J=\frac{3}{2}$ ($x=-0.03$ and 3.7) and Khan and Rasmussen:[29] $J=\frac{5}{2}$ ($-1.52\leqslant x\leqslant -1.22$, $-0.47\leqslant x\leqslant -0.31$ and $0.96\leqslant x\leqslant 1.96$) or $J=\frac{3}{2}$ ($0.34\leqslant x\leqslant 1.46$). Where necessary the phases of the quoted mixing ratios have all been "corrected" so that the same convention is used for them all. Khan and Rasmussen have also measured the lifetime of the 2.98-MeV level by resonance fluorescence. Values of $x\geqslant 0.85$ can consequently be eliminated since they correspond to an enhancement of the $E2$ transition rate of at least 50 times the Weisskopf single-particle estimate of Wilkinson.[30] Of the remaining combinations of spin and mixing ratio (x) there is no agreement between the present work and that of Ref. 29 while agreement can be obtained with Ref. 4 only for $J=\frac{3}{2}$. The values quoted in Ref. 23 probably overlap with those of the present work when reasonable errors are assigned to them. In summary,

TABLE II. Decay modes of the levels below 3 MeV in Na^{23}. The decays are given as percentages. In assigning the decay modes of the 2.64- and 2.71-MeV levels, only the major decay modes seen in this work are listed. Weak branches from either level could have been missed.

Level (MeV)	to ground state	to 0.44	to 2.08
0.44	100	...	...
2.08	9±2	91±2	...
2.39	67±4	33±4	...
2.64	100	...	...
2.71	...	68±4	32±4
2.98	60±5	40±5	...

the results of three of the reported experiments are consistent with $J=\frac{3}{2}$, $x=0.11\pm0.05$ while if $J=\frac{5}{2}$, only two of the experiments are consistent with $x\sim 0.5$. We conclude that no rigorous assignment can be made to this level at 2.98 MeV though $J=\frac{3}{2}$ is slightly favored. This conclusion modifies an earlier deduction by us. At that time we did not know of the work of Khan and Rasmussen.[29] Since there is a definite discrepancy between the present work and that of Ref. 29 the assignment previously made can not be considered definite. From the measured width of the level if $J=\frac{3}{2}$, $x=0.11\pm0.05, 0.3<|M(E2)|^2<2.4$ while if $J=\frac{5}{2}$, $x=0.54\pm0.11$ the enhancement over the $E2$ Weisskopf single-particle estimate is at least 20 (which is just the observed enhancement for the $E2$ decay of the 0.439-MeV level). In either case the inhibition of the $M1$ transition rate

[29] N. A. Khan and V. K. Rasmussen, Conference on Bases for Nuclear Spin-Parity Assignments, Gatlinburg, 1965 (to be published).

[30] D. H. Wilkinson in *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), Part B, p. 862 ff.

TABLE III. Mixing parameter assignments obtained in the present work for electromagnetic transitions between low-lying levels in Na^{23}. The phase of the mixing ratio is as defined by Thomas *et al.* (Ref. 19) (convention II). The comparison of phases as quoted by different authors is discussed further below.

Transition	Spin combination	Mixing parameter x
$0.44\rightarrow 0$	$\frac{5}{2}\rightarrow\frac{3}{2}$	$+0.08\pm0.02$
$2.08\rightarrow 0.44$	$\frac{7}{2}\rightarrow\frac{5}{2}$	$+0.20\pm0.03$
	$(\frac{3}{2})\rightarrow\frac{5}{2}$	-0.14 ± 0.08
$2.39\rightarrow 0$	$\frac{1}{2}\rightarrow\frac{3}{2}$	not determined
	$(\frac{3}{2})\rightarrow\frac{3}{2}$	-0.20 ± 0.05 or $x>10$
$2.39\rightarrow 0.44$	$\frac{1}{2}\rightarrow\frac{5}{2}$	not determined
	$(\frac{3}{2})\rightarrow\frac{5}{2}$	$-4.6\ \pm1.6$
$2.64\rightarrow 0$	$\frac{1}{2}\rightarrow\frac{3}{2}$	not determined
	$(\frac{3}{2})\rightarrow\frac{3}{2}$	-0.26 ± 0.05 or $\lvert x\rvert\geqslant 30$
	$(\frac{5}{2})\rightarrow\frac{3}{2}$	-0.15 ± 0.04
$2.71\rightarrow 0.44$	$\frac{9}{2}\rightarrow\frac{5}{2}$	-0.05 ± 0.07
	$(\frac{5}{2})\rightarrow\frac{5}{2}$	$+1.80\pm0.35$
$2.71\rightarrow 2.08$	$\frac{9}{2}\rightarrow\frac{7}{2}$	$+0.12\pm0.04$
	$(\frac{5}{2})\rightarrow\frac{7}{2}$	-0.04 ± 0.05 or -5.10 ± 1.20
	$(\frac{5}{2})\rightarrow(\frac{3}{2})$	$+0.16\pm0.05$
$2.98\rightarrow 0$	$\frac{3}{2}\rightarrow\frac{3}{2}$	$+0.11\pm0.05$ or 2.70 ± 0.40
	$(\frac{5}{2})\rightarrow\frac{3}{2}$	$+0.54\pm0.11$
$2.98\rightarrow 0.44$	$\frac{3}{2}\rightarrow\frac{5}{2}$	$-0.3\ \pm0.3$ or $-3.0\ \pm2.0$

is that which is generally observed (unless forbidden by some selection rule)—a factor of 7 or 10.

DISCUSSION

The results of the present work are collected in Tables I to III. Table IV summarizes the known properties of the levels in Na^{23} below 3 MeV. The spin assignments are discussed in the text. The spins assigned to the excited states in column 2 of Table IV are

TABLE IV. Known properties of levels in Na^{23} below 3 MeV. The spin-parity assignments in column 2 are rigorous in the sense discussed in the text.

Level (MeV)	J^π	Mean lifetime (sec)	Refs. for lifetimes
0	$\frac{3}{2}^+$		
0.439	$\frac{5}{2}^+$	$(1.6\pm0.2)\times10^{-12}$, total[a]	15–18
		6.4×10^{-10} ($\pm20\%$), $E2$	11, 12
2.08	$\frac{7}{2}^{(+)}$ or $(\frac{3}{2}^+)$		
2.39	$\frac{1}{2}^{(+)}$ or $(\frac{3}{2})$		
2.64	$\frac{1}{2}$ or $(\frac{3}{2},\frac{5}{2})$		
2.71	$\frac{9}{2}$ or $(\frac{5}{2})$		
2.98	$\frac{3}{2}$ or $(\frac{5}{2})$	$(5.0\pm0.8)\times10^{-15}$[b]	29

[a] Averaged results of Refs. 15–18.

[b] If width of ground-state transition is $(80\pm12)\times10^{-3}$ eV (Ref. 29) and branching is (60 ± 5) % to ground state.

rigorous assignments made mainly on the basis of the present work. (By rigorous, we mean in the present case that the spin has been assigned as a result of a gamma-ray angular-correlation measurement in which all spin possibilities that do not violate sum rules for transition strengths have been considered. The whole question of rigor in spin and parity assignments has been recently discussed.)[31] It can be seen that on this basis only the first excited state has a firm assignment. For each of the other levels, however, there is evidence[32] of varying reliability which shows that the unbracketed spin assignment in Table I is indeed the correct one. In the following discussion it will be assumed that the spins of the levels are indeed these unbracketed ones.

The 2.39- and 2.64-MeV States

The most striking feature of the decay modes of the low-lying levels is the very different decay modes of the two spin-$\frac{1}{2}$ states at 2.39 MeV and 2.64 MeV. The $2.39\rightarrow 0.44$-MeV transition ($\frac{1}{2}\rightarrow\frac{5}{2}$) is $E2$ if the parity of the 2.39-MeV level is even, hence even if there is no $E2$ component in the ground-state transitions the $M1$ $2.39\rightarrow 0$-MeV transition must be quite strongly inhibited in order to explain the observed branching ratio.

Specifically, in terms of the Nilsson[1] model, the 2.39-MeV and 2.64-MeV states can be identified as the predicted single particle level[9] and single hole level 6, respectively. Evidence for these assignments comes from the $Ne^{22}(d,n)Na^{23}$ reaction (Paul and Montague[26]) in which the 2.39-MeV level displays a strong $l=0$ stripping pattern while the 2.64-MeV state is weakly excited and shows no identifiable stripping pattern. Further evidence is provided by the work of Clegg and Foley[3] who identify a 2.64-MeV gamma ray observed in the bombardment of Mg^{24} by 150-MeV protons as due to the $Mg^{24}(p,2p)Na^{23}$ reaction which would strongly excite hole states. They identify this gamma ray as due to the ground-state decay of the state at 2.64 MeV in Na^{23}.

In terms of the above mode the $M1$ transition probabilities from either of these states to the ground state can be written[1]

$$T_{1/2\rightarrow 3/2}(M1)=1/3\hbar(E_\gamma/\hbar c)^3(e\hbar/2Mc)^2G_{M1}^2\ \text{sec}^{-1}$$
$$=1.05\times 10^{12}E_\gamma^3G_{M1}^2\ \text{sec}^{-1}, \quad (1)$$

where E_γ is measured in MeV and

$$G_{M1}=-\sqrt{2}[g_s^*a_{21}'a_{21}+g_l^*(\sqrt{6}a_{21}'a_{20}+2a_{22}'a_{21})]. \quad (2)$$

$a_{l\Lambda}$ are the normalized Nilsson coefficients, while $g_s^*=g_s-g_R$, $g_l^*=g_l-g_R$, where g_R is the core gyromagnetic ratio.

The $E2$ transition probabilities for the decay to the ground or first excited states are, similarly,

$$T_{1/2\rightarrow 3/2}(E2)=A\,|1-2b_{E2}|^2G_{E2}^2\ \text{sec}^{-1}, \quad (3)$$

$$T_{1/2\rightarrow 5/2}(E2)=A\,|2+b_{E2}|^2G_{E2}^2\ \text{sec}^{-1}, \quad (4)$$

where

$$A=\frac{1}{75}(1+Z/A^2)^2\frac{e^2}{\hbar}\left(\frac{E_\gamma}{\hbar c}\right)^5\left(\frac{\hbar}{M\omega_0}\right)^2=1.15\times 10^9E_\gamma^5$$

$\hbar\omega_0=12.3$ MeV (from Nilsson,[1] Eqs. (4) and (12c), $\hbar\omega_0^0=41A^{-1/3}$; we have taken $\kappa=0.10$ and $\eta=4$);

$$G_{E2}=\tfrac{1}{2}a_{21}'a_{20}+(\sqrt{\tfrac{3}{2}})a_{22}'a_{21}+\sqrt{2}a_{21}'a_{00};$$
$$b_{E2}=\frac{1}{G_{E2}}[-a_{22}'a_{20}-(\sqrt{\tfrac{3}{2}})a_{21}'a_{21}+\sqrt{2}a_{22}'a_{00}]. \quad (5)$$

The branching ratios predicted by these expressions are approximately the same for both initial states: $T_{1/2\rightarrow 3/2}/T_{1/2\rightarrow 5/2}\sim 2\times 10^3$, where T is the transition probability, in comparison with the experimental values (2 ± 0.2) and >24 for the 2.39- and 2.64-MeV states, respectively. The model predicts one decay mode successfully but not the other. It is, however, of interest to note that for the 2.39-MeV level the ratio of the $E2$ transition probabilities is $T_{1/2\rightarrow 3/2}(E2)/T_{1/2\rightarrow 5/2}(E2)\sim 2.3$. The decay mode of this state could then be understood if some nuclear structure effect (so far unconsidered) inhibited the $M1$ transition to the ground state by a factor of at least 2×10^3.

Pelte, Povh, and Schürlein[33] have recently shown that a similar experimental situation exists in Ne^{21}, in which the two spin-$\frac{1}{2}$ levels are within 10 keV of each other at 2.80 MeV. However, in this case, it is the "hole" state which decays $\leqslant 40$, $\geqslant 60$ to the ground state and first excited state while the "particle" state decays predominantly to ground. One solution to this problem could be that, because of their proximity, the two states $|9\rangle$ and $|6\rangle$ are indeed mixed. ($|9\rangle$ and $|6\rangle$ mean, respectively a particle on level 9 and a hole in level 6.) The two perturbed states would then be $a|9\rangle\pm b|6\rangle$ so that, depending on the various matrix elements concerned, for one of the perturbed states the reduced matrix element for the ground-state transition could become very small (the other one remaining more or less normal). For instance, in the case of Ne^{21} for $\eta=4$, $a\sim 0.50$, $b\sim 0.866$ the $M1$ transition from the state $a|9\rangle-b|6\rangle$ becomes very small while the state still remains mainly a hole state in agreement with experiment: $b^2=0.75$. However, although in the case of Na^{23} for $\eta=4$, $a\sim 0.55$, $b\sim 0.84$ an identical situation exists (i.e., the $M1$ transition is very small), it is still the hole state whose $M1$ decay is inhibited, in disagreement with experiment. In both cases the $E2$ transition to the ground state would be inhibited, but only by a factor of about 10.

[31] A discussion of this appears in a topical conference, Ref. 29.

[32] For example, from the $2I+1$ rule, see O. Hansen, E. Koltay, N. Lund, and B. S. Madsen, Nucl. Phys. **51**, 307 (1964). This method, while not rigorous, in most cases gives a good indication of the spin.

[33] D. Pelte, B. Povh, and B. Schürlein, Nucl. Phys. **73**, 481 (1965).

The 0.44-MeV Level

There is a slight discrepancy between the magnitude of the mixing ratio as deduced from the $E2$[11,12] and total lifetimes[15–18] for this level and that measured in the present work. We do not consider this a serious disagreement in view of the very different systematic errors of the two measurements.

The 2.08-MeV Level

The value of the mixing ratio determined in the present work gives a much narrower estimate than that of Howard *et al.*[6,24] Howard, Allen, and Bromley,[6] using the collective model, have given a theoretical estimate of this quantity as well as the branching ratio for the decay of this level to the ground and first excited states. The results of their calculations are reproduced in Fig. 14 together with the branching ratio and mixing ratio as determined in the present work. The upper two curves are the calculated ratio of the total transition probabilities to the first excited and ground state for values of the core gyromagnetic ratio $G_R=0.48$ and 0.23, while the lower two curves are the ratio of the $E2$ and $M1$ transition probabilities for the decay to the first excited state for the same values of G_R. In both cases the favored range for η is $3<\eta<5$ which is in accord with other estimates of the deformation.

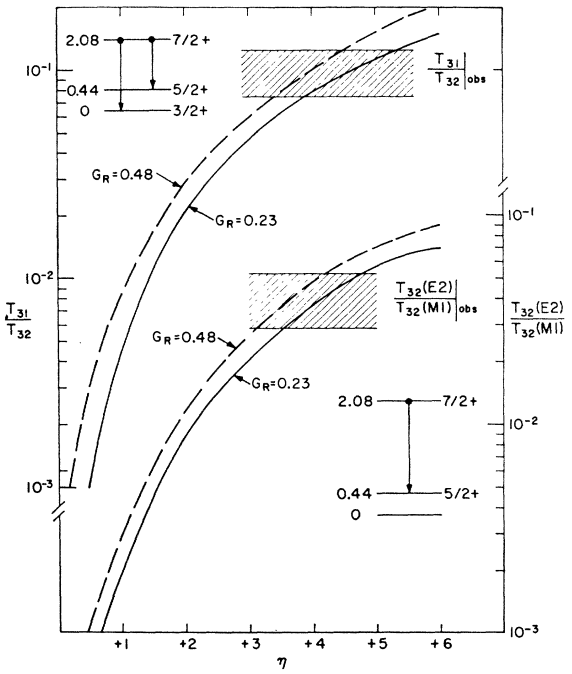

FIG. 14. The 2.08-MeV level: comparisons of results obtained in the present work with the theoretical predictions of Ref. 6. The upper two curves give the calculated branching ratio T_{31}/T_{32} for values of the core gyromagnetic ratio $G_R=0.48$ and 0.23 for the decays from the 2.08-MeV level to the ground-state and 0.44-MeV level. The cross-hatched region lying on the curves is the branching ratio obtained in the present work. The scale for this case is to the left. The lower two curves give the calculated ratio of the $E2$ to $M1$ transition probabilities for the decay to the first excited state. The cross-hatched region gives the results obtained in the present work. (Scale to the right.)

REMARKS ON RELATIVE PHASE CALCULATIONS, PREDICTIONS FOR Mg^{23}

The calculation of Howard, Allen, and Bromley[6] of the relative phases (the sign of x) of the $E2$ and $M1$ matrix elements in the intraband mixed transitions of the N or $Z=11$ nuclei is a very interesting one. This calculation and the comparison between different authors and between different experimental methods as well as between experiment and theoretical predictions is difficult for three reasons:

1. In Biedenharn's[34] formulation of the γ-γ correlation formula, the mixing ratio (x) is written as $x_\alpha=(J_\alpha\|L'\|J_{\rm int})/(J_\alpha\|L\|J_{\rm int})$ where J_α is either the first or last state and $J_{\rm int}$ is the intermediate state. This leads to greater symmetry in the theoretical formula, however, the sign of the mixing ratio depends now on whether the transition is the first or second one in a cascade, since (Lloyd,[25] Brink, and Rose[35]) "emission" and "absorption" matrix elements are connected by the following relation:

$$(J_f\|L'\|J_i)/(J_f\|L\|J_i)=(-)^{L'-L}(J_i\|L'\|J_f)/(J_i\|L\|J_f).$$

Furthermore it is now possible to measure a mixing ratio in a number of different ways (γ-γ, particle-γ, triple-γ correlation, etc.) and the use of the above convention is much more confusing than the definition adopted by Litherland and Ferguson[8] (1961), Ferguson and Rutledge[36] (1962) and Smith[37] (1962). These authors defined the mixing ratio (x) as $x=(J_2\|L'\|J_1)/(J_2\|L\|J_1)$ where J_2 is obtained from J_1 by operating upon it with the operator L, i.e., the matrix elements are always written in normal (temporal) order. Using this definition, the ordinary γ-γ correlation from an unaligned source (spin sequence $a(L_1,L_1')b(L_2,L_2')c$ where $(L'=L+1)$ is given by

$$W(\theta)=\sum_k A_k^{(1)}A_k^{(2)}Q_k^{(1)}Q_k^{(2)}P_k(\cos\theta), \qquad (6)$$

where

$$A_k^{(1)}=F_k(L_1L_1ba)+(-)^{L_1'-L_1}2xF_k(L_1L_1'ba)+x^2F_k(L_1'L_1'ba),$$

and

$$A_k^{(2)}=F_k(L_2L_2cb)+2xF_k(L_2L_2'cb)+x^2F_k(L_2'L_2'cb).$$

[34] See, e.g., L. C. Biedenharn in *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), Part B, p. 772.

[35] D. Brink and H. J. Rose (private communication).

[36] A. J. Ferguson and A. R. Rutledge, Chalk River Report No. CRP-615, Chalk River, Ontario, 1962 (unpublished). See also the 1957 report by these authors in which a different phase is used.

[37] P. B. Smith, in *Nuclear Reactions*, edited by P. M. Endt and P. B. Smith (North-Holland Publishing Company, Amsterdam, 1962), p. 248.

TABLE V. A comparison of the phases of the mixing ratios which are quoted according to a given definition. The relative phases, only, are significant both between the two entries in the same row and within each column.

First transition	Second transition	Definition
−	+	Biedenharn[a]
+	+	II[b]
−	+	Ferguson and Rutledge (1957)[c]
−	−	I[d]

[a] Used in Refs. 34 and 38 and generally in γ-γ correlations from sources.
[b] The definition adopted in the present work.
[c] Ferguson and Rutledge (Ref. 36) (1962) use phase I.
[d] Used in Refs. 8, 36, 37, and 40.

The function $F_k(LL'ba)$ is tabulated in Refs. 38 and 20. The Q_k's are attenuation coefficients.[8] The adoption of this "normal order" definition would mean that the sign and magnitude of a mixing ratio determined in two different circumstances could be compared directly. There is unfortunately a further problem:

2. It was thought that there was an inherent arbitrariness in the relative phases of the vector potentials which occur in the reduced matrix elements (see Devons and Goldfarb,[39] p. 393) so that there have arisen two phase conventions: that quoted in Refs. 8, 36, 37, 40 (we will call this convention I) and that used in writing down Eq. (6) above (convention II). There is still another complication—because of "hidden" phases which are sometimes left in the reduced matrix elements and sometimes put in the angular correlation coefficients. The phase quoted by some authors[8,36] depends upon whether the mixture is a "natural" one ($M1+E2$) or "unnatural" ($E1+M2$). Because the relative parities of the nuclear levels involved are often not known when a mixing ratio is quoted the result is given assuming (generally implicitly) that the mixture is a natural one. In compiling Table V we specifically take the "natural" case of a mixed $ML+E(L+1)$ transition, hence the phase quoted according to convention I is the opposite of that quoted according to convention II. We can summarize the relationships between the different definitions in Table V.

3. There is still, however, the problem of comparing the phase of the $E2/M1$ mixing ratio as calculated according to some nuclear model and that determined experimentally. This is a much more difficult problem. One thing that can be done is to normalize the phase for one measured transition and then try to predict the phase for some other transition which is linked to it. Returning now to the N or $Z=11$ nuclei, this is essentially what Howard, Allen, and Bromley[6] did.

There were two experimental results quoted in this work which apparently disagreed with the predictions. It turns out that there is no disagreement. A careful examination of the experimental results gives Table VI. Except for experimental inadequacies the correctness of the predictions is striking: four phases are all correctly predicted after one is normalized, and predicted magnitudes are all very close to the observed magnitudes except in the case of the result of Deuchers and

TABLE VI. Predicted and experimental amplitudes and phases of mixing ratios. Where necessary, the quoted phases have been changed to conform with convention II. The quantity x_{21} refers to the mixing ratio for the transition from the first excited state to the ground state, x_{32} to that from the second to the first excited state.

	x_{21}			x_{32}			
		Predicted[a]			Predicted[a]		
Nucleus	Observed	$g_R=0.23$	$g_R=0.48$	Observed	$g_R=0.23$	$g_R=0.48$	Ref.
Ne^{21}	-0.08 ± 0.03	-0.08	-0.07	-0.18 ± 0.03	-0.22	-0.19	42
	$-0.03 \leqslant x \leqslant 0.044$[b]						41
	$\lvert x\rvert < 0.03$						f
				-0.11 ± 0.04[c]			g
				$-0.11 < x < 0.17$[d]			h
Na^{21}	$+0.05 \pm 0.05$	$+0.05$	$+0.06$		$+0.15$	$+0.17$	i
Na^{23}	$+0.045 \pm 0.015$	$+0.075$	$+0.085$				14
	$+0.08 \pm 0.02$			$+0.20 \pm 0.03$	$+0.19$	$+0.22$	present work
				$0.23 < x < 0.51$[e]			24
				or			
				$1.5 < x < 2.5$[e]			

[a] Howard, Allen, and Bromley, Ref. 6.
[b] Actually Deuchars and Dandy (Ref. 41) quote $-0.03 \leqslant x \leqslant -0.004$ (phase changed to convention II) but their quoted value of $a_2/a_0 = -0.43 \pm 0.04$ overlaps with the value $a_2/a_0 = 0.40$ expected for $x = 0$.
[c] Used $x_{21} = -0.02$.
[d] Used $\lvert x_{21}\rvert < 0.03$.
[e] Used $x_{21} = +0.045 \pm 0.015$.
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TABLE VII. Gamma-ray transitions and phases in Na^{23} and Mg^{23}. The ground state, 0.44 (0.45) MeV, and 2.08 (2.04) MeV states of Na^{23} (Mg^{23}) are labelled 1, 2, 3 respectively so that x_{21} is the mixing ratio for the decay from the first excited state to the ground state, etc. There is, at present, no experimental information available on Mg^{23}. Convention II (see text) is used in quoting the phases of the mixing ratios x_{21} and x_{31}.

	Na^{23}		Mg^{23}
	Theory	Expt.	Theory
$T(M1)_{21}$ (sec^{-1})	10^{12}	$(0.63\pm0.08)\times10^{12}$	4.55×10^{11}
x_{21}	$+0.08$	$+(0.08\pm0.02)$	-0.14
$T(M1)_{32}$ (sec^{-1})	1.5×10^{13}		0.67×10^{13}
x_{32}	$+0.23$	$+(0.20\pm0.03)$	-0.36
$T(E2)_{31}/T_{32}$ (total)	0.11	(0.10 ± 0.03)	0.26
$T_{31}+T_{32}$ (total) (sec^{-1})	1.9×10^{13}		10^{13}

Dandy,[41] which disagrees with the more recent work of Pronko, Olsen, and Sample.[42] It is therefore interesting to try to predict the mixing ratios for the $\frac{5}{2}\rightarrow\frac{3}{2}$ and $\frac{7}{2}\rightarrow\frac{5}{2}$ transitions in Mg^{23} as well as the branching ratio of the 2.04-MeV level of Mg^{23} (assumed to be the mirror state of the Na^{23} 2.08-MeV level). To do this we have used the formulas given by Howard, Allen, and Bromley,[6] taking $\eta=4$, $\hbar\omega_0=12.3$ MeV, $g_R=0.30$. With these assumptions we get the results listed in Table VII. In view of the general agreement between the experimental measurements and theoretical calculations in the case of Na^{23} we feel that the predictions for Mg^{23} should be quite accurate. In particular it would be very surprising if the predicted signs of the mixing ratios x were found to be incorrect.

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