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Nonadiabatic Effects of Nuclear Rotational Spectra*

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Based on the cranking and the pairing-plus-quadrupole force models, a description of the centrifugal stretching and the Coriolis-force antipairing effects is derived by using the number-conserved wave function in treating the pairing correlations. Detailed numerical calculations are performed to obtain the lowest order nonadiabatic parameter. The observed deviations of the rotational spectra from the $I(I+1)$ formula are approximately accounted for in terms of the centrifugal stretching and the Coriolis-force antipairing (CAP) effects. In general, the CAP effects are more important than the centrifugal stretching effects except for nuclei in the beginning of the deformed region. There centrifugal stretching effects become comparable with the CAP effects. The dependence of both of these effects on the parametric values involved in the theory is discussed.

1. INTRODUCTION

IT is well established that heavy nuclei in the mass region $150<A<190$, $A>224$, have stable axial-symmetric spheroidal deformations and exhibit rotational sequences of levels in their excitation spectra. In recent experimental studies of a number of even-even rare-earth nuclei, rather high-lying members of the ground-state rotational bands have been successfully observed.[1,2] In particular, Stephens, Lark, and Diamond[2] have accurately determined the energies of these rotational levels, in some cases, up to states with spin 16 or 18. The energy systematics of these rotational levels clearly demonstrate that the excitation energy E_I of high-spin states becomes increasingly smaller than that predicted from the $I(I+1)$ formula. This indicates that as the frequency Ω of the rotation increases, nonadiabatic effects of the nuclear rotation (such as changes of the nuclear shape due to the centrifugal forces and disturbance of the intrinsic structure due to the Coriolis forces) become more important.

The deviations of the rotational spectra from the simple $I(I+1)$ formula have usually been described by the higher order terms in the expansion

$$E_I=E_0^{(0)}+(1/2\mathfrak{F}_0)I(I+1)+BI^2(I+1)^2+CI^3(I+1)^3+\cdots. \quad (1)$$

$E_0^{(0)}$ and $\mathfrak{F}_0$ are the intrinsic energy and the moment of inertia, respectively. The coefficients B and C, etc., are parameters characterizing the nonadiabaticity of the nuclear rotation. In cases where only relatively low spin states are involved, expansion (1) terminated at the quadratic or cubic terms can reproduce fairly well the experimental energies. However, when the high-spin data are utilized, such a simple analysis usually does not fit experiment satisfactorily.[2]

An alternative approach to describe the rotational spectrum has been considered by Harris.[3] This is based on the following two sets of equations:

$$E_I=E_0^{(0)}+\tfrac{1}{2}\Omega^2[\mathfrak{F}_0+\sum_{n=1}(2n+1)A_n\Omega^{2n}] \quad (2a)$$

and

$$[I(I+1)]^{1/2}=\Omega[\mathfrak{F}_0+\sum_{n=1}(n+1)A_n\Omega^{2n}], \quad (2b)$$

where Ω is the frequency of the rotation. Retaining only the lowest order nonadiabatic term and thus assuming

$$E_I=E_0^{(0)}+\tfrac{1}{2}\Omega^2(\mathfrak{F}_0+3A_1\Omega^2) \quad (3a)$$

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[1] H. Morinaga and P. C. Gugelot, Nucl. Phys. **46**, 210 (1963); K. Kotajima and D. Vinciguerra, Phys. Letters **8**, 68 (1964); G. B. Hansen, B. Elbek, K. A. Hagemann, and W. F. Hornyak, Nucl. Phys. **47**, 529 (1963).

[2] F. S. Stephens, N. Lark, and R. M. Diamond, Phys. Rev. Letters **12**, 225 (1964); Nucl. Phys. **63**, 82 (1965).

[3] S. M. Harris, Phys. Rev. Letters **13**, 663 (1964); Phys. Rev. **138**, B509 (1965).

and

$$[I(I+1)]^{1/3}=\Omega(\mathfrak{F}_0+2A_1\Omega^2). \quad (3b)$$

Harris has analyzed the data of Ref. 2 and found that all the rotational energies can be reproduced strikingly well when the two parameters $\mathfrak{F}_0$ and A_1 are determined by an empirical least-squares fit to the experimental energies. A more extended analysis, including one higher order term, has very recently been carried out by the same author[3] and reliable values of A_1 have been determined.

The aim of the present work is to calculate this nonadiabatic parameter A_1, based on the cranking model of Inglis[4] and on the pairing-plus-quadrupole force model. Two important nonadiabatic effects are taken into account; one is the centrifugal stretching effect and the other is the Coriolis force antipairing (CAP) effect originally suggested by Mottelson and Valatin.[5] Although a number of studies[6] investigating the above effects have been reported, a quantitative understanding of these effects has not yet been obtained. This is mainly due to the fact that previous investigations introduced somewhat rough approximations, particularly in obtaining numerical estimates of these effects. (See *Note added in proof*, Ref. 11.) We have attempted in this paper to perform more realistic numerical calculations in order to make the quantitative aspect of these effects more completely clear.

In this calculation, we essentially follow the method given in our previous note,[7] in which an approach for describing the CAP effect was devised within the framework of the cranking model. In the present paper we have taken into account the conservation of the number of particles in dealing with the pairing correlation by following the method given by Dietrich, Mang, and Pradal.[8] We have also treated the centrifugal stretching effect.

It should be noted here that the essential idea of the method described in the present paper is quite analogous to that in the Hartree-Fock-Bogoliukov (HFB) approach of Marshalek.[9] In our treatment, however, the conservation of the number of particles is taken into account explicitly although to do this we have neglected for the sake of simplicity the effect of the centrifugal stretching of the γ deformation as well as the Coriolis-force effect on the independent quasiparticle motions. The consideration of the conservation of the particle number, however, may be important when studying the behavior of the rotational spectra in the region of the very high spin states.[10] Although in the present work we have not tried to deal with this problem, our method should be applicable.

Our approach is based on the variational method. Following Ref. 7, we introduce two parameters in order to describe the CAP effect, the amounts λ_P and λ_N of the reduction of the coupling strength of the pairing force of the proton and the neutron. In the present investigation we introduce an additional parameter to describe the centrifugal stretching effect; the deviation Q of the mass quadrupole distribution of nucleons in a nucleus from the equilibrium value Q_0 in the absence of the rotation. Treating them as variational parameters, we first construct a trial wave function describing the state of a rotating nucleus in the rotating frame of reference. Then, we require that the total energy in the rotating system must have a minimum value. From this requirement, we obtain a set of equations which determine Q and $\lambda\tau$ ($\tau=P$ and N) as functions of Ω (Sec. 2). The total energy E_I can then be written as a simple sum of the intrinsic energy E_0 and the kinetic energy of the rotation, although E_0 and the moment of inertia $\mathfrak{F}$ both become functions of Ω. The functional dependence of E_I and $\mathfrak{F}$ is then identical with that given in Eqs. (2a) and (2b).

In Sec. 3, equations for Q and $\lambda\tau$ are solved and the formula for calculating the lowest nonadiabatic parameter A_1 is derived. In addition to the contributions from the centrifugal stretching and the CAP effects, an interference term between these effects also results. The contribution from the centrifugal stretching effect is expressed in terms of a parameter characterizing the restoring force with respect to the change of the mass quadrupole moment and the first derivative of the moment of inertia $\mathfrak{F}$ with respect to Q. The restoring-force parameter is given in terms of the second derivative of the intrinsic energy E_0 with respect to Q. The contribution from the CAP effect is given by similar quantities, but in this case the derivatives must be taken with respect to $\lambda\tau$. In our treatment, the interference effect is characterized by the change of the mass quadrupole moment induced by the CAP effect.

For each of these quantities defining A_1, formulas suitable for numerical evaluation are derived.

In Sec. 4, some details of our numerical calculations are discussed. The actual calculations were performed for nuclei for which the empirical values of A_1 are

[4] D. R. Inglis, Phys. Rev. **96**, 1059 (1954).

[5] B. R. Mottelson and P. G. Valatin, Phys. Rev. Letters **5**, 511 (1960).

[6] A systematic review on the previous research has been given in Ref. 9. This paper should be consulted for further references.

[7] T. Udagawa and R. K. Sheline, Phys. Letters **15**, 172 (1965).

[8] K. Dietrich, H. J. Mang, and J. H. Pradal, Phys. Rev. **135**, B22 (1964). This paper will be referred to here as DMP.

[9] E. R. Marshalek, Phys. Rev. **139**, B770 (1965). In this work, in addition to the centrifugal stretching of the β deformation and the CAP effects, Coriolis-force effects on the quasiparticle motion as well as the centrifugal stretching effect of the γ deformation have been included. See also Ref. 11.

[10] This is suggested by results of the work of Rho and Rasmussen. They showed that, although the BCS approximation is reasonably good for the case where the strength of the pairing force is relatively large, it becomes inaccurate as the pairing force strength decreases; M. Rho and J. O. Rasmussen, Phys. Rev. **135**, B1285 (1964). It has also been argued that the rather abrupt phase transition between superfluid and normal states predicted from the BCS theory may be a spurious result for the nuclear problem. See Ref. 9 and A. Faessler, W. Greiner, and R. K. Sheline, Nucl. Phys. **62**, 241 (1965).

available.[3] These results are presented in Sec. 5 together with the discussion.[11]

2. FORMULATION OF THE METHOD

2.1. Hamiltonian

The Hamiltonian of the pairing-plus-quadrupole force model[12] consists of three parts:

$$\mathcal{H}=H_s+H_p+H_Q. \tag{4}$$

The first part is the spherical single-particle energy. In the second quantization formalism it is

$$H_s=\sum_\alpha \epsilon_\alpha C_\alpha^\dagger C_\alpha. \tag{5}$$

$C_\alpha^\dagger$ and C_α are the creation and annihilation operators for the spherical single-particle state $|\alpha\rangle$, and ϵ_α is the single-particle energy. The second part represents the pairing force

$$H_p=-\sum_\tau G_\tau^{(0)} V_\tau, \tag{6a}$$

where

$$V_\tau=\tfrac{1}{4}\sum_{\alpha\beta} C_\alpha^\dagger C_{T\alpha}^\dagger C_{T\beta} C_\beta, \tag{6b}$$

$T\alpha$ denotes the time-reversed state of α and $G_\tau^{(0)}$ are the strengths of the pairing force for the proton ($\tau=P$) and the neutron ($\tau=N$). The third part of $\mathcal{H}$ is a quadrupole force that is responsible for the deformation of nuclei:

$$H_Q=-\tfrac{1}{2}k\sum_{\alpha\beta\gamma\delta}\sum_\mu \langle\alpha|4(\tfrac{1}{5}\pi)^{1/2}r^2Y_{2\mu}|\gamma\rangle^* \times\langle\beta|4(\tfrac{1}{5}\pi)^{1/2}r^2Y_{2\mu}|\delta\rangle C_\alpha^\dagger C_\beta^\dagger C_\delta C_\gamma, \tag{7}$$

where k is the coupling constant.

The deformation caused by this quadrupole force is in general twofold; one is the axial symmetric β deformation and the other is the unsymmetric γ deformation. In the present work, we will not consider any effect of the γ deformation. Furthermore, throughout this work we assume that the quadrupole force can be treated in the deformed Hartree approximation. Then, when $\Omega=0$, the internal excitation of our system is described by the following Hartree Hamiltonian H_0:

$$H_0=H_s+H_p+H_d. \tag{8}$$

H_d, the Hartree potential of the quadrupole force, is given by

$$H_d=-kQ_0\hat{Q}, \tag{9}$$

where

$$\hat{Q}=\sum_{\alpha\beta} q_{\alpha\beta}C_\alpha^\dagger C_\beta \tag{10a}$$

and

$$q_{\alpha\beta}=4(\tfrac{1}{5}\pi)^{1/2}\langle\alpha|r^2Y_{20}|\beta\rangle. \tag{10b}$$

In (9) Q_0, the strength parameter of the quadrupole field, must be determined self-consistently; it must coincide with the mass quadrupole distribution of the nucleons in the nucleus:

$$Q_0=\langle\Phi_0^{(0)}|\hat{Q}|\Phi_0^{(0)}\rangle. \tag{11}$$

Here $|\Phi_0^{(0)}\rangle$ is the ground-state wave function of the system and is obtained in turn as the lowest eigensolution ($n=0$) of the Hamiltonian H_0:

$$H_0|\Phi_n^{(0)}\rangle=\epsilon_n^{(0)}|\Phi_n^{(0)}\rangle. \tag{12}$$

2.2. Treatment of the Centrifugal Stretching and the CAP Effects

Consider a rotating nucleus with a constant angular frequency Ω about an axis chosen perpendicular to the symmetric axis. Then, if this nucleus were in the lowest intrinsic state $|\Phi_0^{(0)}\rangle$ in the absence of the rotation, the time-dependent perturbations due to the rotation will distort the wave function and admix with it states of higher energy. In the rotating frame of reference, the effects of these perturbations can be described by the well-known static Coriolis force $-\Omega J_x$. In the usual cranking model calculations, in which the rotation is assumed to be slow as compared to the internal motion, the effects of this Coriolis force are estimated by the lowest order perturbation method. The disturbed wave function is then given by

$$|\Psi(\Omega)\rangle =\frac{1}{N_0}\left(|\Phi_0^{(0)}\rangle+\Omega\sum_n \frac{\langle\Phi_n^{(0)}|\hat{J}_x|\Phi_0^{(0)}\rangle}{\epsilon_n^{(0)}-\epsilon_0^{(0)}}|\Phi_n^{(0)}\rangle\right), \tag{13}$$

where N_0 is the normalization constant. This corresponds to the adiabatic approximation. However, we may include the centrifugal stretching and the CAP effects by changing the intrinsic wave functions $|\Phi_n^{(0)}\rangle$ and the energies $\epsilon_n^{(0)}$ in the following way.

First we assume that the mass quadrupole distribution of nucleons in the nucleus is stretched out by the Q due to the centrifugal stretching effect and that due to the CAP effect, the effective strengths of the pairing force of the proton and the neutron are reduced[5] by amounts $\lambda\tau$ ($\tau=P$ for the protons and $\tau=N$ for the neutrons). With this change in the intrinsic structure of

[11] *Note added in proof.* After submitting this manuscript for publication, the results of the numerical calculations on the B value which is related with our A_1 value through the equation $B=-A_1/2\mathfrak{I}_0^4$ (in units of $\hbar=1$) were reported by E. R. Marshalek and J. B. Milazzo, Phys. Rev. Letters **16**, 190 (1966). They have calculated the effects of the γ deformation and the Coriolis-force effects on the independent quasiparticle motions in addition to the effects considered in this paper. As their calculations do not consider the nuclei treated in the present work, it is somewhat difficult to compare their numerical results with ours. The general trend of the results for the centrifugal stretching and the CAP effect seems to be very similar. An important consequence implied in their calculation, however, is that the Coriolis-force effects on the quasiparticle motion give rise to effects on the B values which are as large or larger than the CAP effect. In spite of the fact that this paper does not include the Coriolis-force effects on the quasiparticle motion (or perhaps because of it) the calculated results agree much more closely with experiment than those of Marshalek and Milazzo. For this reason, we hope to include these effects in future calculations. See also M. Richard and J. J. Griffin, Bull. Am. Phys. Soc. **11**, 103 (1966).

[12] L. S. Kisslinger and R. A. Sorensen, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **32**, No. 9 (1960).

the nucleus, the intrinsic wave functions and the energies will be disturbed from the original $|\Phi_n{}^{(0)}\rangle$ and $\epsilon_n{}^{(0)}$. The disturbed wave functions and energies, $|\Phi_n\rangle$ and ϵ_n, may be given as solutions of a Hamiltonian H, whose deformed potential is stretched out by $(Q+\hat{\mu})$, and whose pairing force strengths are reduced by λ_P and λ_N:

$$H|\Phi_n\rangle=\epsilon_n|\Phi_n\rangle \tag{14}$$

and

$$H=H_0-kQ\hat{Q}+\sum_\tau \lambda_\tau V_\tau-\hat{\mu}\hat{Q}. \tag{15}$$

The Lagrangian multiplier $\hat{\mu}$, introduced in the above equation, is then fixed by the following self-consistency condition:

$$Q\equiv Q_0+Q=\langle\Phi_0|\hat{Q}|\Phi_0\rangle, \tag{16}$$

where $|\Phi_0\rangle$ denotes the ground-state solution ($n=0$) of Eq. (14).

Now, replacing $|\Phi_n{}^{(0)}\rangle$ and $\epsilon_n{}^{(0)}$ in Eq. (13) by the disturbed wave functions $|\Phi_n\rangle$ and energies ϵ_n, we get

$$|\Psi(\Omega)\rangle=\frac{1}{N_0}\left\{|\Phi_0\rangle+\Omega\sum_n\frac{\langle\Phi_n|\hat{J}_x|\Phi_0\rangle}{\epsilon_n-\epsilon_0}|\Phi_n\rangle\right\}. \tag{17}$$

This wave function may be used to describe our rotating nucleus influenced by the centrifugal stretching and the CAP effects. It contains, however, three unknown parameters Q, λ_P, and λ_N. In order to fix these, we minimize the total energy $\mathcal{E}_0$ in the rotating system,

$$\begin{aligned}&\frac{\partial}{\partial Q}\mathcal{E}_0(Q,\lambda_\tau;\Omega)=0,\\ &\frac{\partial}{\partial\lambda_\tau}\mathcal{E}_0(Q,\lambda_\tau;\Omega)=0. \quad (\tau=P \text{ and } N).\end{aligned} \tag{18}$$

$\mathcal{E}_0$ is given by

$$\begin{aligned}&\mathcal{E}_0(Q,\lambda_\tau;\Omega)\\ &=\langle\Psi(\Omega)|\mathcal{H}-\Omega\hat{J}_x|\Psi(\Omega)\rangle\\ &=\langle\Psi(\Omega)|H+\tfrac{1}{2}kQ^2\\ &\qquad+\sum_\tau\lambda_\tau V_\tau+\hat{\mu}Q-\Omega\hat{J}_x|\Psi(\Omega)\rangle.\end{aligned} \tag{19}$$

In calculating this energy expectation value, neglecting a small correction term proportional to $\lambda_\tau\Omega^2$,[13] we can write $\mathcal{E}_0$ as a sum of two terms:

$$\mathcal{E}_0(Q,\lambda_\tau;\Omega)=\mathrm{E}_0(Q,\lambda_\tau)-\tfrac{1}{2}\mathcal{F}(Q,\lambda_\tau)\Omega^2. \tag{20}$$

The intrinsic energy $E_0(Q,\lambda_\tau)$ and the moment of inertia $\mathcal{F}(Q,\lambda_\tau)$ are, respectively, given by

$$E_0(Q,\lambda_\tau)=\langle\Phi_0|H+\tfrac{1}{2}kQ^2+\sum_\tau\lambda_\tau V_\tau+\hat{\mu}Q|\Phi_0\rangle \tag{21}$$

and

$$\mathcal{F}(Q,\lambda_\tau)=2\sum_n|\langle\Phi_n|\hat{J}_x|\Phi_0\rangle|^2/(\epsilon_n-\epsilon_0). \tag{22}$$

[13] This term contributes the moment of inertia $\mathcal{F}$. If it is retained, the first-order dependence on λ_τ of $\mathcal{F}$ will be modified from that given by Eq. (22). However, this term consists of a sum of terms with random sign, while the term proportional to λ_τ in Eq. (22) is given by a coherent sum.

Equations (18) and (20) with (21) and (22) are a basic set of equations which determine Q and λ_τ. The solutions will be given as functions of Ω. Then, both E_0 and $\mathcal{F}$ become functions of Ω only.

2.3. Total Energy in the Laboratory System

The total energy of the state with spin I in a ground-state rotational band is given by

$$E_I=\mathcal{E}_0+\Omega\langle\Psi(\Omega)|\hat{J}_x|\Psi(\Omega)\rangle. \tag{23}$$

In order to relate Ω to I, we impose the usual condition,

$$[I(I+1)]^{1/2}=\langle\Psi(\Omega)|\hat{J}_x|\Psi(\Omega)\rangle. \tag{24}$$

The right-hand side of the above equation can be rewritten as

$$\langle\Psi(\Omega)|\hat{J}_x|\Psi(\Omega)\rangle=\Omega\mathcal{F}(\Omega). \tag{25}$$

Then, inserting this into (23) and (24), we obtain

$$E_I=E_0(\Omega)+\tfrac{1}{2}\Omega^2\mathcal{F}(\Omega) \tag{26a}$$

and

$$[I(I+1)]^{1/2}=\Omega\mathcal{F}(\Omega), \tag{26b}$$

where we used the notations $E_0(\Omega)$ and $\mathcal{F}(\Omega)$ instead of $E_0(Q,\lambda\tau)$ and $\mathcal{F}(Q,\lambda\tau)$ in order to express explicitly the fact that both quantities are functions of Ω. When Ω is eliminated from the second equation (26b), we obtain the total energy E_I as a function of spin I. E_I has a form similar to that obtained in the usual cranking-model calculations; it is given as a sum of the intrinsic energy and the kinetic energy of the rotation. It should be remembered, however, that in the present case, both E_0 and $\mathcal{F}$ are functions of Ω.

In the rest of this section, we shall briefly show that if we expand E_0 and $\mathcal{F}$ as power series in Ω, Eqs. (26a) and (26b) reduce to Eqs. (2a) and (2b). For this purpose, we should notice that the A_n's in Eqs. (2a) and (2b) are quite arbitrary, and we may expand as follows:

$$\mathcal{F}(\Omega)=\mathcal{F}_0+\sum_n(n+1)A_n\Omega^{2n}. \tag{27}$$

Then, Eq. (26b) becomes (2b). Using Eq. (18), we can easily obtain a power series expansion of E_0. From Eq. (18), we obtain

$$\delta E_0/\delta\Omega-\tfrac{1}{2}\Omega^2\delta\mathcal{F}/\delta\Omega=0. \tag{28}$$

Using Eqs. (27) and (28), one can fix the coefficients in the expansion of E_0 except for the constant term. Denoting this constant term by $\mathrm{E}_0{}^{(0)}$, the resulting expansion is

$$E_0=E_0{}^{(0)}+\tfrac{1}{2}\Omega^2(\mathcal{F}_0+\sum_n nA_n\Omega^{2n}). \tag{29}$$

Then, if (27) and (29) are substituted into (26a), we immediately obtain (2a).

3. NONADIABATIC PARAMETER A_1

Having discussed the general problem, we will in this section concentrate our attention on the lowest order

nonadiabatic parameter A_1 and present an explicit technique for performing the numerical calculations.

In Sec. 3.1, we will first give a brief description of the solution of the Hartree and pairing force problem defined by Eqs. (14), (15), and (16). In order to treat the pairing force, the DMP method will be introduced. Then, in Secs. 3.2 and 3.3, we will calculate E_0 and $\mathfrak{F}$ as functions of Q and $\lambda\tau$ using a power series expansion. It is, however, only necessary to calculate the power series up to the lowest order terms in Q and $\lambda\tau$, the second-order terms for E_0 and the first-order terms for $\mathfrak{F}$, because these terms completely determine A_1. Finally, in Sec. 3.4, we give the final formula for calculating A_1.

3.1. Perturbation Solutions of the Deformed Hartree Equation and Ground-State Wave Function of the Intrinsic System

The single-particle part of the Hamiltonian (15) consists of two parts; the spherical part and the deformed part. The deformed part of the potential is the Hartree potential of the quadrupole force, which is not necessarily diagonalized in the starting spherical representation. In order to diagonalize both parts at the same time, we introduce the following Hartree transformation from the spherical to the deformed representation:

$$a_i^{\dagger}=\sum_\alpha W_\alpha{}^i C_\alpha^{\dagger}, \tag{30a}$$

$$a_i=\sum_\alpha W_\alpha{}^{i*} C_\alpha, \tag{30b}$$

where the transformation amplitudes $W_\alpha{}^i$ satisfy the condition

$$\sum_\alpha W_\alpha{}^i W_\alpha{}^{j*}=\delta_{ij}, \tag{31a}$$

$$\sum_i W_\alpha{}^i W_\beta{}^{i*}=\delta_{\alpha\beta}. \tag{31b}$$

We will define the amplitudes $W_\alpha{}^i$ and the single energy ϵ_i of the state i in the new representation by the following equation:

$$\epsilon_i W_\alpha{}^i=\epsilon_\alpha W_\alpha{}^i-(kQ_0+\mu)\sum_\beta q_{\alpha\beta}W_\beta{}^i, \tag{32}$$

where μ is defined as

$$\mu=kQ+\hat{\mu}.$$

μ describes the change of the deformation of the potential caused essentially by the centrifugal stretching effect. It reduces to zero in the absence of rotation. Now we introduce the solutions of Eq. (32) for the case of $\mu=0$. The solutions corresponding to $W_\alpha{}^i$ and ϵ_i are designated $X_\alpha{}^i$ and $\epsilon_i{}^{(0)}$ and are defined as

$$\epsilon_i{}^{(0)}X_\alpha{}^i=\epsilon_\alpha X_\alpha{}^i-kQ_0\sum_\beta q_{\alpha\beta}X_\beta{}^i. \tag{33}$$

This equation is quite analogous to that of the Nilsson Model.[14] Thus, we may call the state i defined by this equation the Nilsson state and $X_\alpha{}^i$ and $\epsilon_i{}^{(0)}$ the Nilsson amplitude and single-particle energy, respectively. Starting from these solutions, one can write down the perturbation solutions of $W_\alpha{}^i$ and ϵ_i. They are given to first order for $W_\alpha{}^i$ and to second order for ϵ_i by

$$W_\alpha{}^i=X_\alpha{}^i+\mu\sum_j(q_{ij}{}^{(0)}/(\epsilon_j{}^{(0)}-\epsilon_i{}^{(0)}))+\cdots \tag{34a}$$

and

$$\epsilon_i=\epsilon_i{}^{(0)}-\mu q_{ii}{}^{(0)}-\tfrac{1}{2}\mu^2 q_{ii}{}^{(1)}+\cdots, \tag{34b}$$

where

$$q_{ij}{}^{(0)}=\sum_{\alpha\beta}X_\alpha{}^{i*}q_{\alpha\beta}X_\beta{}^j \tag{35}$$

and

$$q_{ii}{}^{(1)}=\sum_j(|q_{ij}{}^{(0)}|^2/(\epsilon_i{}^{(0)}-\epsilon_j{}^{(0)})). \tag{36}$$

Now rewriting the Hamiltonian (15) in terms of the new operators $a_i^{\dagger}$ and a_i, we obtain

$$H=\sum_i \epsilon_i a_i^{\dagger}a_i-\sum_\tau G_\tau V_\tau, \tag{37}$$

where

$$G_\tau=G_\tau{}^{(0)}-\lambda_\tau, \tag{38}$$

$$V_\tau=\tfrac{1}{4}\sum_{ij} a_i^{\dagger}a_{Ti}^{\dagger}a_{Tj}a_j. \tag{39}$$

Ti is the degenerate time-reversed state corresponding to state i. We follow the DMP method[8] to obtain the ground-state wave function including the correlation due to the pair interaction. In this approximation it is assumed that

$$|\Phi_0\rangle=(1/N_0)P_n\prod_i(1+f_i a_i^{\dagger}a_{Ti}^{\dagger})|0\rangle, \tag{40}$$

where N_0 is the normalization constant and P_n is an operator that projects out from the BCS-type wave function only those components containing the correct numbers of particles, n. $|0\rangle$ denotes the vacuum state.

Defining the wave function $|m\rangle$ by

$$|m\rangle=(1/m!)(\sum_i f_i a_i^{\dagger}a_{Ti}^{\dagger})^m|0\rangle, \tag{41}$$

$|\Phi_0\rangle$ can be expressed in a more compact way as

$$|\Phi_0\rangle=(1/N_0)|n\rangle. \tag{40a}$$

For the sake of convenience, we introduce the following symbolism:

$$B^{(m)}=\langle m|m\rangle, \tag{42}$$

$$B_{i_1 i_2\cdots i_P}{}^{(m)}=\langle m|a_{i_1}{}^{\dagger}a_{T_1}{}^{\dagger}\times a_{i_2}{}^{\dagger}a_{T_2}{}^{\dagger}\cdots a_{i_P}{}^{\dagger}a_{Ti_P}{}^{\dagger}|m-P\rangle. \tag{43}$$

The pair correlation amplitudes f_i in the ground-state wave function are determined from the requirement

$$\delta\langle\Phi_0|H|\Phi_0\rangle=0. \tag{44}$$

From this, one obtains

$$\Gamma_i f_i-\Delta_i=0. \tag{45}$$

[14] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **29**, No. 16 (1955).

The quantities Γ_i and Δ_i are defined by

$$\Gamma_i=(\epsilon_i-\tfrac{1}{2}G_\tau)\frac{B_i^{(n)}}{B^{(n)}} +\sum_j^{(\tau)}(\epsilon_j-\tfrac{1}{2}G_\tau)f_j\frac{B_{ij}^{(n)}}{B^{(n)}}-\tfrac{1}{2}\epsilon_{0\tau}\frac{B_i^{(n)}}{B^{(n)}}, \quad (45a)$$

$$\Delta_i=2G_\tau\kappa_i, \quad (45b)$$

$$\epsilon_{0\tau}=2\sum_i{}^{(\tau)}(\epsilon_i-\tfrac{1}{2}G_\tau)\rho_i-2\sum_i{}^{(\tau)}G_\tau\kappa_i, \quad (46)$$

$$\rho_i=\langle\Phi_0|a_i^\dagger a_i|\Phi_0\rangle=f_iB_i^{(n)}/B^{(n)}, \quad (47)$$

and

$$\kappa_i=\tfrac{1}{2}\sum_j{}^{(\tau)}B_{ij}^{(n+1)}/B^{(n)}. \quad (48)$$

The symbol $\sum_i{}^{(\tau)}$ in the above equations means that the sum should be taken only over the proton or the neutron orbitals i, according to whether $\tau=P$ or N. Also $\epsilon_{0\tau}$ given by Eq. (46) is the proton or the neutron part of the ground-state energy ϵ_0. Then, ϵ_0 will be given by

$$\epsilon_0=\langle\Phi_0|H|\Phi_0\rangle=\sum_\tau\epsilon_{0\tau}. \quad (49)$$

Using the explicit form of $|\Phi_0\rangle$, we can rewrite the self-consistent condition (16) as follows:

$$Q=2\sum_i\rho_iq_{ii}, \quad (50)$$

where

$$q_{ii}=\sum_{\alpha\beta}W_\alpha{}^{i*}q_{\alpha\beta}W_\beta{}^i. \quad (51)$$

3.2. Calculation of the Intrinsic Energy E_0

Inserting $|\Phi_0\rangle$ given by Eq. (37) into (21), one can rewrite the intrinsic energy E_0 as follows:

$$E_0(Q,\lambda_\tau)=2\sum_\tau[\sum_i{}^{(\tau)}(\epsilon_i-\tfrac{1}{2}G_\tau{}^{(0)})\rho_i-G_\tau{}^{(0)}\sum_i{}^{(\tau)}\kappa_i] +\tfrac{1}{2}kQ^2+(\mu-kQ_0)Q. \quad (52)$$

In this section, we seek a power-series expansion of E_0 in Q and λ_τ. For this purpose, we first expand ρ_i and κ_i as power series in μ and λ_τ,

$$\rho_i=\rho_i{}^{(0)}+\mu\rho_{i,\mu}{}^{(1)}+\lambda\rho_{i,\lambda}{}^{(1)} +\tfrac{1}{2}\mu^2\rho_{i,\mu\mu}{}^{(2)}+\tfrac{1}{2}\lambda^2\rho_{i,\lambda\lambda}{}^{(2)}+\lambda\mu\rho_{i,\lambda\mu}{}^{(2)}+\cdots \quad (53a)$$

and

$$\kappa_i=\kappa_i{}^{(0)}+\mu\kappa_{i,\mu}{}^{(1)}+\lambda\kappa_{i,\lambda}{}^{(1)} +\tfrac{1}{2}\mu^2\kappa_{i,\mu\mu}{}^{(2)}+\tfrac{1}{2}\lambda^2\kappa_{i,\lambda\lambda}{}^{(2)}+\lambda\mu\kappa_{i,\lambda\mu}{}^{(2)}+\cdots, \quad (53b)$$

where we simply denote λ_τ by λ. The various coefficients in the expansions will be defined by the partial derivatives of ρ_i and κ_i with respect to μ and λ. Before inserting the above expansion into E_0, we must notice that there exist relations between the coefficients in the expansion of ρ_i and κ_i, stemming from the fact that the ground-state energy ϵ_0 is minimized in f_i. From this fact, it can be shown that

$$\sum_i^{(\tau)}(\epsilon_i-\tfrac{1}{2}G_\tau)\frac{\delta\rho_i}{\delta\lambda_\tau}-G_\tau\sum_i^{(\tau)}\frac{\delta\kappa_i}{\delta\lambda_\tau}=0 \quad (54a)$$

and

$$\sum_i^{(\tau)}(\epsilon_i-\tfrac{1}{2}G_\tau)\frac{\delta\rho_i}{\delta\mu}-G_\tau\sum_i^{(\tau)}\frac{\delta\kappa_i}{\delta\mu}=0. \quad (54b)$$

These equations in turn lead to relations between the coefficients in the expansions of ρ_i and κ_i. Now, using the relations thus obtained and inserting the expansions of ρ_i and κ_i into E_0, we obtain

$$E_0=E_0{}^{(0)}+\tfrac{1}{2}(2\mu Q-B\mu^2-kQ^2) +\tfrac{1}{2}\sum_\tau D_\tau\lambda_\tau{}^2+\cdots. \quad (55)$$

Here $E_0{}^{(0)}$, the intrinsic energy in the absence of the rotation, is given by

$$E_0{}^{(0)}=\circ\sum_\tau[\sum_i{}^{(\tau)}(\epsilon_i{}^{(0)}-\tfrac{1}{2}G_\tau{}^{(0)})\rho_i{}^{(0)} -G_\tau{}^{(0)}\sum_i{}^{(\tau)}\kappa_i{}^{(0)}]+\tfrac{1}{2}kQ_0{}^2 \quad (56)$$

and the quantities B and D_τ are defined by

$$B=2\sum_i(q_{ii}{}^{(0)}\rho_{i,\mu}{}^{(1)}+2q_{ii}{}^{(1)}\rho_i{}^{(0)}), \quad (57)$$

$$D_\tau=-2\sum_i{}^{(\tau)}\kappa_{i,\lambda}{}^{(1)}. \quad (58)$$

In order to express E_0 in terms of Q and λ_τ only, it is still necessary to obtain μ as a function of Q and λ_τ. For this purpose, we solve the self-consistent equation (50). Using again the expansion of ρ_i and q_{ii} given by

$$q_{ii}=q_{ii}{}^{(0)}+2\mu q_{ii}{}^{(1)}+\cdots,$$

one obtains to first order in Q and λ_τ,

$$\hat{\mu}=-kQ+(1/B)(Q-\sum_\tau D_\tau'\lambda_\tau). \quad (59)$$

From this, we obtain

$$\mu=(1/B)(Q-\sum_\tau D_\tau'\lambda_\tau), \quad (60)$$

where

$$D_\tau'=2\sum_i{}^{(\tau)}q_{ii}{}^{(0)}\rho_{i,\lambda}{}^{(1)}. \quad (61)$$

Substituting (60) into (55), we have to second order in Q and λ_τ

$$E_0=E_0{}^{(0)}+(1/2B)(1-kB)Q^2 +\tfrac{1}{2}\sum_\tau D_\tau\lambda_\tau{}^2-(1/2B)(\sum_\tau D_\tau'\lambda_\tau)^2. \quad (62)$$

The physical significance of each term in the above expression is clear. The first term, as already noted, is the intrinsic energy in the absence of rotation. The second and third terms represent the increase in the intrinsic energy due to the centrifugal stretching and the CAP effects, respectively. The fourth term describes the energy gain coming from the change in the potential energy versus deformation surface caused by the CAP-induced reduction in the pairing correlations. Thus, although this term depends only on λ_τ we may call it the interference effect between the centrifugal stretching and the CAP effects.

Equation (62) can then be rewritten in a more compact form:

$$E_0=E_0{}^{(0)}+\tfrac{1}{2}C_QQ^2+\tfrac{1}{2}\sum_\tau C_\tau\lambda_\tau{}^2-C_{PN}\lambda_P\lambda_N, \quad (63)$$

where

$$C_Q=(1/B)(1-kB), \tag{64a}$$

$$C_\tau=D_\tau-(1/B)D_\tau'^2, \tag{64b}$$

and

$$C_{PN}=(1/B)D_P'D_N'. \tag{64c}$$

B, D_τ, and D_τ' in the above equations are defined by Eqs. (57), (58), and (61). They are essentially given in terms of the partial derivatives $\rho_{i,\mu}{}^{(1)}$, $\rho_{i,\lambda}{}^{(1)}$, and $\kappa_{i,\lambda}{}^{(1)}$ of ρ_i and κ_i with respect to μ and λ at their zero point. In the actual numerical calculation in Sec. 5, we will evaluate D_τ and D_τ' by calculating $\rho_{i,\lambda}{}^{(1)}$ and $\kappa_{i,\lambda}{}^{(1)}$ numerically, but in obtaining the values of B, we use the following expression:

$$\rho_{i,\mu}{}^{(1)}=(q_{ii}{}^{(0)}/G_\tau{}^{(0)}\kappa_i{}^{(0)})[\rho_i{}^{(0)}(-1\rho_i{}^{(0)})]^2, \tag{65}$$

which is derived by solving the equation for the pair amplitude f_i approximately. The detailed derivation of this expression is presented in the Appendix.

So far, we have neglected any effect arising from the Coulomb interaction between protons. It is, however, important to take this effect into account when calculating the restoring force parameter C_Q. The discussion of this point, however, will be delayed until Sec. 4.4.

3.3. Calculation of the Moment of Inertia

The moment of inertia is defined by Eq. (22). Within the DMP approximation, the wave functions $|\Phi_n\rangle$ of the excited states n that couple to the ground state through the operator $\hat{J}_x$ will be expressed by

$$|\Phi_n\rangle\equiv|\Phi_{ij}\rangle=(1/N_{ij})a_i{}^\dagger a_{Tj}P_n\prod_k(1+f_k a_k{}^\dagger a_{Tk}{}^\dagger)|0\rangle.$$

The pair amplitude f_k is then determined from the condition

$$\delta\langle\Phi_{ij}|H|\Phi_{ij}\rangle=0.$$

It is evident that, if we follow this procedure, we must solve f_k for each state $n=(i,j)$. This is practically impossible to carry out. Thus, in the present calculation, we introduce the following approximation:

$$|\Phi_{ij}\rangle=(1/N_{ij})a_i{}^\dagger a_{Tj}|\Phi_0\rangle, \tag{66}$$

where N_{ij} is the normalization constant. This approximation, as is obvious, corresponds to assuming that even if the orbits i and j are occupied by unpaired particles, the pair amplitudes of the other orbits are unchanged as compared to those in the ground state. The energy ϵ_n is then given by

$$\begin{aligned}\epsilon_n\equiv\epsilon_{ij}&=\langle\Phi_{ij}|H|\Phi_{ij}\rangle\\&=\epsilon_i+\epsilon_j+2\textstyle\sum_\tau\{\sum_k{}^{(\tau)}(\epsilon_k-\frac{1}{2}G_\tau)f_kB_{ijk}{}^{(n+1)}\\&\qquad-G_\tau\textstyle\sum_{kl}{}^{(\tau)}B_{ijkl}{}^{(n+2)}\}/B_{ij}{}^{(n+1)}.\end{aligned} \tag{67}$$

Using (66), one can rewrite $\mathfrak{F}$ as follows:

$$\mathfrak{F}=2\sum_{ij}\frac{|\langle j|j_x|i\rangle|^2}{(\epsilon_{ij}-\epsilon_0)}\frac{(f_i-f_j)}{(f_i+f_j)}(\rho_i-\rho_j), \tag{68}$$

where $\langle j|j_x|i\rangle$ is the single-particle matrix element of the angular momentum operator j_x taken between the deformed orbits i and j. Expanding $\mathfrak{F}$ in a power series in μ and λ_τ and inserting μ given by Eq. (60), one obtains to lowest order,

$$\mathfrak{F}=\mathfrak{F}_0+\mathfrak{A}_Q Q+\textstyle\sum_\tau\mathfrak{A}_\tau\lambda_\tau, \tag{69}$$

where $\mathfrak{F}_0$, $\mathfrak{A}_Q$, and $\mathfrak{A}_\tau$ are defined by

$$\mathfrak{F}_0=[\mathfrak{F}]_{\mu=0,\lambda=0}, \tag{70a}$$

$$\mathfrak{A}_Q=\frac{1}{B}\left[\frac{\partial\mathfrak{F}}{\partial\mu}\right]_{\mu=0,\lambda=0}, \tag{70b}$$

$$\mathfrak{A}_\tau=\left[\frac{\partial\mathfrak{F}}{\partial\lambda_\tau}\right]_{\mu=0,\lambda=0}-\frac{D_\tau'}{B}\left[\frac{\partial\mathfrak{F}}{\partial\mu}\right]_{\mu=0,\lambda=0}. \tag{70c}$$

In the actual numerical calculation, both partial derivatives $[\partial\mathfrak{F}/\partial\mu]_{\mu=0,\lambda=0}$ and $[\partial\mathfrak{F}/\partial\lambda_\tau]_{\mu=0,\lambda=0}$ will be evaluated numerically. This point will be discussed in additional detail in the next section and the matrix element $\langle j|j_x|i\rangle$ calculated.

3.4. Formula for Calculating the Nonadiabatic Parameter A_1

In the lowest order approximation (63) and (69), it is straightforward to solve (18). Only the results are quoted:

$$Q=\Lambda_Q\Omega^2 \tag{71a}$$

and

$$\lambda_\tau=\Lambda_\tau\Omega^2\quad(\tau=P\text{ and }N), \tag{71b}$$

where Λ_Q and Λ_τ are defined by

$$\Lambda_Q=\mathfrak{A}_Q/2C_Q \tag{72a}$$

and

$$\Lambda_\tau=(\mathfrak{A}_\tau/2C_\tau)\sigma_\tau\quad(\tau=P\text{ and }N), \tag{72b}$$

with

$$\begin{aligned}\sigma_\tau&=(1+C_{PN}/\mathfrak{A}_PC_N)/(1-C_{PN}{}^2/C_PC_N)\text{ for }\tau=P\\&=(1+C_{PN}/\mathfrak{A}_NC_P)/(1-C_{PN}{}^2/C_PC_N)\text{ for }\tau=N.\end{aligned} \tag{73}$$

If we now use (71a) and (71b) in (69), and examine the coefficient of Ω^4, we obtain the desired expression for the nonadiabatic parameter A_1:

$$A_1=A_Q+\textstyle\sum_\tau A_\tau, \tag{74}$$

with

$$A_Q=\mathfrak{A}_Q{}^2/4C_Q, \tag{75a}$$

$$A_\tau=(\mathfrak{A}_\tau{}^2/4C_\tau)\sigma_\tau\quad(\tau=P\text{ and }N). \tag{75b}$$

Clearly, A_Q describes the contribution from the centrifugal stretching effect while A_τ describes the contributions from the CAP and the interference effect

TABLE I. μ values and shifts of the single-particle energy levels.

	N	μ	Additional shifts (in units of $\hbar\tilde{\omega}_0$)
Protons	0	0.00	
	1	0.00	
	2	0.00	
	3	0.45	-0.23
	4	0.55	
	5	0.45	$h_{11/2}$: -0.20; others: $+0.17$
	6	0.45	$+0.17$
	7	0.45	
	8	0.45	
Neutrons	0	0.00	
	1	0.00	
	2	0.00	
	3	0.35	
	4	0.45	-0.37
	5	0.45	
	6	0.45	$i_{13/2}$: unchanged; others: $+0.21$
	7	0.45	
	8	0.45	

discussed in Sec. 3. As this interference effect is characterized by the quantity D_τ', we can obtain the contribution from the CAP effect only by setting $D_\tau'=0$ in the expression (75b). If we take the difference between A_τ and the contribution from the CAP effect, we can also extract the contribution from the interference effect. As will be shown in Sec. 5, however, this effect is in general very small. Thus, in what follows, we will call A_τ the CAP effect unless otherwise noted.

4. DETAILS OF CALCULATION

4.1. Single-Particle Energies and Wave Functions

The single particle energies $\epsilon_i^{(0)}$ and the wave functions $X_\alpha{}^i$ are basic quantities required in our calculations. The problem of obtaining these quantities from the set of equations already given in Secs. 2 and 3, however, is very complicated. It involves solving a self-consistency problem. The mass quadrupole moment Q_0 in the deformed potential that defines $\epsilon_i^{(0)}$ and $X_\alpha{}^i$ must be chosen so that it coincides with the corresponding quantity obtained from the wave function which also is determined by the same potential and pairing forces. Thus, in the present calculation, to avoid solving such a self-consistency problem, we simply take these values from the Nilsson model. However, as will be discussed in detail in the next subsection, we will take into account the self-consistency requirement by suitably choosing the strength of the quadrupole force. In doing this, we have carefully included the effect arising from the *ad hoc* assumption of the volume conservation introduced in the Nilsson model.

Furthermore, for the purpose of our calculations, it is important to include the interaction between the spherical harmonic oscillator shells N and $N+2$ in diagonalizing the Nilsson Hamiltonian. For this reason, we use the alternative representation described in Appendix A of Nilsson's paper.[14] However, we have neglected effects arising from the difference between $\bar{l}$ and $\bar{l}_t$ in diagonalizing the Nilsson Hamiltonian.

There are many parameters in the Nilsson model. We have, however, fixed the values of these parameters following the suggestion of Bes and Szymanski[15]:

(1) The frequency $\tilde{\omega}_0$ of the harmonic oscillator potential is fixed as $\hbar\tilde{\omega}_0=41A^{-1/3}$ MeV.

(2) The strength κ of the spin-orbit potential is chosen to be 0.05.

(3) In the actual calculation, we will take into account all the levels belonging to the oscillator shells up to $N=6$ for both protons and neutrons. In the case when $q_{ii}{}^{(1)}$ defined by Eq. (58) is evaluated, however, we have included the effects of the levels belonging to the $N=7$ and 8 shells.

(4) The values of the strength of the $\bar{l}^2$ term are the same values used by Bes and Szymanski.

(5) Following Bes and Szymanski, we introduce the additional level shift which is necessary in order to reproduce the empirical level ordering in odd-mass nuclei. In Ref. 14, three sets of shifts have been proposed for each proton and neutron orbital. We have chosen set IV for protons and set VI for neutrons as recommended by the authors. All the parametric values defining the single-particle level spectrum employed in this calculation are listed in Table I except the deformation parameter.

4.2. Deformation Parameter and Strength of Quadrupole Force

For the sake of self-consistency, the equilibrium deformation ϵ_0 should be determined for a given value of the strength k of the quadrupole force. In the present calculation, however, ϵ_0 is treated as a given parameter and the strength of the quadrupole force is fixed so that the self-consistency requirement (11) is satisfied. Such a self-consistent determination of the strength of the quadrupole force was first considered by Bes[16] and Bes and Szymanski.[15] These authors however disregarded the effect of the volume conservation imposed in the Nilsson model in deriving the relation between ϵ_0 and k.

In order to clarify this point, consider first the part of the Nilsson potential which is responsible for the deformation energy of the system. Denoting this as $V_D{}^{(N)}$, it is given by

$$V_D{}^{(N)}\simeq-\tfrac{1}{3}M\omega_0{}^2\epsilon\hat{Q}+\tfrac{1}{2}M(\omega_0{}^2-\tilde{\omega}_0{}^2)r^2, \qquad (76)$$

where ϵ is the deformation parameter and

$$\omega_0=\tilde{\omega}_0[1+\tfrac{1}{9}\epsilon^2+O(\epsilon^3)]. \qquad (77)$$

It is clear that the second term of the right-hand side of Eq. (76) comes from the volume conservation condition. As is known, this term plays an essential role in

[15] D. R. Bes and Z. Szymanski, Nucl. Phys. **28**, 42 (1961).
[16] D. R. Bes, Kgl. Danske Videnskab Selskab, Medd. Fys. Mat. **33**, No. 2 (1961).

explaining the strong restoring force effect of the closed shell core against the β deformation.

If this term is neglected, one can identify $V_D^{(N)}$ with the Hartree potential of the quadrupole force, $V_D^{(H)}$, as suggested by Bes and Szymanski,[15,16]

$$V_D^{(H)} = -kQ_0\hat{Q}, \tag{78}$$

and obtain

$$kQ_0 \simeq \tfrac{1}{3}M\omega_0^2\epsilon. \tag{79}$$

Then

$$k \simeq \tfrac{1}{3}M\omega_0^2\epsilon/Q_0(\epsilon). \tag{80}$$

This is the relation used to determine the value of k in Ref. (15) and (16).

However, it is important to include the effect of the second term in calculating the β-deformation energy or the restoring force effect against β deformation. We have taken into account the effect in the following way. Using (77), we can write

$$\tfrac{1}{2}M(\omega_0^2-\tilde{\omega}_0^2)r^2 \simeq \tfrac{1}{9}M\tilde{\omega}_0^2\epsilon^2r^2. \tag{81}$$

Taking the expectation value of the right-hand side of this equation with respect to the ground state $|\Phi_0^{(0)}\rangle$, we get

$$\langle\Phi_0^{(0)}|\tfrac{1}{9}M\tilde{\omega}_0^2\epsilon^2r^2|\Phi_0^{(0)}\rangle \simeq \tfrac{1}{12}M\tilde{\omega}_0^2\epsilon Q_0, \tag{82}$$

where we used the well-known relations

$$Q_0 \simeq \tfrac{4}{5}AR_0^2\epsilon \tag{82a}$$

and

$$\langle\Phi_0^{(0)}|r^2|\Phi_0^{(0)}\rangle \simeq \tfrac{3}{5}AR_0^2. \tag{82b}$$

Then, it follows that the contribution of the second term can effectively be written as

$$\tfrac{1}{2}M(\omega_0^2-\tilde{\omega}_0^2)r^2 \simeq \tfrac{1}{12}M\tilde{\omega}_0^2\epsilon\hat{Q}. \tag{83}$$

Adding this to the first term, we obtain

$$V_D^{(N)} \simeq -\tfrac{1}{4}M\omega_0^2\epsilon\hat{Q}. \tag{84}$$

It is then straightforward to show that the relation between k and ϵ becomes

$$k \simeq \tfrac{1}{4}M\omega_0^2\epsilon/Q_0(\epsilon), \tag{85}$$

i.e., the effect of the correction term arising from the volume conservation changes the value of k by a factor of $\frac{3}{4}$. It must be noted here, however, that the above relation should be used only in order to obtain the value of k employed in calculating the β-deformation energy. Since the second term of Eq. (76) does not contribute, for example, to the γ-deformation energy, for the purpose of the calculation of this energy we should use relation (80) instead of (85). This suggests that one should use for the calculation of the γ-deformation energy a larger value of k by about a factor of 1.3 compared to the case of the β-deformation energy. The values of k, however, have already been extracted for both cases from the experimental information on the β and γ vibrations. Introducing the parameter k_0 defined by

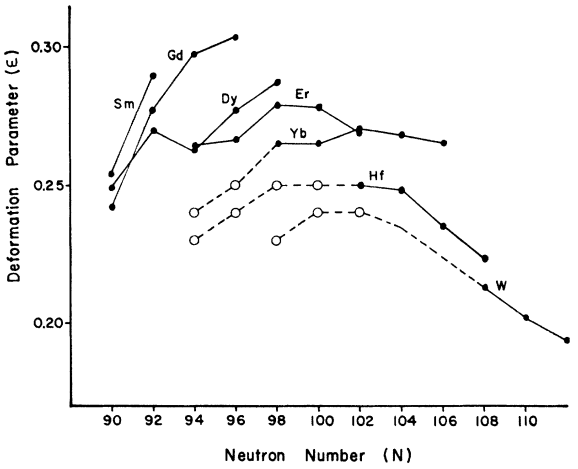

FIG. 1. Systematics of the equilibrium deformation parameter ϵ_0. The points connected by a solid line refer to the experimental values of ϵ_0. These experimental values are taken from B. Elbek, *Determination of Nuclear Transition Probabilities by Coulomb Excitation* (Ejnar Munksgaard, Copenhagen, 1963). The open circles are extrapolated values used in the present calculations.

$$k = k_0(M\omega_0/\hbar)^2\hbar\omega_0A^{-4/3}, \tag{86}$$

it has been shown by Bes[17] that the value of k_0 obtained from the analysis of the β vibration is $k_0 \simeq 0.25$ while the γ vibration gives $k_0 \simeq 0.36$. The ratio is 1.4, which is quite consistent with our prediction.

If experimental values of the equilibrium deformation ϵ_0 were available, it would be desirable to use them. Unfortunately, for nuclei with which we are concerned in the present calculation, there are no available data. Therefore it was necessary to fix these values in a rather arbitrary way. We did, however, take into consideration the systematics of the neutron-member dependence of ϵ_0, as shown in Fig. 1.

4.3. Coulomb Correction

So far, we have neglected any effect arising from the Coulomb interaction between protons. In calculating the restoring force parameter C_Q, however, it is important to include the effect. Assuming a uniformly charged spheroid, the Coulomb energy of a nucleus is given by

$$E_C = \tfrac{3}{5}[(Ze)^2/R_0][1-\epsilon^2/45+O(\epsilon^3)]. \tag{87}$$

Z is the proton number of the nucleus and R_0 is the nuclear radius. Then, the correction ΔC_Q for the restoring force parameter arising from the above Coulomb energy will be given by

$$\Delta C_Q = -(8/75)[\epsilon/Q_0(\epsilon)]^2(Ze)^2/R_0. \tag{88}$$

In the actual numerical calculation in Sec. 5, we will include the above correction by simply adding it to the C_Q calculated from Eq. (64a).

[17] D. R. Bes, Nucl. Phys. **49**, 544 (1963).

TABLE II. Experimental and theoretical values of the nonadiabatic parameter A_1 and the moments of inertia. The nuclides are identified in column one. Column two shows the values of ϵ_0 and column three gives the values of k_0 defined by Eq. (86) and determined from Eq. (85). Columns eight and ten give the final theoretical values of the nonadiabatic parameter A_1 and the moments of inertia. The corresponding experimental values are listed in columns nine and eleven.

Nucleus	ϵ_0	k_0	A_Q	A_P	A_N	ΔA (10^{-8} keV^{-3})	A_{th}	A_{exp}	$\mathfrak{F}_0^{th}$ (MeV^{-1})	$\mathfrak{F}_0^{exp}$ (MeV^{-1})
70 Yb^{164}	0.24	0.184	6.93	0.76	2.05	−0.46	9.80	8.22	22.86	23.72
Yb^{166}	0.25	0.184	3.26	0.93	2.81	−0.41	7.00	5.59	26.00	29.10
72 Hf^{166}	0.23	0.188	5.97	0.36	2.17	−0.14	8.49	8.35	20.32	17.62
Hf^{168}	0.24	0.188	2.51	0.38	2.67	−0.14	5.56	8.95	22.94	23.37
Hf^{170}	0.25	0.188	0.48	0.39	6.41	−0.05	7.28	7.26	27.45	29.54
Hf^{172}	0.25	0.188	0.61	0.40	3.22	−0.08	4.23	4.77	25.91	31.49
74 W^{172}	0.23	0.191	1.05	0.24	7.19	−0.12	8.48	10.74	24.72	23.36
W^{174}	0.24	0.192	0.63	0.23	3.35	−0.04	4.21	7.82	24.00	26.18
W^{176}	0.24	0.192	1.77	0.26	5.16	−0.24	7.19	5.61	25.97	27.19

4.4. Evaluations of the Derivative of the Moment of Inertia with Respect to μ and the Single-Particle Matrix Element of j_x

In order to calculate the derivative of the moment of inertia with respect to μ in Eqs. (70b) and (70c), we must give a relation between μ and the change of the deformation parameter of the Nilsson model, $\delta\epsilon$. It is, however, somewhat difficult to derive rigorously such a relation, because in the present investigation without solving the Hartree problem exactly we have introduced the Nilsson model.

Thus, in the following, we assume

$$Q \simeq [Q_0(\epsilon_0)/\epsilon_0]\epsilon. \tag{89}$$

Then, one may calculate the derivative from the following expression:

$$(1/B)(\partial\mathfrak{F}/\partial\mu)_{\lambda=0} \simeq [\epsilon_0/Q_0(\epsilon_0)](\partial\mathfrak{F}/\partial\epsilon)_{\lambda=0}. \tag{90}$$

When the Nilsson model is used, the moment of inertia is obtained as a function of ϵ. Then, it is easy to evaluate the derivative in the above equation numerically.

In obtaining the moment of inertia $\mathfrak{F}$, however, it is necessary to calculate the single-particle matrix element of the operator j_x by using the single-particle wave functions in the representation described in the Appendix of Ref. 14. A simple procedure to calculate the matrix element, however, has already been derived by Nilsson and Prior.[18] We follow this procedure in the present calculation. The moment of inertia is then obtained

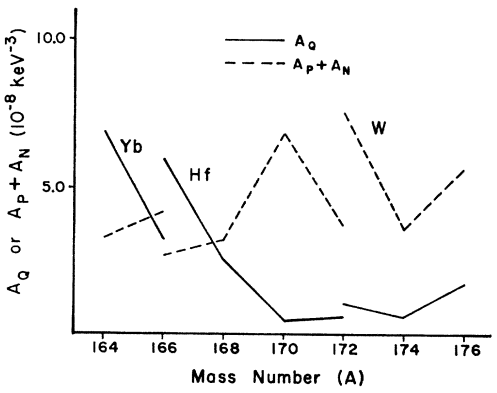

FIG. 2. The calculated values of A_Q and A_P+A_N in units of 10^{-8} keV^{-3}.

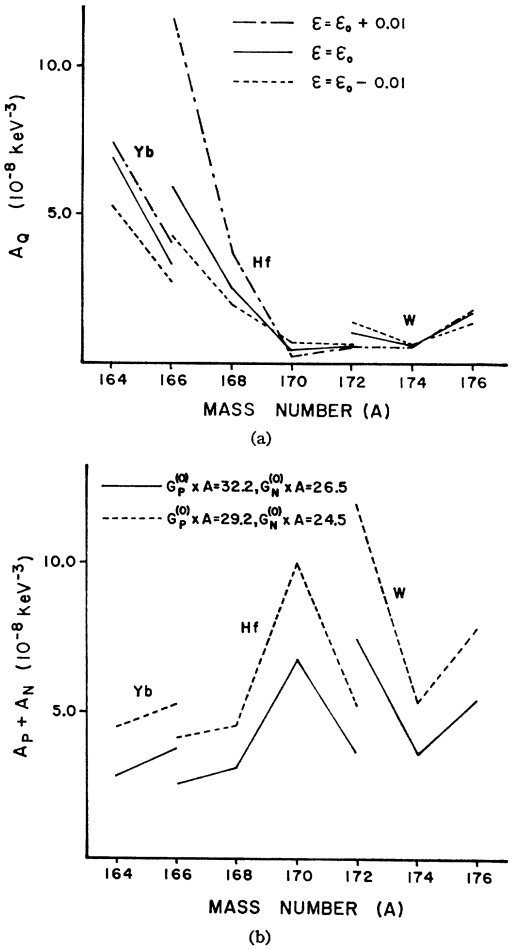

FIG. 3. The dependence of the calculated values (a) of A_Q on ϵ; and (b) of A_P+A_N on $G_\tau^{(0)}$.

[18] S. G. Nilsson and O. Prior, Kgl. Danske Videnskab Selskab, Mat. Fys. Medd. 32, No. 16 (1961).

from the following expression:

$$\mathfrak{F}=\mathfrak{F}^{sp}(1+\tfrac{1}{4}\epsilon^2)+\tfrac{1}{4}\epsilon^2\mathfrak{F}_{rig}, \qquad (91)$$

where $\mathfrak{F}^{sp}$ is the moment of inertia obtained when using the single-particle wave functions in the ordinary representation and $\mathfrak{F}_{rig}$ is the rigid value of the moment of inertia.

4.5. Strength Parameters of Pairing Force

It has been shown that the pairing force strength $G_\tau^{(0)}$ is inversely proportional to the mass number A. The proportionality factor has also been determined by many authors from the analysis of the experimental even-odd mass difference and other experimental data.

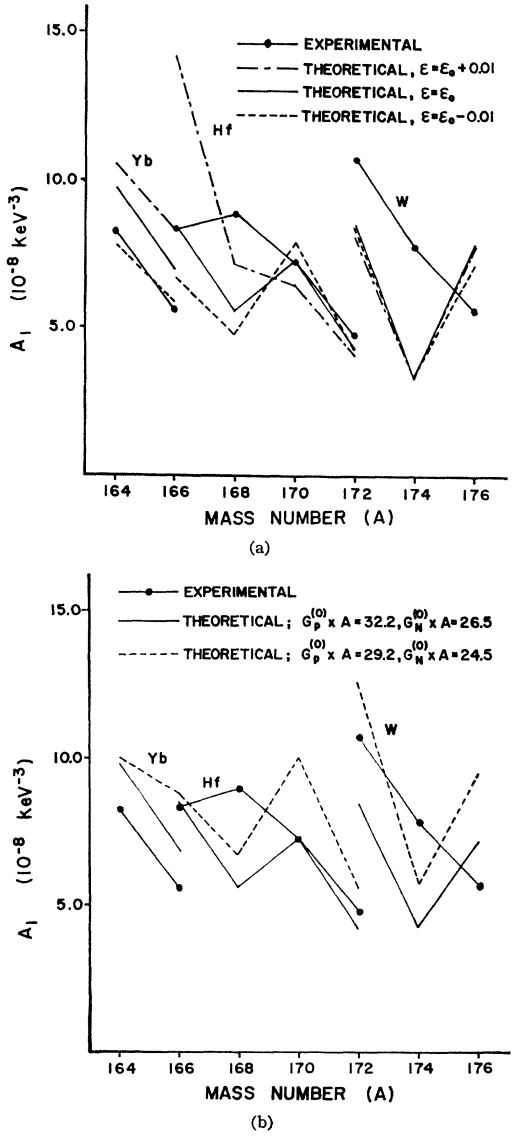

FIG. 4. The dependence of the calculated values of A_1 (a) on ϵ and (b) on $G_\tau^{(0)}$.

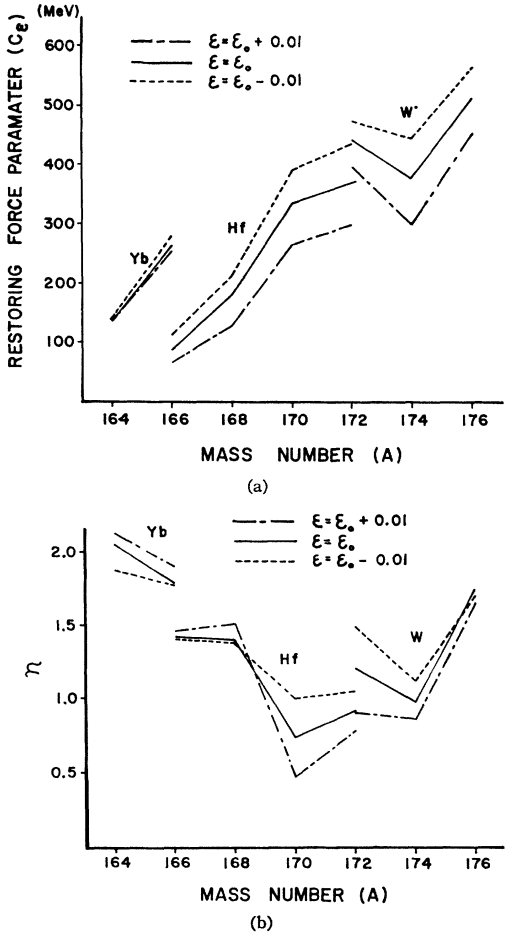

FIG. 5. The calculated values (a) of C_ϵ and (b) of n.

The actual value, however, depends on the number of single-particle states into which the nucleons pairs are allowed to scatter.

In the present calculation, we allow the pairing force to scatter only among the 24 single-particle states nearest to the Fermi surface. This choice is the same as that of Bes and Szymanski.[15] Thus, we use the same value of $G_\tau^{(0)}$ determined by them, namely,

$$G_P^{(0)}=32.2/A \text{ MeV},$$
$$G_N^{(0)}=26.5/A \text{ MeV}. \qquad (92)$$

5. RESULTS AND DISCUSSIONS

The calculated values of the nonadiabatic parameter A_1, including contributions from the centrifugal stretching, the CAP and the interference effects are listed in Table II and compared with the appropriate experimental values. The values of the deformation parameter and the strength of the quadrupole force used in the calculations are also presented. Both the calculated and experimental values of the moments of inertia are also tabulated for the purpose of reference. As noted in Sec. 3,

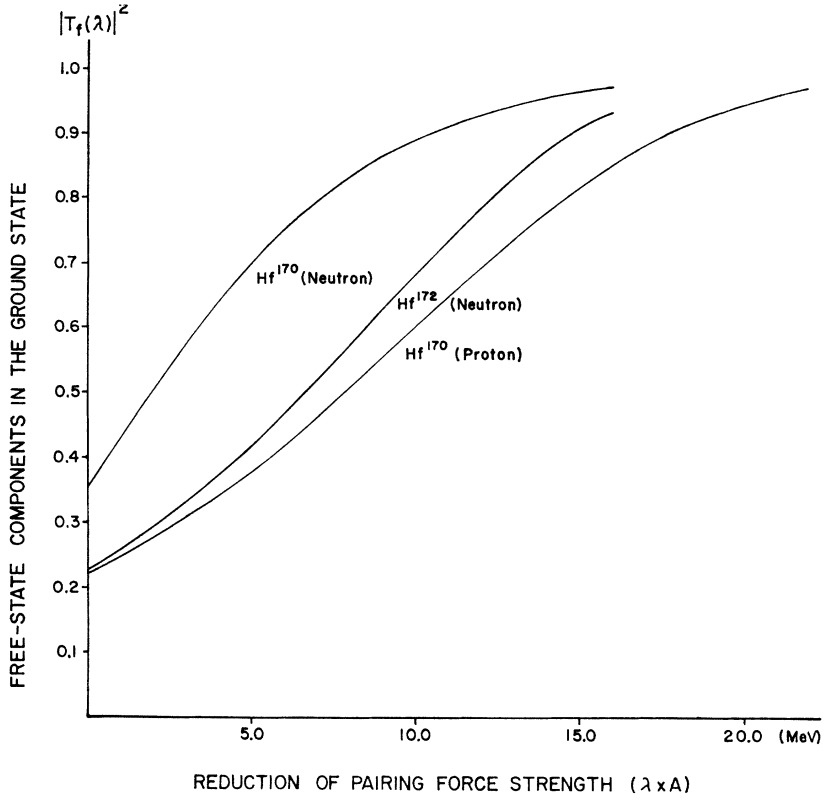

FIG. 6. The squared amplitudes of the free state admixed in the ground state as a function of λ_τ.

the interference effects ΔA given in column seven of this table are obtained by taking the difference between A_P+A_N and the corresponding value calculated by assuming $D_\tau'=0$. It is seen that the values of ΔA are, in general, very small.

To show the relative importance of the centrifugal stretching and the CAP effect, we have plotted both the values of A_Q and A_P+A_N as functions of the mass number in Fig. 2.

Figures 3(a) and 3(b) show the dependence of the values of A_Q on the deformation parameter ϵ and of A_P+A_N on the pairing force strengths $G_P{}^{(0)}$ and $G_N{}^{(0)}$. In addition to that dependence, we have also investigated the dependence of A_Q on $G_P{}^{(0)}$ and $G_N{}^{(0)}$ and of A_P+A_N on ϵ. It is found that such dependence is, in general, small. To show the effects of ϵ, $G_P{}^{(0)}$, and $G_N{}^{(0)}$ on A_1, we have also plotted in Fig. 4(a) and 4(b) the values of A_1 for various parametric values employed in Fig. 3(a) and 3(b).

From the results presented above, it is obvious that the centrifugal stretching effect is important for nuclei close to the beginning of the deformed region and is rather sensitive to the deformation parameter. In the cases of Yb^{164} and Hf^{166}, which have the least number of neutrons—94—of any nuclei considered here, the effect exceeds the CAP effect and accounts for more than half of the experimental value. In the next nuclei, with 96 neutrons, Yb^{166} and Hf^{168}, the effect decreases but still gives an appreciable amount of the total contribution. For nuclei with more neutrons ($N\geq 98$), the contribution is in general very small.

The large contribution from the centrifugal stretching effect for 94- and 96-neutron nuclei results from the fact that the restoring force parameters C_Q are small and the derivatives of the moment of inertia with respect to ϵ are large for these nuclei as compared with other nuclei. To see this, we have plotted in Fig. 5(a) and 5(b) the calculated values of C_ϵ and n defined by

$$C_\epsilon=[\boldsymbol{Q}_0(\epsilon_0)/\epsilon_0]^2 C_Q\,, \tag{93a}$$

$$n=(\epsilon_0/\mathfrak{F}_0)(\partial\mathfrak{F}/\partial\epsilon)_{\epsilon=\epsilon_0,\lambda=0}\,, \tag{93b}$$

respectively. The three different choices of ϵ considered in Fig. 3(a) have been used in evaluating C_ϵ and n in Fig. 5(a) and 5(b).

If the experimental values of C_ϵ were available, we could make an independent test of the present calculation by comparing them with the calculated ones. Unfortunately, however, there are no such data available for nuclei considered here. We have data for other nuclei, such as Sm^{152}, Sm^{154}, Gd^{154}, and Gd^{156}. It is very difficult to test the theory with these data, because the calculated values of C_ϵ are extremely sensitive to the value of the quadrupole force strength k_0, while the expression (85) that determines the value of k_0 is an approximate one, as discussed in Sec. 4.2. This sensitivity might be due to

the fact that, as these nuclei are just at the beginning of the deformed region, the deformations are not so stable. Thus, it would be desirable to measure the values of C_ϵ for nuclei where the deformations become more stable.

If we assume, following the hydrodynamical model, that

$$\mathfrak{F} \propto \epsilon^2,$$

then the value of n defined by Eq. (93b) becomes 2. As is seen from Fig. 5(b), however, our calculated values of n are, in most cases, smaller than this value. However, in the cases of the two Yb isotopes, the values of n are nearly equal to 2.

In contrast to A_Q, the total contribution A_P+A_N from the CAP effect is important for all nuclei. Taking, as examples, the cases of Hf^{170} and Hf^{172}, it accounts for more than 90% of the total experimental values. The main contribution from the CAP effect, however, comes from the neutron configuration. The contribution from the protons is generally small and accounts for only about 10–30% of the total contribution.

It is interesting to note that the CAP effect has a pronounced peak at the point where the nucleus has 98 neutrons.

In order to obtain a deeper physical understanding of the nature of the CAP effect, we have calculated the component of the free state admixed into our lowest intrinsic wave function $|\Phi_0(0,\lambda_\tau)\rangle$ obtained by setting $Q=0$ as a function of λ_τ. The calculations have been made for each proton and neutron part of the wave function separately. Denoting the separated parts of the wave function separately. Denoting the separated parts of the wave function by $|\Phi_{0\tau}(\lambda_\tau)\rangle$, we can define the above component of the free state as

$$T_f(\lambda_\tau)=\langle\Phi_{0\tau}(\lambda_\tau)|\Phi_{f\tau}\rangle, \tag{94}$$

where $|\Phi_{f\tau}\rangle$ is the free state having no pair correlation and is given by

$$|\Phi_{f\tau}\rangle=|\Phi_{0\tau}(G_\tau{}^{(0)})\rangle, \tag{95}$$

since $\lambda_\tau=G_\tau{}^{(0)}$ corresponds to zero pairing force strength.

In Fig. 6, we have plotted the squared amplitude $|T_f(\lambda_\tau)|^2$ as a function of $\lambda_\tau\times A$ for three cases; for the proton part of the wave function of Hf^{170} and the neutron parts of Hf^{170} and Hf^{172}. One finds that even in the original ground state (i.e. $\lambda=0$) there is about 20%–35% of the free state. Furthermore, this initial mixing of the free state is, in general, larger for the case where the CAP effect is more important. A more remarkable feature, however, is the fact that the wave function approaches the free state more rapidly in the case where the values of A_P+A_N is larger.

Finally, in order to give a measure of the changes of the values of $\delta\epsilon$ and λ_τ as functions of the value of spin I, we have plotted in Fig. 7(a) $\delta\epsilon$ for Hf^{166}, H^{168}, and Hf^{170} and in Fig. 7(b) the $\lambda_P\times A$ for Hf^{170} and $\lambda_N\times A$ for Hf^{170} and Hf^{172} as functions of $I(I+1)$. In calculating ϵ

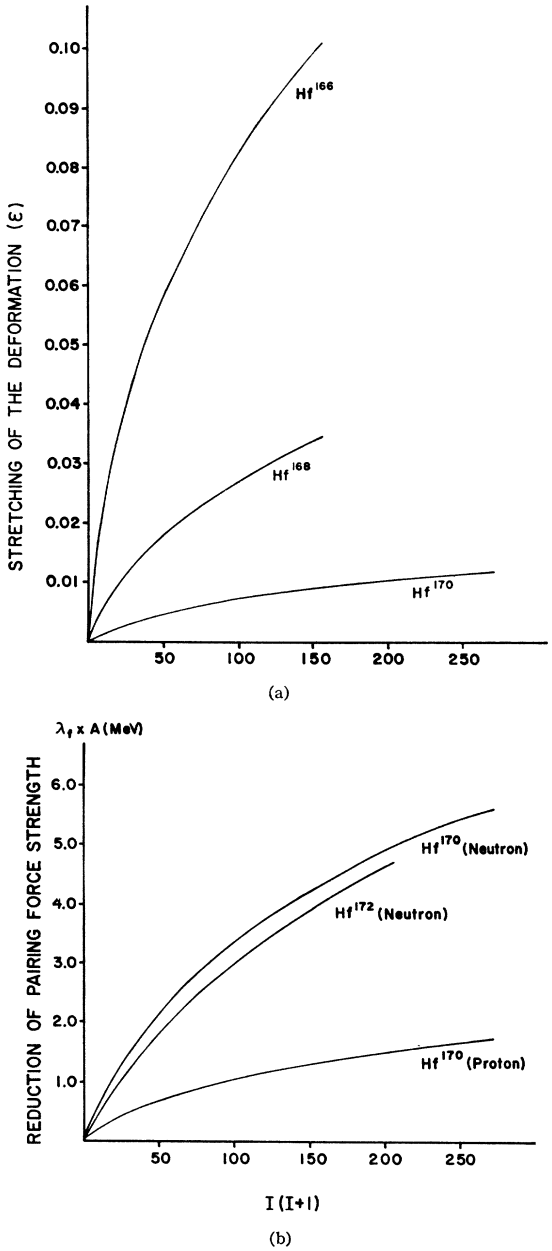

FIG. 7. (a) The changes of the deformation parameter and (b) the reduction of the pairing force strength as a function of $I(I+1)$.

we used the relation:

$$\delta\epsilon=[\epsilon_0/Q_0(\epsilon_i)]\Lambda_Q\Omega^2. \tag{96}$$

We have used the values of Ω obtained from Eq. (3b) and also the experimental values of A_1 determined in Ref. 3 in calculating these quantities.

6. CONCLUDING REMARKS

Based on the cranking model and the pairing-plus-quadrupole force model, a description of the centrifugal

stretching and the CAP effects is derived by using the number-conserved wave function in treating the pairing correlations. Detailed numerical calculations were performed on the lowest order nonadiabatic parameter A_1. The results indicate that the experimental values of A_1 can be almost accounted for if both the centrifugal stretching and the CAP effects are taken into account.[19] However, we have found that the centrifugal stretching effect plays an important role only for nuclei near the beginning of the deformed region. On the other hand, for nuclei in the middle of the deformed region, the CAP effect is in general considerably more important than the centrifugal stretching effect.

The calculated values of A_1 are rather sensitive to the parameters involved in the present theory. In particular, it has been shown that the centrifugal stretching effects are sensitive to the deformation parameter while the CAP effects depend strongly on the pairing force strength. Furthermore, it may be of value to note here that according to the preliminary results of our calculation, both effects depend rather sensitively on the position of the single-particle orbitals. For a more detailed fit with experiments than that obtained here, it would be necessary to perform a more systematic calculation, including nuclei which are not considered here. Such a systematic calculation is now underway.

APPENDIX

The density ρ_i of the orbit i is defined by Eq. (47) in the text:

$$\rho_i = f_i B_i^{(n)}/B^{(n)}. \tag{A1}$$

The partial derivative of this density with respect to μ is

$$\rho_{i,\mu}^{(1)} = 2\frac{f_{i,\mu}^{(1)}}{f_i^{(2)}}\rho_i^{(0)}(1-\rho_i^{(0)}) + 2f_i^{(0)}\sum_{i\neq j} f_{j,\mu}^{(1)} \times\left\{\left[\frac{B_i^{(n)}}{B^{(n)}}\right]_{\mu=0,\lambda=0} - \frac{\rho_i^{(0)}\rho_j^{(0)}}{f_i^{(0)}f_j^{(0)}}\right\}, \tag{A2}$$

where $f_i^{(0)}$ and $f_{i,\mu}^{(1)}$ are defined by

$$f_i = f_i^{(0)} + \mu f_{i,\mu}^{(1)} + \cdots, \tag{A3}$$

while $\rho_i^{(0)}$ is $[\rho_i]_{\lambda=0,\mu=0}$.

If we take the BCS limit of the quantities ρ_i and $f_i f_j B_{ij}^{(n)}/B^{(n)}$,

$$\rho_i \equiv \langle n|a_i^\dagger a_i|n\rangle \tag{A4a}$$

and

$$f_i f_j B_{ij}^{(n)}/B^{(n)} \equiv \langle n|a_i^\dagger a_i a_j^\dagger a_j|n\rangle, \quad (i\neq j); \tag{A4b}$$

by replacing the number-conserved wave function $|n\rangle$ by the BCS wave function, we obtain

$$\rho_i \longrightarrow v_i^2, \tag{A5a}$$

and

$$f_i f_j B_{ij}^{(n)}/B^{(n)} \longrightarrow v_i^2 v_j^2, \tag{A5b}$$

In this BCS limit, it is obvious that the second term in the right-hand side of Eq. (A2) vanishes. Neglecting this as a small correction, we obtain

$$\rho_{i,\mu}^{(1)} \simeq 2(f_{i,\mu}^{(1)}/f_i^{(0)})\rho_i^{(0)}(1-\rho_i^{(0)}). \tag{A6}$$

In order to solve $f_{i,\mu}^{(1)}$, we expand Γ_i and Δ_i as power series in μ and λ,

$$\Gamma_i = \Gamma_i^{(0)} + \mu\Gamma_{i,\mu}^{(1)} + \lambda\Gamma_{i,\lambda}^{(1)} + \cdots \tag{A7a}$$

and

$$\Delta_i = \Delta_i^{(0)} + \mu\Delta_{i,\mu}^{(1)} + \lambda\Delta_{i,\lambda}^{(1)} + \cdots. \tag{A7b}$$

Then, from Eq. (45) which defines f_i we get

$$f_i^{(0)} = \Delta_i^{(0)}/\Gamma_i^{(0)} \tag{A8a}$$

and

$$f_{i,\mu}^{(1)} = (\Delta_{i,\mu}^{(1)} - f_i^{(0)}\Gamma_{i,\mu}^{(1)})/\Gamma_i^{(0)}. \tag{A8b}$$

Here, again neglecting the terms which tend to zero when taking the BCS limit, we can easily show that

$$\Delta_{i,\mu}^{(1)} \simeq 0 \tag{A9a}$$

and

$$\Gamma_{i,\mu}^{(1)} \simeq -(q_{ii}^{(0)}/f_i^{(0)})\rho_i^{(0)}(1-\rho_i^{(0)}), \tag{A9b}$$

from which we obtain

$$f_{i,\mu}^{(1)} \simeq -\frac{\Gamma_{i\,\mu}^{(1)}}{\Gamma_i^{(0)}} f_i^{(0)} = \frac{f_i^{(0)}}{\Delta_i^{(0)}} q_{ii}^{(0)}\rho_i^{(0)}(1-\rho_i^{(0)}) = \frac{f_i^{(0)} q_{ii}^{(0)}}{G_\tau^{(0)}\kappa_i^{(0)}}\rho_i^{(0)}(1-\rho_i^{(0)}). \tag{A10}$$

Inserting this into Eq. (A6), we get

$$\rho_{i,\mu}^{(1)} = (q_{ii}^{(0)}/G_\tau^{(0)}\kappa_i^{(0)})[\rho_i^{(0)}(1-\rho_i^{(0)})]^2. \tag{A11}$$

[19] In the present work, the effect of the γ deformation on the A_1 value has not been considered. It has, however, been shown that the effect is in general very small, amounting to only 5–10% of the total experimental A_1. See O. B. Nielsen, *Rutherford Jubilee International Conference, Manchester, 1961* (Heywood and Company, Ltd., London, 1961) p. 317 and O. Nathan and S. G. Nilsson, *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1965), p. 676.