The density of states was finally approximated by

$$\left.\begin{aligned}\rho_z(C)&=10.20-4.16C+0.765C^2, & -0.5<C<3,\\ &=-1.80-28.16C+0.765C^2, & -1<C<-0.5,\\ &=52.80-24C, & -1.5<C<-1.\end{aligned}\right\} \tag{C14}$$

Equation (C14) is a good approximation to the two-dimensional density of states for an hexagonal tight-binding system. It has been arbitrarily normalized such that

$$\int_{-1.5}^{+3}\rho_z(C)dC=45.70. \tag{C15}$$

The quantities of interest are the amplitudes of the Fourier components of this two-dimensional density of states, i.e.,

$$\begin{aligned}\rho_n(q)&=\frac{1}{10\pi}\int_{-5\pi}^{5\pi}\rho_z(x)\exp[-inx/5]dx\\ &=\frac{1}{5\pi}\int_0^{5\pi}\rho_z(C)\frac{dC}{dx}\cos[nx/5]dx.\end{aligned} \tag{C16}$$

In particular, for the free electron mass, $m_0=1$, $n=216$.

The numerical calculations involved in the determination of the energy eigenvalues (C1) to (C11) as well as the integrations (C16) for the Fourier components ρ_n were carried out in the IBM 7094–7044 complex of the University of Chicago Computation Center.

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Radiation from Thick Silver Foils Bombarded by Grazing-Incidence Electrons*

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Silver foils several thousand angstroms in thickness were bombarded by electrons at grazing incidence. A peak was observed at approximately 3550 Å in the optical emission spectrum. For an angle of incidence of 89° from the foil normal, the peak intensity was approximately ten times as great as that of previously observed intensities of transition radiation at 3300 Å from silver foils bombarded with normally incident electrons. The intensity was found to be directly proportional to the electron energy from 40 to 80 keV, and showed a marked dependence on the quality of the foil surface. The results are in agreement with the findings of Boersch *et al.*

INTRODUCTION

It was shown by Ritchie[1] that in addition to collective oscillations by the bulk electrons in a conductor, there should exist plasma oscillations on the conductor surface. Ferrell[2] predicted that bulk plasma oscillations induced by charged particle excitation should decay by the emission of monochromatic photons at the plasma frequency. Since Ferrell's prediction, many investigators[3] have searched for plasma radiation. Most of these investigators have bombarded metal foils with normally incident electrons and have interpreted the emission spectra in terms of the decay of volume plasmons. Recently Boersch *et al.*[4] bombarded silver foils with 30-keV electrons at grazing incidence and found an intense peak at 3500 Å, which was thought to be due to the decay of surface plasma oscillations. However, Ferrell's model of a conductor bounded by a plane surface forbids emission by the decay of surface plasma oscillations.

This paper presents the results of a further attempt to determine whether or not surface plasma oscillations decay by emission of electromagnetic radiation. Thick silver foils were bombarded by grazing-incidence electrons. The emission spectrum was studied as a function of the angle of incidence of the electron beam, the electron energy, and the quality of the foil surface.

EXPERIMENTAL

The techniques used in measuring the optical emission from electron-bombarded foils have been described

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previously.[5] Briefly, silver films several thousand angstroms in thickness were bombarded by an electron beam whose energy was varied from 40 to 80 keV. The beam current was held below about 1.0 μA to avoid excessive heating of the foil. The light emitted at 30° from the foil normal was analyzed with a Seya-Namioka vacuum ultraviolet spectrometer using an EMI 6256B photomultiplier as a detector. The grating was a Bausch and Lomb aluminized replica grating blazed at 3500 Å. A Glan prism polarizer placed in the exit arm was adjusted so that the light transmitted was polarized either parallel or perpendicular to the plane formed by the electron beam and the emitted light. Silver films were prepared by evaporating silver onto tantalum substrates. The films were at least 3000 Å in thickness and were assumed to be infinitely thick. The substrates

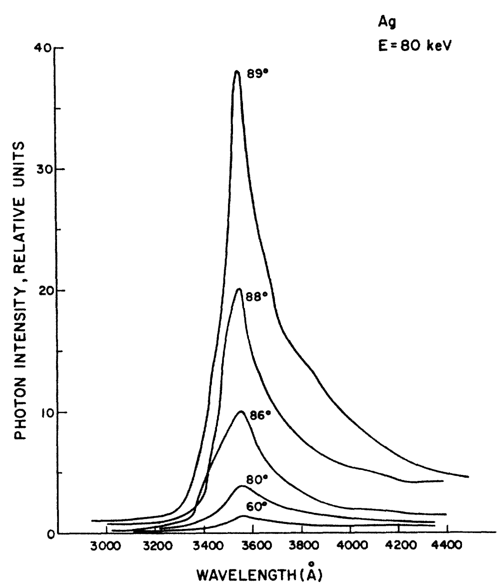

FIG. 1. Spectra of light emitted in the parallel plane of polarization from thick Ag foils bombarded by 80-keV electrons at various angles of incidence.

were made narrow ($\frac{1}{8}$ in.) to ensure that the source of radiation was within the acceptance angle of the spectrometer. The preparation could be performed under vacuum inside the spectrometer when it was desired to avoid atmospheric contamination of the foil surface. A calibrated foil holder was constructed so that, by rotating the foil, the angle of beam incidence could be easily varied.

RESULTS AND DISCUSSION

Spectra of light emitted in the parallel plane of polarization for several angles of electron beam incidence are shown in Fig. 1. The photon intensities have been corrected for the change in the component of foil

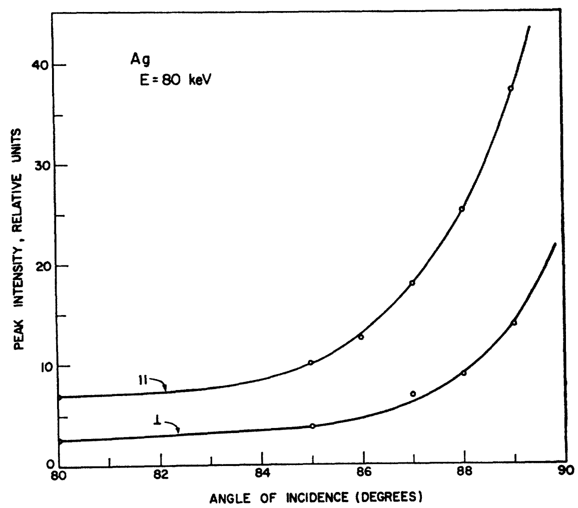

FIG. 2. Peak radiation intensity as a function of the angle of electron beam incidence for a freshly evaporated foil. Parallel and perpendicular polarization components are shown for 80-keV electrons.

area perpendicular to the electron beam for changing angles of incidence. In addition, a correction has been made for the spectral response of the spectrometer. The spectrometer response was determined using a tungsten filament lamp calibrated by the National Bureau of Standards as a source of known spectral radiance. It was not possible to determine the exact number of electrons contributing to the emission of radiation because of the large number of scattered electrons inside the irradiation chamber. The photon intensities are therefore given in relative units, and an approximate comparison can be made with the results of experiments on foils bombarded by normally incident electrons of equal beam current.

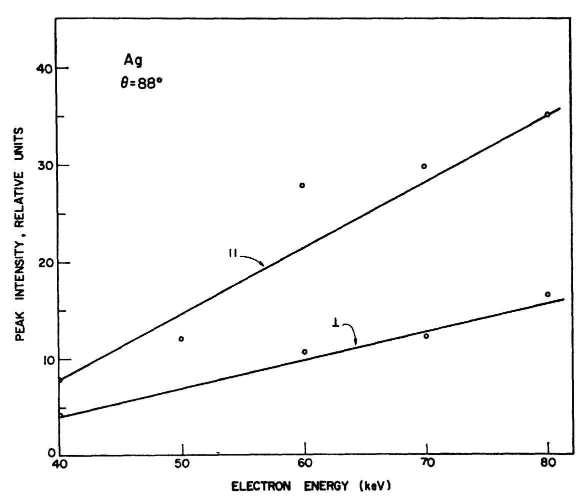

FIG. 3. Peak radiation intensity as a function of electron energy for the parallel and perpendicular polarization components. The angle of electron incidence is 88°.

[5] E. T. Arakawa, N. O. Davis, L. C. Emerson, and R. D. Birkhoff, J. Phys. Radium 25, 129 (1964).

An intense peak was observed at approximately 3550 Å for angles of incidence near 90°, as measured from the normal to the foil surface. At 89° the peak intensity was approximately ten times as great as that of previously observed intensities of transition radiation at 3300 Å from silver foils bombarded by normally-incident electrons.[5] The peak intensity decreased rapidly as the angle of incidence was decreased to 85°. Below 85° the peak intensity decreased more slowly, and no peak at all was observed for angles of incidence below approximately 30°.

The observed peak intensities in the parallel and perpendicular planes plotted as a function of the angle of beam incidence for electron energies of 80 keV are shown in Fig. 2. Transition radiation from thin silver foils bombarded by electrons at normal incidence has been observed to be polarized parallel to the plane formed by the electron beam and the emitted light.[5] The polarization at the peak wavelength of the radiation studied in this investigation was observed to be less than one-third that of transition radiation produced by silver foils bombarded at normal incidence. As in the case of transition radiation, however, the peak intensity was found to be directly proportional to the electron energy. Figure 3 shows the peak intensity as a function of electron beam-energy from 40 to 80 keV. This indicates that the observed radiation is not the long-wavelength component of bremsstrahlung, which varies in intensity inversely with electron energy. In addition, the peak intensity from bremsstrahlung would occur at the transparency in Ag, which is at 3250 Å. The wavelength at which the peak intensity was observed (3550 Å) was not noticeably shifted by a change in the angle of electron incidence (Fig. 4).

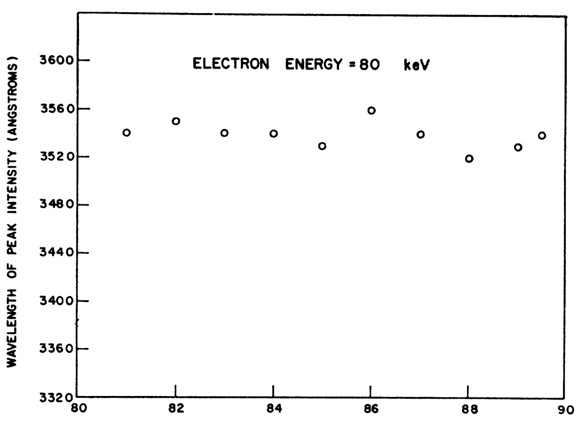

FIG. 4. Wavelength of peak intensity versus angle of electron beam incidence for 80-keV electrons.

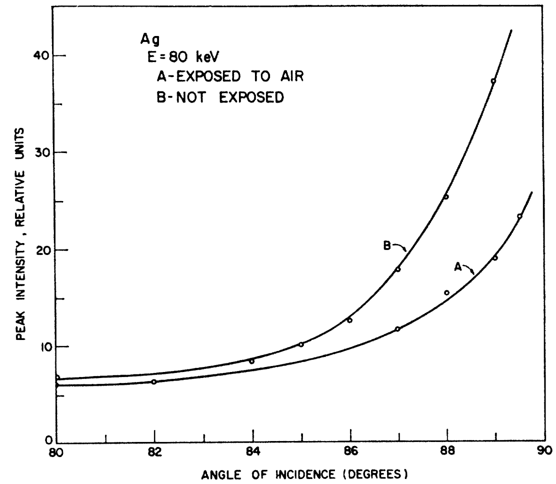

FIG. 5. Comparison of the peak intensity with angle of incidence for a fresh and a contaminated foil surface.

If the observed radiation is due to a surface phenomenon, the quality of the foil surface would be expected to have a marked effect on the intensity of the radiation peak. To determine if this was the case, foils were evaporated and exposed to air before bombardment, while others were evaporated inside the spectrometer and bombarded immediately. A comparison of the results of the two procedures is shown in Fig. 5. For angles of incidence close to 90°, radiation from foils evaporated in the spectrometer under a vacuum of 10^{-5} Torr was approximately twice as intense as that from foils which were exposed to air before bombardment.

The wavelength of the peak intensity and the dependence of the peak intensity on incident angle, electron energy, and quality of the foil surface observed in this investigation are in agreement with the results of Boersch *et al.* However, these results are not conclusive evidence that the observed radiation results from the decay of surface plasmons. More recent work on thin Ag foils casts some doubt upon this interpretation. In brief, Ag foils of thickness 700 Å bombarded by an electron beam at grazing incidence did not show the intense emission at 3550 Å. These results will be presented in a subsequent publication.

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