Weak Interactions in $SU(6)$ and $\tilde{U}(12)$†

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Weak interactions are studied in $SU(6)$ and $\tilde{U}(12)$. The weak currents are assigned to the 35-dimensional representation of $SU(6)$ [or the **143** in $\tilde{U}(12)$], and the predictions of a Cabibbo-type theory are derived. For nonleptonic decay, the **35** has been shown to be inconsistent with experiment, and so we assign the effective Hamiltonian to an admixture of **35** and **405**. This assignment is now consistent with experiment, and, when one extra assumption is made, it leads to the Lee-Sugawara triangle and the vanishing of $\langle\Sigma^+|n\pi^+\rangle$ for parity-violating amplitudes. In $\tilde{U}(12)$, the assignment **143**+**5940** yields the same results. Amplitudes for Ω^- decay modes are related to amplitudes of known decays, and the corresponding rates are calculated.

I. INTRODUCTION

THE group $SU(6)$ has been proposed[1] as a possible symmetry of strongly interacting particles. In analogy with Wigner's supermultiplet scheme for nuclei,[2] the theory is developed from the identification of an $SU(2)_s \times SU(3)$ subgroup with the direct product of intrinsic spin and unitary spin. It has had some success in explaining the static properties,[3] both strong and electromagnetic, of hadrons, and so we propose to examine the consequences of $SU(6)$ for weak interactions.

Some of these consequences have already been discussed in an earlier paper.[4,5] It was shown that the assignment of the weak Hamiltonian to the **35**-dimensional representation of $SU(6)$ is consistent with the data on leptonic hyperon decay, but not with the data on nonleptonic decay. On the basis of this result, it was argued that the nonleptonic part of the weak Hamiltonian must be an admixture of the **35** and **405** representations. Here we shall show that the inclusion of the **405** terms removes the inconsistency with experiment and enables us to apply CP invariance and T-L invariance[6] to the Hamiltonian. Among the interesting results which follow from these weak symmetries are: (i) the Lee-Sugawara triangle[7] for S waves, and (ii) the vanishing of the S-wave amplitude for $\Sigma^+ \rightarrow n+\pi^+$.

Because $SU(6)$ is a nonrelativistic theory, it contains no definite prescription for dealing with orbital angular momentum, and hence it becomes ambiguous when applied to P-wave nonleptonic decay. Two plausible alternatives are to assume either that a P-wave pion behaves like the vector meson ρ, or that it transforms under $SU(6)$ in exactly the same way as an S-wave pion; the former implies a close relation between angular momentum and spin in $SU(6)$, and the latter corresponds to the assumption that orbital angular momentum is independent of $SU(6)$. As we have no *a priori* criteria for choosing between these alternatives, we shall determine the consequences of both of them, and leave the choice to experiment and to higher symmetries.

One of the higher symmetries we may consider is the relativistic generalization of $SU(6)$ known as $\tilde{U}(12)$.[8] In this scheme, leptonic decays can be assigned to the **143** representation, but nonleptonic decays must be an admixture of the **143** and the **5940**. As expected, the consequences of this assignment for S-wave nonleptonic decay are identical with those of $SU(6)$; in addition, the collinear subgroup $SU(6)_W$ can be used to impose extra constraints upon the interaction, and it yields the same results as were derived from T-L invariance in the nonrelativistic theory. The consequences of $\tilde{U}(12)$ for P-wave decays can be derived from $SU(6)$ provided that a P-wave pion is treated like the vector meson ρ.

A detailed discussion of leptonic decays is given in Sec. II. The effective Hamiltonian is assigned to a 35-plet in $SU(6)$, and to a **143** in $\tilde{U}(12)$; it is found that both assignments yield exactly the same results. Section III is devoted to a general discussion of nonleptonic decays in $SU(6)$, and in Sec. IV, we make use

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[5] M. Suzuki, Phys. Letters 14, 64 (1965); G. Altarelli, F. Buccella, and R. Gatto, *ibid.* 19, 70 (1965); P. Babu, Phys. Rev. Letters 14, 166 (1965).

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TABLE I. Predictions for matrix elements and branching ratios for leptonic decays based on $SU(6)$ (Ref. 12).

Decay	Matrix element	Branching ratio Predicted	Branching ratio Observed
$n\to p+e^-+\nu_e$	$G\cos\theta(V-1.15A)$		
$\Sigma^+\to\Lambda+e^++\nu_e$	$\frac{1}{5}(\sqrt{6})G\cos\theta(1.15A)$	0.33×10^{-4}	$(0.32\pm0.17)\times10^{-4}$
$\Sigma^-\to\Lambda+e^-+\nu_e$	$-\frac{1}{5}(\sqrt{6})G\cos\theta(1.15A)$	0.54×10^{-4}	$(0.75\pm0.28)\times10^{-4}$
$\Sigma^-\to n+e^-+\bar{\nu}_e$	$-G\sin\theta(V+0.23A)$	11.9×10^{-4}	$(12\pm2)\times10^{-4}$
$\Sigma^-\to n+\mu^-+\nu_\mu$	$-G\sin\theta(V+0.23A)$	5.5×10^{-4}	$(6.6\pm1.4)\times10^{-4}$
$\Lambda\to p+e^-+\bar{\nu}_e$	$-(\sqrt{\frac{3}{2}})G\sin\theta(V-0.69A)$	9.1×10^{-4}	$(8.8\pm0.8)\times10^{-4}$
$\Lambda\to p+\mu^-+\nu_\mu$	$-(\sqrt{\frac{3}{2}})G\sin\theta(V-0.69A)$	1.5×10^{-4}	$(1.3\pm0.6)\times10^{-4}$
$\Xi^-\to\Lambda+e^-+\bar{\nu}_e$	$-(\sqrt{\frac{3}{2}})G\sin\theta(V-0.23A)$	6×10^{-4}	$(14\pm8)\times10^{-4}$
$\Xi^-\to\Sigma^0+e^-+\bar{\nu}_e$	$-(1/\sqrt{2})G\sin\theta(V-1.15A)$	0.66×10^{-4}	
$\Xi^0\to\Sigma^++e^-+\bar{\nu}_e$	$-G\sin\theta(V-1.15A)$	2.24×10^{-4}	$<13\times10^{-4}$

of T-L invariance and CP invariance. The $\tilde{U}(12)$ theory is applied to nonleptonic decay in Sec. V. Mathematical details are relegated to the appendices: Appendix A contains the identification of the generators of $SU(6)$ with spin and unitary spin; and Appendix B contains some of the tensor components used in the body of the paper.

II. LEPTONIC DECAYS IN $SU(6)$

We assume that leptonic decays of hadrons are engendered by a current-current interaction of the form

$$H_L=J_\mu L_\mu^\dagger+\text{H.c.}, \tag{2.1}$$

where J_μ is the hadronic weak current and L_μ is the leptonic current:

$$L_\mu=\bar{\psi}_l\gamma_\mu(1+\gamma_5)\psi_{\nu_l}.\quad (l=e,\mu)\,. \tag{2.2}$$

Since we consider leptons to be outside the group $SU(6)$ [i.e., L_μ transforms as a singlet under $SU(6)$], H_L transforms in exactly the same way as J_μ.

In order to extend Cabibbo's theory of leptonic decays,[9] to $SU(6)$, we make the following assumptions about J_μ: (i) J_μ transforms as **35** under $SU(6)$; and (ii) J_μ has unit length. Since the $SU(3)$ content of the **35** consists of octets and a singlet (which does not contribute to leptonic decays), assumption (i) leads automatically to the $|\Delta\mathbf{T}|=1$ rule for strangeness-conserving decays, and to the $\Delta S=\Delta Q$, $|\Delta\mathbf{T}|=\frac{1}{2}$ rule for strangeness-violating decays. Assumption (ii) allows us to write J_μ in the form

$$J_\mu=(j_\mu^{(0)}+g_\mu^{(0)})\cos\theta+(j_\mu^{(1)}+g_\mu^{(1)})\sin\theta\,, \tag{2.3}$$

where j_μ and g_μ represent vector and axial-vector current, respectively, and the superscript indicates the value of $|\Delta S|$. By comparing $K^\pm\to\mu^\pm\nu$ and $K^\pm\to\pi^0e^\pm\nu$ to the corresponding pionic decay modes, Cabibbo found a consistent value of $\theta\simeq\sin\theta\simeq0.26$. We shall adopt this value of θ in our calculations.

In the nonrelativistic limit, the vector and axial vector parts of J_μ become Fermi and Gamow-Teller type matrix elements, respectively.[10] In Fermi-type matrix elements the intrinsic spins of the baryons are coupled to a resultant zero, and in the Gamow-Teller type they are coupled to a resultant unity. With our assignment of J_μ to **35**, this means that the Fermi-type matrix element is assigned to the (**8**, **1**) part of a **35** and the Gamow-Teller type is assigned to the (**8**, **3**) part. At this point we assume that the Fermi type and the Gamow-Teller type matrix elements belong to two different **35**'s.

Because the direct product

$$\mathbf{56}^*\otimes\mathbf{56}=\mathbf{1}\oplus\mathbf{35}\oplus\mathbf{405}\oplus\mathbf{2695} \tag{2.4}$$

contains only one **35**, the form of the baryon current is unique and the D to F ratios are fixed; Fermi matrix elements are pure F type and Gamow-Teller ones have a D/F ratio of $\frac{3}{2}$. In Cabibbo's original theory, F-type coupling for Fermi transitions is a consequence of the conserved vector current hypothesis; the value of $D/F=\frac{3}{2}$ for Gamow-Teller transitions is in good agreement with a recent analysis of experimental data.[11]

We now turn to a more detailed treatment of J_μ in order to evaluate the hyperon decay rates explicitly, and to predict the leptonic decay rates of Ω^-. The assignment of **35** for J_μ leads to the following expressions for $j_\mu^{(i)}$ and $g_\mu^{(i)}$:

$$j_4^{(0)}=T_2^4+T_1^3 \tag{2.5a}$$

$$g_3^{(0)}=T_2^4-T_1^3 \tag{2.5b}$$

$$j_4^{(1)}=T_2^6+T_1^5 \tag{2.5c}$$

$$g_3^{(1)}=T_2^6-T_1^5. \tag{2.5d}$$

These are based on our identification of the **35** tensor in Appendix A. Each term T_ν^μ is constructed from baryon-antibaryon states

$$T_\nu^\mu=\bar{B}^{\alpha\beta\mu}B_{\alpha\beta\nu}. \tag{2.6}$$

[9] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1965).

[10] E. J. Konopinski, Ann. Rev. Nucl. Sci. **9**, 99 (1965).

[11] W. J. Willis *et al.*, Phys. Rev. Letters **13**, 291 (1965). See also W. J. Willis, Proceedings of the International Conference on Weak Interactions, Argonne National Laboratory Report No. ANL-7130, Argonne, Illinois, 1965, p. 159 (unpublished).

TABLE II. Decay rates for Ω^--decay modes predicted in $SU(6)$.

Decay mode	Rate in 10^{10} sec^{-1}
$\Omega^- \to \Lambda + K^-$	3.56
$\Omega^- \to \Xi^0 + \pi^-$	2.23
$\Omega^- \to \Xi^- + \pi^0$	1.12
$\Omega^- \to \Xi^{*0} + \pi^-$	0.22
$\Omega^- \to \Xi^{*-} + \pi^0$	0.11
$\Omega^- \to \Xi^0 + e^- + \bar{\nu}_e$	0.0047
$\Omega^- \to \Xi^0 + \mu^- + \bar{\nu}_\mu$	0.0032
$\Omega^- \to \Xi^{*0} + e^- + \bar{\nu}_e$	0.0017
$\Omega^- \to \Xi^{*0} + \mu^- + \bar{\nu}_\mu$	0.0001

The matrix elements and decay rates for all octet hyperon leptonic decays are listed in Table I.[12] We have compared our predictions with experiment where available and find good agreement. The experimental estimate[11] for A/V in $\Lambda \to p + e^- + \bar{\nu}_e$ is $-0.9_{-0.3}^{+0.25}$ which is to be compared with the predicted value of -0.69. We have used

$$G_A/G_V(n \to p) = -1.15\,,\quad \sin\theta \simeq \theta = 0.26\,,\quad G = 1.025/m_p^2\,. \tag{2.7}$$

In the case of Ω^- decay, the matrix elements for

$$\Omega^- \to \Xi^0 + l^- + \nu_l \tag{2.8a}$$

$$\Omega^- \to \Xi^{*0} + l^- + \nu_l \tag{2.8b}$$

are found to be[13]

$$\langle \Omega^- | \Xi^0 \rangle_A = (2\sqrt{6/5}) 1.15 G \sin\theta \tag{2.9}$$

$$\langle \Omega^- | \Xi^0 \rangle_V = (2\sqrt{6/5}) G \sin\theta [(\mu_p - \mu_n)/2m_p] \tag{2.10}$$

$$\langle \Omega^- | \Xi^{*0} \rangle = -\sqrt{3} G \sin\theta (V - 0.69A) \tag{2.11}$$

and the corresponding decay rates are given in Table II. The predicted rate for (2.8a) is somewhat smaller than the estimate of Glashow and Socolow,[14] but the rate for (2.8b) is in good agreement with theirs. The vector matrix element in (2.10) is derived from the conserved-vector current hypothesis, and its contribution to the $\Omega^- \to \Xi^0$ transition rate is negligible.

We conclude from this analysis that the assignment of J_μ to the **35** in $SU(6)$ is quite successful in predicting the leptonic decay rates of hyperons. The detailed predictions about the V/A ratios in each case remain to be tested. We also note that had the vector and axial vector currents been assigned to the same **35**, then[15]

$$G_A/G_V(n \to p) = -5/3 \tag{2.12}$$

which is to be compared with the experimental value of Eq. (2.7).

The assignment **143** in $\tilde{U}(12)$ for weak currents was examined by Horn *et al.*,[16] and in the low-momentum limit, i.e., when we confine our attention to vector and axial vector matrix elements, it leads to the same results as were found above in $SU(6)$.

III. NONLEPTONIC DECAYS IN $SU(6)$

The simplest assignment for the effective nonleptonic decay Hamiltonian is the **35**.[17] As shown elsewhere[4,5] this leads to a serious discrepancy with experiment, and so it was suggested that the nonleptonic decays transform as a linear combination of **35** and **405**:

$$H_{\rm NL} = H(\mathbf{35}) + H(\mathbf{405})\,. \tag{3.1}$$

There are several arguments which seem to make the assignment in (3.1) appear reasonable. One is the heuristic argument given in I; another is to consider the nonleptonic decays as being engendered by a current-current interaction[18] with the current transforming as **35**. Then the nonleptonic decays can only transform as representations symmetric in the two currents,[19] i.e., **1**, **35**, **189**, and **405**. Of these, **1** does not contribute to observable decays and, with the assumption of octet dominance, the contribution from **189** is proportional to one of the terms in **35**. Hence we are left with **35**+**405** for nonleptonic decays.

If the current-current interaction is engendered by an intermediate boson, it is possible to build a model which leads to the assignment (3.1). The simplest $SU(6)$ transformation properties for an intermediate boson are **15** and **21**. Under $SU(3)\times SU(2)_s$ these decompose into

$$\mathbf{15} = (\mathbf{6},\mathbf{1}) + (\mathbf{3}^*,\mathbf{3})\,, \tag{3.2a}$$

$$\mathbf{21} = (\mathbf{6},\mathbf{3}) + (\mathbf{3}^*,\mathbf{1})\,. \tag{3.2b}$$

We now assume that the weak interaction Lagrangian transforms as the **21**. Then since

$$\mathcal{L} = J_i W_i^\dagger + J_i^\dagger W_i + L_i W_i^\dagger + L_i^\dagger W_i\,, \tag{3.3}$$

where W_i are the intermediate boson field operators, our assumption about $\mathcal{L}$ fixes the W_i to belong to **21**.[20] From (3.3) we can form the effective Hamiltonians for both leptonic and nonleptonic decays of hyperons. Since this involves taking $\mathcal{L}$ to second order, we obtain

[12] The experimental values are based on W. J. Willis, Ref. 11, and I. V. Chuvilo, *in Proceedings of the International Conference on High Energy Physics, Dubna, 1961* (Atomizdat, Moscow, 1965).

[13] $\Omega^- \to \Xi^0 + l^- + {}_e\bar{\nu}$ was also considered by M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 51 (1965); and by I. J. Muzinich, Phys. Letters **14**, 252 (1965).

[14] S. L. Glashow and R. Socolow, Phys. Letters **10**, 143 (1964).

[15] R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters **13**, 687 (1964). See also M. A. B. Bég and A. Pais, Ref. 12.

[16] D. Horn, M. Kugler, H. J. Lipkin, S. Meshkov, J. C. Carter, and J. J. Coyne, Phys. Rev. Letters **14**, 717 (1965).

[17] This assignment automatically restricts the interaction to transform like an octet under $SU(3)$ and so the $\Delta \mathbf{T} = \frac{1}{2}$ rule is guaranteed.

[18] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[19] This is analogous to only **1**, **8** and **27** appearing in an $SU(3)$ version. See M. Gell-Mann, Phys. Rev. Letters **12**, 155 (1964).

[20] This scheme is very similar to that of d'Espagnat in $SU(3)$. B. D'Espagnat, Phys. Letters **7**, 166 (1963); B. D'Espagnat, and Y. Villachon, Nuovo Cimento **33**, 948 (1964).

the result that both H_L and H_{NL} transform as

$$\mathbf{21}^*\otimes\mathbf{21}=\mathbf{1}\oplus\mathbf{35}\oplus\mathbf{405}. \qquad (3.4)$$

For leptonic decays it is possible to drop the contribution from **405**, whereas for nonleptonic decays we are forced to keep the **405**.

To determine possible coupling schemes arising out of the assignment $H_{NL}\sim\mathbf{35}+\mathbf{405}$ we consider the following:

$$H_1(\mathbf{35})=[(\bar{B}B)_{35}\otimes\varphi_{35}]_{35} \qquad (3.5a)$$

$$H_2(\mathbf{35})=[(\bar{B}B)_{405}\otimes\varphi_{35}]_{35} \qquad (3.5b)$$

$$H_3(\mathbf{405})=[(\bar{B}B)_{405}\otimes\varphi_{35}]_{405} \qquad (3.5c)$$

$$H_3'(\mathbf{405})=[(\bar{B}B)_{405}\otimes\varphi_{35}]_{405} \qquad (3.5d)$$

$$H_4(\mathbf{405})=[(\bar{B}B)_{2695}\otimes\varphi_{35}]_{405}. \qquad (3.5e)$$

Equation (3.5) exhausts[21] the possible ways in which **35** and **405** can be constructed out of $\mathbf{56}^*\otimes\mathbf{56}\otimes\mathbf{35}$. The contributions from H_1 and H_2 are written down in I and in several other papers. H_3 and H_3' correspond to the two ways in which one can construct a **405** out of $\mathbf{405}\otimes\mathbf{35}$, and are given in detail in Appendix B.

We now consider the S-wave amplitudes arising from couplings in Eq. (3.5). For the **405** part we assume octet dominance[22] and so pick out the **(8,1)** component. Since this component is unique and since the contribution of H_3' is identically zero, there are only four independent S-wave amplitudes. From these amplitudes we obtain the following matrix elements:

$$\langle\Lambda|p\pi^-\rangle_S=\sqrt{3}(S_1+\tfrac{1}{12}S_3+\tfrac{1}{6}S_4) \qquad (3.6a)$$

$$\langle\Xi^-|\Lambda\pi^-\rangle_S=\sqrt{3}(S_1-S_2+\tfrac{1}{6}S_3) \qquad (3.6b)$$

$$\langle\Sigma^+|p\pi^0\rangle_S=(S_1-2S_2+\tfrac{1}{4}S_3-\tfrac{1}{2}S_4) \qquad (3.6c)$$

$$\langle\Sigma^+|n\pi^+\rangle_S=\sqrt{2}(S_2+\tfrac{1}{2}S_4) \qquad (3.6d)$$

$$\langle\Sigma^-|n\pi^-\rangle_S=\sqrt{2}(S_1-S_2+\tfrac{1}{4}S_3) \qquad (3.6e)$$

$$\langle\Omega^-|\Xi^{*0}\pi^-\rangle_S=-(\sqrt{6})S_1. \qquad (3.6f)$$

The amplitudes $\langle\Omega^-|\Xi^{*-}\pi^0\rangle$ and $\langle\Omega^-|\Xi^{*0}\pi^-\rangle$ are related by the $|\mathbf{\Delta T}|=\frac{1}{2}$ rule:

$$\langle\Omega^-|\Xi^{*0}\pi^-\rangle=-\sqrt{2}\langle\Omega^-|\Xi^{*0}\pi^0\rangle. \qquad (3.7)$$

We note that the Lee-Sugawara sum rule[7]

$$\sqrt{3}\langle\Sigma^+|p\pi^0\rangle+\langle\Lambda|p\pi^-\rangle=2\langle\Xi^-|\Lambda\pi^-\rangle \qquad (3.8)$$

is satisfied only if $S_4=0$.

Turning to the P-wave amplitudes we recall that $SU(6)$ contains no clear-cut prescription for dealing with orbital angular momentum.[23] If the gradient operator $\boldsymbol{\nabla}$ is independent of $SU(6)$, then a P-wave pion transforms under $SU(6)$ in the same way as an S-wave pion, and the P-wave amplitude for nonleptonic decay transforms as **(8,3)** under $SU(3)\times SU(2)$. If, on the other hand, orbital angular momentum and intrinsic spin are intimately related, it is reasonable to suppose that a P-wave pion behaves like the vector meson ρ under $SU(6)$.[24] In this case, the P-wave amplitude transforms as **(8,1)**. We shall refer to amplitudes arising from the **(8,3)** as type 1, and those arising from **(8,1)** as type 2.

The **405** contains two **(8,3)**'s, and so the terms H_3, H_3' and H_4 each contribute two independent amplitudes of type 1. We choose the two **(8,3)**'s in **405** to be orthogonal and distinguish between them by subscripts a and b. In all, we have eight independent amplitudes. The P-wave matrix elements of type 1 are

$$\langle\Lambda|p\pi^-\rangle_P=\frac{1}{\sqrt{3}}[-3P_1+(P_{3a}-P_{3a}') -2(P_{3b}+P_{3b}')-(P_{4a}-P_{4b})], \qquad (3.9a)$$

$$\langle\Xi^-|\Lambda\pi^-\rangle_P=\frac{1}{\sqrt{3}}[-P_1+3P_2+(P_{3a}+P_{3a}') -(P_{3b}+P_{3b}')-(P_{4a}-P_{4b})], \qquad (3.9b)$$

$$\langle\Sigma^+|p\pi^0\rangle_P=(\tfrac{1}{3}P_1-2P_2-P_{3a}-P_{4a}), \qquad (3.9c)$$

$$\langle\Sigma^-|n\pi^-\rangle_P=\sqrt{2}(\tfrac{1}{3}P_1-P_2+P_{3a}'+P_{3b} +P_{3b}'-P_{4a}-P_{4b}), \qquad (3.9d)$$

$$\langle\Sigma^+|n\pi^+\rangle_P=\sqrt{2}(P_2+P_{3a}+P_{3a}'+P_{3b} +P_{3b}'-P_{4b}), \qquad (3.9e)$$

$$\langle\Omega^-|\Xi^{*0}\pi^-\rangle_P=-\sqrt{2}P_1, \qquad (3.9f)$$

$$\langle\Omega^-|\Xi^0\pi^-\rangle_P=\frac{1}{\sqrt{3}}(-4P_1-3P_{3a}'+3P_{3b}'), \qquad (3.9g)$$

$$\langle\Omega^-|\Lambda k^-\rangle_P=\sqrt{2}(-6P_1-2P_{3a}-2P_{3a}' +2P_{3b}+2P_{3b}'-P_{4a}+P_{4b}). \qquad (3.9h)$$

Again the matrix elements $\langle\Omega^-|\Xi^-\pi^0\rangle$ and $\langle\Omega^-|\Xi^0\pi^-\rangle$ are related by $\mathbf{\Delta T}=\frac{1}{2}$:

$$\langle\Omega^-|\Xi^0\pi^-\rangle=-\sqrt{2}\langle\Omega^-|\Xi^-\pi^0\rangle. \qquad (3.10)$$

We note that the sum rule (3.8) is satisfied individually by the amplitudes P_1, P_{3a}, P_{3b}, P_{3a}', and P_{4b}.

With the above treatment of orbital angular momentum, we expect a D-wave octet to transform like

[21] Actually there are two more. One is $H_0(\mathbf{35})\sim[(\bar{B}B)_1\otimes\varphi_{35}]_{35}$ which obviously does not contribute to decay modes of interest and the other is $H_x(\mathbf{405})\sim[(BB)_{35}\otimes\varphi_{35}]_{405}$ which as stated in Ref. 4, gives amplitudes proportional to $H_1(35)$ when octet dominance is assumed.

[22] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964); R. Dashen, S. Frautschi, M. Gell-Mann, and Y. Hara, *Eightfold Way* (W. A. Benjamin and Company, Inc., New York, 1964).

[23] This ambiguity was also discussed earlier in S. P. Rosen, Phys. Rev. Letters **14**, 758 (1965).

[24] It is interesting to note that it is only with this type of treatment of P-wave pion that an $SU(6)$-invariant strong baryon-meson interaction can be written down.

(8,5). **405** contains one (**8**,**5**) and we obtain for the D-wave amplitudes of Ω^- decays

$$\langle\Omega^-|\Xi^0\pi\rangle_D=(1/3\sqrt{3})D_3' \quad (3.11a)$$

$$\langle\Omega^-|\Lambda K^-\rangle_D=-(1/6\sqrt{2})D_4. \quad (3.11b)$$

Since Eq. (3.11) relates the two decay modes neither to one another nor to any other better known amplitude, it might seem academic to write down Eq. (3.11). However, the equation does turn out to be useful, as we shall see later.

Next, we consider P-wave amplitudes of type 2, i.e., treat a P-wave pion like a ρ meson under $SU(6)$. As discussed before, in this case the effective interaction transforms as (**8**,**1**) and there are five independent amplitudes in all (unlike the S wave, the contribution form H_3' is not zero). The matrix elements are then

$$\langle\Lambda|p\pi^-\rangle_P=-(1/\sqrt{3})(3P_1+P_2'+3P_3+2P_3'+P_4), \quad (3.12a)$$

$$\langle\Xi^-|\Lambda\pi^-\rangle_P=-(1/\sqrt{3})(P_1+\tfrac{1}{2}P_2'+2P_3), \quad (3.12b)$$

$$\langle\Sigma^+|p\pi^0\rangle_P=(\tfrac{1}{3}P_1+\tfrac{2}{3}P_2'+P_3+2P_3'+P_4), \quad (3.12c)$$

$$\langle\Sigma^-|n\pi^-\rangle_P=\sqrt{2}(\tfrac{1}{3}P_1-\tfrac{1}{6}P_2'+P_3), \quad (3.12d)$$

$$\langle\Sigma^+|n\pi^+\rangle_P=-\sqrt{2}(\tfrac{5}{6}P_2'+2P_3'+P_4), \quad (3.12e)$$

$$\langle\Omega^-|\Xi^{*0}\pi^-\rangle_P=-(\sqrt{2}/\sqrt{3})P_1, \quad (3.12f)$$

$$\langle\Omega^-|\Xi^0\pi^-\rangle_P=-(4/\sqrt{3})P_1, \quad (3.12g)$$

$$\langle\Omega^-|\Lambda K^-\rangle_P=\sqrt{2}(P_2'+4P_3). \quad (3.12h)$$

We notice that the amplitude engendered by the coupling H_1 (i.e., P_1) is the same for either type of treatment of P wave. Furthermore P_1 is the only amplitude that satisfies the sum rule of Eq. (3.8).

IV. *T-L* AND OTHER SYMMETRIES

To reduce the number of amplitudes appearing in Eqs. (3.6), (3.9), (3.11), and (3.12) we look for further restrictions that can be imposed on $H_{\rm NL}$. From $SU(3)$ considerations we know that only two weak symmetries are available—T-L symmetry and R invariance.[25] The concept of T-L invariance can be easily generalized to $SU(6)$, and the $TL(1)$ and $TL(2)$ symmetries can be defined by:

$TL(1)$ invariance: $H_{\rm NL}$ is even under the interchange of $SU(6)$ indices

$$(6\leftrightarrow 4 \quad\text{and}\quad 5\leftrightarrow 3).$$

$TL(2)$ invariance: $H_{\rm NL}$ is odd under the interchange of $SU(6)$ indices

$$(6\leftrightarrow 4 \quad\text{and}\quad 5\leftrightarrow 3).$$

It has been shown that a current-current theory of nonleptonic decays with Cabibbo-type currents and CP invariance leads to a $TL(1)$ invariant Hamiltonian.[25,26] Thus, we shall obtain the consequences of a CP-invariant current-current theory when we impose $TL(1)$ invariance. We would like to emphasize this point because there exists a misconception that certain results follow from CP invariance when, in fact, they really follow from $TL(1)$ invariance.

The requirement of this extended $TL(1)$ invariance leads[27] to the following restrictions on the amplitudes in Eqs. (3.6) to (3.12):

$$\begin{aligned} &S_2=S_4=0,\\ &D_3-D_3'=D_4=0,\\ &P_{3a}-P_{3a}'=P_{3b}+P_{3b}'=P_{4b}=0,\\ &P_3-P_3'=0. \end{aligned} \quad (4.1)$$

On the other hand, $TL(2)$ invariance requires

$$\begin{aligned} &P_2=P_{3a}+P_{3a}'=P_{3b}-P_{3b}'=P_{4a}=0,\\ &P_2'=P_3+P_3'=P_4=0,\\ &D_3+D_3'=0. \end{aligned} \quad (4.2)$$

As expected from an $SU(3)$ analysis, the Lee-Sugawara sum rule is a consequence of $TL(1)$ invariance in the case of S waves and of $TL(2)$ invariance in the case of P waves; in other words, $TL(2)\times P$ invariance gives the sum rule for both S and P waves.[25] Furthermore $TL(1)$ invariance predicts $\langle\Sigma^+|n\pi^+\rangle_S=0$. It is amusing to note that one can define an "R" (R_6) transformation in $SU(6)$ such that $R_6\times P$ invariance is the same as $TL(2)\times P$ invariance for $H_{\rm NL}$. R_6 is defined by $R_6(T_\nu{}^\mu)R_6^{-1}=-T_\mu{}^\nu$. This R_6P-invariant $H_{\rm NL}$ leads to the attractive predictions given above and does not lead to any violent disagreement with experiment. However, since we have based our assignment of $H_{\rm NL}$ to **35**+**405** on a possible current-current origin of $H_{\rm NL}$, we shall, in the following, confine our attention to $TL(1)$ invariance for $H_{\rm NL}$.

For S-wave amplitudes $TL(1)$ invariance yields

$$\langle\Lambda|p\pi^-\rangle_S=\sqrt{3}(S_1+\tfrac{1}{12}S_3), \quad (4.3a)$$

$$\langle\Xi^-|\Lambda\pi^-\rangle_S=\sqrt{3}(S_1+\tfrac{1}{6}S_3), \quad (4.3b)$$

$$\langle\Sigma^+|p\pi^0\rangle_S=(S_1+\tfrac{1}{4}S_3), \quad (4.3c)$$

$$\langle\Sigma^-|n\pi^-\rangle_S=\sqrt{2}(S_1+\tfrac{1}{4}S_3), \quad (4.3d)$$

$$\langle\Sigma^+|n\pi^+\rangle_S=0, \quad (4.3e)$$

$$\langle\Omega^-|\Xi^{*0}\pi^-\rangle_S=-(\sqrt{6})S_1. \quad (4.3f)$$

Hence, as noted earlier we predict the sum rule (3.8) for S-wave amplitudes and also predict that $\Sigma^+\to n+\pi^+$

[25] S. P. Rosen, Phys. Rev. **137**, B431 (1965).

[26] It is crucial that the currents be Cabibbo-type as opposed to pseudo-Cabibbo type i.e., $\theta_v=\theta_A$ and not $\theta_v=-\theta_A$. In the latter case one gets $TL(1)$ invariance for P waves and $TL(2)$ invariance for S waves; this does not yield the nice predictions that $TL(1)$ invariance yields for S waves. See also S. Coleman, S. L. Glashow, and B. W. Lee, Ann. Phys. (N. Y.) **30**, 348 (1964).

[27] Throughout this paper we have used an effective nonderivative coupling. See Ref. 25 for a discussion on this point.

is a pure P-wave transition. In view of recent experimental evidence,[28] this is an attractive result. We note that it is obtained here from weaker assumptions than in $SU(3)$ (Ref. 24) and in $SU(6)$ with **35** dominance.[5]

For amplitudes of Type 1 [i.e., ∇ outside $SU(6)$] we note the following: $TL(1)$ invariance requires $D_4=0$ so that

$$\langle\Omega|\Lambda K^-\rangle_D=0. \tag{4.4}$$

After imposing $TL(1)$ invariance on the P-wave amplitudes, we obtain the following interesting relation[29]:

$$\langle\Omega^-|\Lambda K^-\rangle_P=(\sqrt{\tfrac{3}{2}})[\sqrt{3}\langle\Sigma^+|p\pi^0\rangle_P+\langle\Lambda|p\pi^-\rangle_P -2\langle\Xi^-|\Lambda\pi^-\rangle_P]. \tag{4.5}$$

Comparing this result with the Lee-Sugawara sum rule, (3.8), we see that

$$\langle\Omega^-|\Lambda K^-\rangle_P=0 \tag{4.6}$$

if (3.8) holds. Empirically the Lee-Sugawara sum rule is satisfied[30] quite well for both S- and P-wave amplitudes, and so $TL(1)$ invariance implies that the decay $\Omega^-\to\Lambda+K^-$ is almost forbidden. Since three out of the seven observed events of Ω^- exhibit this decay mode,[30] the above result clashes badly with experiment. We conclude that type-I matrix elements do not provide a satisfactory way of dealing with P-wave decays in $SU(6)$.

We turn to the P-wave amplitude of type 2 [i.e., $\nabla\pi\sim\varrho$ in $SU(6)$]. We find that $TL(1)$ invariance gives

$$\langle\Lambda|p\pi^-\rangle_P=-(1/\sqrt{3})(3P_1+P_2'+5P_3+P_4), \tag{4.7a}$$

$$\langle\Xi^-|\Lambda\pi^-\rangle_P=-(1/\sqrt{3})(P_1+\tfrac{1}{2}P_2'+2P_3), \tag{4.7b}$$

$$\langle\Sigma^+|p\pi^0\rangle_P=(\tfrac{1}{3}P_1+\tfrac{2}{3}P_2'+3P_3+P_4), \tag{4.7c}$$

$$\langle\Sigma^-|n\pi^-\rangle_P=\sqrt{2}(\tfrac{1}{3}P_1-\tfrac{1}{6}P_2'+P_3), \tag{4.7d}$$

$$\langle\Sigma^+|n\pi^+\rangle_P=-\sqrt{2}(\tfrac{5}{6}P_2'+2P_3+P_4), \tag{4.7e}$$

$$\langle\Omega^-|\Xi^{*0}\pi^-\rangle_P=-(\sqrt{\tfrac{2}{3}})P_1, \tag{4.7f}$$

$$\langle\Omega^-|\Xi^0\pi^-\rangle_P=-(4/\sqrt{3})P_1, \tag{4.7g}$$

$$\langle\Omega^-|\Lambda K^-\rangle_P=\sqrt{2}(P_2'+4P_3). \tag{4.7h}$$

In this case the relation (4.5) does not hold and so there are no unsatisfactory predictions for Ω^- decays. When Eq. (4.3) and Eq. (4.7) are taken together, it is easy to show that the empirical relations[30]

$$\alpha_0\alpha_\Lambda<0,\ \alpha_\Lambda\alpha\ <0$$

can be satisfied. We recall that this was the crucial point on which **35** dominance for H_{NL} failed.[4] Furthermore, it is possible to satisfy $\alpha_-\approx0$ by setting:

$$\tfrac{1}{3}P_1-\tfrac{1}{6}P_2'+P_3=0. \tag{4.8}$$

This creates no difficulties.

We can express Ω^--decay amplitudes in terms of the better known octet decay amplitudes. For S wave we obtain

$$-\langle\Omega^-|\Xi^{*0}\pi^-\rangle_S=\sqrt{2}[2\langle\Lambda|p\pi^-\rangle_S-\langle\Xi^-|\Lambda\pi^-\rangle_S] \tag{4.9}$$

and for P waves

$$\langle\Omega^-|\Xi^{*0}\pi^-\rangle_P=\tfrac{1}{4}\sqrt{2}\langle\Omega^-|\Xi^0\pi^-\rangle_P, \tag{4.10a}$$

$$\langle\Omega^-|\Xi^0\pi^-\rangle_P=2/13\sqrt{6}[7\langle\Sigma^-|n\pi^-\rangle_P -5\langle\Sigma^+|n\pi^+\rangle_P+5(\sqrt{6})\langle\Lambda|p\pi^-\rangle_P -4(\sqrt{6})\langle\Xi^-|\Lambda\pi^-\rangle_P], \tag{4.10b}$$

$$\langle\Omega^-|\Lambda K^-\rangle_P=(1/13)[21\langle\Sigma^-|n\pi^-\rangle_P -15\langle\Sigma^+|n\pi^+\rangle_P+15(\sqrt{6})\langle\Lambda|p\pi^-\rangle_P -38(\sqrt{6})\langle\Xi^-|\Lambda\pi^-\rangle_P]. \tag{4.10c}$$

The P-wave amplitudes can be simplified by letting

$$\langle\Sigma^-|n\pi^-\rangle_p\approx0.$$

Using the above relations and the experimental values of the octet decay amplitudes, we can calculate the Ω^--decay amplitudes. For $\Omega^-\to\Xi^{0,-}+\pi^{-,0}$ and $\Omega^-\to\Lambda+K^-$, we assume that D-wave contribution to the rate is not significant because of its suppression by phase-space factors. Notice that this assumption is not inconsistent with the D-wave amplitude being of the same order of magnitude as the P wave. The results for Ω^- decay rates[31] are summarized in Table II. We note that our values for the decay rates are higher than those obtained by Glashow and Socolow.[14]

V. WEAK INTERACTIONS IN $\tilde{U}(12)$

It is well-known that $SU(6)$ is a nonrelativistic theory. Several relativistic extensions have been proposed,[8] and the groups $SL(6,c)$[32] and $\tilde{U}(12)$[33] have been put forward as strong-interaction symmetries. The consequences of $SL(6,c)$ for weak interactions have already been studied, leptonic decays by Rühl[34] and nonleptonic decays by Rosen.[23] Here we study the implications of $\tilde{U}(12)$ for weak interactions.

As has been emphasized by Lipkin,[35] it is sufficient for our purposes to use the algebra of the compact group $SU(12)$. This group contains two important and useful subgroups: one is the static subgroup $SU(6)_s$ which corresponds to the nonrelativistic $SU(6)$ theory,

[28] M. Bazin, H. Blumenfeld, U. Nauenberg, L. Seidlitz, R. J. Plano, S. Marateck, and P. Schmidt, Phys. Rev. **140**, B1358 (1965).

[29] This relation was also obtained by C. Iso and M. Kato, Nuovo Cimento **37**, 1735 (1965).

[30] N. P. Samios, Proceedings of the International Conference on Weak Interactions, Argonne National Laboratory, Argonne, 1965, p. 189 (unpublished).

[31] We have used the phase-space estimates of Glashow and Socolow (Ref. 14) and hyperon decay amplitudes calculated in R. H. Graham and S. Pakvasa, Phys. Rev. **140**, B1144 (1965).

[32] B. Sakita and K. C. Wali, Ref. 8; W. Rühl, Ref. 8.

[33] R. Delbourgo, A. Salam and J. Strathdee, Ref. 8; M. A. B. Bég and A. Pais, Ref. 8.

[34] W. Rühl, Phys. Letters **15**, 99 (1965).

[35] H. J. Lipkin, Trieste Seminar, 1965 (unpublished). $SU(12)$ was also arrived at from a different starting point by R. Dashen and M. Gell-Mann, Phys. Letters **17**, 142 (1965).

and the other is the collinear subgroup $SU(6)_w$.[36] Like $SU(6)_s$, $SU(6)_w$ can be decomposed into $SU(3)\times SU(2)_w$ where $SU(3)$ is the unitary symmetry group and $SU(2)_w$ is generated by the relativistic spin operator **W**.[37] Because W spin commutes with the Dirac Hamiltonian, $SU(6)_w$ is a valid symmetry for collinear motion at all momenta both relativistic and nonrelativistic.[38]

To make predictions from this theory, we observe that, in the rest frame of the parent baryon, nonleptonic hyperon decay is a collinear process. Consequently, both $SU(6)_s$ and $SU(6)_w$ are applicable to the decay. In the case of S waves, we follow Lipkin and Meshkov[36] and insist that the predictions of the two subgroups be identical. P-wave amplitudes vanish in the static limit, and so we can derive useful predictions about them only from $SU(6)_w$.

Because of the W-S flip discussed by Lipkin and Meshkov,[36] the pion transforms in $SU(6)_w$ as a member of **(8,3)**, and the ρ meson with $S_3=0$ transforms as **(8,1)**. This means that in $SU(6)_w$ the P-wave nonleptonic decay amplitude transforms as $\mathbf{W}=0$ and the S-wave as $\mathbf{W}=1$, $W_3=0$. It follows that the P-wave matrix elements are of the type 2 discussed in Secs. III and IV, and that the predictions of the $SU(12)$ assignment,[39,40]

$$H_{\rm NL}\simeq \mathbf{143}+\mathbf{5940} \tag{5.1}$$

are those contained in Eqs. (3.12) and (4.1).

The S-wave amplitudes which transform in $SU(6)_w$ as **(8,3)** parts of **35** and are **405** are

$$\langle\Lambda|p\pi^-\rangle_S=\sqrt{3}\bigg(S_1'+\frac{1}{18}S_2'+\frac{1}{36}S_{3a}+\frac{1}{18}S_{3b} +\frac{1}{9}S_{3a}'+\frac{1}{18}S_{3b}'-\frac{1}{36}S_{4a}-\frac{1}{18}S_{4b}\bigg) \tag{5.2a}$$

$$\langle\Xi|\Lambda\pi^-\rangle_S=\sqrt{3}\bigg(S_1'+\frac{1}{12}S_2'+\frac{1}{12}S_{3a}+\frac{1}{12}S_{3b} +\frac{7}{36}S_{3a}'+\frac{5}{36}S_{3b}'+\frac{1}{36}S_{4a}-\frac{1}{36}S_{4b}\bigg) \tag{5.2b}$$

$$\langle\Sigma^+|p\pi^0\rangle_S=\bigg(S_1'+\frac{1}{9}S_2'+\frac{5}{36}S_{3a}+\frac{1}{9}S_{3b} +\frac{5}{18}S_{3a}'+\frac{2}{9}S_{3b}'+\frac{1}{18}S_{4a}\bigg), \tag{5.2c}$$

$$\langle\Sigma^-|n\pi^-\rangle_S=\sqrt{2}\bigg(S_1'+\frac{5}{36}S_2'+\frac{1}{9}S_{3a}+\frac{5}{36}S_{3b} +\frac{1}{4}S_{3a}'+\frac{1}{4}S_{3b}'+\frac{1}{12}S_{4a}+\frac{1}{12}S_{4b}\bigg), \tag{5.2d}$$

$$\langle\Sigma^+|n\pi^+\rangle_S=\sqrt{2}\bigg(\frac{1}{36}S_2'-\frac{1}{36}S_{3a}+\frac{1}{36}S_{3b} -\frac{1}{36}S_{3a}'+\frac{1}{36}S_{3b}'+\frac{1}{12}S_{4b}\bigg) \tag{5.2e}$$

$$\langle\Omega^-|\Xi^{*0}\pi^-\rangle_S=-(\sqrt{6})S_1'. \tag{5.2f}$$

In Eq. (5.2) S_1' and S_2' are the amplitudes from the couplings of the type H_1 and H_2 in Eq. (3.5) and the rest from H_3, H_3' and H_4 with a and b distinguishing between the two **(8,3)**'s contained in **405**. We now impose the requirement that, in the limit of $SU(12)$ symmetry, the predictions of $SU(6)_s$ and $SU(6)_w$ should coincide for S wave (in the zero-momentum limit). Comparing Eq. (3.6) and Eq. (5.2) we note that the amplitude S_1' is proportional to S_1. Also a linear combination of S_{3a}, S_{3b} and S_{3a}', S_{3b}' is proportional to S_3: if we let

$$S_{3a}=S_{3b} \tag{5.3a}$$

$$S_{3a}'=S_{3b}' \tag{5.3b}$$

and define

$$S_3'=S_{3a}+2S_{3a}'=S_{3b}+2S_{3b}' \tag{5.4}$$

then we find that S_3' is proportional to S_3. However, S_2' and S_{4a}, S_{4b} are quite distinct from S_2 and S_4. Hence the only way to reconcile $SU(6)_w$ predictions for S wave with those of $SU(6)_s$ is to use Eqs. (5.3), (5.4) and impose

$$S_2'=S_{4a}=S_{4b}=S_2=S_4=0. \tag{5.5}$$

Equation (5.5) implies that the only independent amplitudes are S_1 and S_3; consequently we predict Eq. (4.3) on the basis of $SU(12)$ and without requiring $TL(1)$ invariance. From this we obtain the Lee-Sugawara sum rule, (3.8), and also

$$\langle\Sigma^+|n\pi^+\rangle_S=0. \tag{5.6}$$

No further restrictions can be imposed on the S-wave amplitudes by means of symmetry arguments. Thus, we find that for both S-wave and P-wave amplitudes, $SU(12)$ confirms the conclusions derived in Sec. IV from $SU(6)_s$ and several reasonable conjectures.

[36] H. J. Lipkin and S. Meshkov, Phys. Rev. Letters **14**, 670 (1965).
[37] M. E. Rose, *Relativistic Electron Theory* (John Wiley & Sons Inc., New York, 1961).
[38] This was noted earlier by K. J. Barnes, P. Carruthers, and F. Von Hippel, Phys. Rev. Letters **14**, 82 (1965). See also R. Dashen and M. Gell-Mann, Phys. Letters **17**, 145 (1965).
[39] This is analogous to the assignment **35**+**405** in $SU(6)$. The assignment **143** for leptonic decays was studied by Horn *et al.* (Ref. 15) and leads to the same results as in the **35** assignment in $SU(6)$. The assignment **143** for nonleptonic decays was studied by several authors (Refs. 15, 40) and found to have the same troubles as those associated with the **35** in $SU(6)$.
[40] M. P. Khanna, Phys. Rev. Letters **14**, 711 (1965); R. Oehme, Phys. Letters **15**, 284 (1965); R. Gatto, L. Maiani and G. Preparata, Phys. Rev. **139**, B1294 (1965); K. Kawarabayashi and R. White, Phys. Rev. Letters **14**, 527 (1965).

VI. CONCLUSION

In our study of weak interactions within the $SU(6)$ scheme we have found that the assumption of minimal transformation properties (**35** in this case) for weak interactions is adequate for leptonic decays but not for nonleptonic decays. The assignment (**35**+**405**) for the nonleptonic decays is based on a current-current origin for nonleptonic decays and several other arguments. We have derived the consequences of this assignment, and have found that it is possible to remove the discrepancy with experiment. Furthermore, using $TL(1)$ invariance (which follows from current-current interaction and CP invariance) we are able to predict (a) the sum rule

$$\sqrt{3}\langle\Sigma^+|p\pi^0\rangle+\langle\Lambda|p\pi^-\rangle=2\langle\Xi^-|\Lambda\pi^-\rangle$$

for the S-wave amplitudes; and (b) that $\Sigma^+\to n+\pi^+$ is a pure P-wave transition. Both of these results are in good agreement with experiment. We relate amplitudes for Ω^- decay to known decay amplitudes and predict decay rates for decays of Ω^- both leptonic and nonleptonic.

In a study of weak interactions in a relativistic version of $SU(6)$, namely, $\tilde{U}(12)$, we find that we are able to deduce results (a) and (b) above without $TL(1)$ invariance. Apart from that the conclusions arrived at in $SU(6)$ are essentially unchanged.

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APPENDIX A: BREAKDOWN OF $SU(6)$ AND IDENTIFICATION OF STATES

In this Appendix we identify the generators of $SU(6)$ in terms of those of $SU(3)\times SU(2)_s$ and also identify the physical states of interest. We denote the generators of $SU(6)$ by $A_\nu{}^\mu$ where μ,ν run from 1 to 6; they satisfy the commutation rules

$$[A_\beta{}^\alpha,A_\nu{}^\mu]=\delta_\beta{}^\mu A_\nu{}^\alpha-\delta_\nu{}^\alpha A_\beta{}^\mu \quad \text{(A1)}$$

are traceless, and obey the unitary restriction $(A_\beta{}^\alpha)^\dagger=A_\alpha{}^\beta$. The breakdown of $SU(6)$ into $SU(3)\times SU(2)_s$ is achieved by identifying the generators of $SU(3)$ and $SU(2)_s$ as follows[41,42]:

$$SU(3):\quad B_j{}^i=(A_{2j}{}^{2i}+A_{2j-1}{}^{2i-1})-\tfrac{1}{3}\delta_j{}^i(A_{2\alpha}{}^{2\alpha}+A_{2\alpha-1}{}^{2\alpha-1}) \quad \text{(A2)}$$
$$(i,j,\alpha=1\text{ to }3)$$

[41] A somewhat different breakdown has been given by M. A. B. Bég and V. Singh, Phys. Rev. Letters **13**, 418 (1964).

[42] Here and throughout the appendices we have used the summation convention for repeated indices.

TABLE III. Identification of baryon states of interest in $SU(6)$.

Baryon	$S_3=\frac{1}{2}$	$S_3=-\frac{1}{2}$
p	$\sqrt{2}(B_{123}-B_{114})$	$\sqrt{2}(B_{223}-B_{124})$
n	$\sqrt{2}(B_{233}-B_{134})$	$\sqrt{2}(B_{234}-B_{144})$
Λ	$\sqrt{3}(B_{145}-B_{235})$	$\sqrt{3}(B_{146}-B_{236})$
Σ^+	$\sqrt{2}(B_{125}-B_{116})$	$\sqrt{2}(B_{225}-B_{126})$
Σ^0	$(B_{145}+B_{235}-2B_{136})$	$(2B_{245}-B_{236}-B_{146})$
Σ^-	$\sqrt{2}(B_{345}-B_{336})$	$\sqrt{2}(B_{445}-B_{336})$
Ξ^0	$\sqrt{2}(B_{156}-B_{255})$	$\sqrt{2}(B_{166}-B_{256})$
Ξ^-	$\sqrt{2}(B_{356}-B_{455})$	$\sqrt{2}(B_{366}-B_{456})$
Ξ^{*0}	$(2B_{156}+B_{255})$	$(2B_{256}+B_{166})$
Ξ^{*-}	$(2B_{356}+B_{455})$	$(2B_{456}+B_{366})$
Ω^-	$-\sqrt{3}B_{556}$	$-\sqrt{3}B_{566}$

where $B_j{}^i$ are generators of $SU(3)$ (Ref. 42).

$$SU(2)_s:\quad \begin{aligned} S_+&=-A_{2\alpha-1}{}^{2\alpha},\\ S_3&=\tfrac{1}{2}(A_{2\alpha}{}^{2\alpha}-A_{2\alpha-1}{}^{2\alpha-1}),\\ S_-&=-A_{2\alpha}{}^{2\alpha-1},\\ &(\alpha=1\text{ to }3).\end{aligned} \quad \text{(A3)}$$

The remaining 24 "off-diagonal" generators which transform as (**8**,**3**) under $SU(3)\times SU(2)_s$ are given by[42]

$$\begin{aligned}(B_+)_j{}^i&=A_{2j-1}{}^{2i}-\tfrac{1}{3}\delta_j{}^iA_{2\alpha-1}{}^{2\alpha},\\ (B_0)_j{}^i&=\tfrac{1}{2}(A_{2j}{}^{2i}-A_{2j-1}{}^{2i-1})-\tfrac{1}{6}\delta_j{}^i(A_{2\alpha}{}^{2\alpha}-A_{2\alpha-1}{}^{2\alpha-1}),\\ (B_-)_j{}^i&=-A_{2j}{}^{2i-1}+\tfrac{1}{3}\delta_j{}^iA_{2\alpha}{}^{2\alpha-1}\\ &(i,j,\alpha=1\text{ to }3).\end{aligned} \quad \text{(A4)}$$

Since the mesons are assigned to **35**, we can identify all **35** meson states from Eqs. (A2)-(A4). The pseudoscalar meson states and the corresponding vector meson states with $S_3=0$ are given by

$$\pi^+=(1/\sqrt{2})(\varphi_2{}^4+\varphi_1{}^3), \quad \text{(A5a)}$$

$$\pi^0=\tfrac{1}{2}(\varphi_4{}^4+\varphi_3{}^3-\varphi_1{}^1-\varphi_2{}^2), \quad \text{(A5b)}$$

$$\pi^-=-(1/\sqrt{2})(\varphi_4{}^2+\varphi_3{}^1), \quad \text{(A5c)}$$

$$K^-=(1/\sqrt{2})(\varphi_6{}^2+\varphi_5{}^1), \quad \text{(A5d)}$$

$$\rho_0{}^+=(1/\sqrt{2})(\varphi_2{}^4-\varphi_1{}^3), \quad \text{(A5e)}$$

$$\rho_0{}^0=\tfrac{1}{2}(\varphi_4{}^4+\varphi_1{}^1-\varphi_2{}^2-\varphi_3{}^3), \quad \text{(A5f)}$$

$$\rho_0{}^-=-(1/\sqrt{2})(\varphi_4{}^2-\varphi_3{}^1), \quad \text{(A5g)}$$

$$K_0{}^{*-}=(1/\sqrt{2})(\varphi_6{}^2-\varphi_5{}^1), \quad \text{(A5h)}$$

where $\varphi_\nu{}^\mu$ is the traceless tensor transforming as **35**. From above Eq. (A5) it is easy to obtain the corresponding classification under $SU(6)_w$ by interchainging π^+ with $\rho_0{}^+$ etc.

The baryon states in **56** are identified with a tensor $B_{\alpha\beta\gamma}$ which is totally symmetric in α, β, and γ. We list the resulting identification for states of interest in Table III.

APPENDIX B: SOME RELEVANT RESULTS OF TENSOR ANALYSIS OF $SU(6)$

In this Appendix we list the relevant components of various tensors of $SU(6)$ which were used in the text. These components are the octet parts of various $SU(6)$ representations with intrinsic spins 0, 1 or 2. In $SU(3)$ space the octet is chosen to have the quantum numbers corresponding to the nonleptonic decays of hyperons i.e., $Y=1$, $|\Delta \mathbf{T}|=-\Delta T_3=\frac{1}{2}$; and in $SU(2)_s$ space for simplicity the $S_3=0$ component is picked out.

(i) **35**:

The breakdown of **35** under $SU(3)\times SU(2)_s$ is

$$\mathbf{35}\sim(\mathbf{8},\mathbf{1})+(\mathbf{8},\mathbf{3})+(\mathbf{1},\mathbf{3})\,. \qquad \text{(B1)}$$

The tensor component for $(\mathbf{8},\mathbf{1})$ is

$$(\mathbf{8},\mathbf{1})\sim T_4{}^6+T_3{}^5 \qquad \text{(B2a)}$$

and that for $(\mathbf{8},\mathbf{3})$ is

$$(\mathbf{8},\mathbf{3})\sim(T_4{}^6-T_3{}^5)\,. \qquad \text{(B2b)}$$

(ii) **405**:

The breakdown of **405** is given by

$$\mathbf{405}\sim(\mathbf{1},\mathbf{1})+(\mathbf{8},\mathbf{1})+(\mathbf{27},\mathbf{1})+2(\mathbf{8},\mathbf{3})+(\mathbf{10}^*,\mathbf{3})+(\mathbf{10},\mathbf{3})+(\mathbf{27},\mathbf{3})+(\mathbf{1},\mathbf{5})+(\mathbf{8},\mathbf{5})+(\mathbf{27},\mathbf{5})\,. \qquad \text{(B3)}$$

The tensor $T_{\alpha\beta}{}^{\mu\nu}$ transforming as **405** is symmetric between both μ and ν, and α and β. The component for $(\mathbf{8},\mathbf{1})$ is

$$(\mathbf{8},\mathbf{1})\sim(T_{32}{}^{52}+T_{41}{}^{61})-(T_{14}{}^{52}+T_{23}{}^{61}) \qquad \text{(B4a)}$$

and the two orthogonal $(\mathbf{8},\mathbf{3})$'s, which we have distinguished by the subscripts a and b in the text, are

$$(\mathbf{8},\mathbf{3})_a\sim T_{41}{}^{61}+T_{43}{}^{63}+T_{45}{}^{65}-T_{32}{}^{52}-T_{34}{}^{54}-T_{36}{}^{56}\,, \qquad \text{(B4b)}$$

$$(\mathbf{8},\mathbf{3})_b\sim T_{23}{}^{61}-T_{14}{}^{52}+T_{43}{}^{63}-T_{45}{}^{65}+T_{36}{}^{56}-T_{34}{}^{54}\,. \qquad \text{(B4c)}$$

The $(\mathbf{8},\mathbf{5})$ is given by

$$(\mathbf{8},\mathbf{5})\sim 2T_{32}{}^{52}+3T_{34}{}^{54}+2T_{36}{}^{56}+2T_{41}{}^{61}+3T_{43}{}^{63}+2T_{45}{}^{65}+T_{14}{}^{52}+T_{23}{}^{61}\,. \qquad \text{(B4d)}$$

We also give below the way in which the amplitudes corresponding to H_1 to H_4 of Eq. (3.5) are constructed out of $\mathbf{56}^*\otimes\mathbf{56}\otimes\mathbf{35}$.

$$H_1\sim\bar{B}^{\alpha\beta\mu}B_{\alpha\beta\delta}\varphi_\nu{}^\delta\,, \qquad \text{(B5a)}$$

$$H_2\sim\bar{B}^{\alpha\beta\mu}B_{\alpha\delta\nu}\varphi_\beta{}^\delta\,, \qquad \text{(B5b)}$$

$$H_3\sim\bar{B}^{\alpha\delta\mu}B_{\alpha\nu\gamma}\varphi_\delta{}^\beta+\bar{B}^{\alpha\delta\beta}B_{\alpha\nu\gamma}\varphi_\delta{}^\mu\,, \qquad \text{(B5c)}$$

$$H_3'\sim\bar{B}^{\alpha\mu\beta}B_{\alpha\nu\delta}\varphi_\gamma{}^\delta+\bar{B}^{\alpha\mu\beta}B_{\alpha\delta\gamma}\varphi_\nu{}^\delta\,, \qquad \text{(B5d)}$$

$$H_4\sim\bar{B}^{\alpha\mu\beta}B_{\delta\nu\gamma}\varphi_\alpha{}^\delta\,. \qquad \text{(B5e)}$$

Errata

***W*-Spin and the Rotation Group in Four Dimensions,** K. AHMED, S. A. DUNNE, M. MARTINIS AND J. R. POSTON [Phys. Rev. **142,** 995 (1966)]. In Ref. 3, insert "H. J. Lipkin, in" before "Proceedings of the Seminar . . ."

The third line of Eq. (2.3) should read

$$|0,0\rangle_S=(\sqrt{\tfrac{1}{2}})(|+-\rangle-|-+\rangle).$$

The fourth line of Eq. (4.1) should read

$$|2,-1\rangle_W=+|1,-1\rangle_S.$$

Polarization of the Σ^0 Particles Produced in the Reaction $\pi^-+p\rightarrow K^0+\Sigma^0$ at 1.5 and 1.8 BeV/*c*, YOUNG S. KIM, G. R. BURLESON, P. I. P. KALMUS, A. ROBERTS, AND T. A. ROMANOWSKI [Phys. Rev. **143,** 1028 (1966)]. There is a misprint in the 3rd line after Eq. (5): "π^-p c.m. frame" should read "Σ^0 rest frame."

We wish to thank Professor F. Crawford for kindly bringing the misprint to our attention.

Nucleon Form Factors and Their Interpretation, L. H. CHAN, K. W. CHEN, J. R. DUNNING, JR., N. F. RAMSEY, J. K. WALKER, AND RICHARD WILSON [Phys. Rev. **141,** 1298 (1966)]. In Table II, Fits Nos. 4 and 5, the mass M_1 should read 752 instead of 760. Under fit No. 3 the annihilation threshold constraint should read "No" instead of "Yes." These were the actual constraints used.

Equations (3) should read:

$$\rho(r)=3.08\exp\{(-4.26\pm0.06)r\}\ (\text{electron charges})\times\text{F}^{-3},$$

$$\mu(r)=8.59\exp\{(-4.26\pm0.06)r\}\ (\text{nuclear magnetons})\times\text{F}^{-3}.$$