Analysis of η^0, $X^0 \to \pi^+\pi^-\gamma$ with a Possible C Violation*

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The asymmetry of the π^+, π^- energy spectra in η^0, $X^0 \to \pi^+\pi^-\gamma$ decays is analyzed under the assumption of a possible C violation in electromagnetic interactions. It is assumed that the P- and D-wave pion-pion phase shifts satisfy an effective-range expansion whose parameters are determined by the position and width of the ρ^0 and f^0 resonances. Within the validity of this assumption the upper limit for the asymmetry parameter in η^0 decay is found to be 1.1%, corresponding to a ratio $R_\eta = \Gamma(\eta_0 \to \pi^0\pi^0\gamma)/\Gamma(\eta^0 \to \pi^+\pi^-\gamma) = 0.25$; the upper limit for the asymmetry parameter in X^0 decay is 18%, corresponding to a ratio $R_X = \Gamma(X^0 \to \pi^0\pi^0\gamma)/\Gamma(X^0 \to \pi^+\pi^-\gamma) = 0.25$. For any given values of the "decay radius" and the ratio of the C-violating to C-conserving coupling constants, the $\pi^+\pi^-$ asymmetry is much larger in X^0 decay than in η^0 decay; hence the notion of maximal C violation in electromagnetic processes can be suitably tested in this decay. This is, moreover, a more sensitive test than a search for the neutral mode $X^0 \to \pi^0\pi^0\gamma$, since the $\pi^+\pi^-$ asymmetry can be as large as 10% for low values of $R_X = 0.04$.

1. INTRODUCTION

THE question of whether C is invariant in electromagnetic interactions of strongly interacting particles has recently been raised by Bernstein, Feinberg, and Lee[1] and also by Barshay[2] in connection with the existence of a small CP-violation amplitude as observed in the $K_2^0 \to \pi^+\pi^-$ experiment.[3] They pointed out that a large C violation in electromagnetic interactions can account for this small CP-violation amplitude. Lee[4] and Bernstein, Feinberg, and Lee[1] discuss the measurement of the asymmetry in the energy distribution of π^+ and π^- in $\eta^0 \to \pi^+\pi^-\gamma$ decay as a possible way to detect such a C-noninvariant effect. Their suggestion, of course applies also to the $\pi^+\pi^-\gamma$ decay mode of the X^0.

The present upper limit for the C-violation amplitude in strong interactions is one to two orders of magnitude smaller than the C-conserving one.[1,4–6] It cannot give rise to an appreciable asymmetry in η^0, $X^0 \to \pi^+\pi^-\gamma$ decay.[4,7] If a sizable asymmetry is found in either decay mode, it is then reasonable to conclude that C and hence T are indeed violated in electromagnetic processes.

The asymmetry in π^+ and π^- energy spectra is due to the interference between odd and even partial waves of the two-pion system. The lowest partial waves are P and D waves, corresponding, respectively, to the C-conserving and C-violating amplitudes. In the absence of strong final-state interactions, such an interference is not possible since these two amplitudes are 90° out of phase as required by parity conservation in electromagnetic processes and CPT invariance.[1] It is clear that the asymmetry depends not only on the D-to-P-wave ratio but also on the pion-pion interactions.[1]

The main purpose of this paper is to give an estimate of the asymmetry expected from η^0 and X^0 decays under various assumptions of the relative strength of the C-violating amplitude and some knowledge of pion-pion interactions. The effects of the strong final-state interaction are studied by using the dispersion relation technique. The formulation of the problem is sufficiently general so that it can be used in the future to get information on pion-pion scattering if such an asymmetry exists.

In Sec. 2 the general forms of the matrix element, spectra, and angular distribution are given. The dispersion relation is discussed in Sec. 3. Numerical results and discussions are given in Sec. 4. In an Appendix the present upper limit of the strength of the C violation in strong interactions is shown to give negligible contribution to the asymmetry in η^0, $X^0 \to \pi^+\pi^-\gamma$ decay.

2. MATRIX ELEMENT, ENERGY, ANGULAR DISTRIBUTION, AND ASYMMETRY

We consider the decay of a pseudoscalar meson (η^0, X^0) of mass m into two charged pions and a photon. Let p_+, p_- and p_γ and e_γ be the four-momentum of π^+, π^- and the photon, respectively, and the polarization vector of the photon. We introduce two phenomenological amplitudes $\mathfrak{M}_1$ and $\mathfrak{M}_2$ corresponding, respectively, to a C-conserving and a C-violating amplitude.

$$\mathfrak{M}_1 = [f(s)/M^3]\epsilon_{\mu\nu\sigma\tau}e_\gamma{}^\mu p_\gamma{}^\nu p_+{}^\sigma p_-{}^\tau, \tag{1}$$

$$\mathfrak{M}_2 = [ig(s)/M^5]\epsilon_{\mu\nu\sigma\tau}e_\gamma{}^\mu p_\gamma{}^\nu p_+{}^\sigma p_-{}^\tau p_\gamma \cdot (p_+ - p_-), \tag{2}$$

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[1] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. 139, B1650 (1965).
[2] S. Barshay, Phys. Letters 17, 78 (1965).
[3] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turley, Phys. Rev. Letters 13, 138 (1964).
[4] T. D. Lee, Phys. Rev. 139, B1415 (1965).
[5] B. Barrett, M. Jacob, M. Nauenberg, and T. N. Truong, Phys. Rev. 141, 1342 (1966).
[6] C. Baltay *et al.*, Phys. Rev. Letters 15, 591 (1965).
[7] S. L. Glashow and C. M. Sommerfield, Phys. Rev. Letters 15, 78 (1965).

where $s=(p_++p_-)^2$ is the total energy squared of the two pions and M is a certain mass which is used here to make $\mathfrak{M}_1$ and $\mathfrak{M}_2$ dimensionless. Henceforth we shall call $1/M$ the "decay radius." We have made the approximation of keeping only a P state for the C-conserving amplitude and D state for the C-violating amplitude. The factor i is included in the definition of $\mathfrak{M}_2$ to make f and g relatively real in the absence of the strong final-state interaction.[1] To the first order in the electromagnetic interaction the C-violating amplitude (D wave) must be in the isospin $I=0$ state since both η^0 and X^0 are isosinglets. Hence the rate for η^0, $X^0\to\pi^0\pi^0\gamma$ is $\frac{1}{2}|\mathfrak{M}_2|^2$ multiplied by the appropriate phase-space factor.

After summing over the polarization of the photon, we have

$$\begin{aligned}|\mathfrak{M}|^2=&|\mathfrak{M}_1+\mathfrak{M}_2|^2\\ =&(1/M^6)(\tfrac{1}{2}(m^2-s))^2(\tfrac{1}{4}(s-4))\sin^2\theta\{|f(s)|^2\\ &-(i/2M^2)(f^*g-g^*f)[(s-4)/s]^{1/2}(m^2-s)\cos\theta\\ &+(1/4M^4)|g(s)|^2[(s-4)/s](m^2-s)^2\cos^2\theta\},\end{aligned}\quad(3)$$

where we have taken the pion mass to be unity. The rate is given by

$$d\Gamma=(2\pi)^4\delta^4(p-p_+-p_--p_\gamma)[16(2\pi)^9mE_+E_-E_\gamma]^{-1}\times d\mathbf{p}_+d\mathbf{p}_-d\mathbf{p}_\gamma|\mathfrak{M}|^2,\quad(4)$$

where p is the 4-momentum of the decay particle; the meaning of the rest of the symbols is obvious. The angle θ is defined as the angle between $\mathbf{p}_+$ and $\mathbf{p}_\gamma$ in the center of mass of the two pions. Using (3) and (4) it is straightforward to compute the energy and angular distributions as a function of s and θ (Ref. 8):

$$\begin{aligned}\Gamma(s,\cos\theta)=&\partial^2\Gamma/\partial s\partial\cos\theta\\ =&(1/NM^6)(m^2-s)^3[(s-4)^3/s]^{1/2}\sin^2\theta\\ &\times\{|f(s)|^2-(i/2M^2)(f^*g-g^*f)\\ &\times[(s-4)/s]^{1/2}(m^2-s)\cos\theta+[|g(s)|^2/4M^4]\\ &\times[(s-4)/s](m^2-s)^2\cos^2\theta\},\end{aligned}\quad(5)$$

where $N=4(16)^2(2\pi)^3m^3$.

The asymmetry parameter α is defined by

$$\alpha=\frac{(\text{number of events with }T_+>T_-)-(\text{number of events with }T_+<T_-)}{\text{total number of events}},\quad(6a)$$

where T_+, T_- are, respectively, the laboratory kinetic energy of the π^+ and π^-.

$$\alpha=\int_4^{m^2}X(s)ds\bigg/\int_4^{m^2}\Gamma(s)ds,\quad(6b)$$

where

$$X(s)=\frac{1}{NM^6}\frac{i}{4M^2}(f^*g-g^*f)(m^2-s)^4\frac{(s-4)^2}{s},\quad(6c)$$

and

$$\Gamma(s)=\frac{1}{NM^6}(m^2-s)^3\left(\frac{(s-4)^3}{s}\right)^{1/2}\left[\tfrac{4}{3}|f(s)|^2+\frac{1}{15}\left(\frac{s-4}{s}\right)\left(\frac{m^2-s}{M^2}\right)^2|g(s)|^2\right].\quad(6d)$$

The rate of η^0, $X^0\to\pi^+\pi^-\gamma$ decay is given by

$$\Gamma(X^0,\eta^0\to\pi^+\pi^-\gamma)=\int_4^{m^2}\Gamma(s)ds,\quad(6e)$$

while that of η^0, $X^0\to\pi^0\pi^0\gamma$ decay has the following value:

$$\Gamma(X^0,\eta^0\to\pi^0\pi^0\gamma)=\frac{1}{2}\int_4^{m^2}\Gamma_2(s)ds,\quad(6f)$$

where $\Gamma_2(s)$ is $\Gamma(s)$ with $f(s)=0$.

3. DISPERSION RELATIONS

We assume that $f(s)$ and $g(s)$ satisfy the following subtracted dispersion relation:

$$f(s)=f_1+\frac{s-4}{\pi}\int_4^\infty ds'\frac{\mathrm{Im}f(s')}{(s'-s-i\epsilon)(s'-4)},\quad(7a)$$

$$g(s)=g_2+\frac{s-4}{\pi}\int_4^\infty ds'\frac{\mathrm{Im}g(s')}{(s'-s-i\epsilon)(s'-4)},\quad(7b)$$

where $f_1=f(4)$ and $g_2=g(4)$ are both real and are defined as the coupling constants. (Throughout this calculation we assume that T invariance is valid for strong interactions.) Using the unitarity condition to calculate Imf and Img and assuming that the two-pion intermediate state dominates the dispersion integral, we arrive at

$$f(s)=f_1+\frac{s-4}{\pi}\int_4^\infty ds'\frac{e^{-i\delta_1}\sin\delta_1(s')f(s')}{(s'-s-i\epsilon)(s'-4)},\quad(8a)$$

$$g(s)=g_2+\frac{s-4}{\pi}\int_4^\infty ds'\frac{e^{-i\delta_2}\sin\delta_2(s')g(s')}{(s'-s-i\epsilon)(s'-4)},\quad(8b)$$

where δ_1 and δ_2 are, respectively, the P- and D-wave phase shift of the two pions. The D amplitude is in the isospin $I=0$. The solution of (8a) and (8b) is well

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[8] In terms of the Dalitz variables, we have

$$\Gamma=\frac{\sqrt{3}Q^2}{16m(2\pi)^3}|\mathfrak{M}|^2dTd\delta T,$$

where $T=T_\gamma/Q$, $\delta T=(T_+-T_-)/\sqrt{3}Q$, $Q=m-2$, T_γ is the energy of the photon and T_+, T_- are, respectively, the laboratory kinetic energy of the π^+ and π^-. In terms of the variables s, $\cos\theta$ we have:

$$4m+4mT_+=(m^2+s)+[(s-4)/s]^{1/2}(m^2-s)\cos\theta,$$
$$4m+4mT_-=(m^2+s)-[(s-4)/s]^{1/2}(m^2-s)\cos\theta,$$
$$4mT_\gamma=2(m^2-s).$$

known[9]:

$$f(s)=f_1 \exp\frac{s-4}{\pi}\int_4^\infty ds'\frac{\delta_1(s')}{(s'-s-i\epsilon)(s'-4)}\equiv\frac{f_1}{D_1(s)}, \quad (9a)$$

$$g(s)=g_2 \exp\frac{s-4}{\pi}\int_4^\infty ds'\frac{\delta_2(s')}{(s'-s-i\epsilon)(s'-4)}\equiv\frac{g_2}{D_2(s)}, \quad (9b)$$

where the D's are the same D functions used to solve the N/D equations for the corresponding partial-wave-amplitude integral equations for pion-pion scattering.[10] It is noticed that $f(s)$ and $g(s)$ have, respectively, the phases of the P- and D-wave phase shifts.

In principle, one can assume some expressions for the phase shifts and calculate $D_1(s)$ and $D_2(s)$. Upon substituting expressions for $f(s)$ and $g(s)$ as given by Eqs. (9a) and (9b) into Eqs. (5) and (6), the asymmetry parameter, two pion spectra and angular distribution can be calculated in terms of one unknown parameter g_2/f_1M^2 and thus can be compared with the future experimental data. Even if the C-violating amplitude is absent, measurement of the two-pion spectra in X^0 decay would allow an accurate determination of the parameters used to describe the P-wave phase shift.

For our present purpose, we shall assume that we have some rough knowledge of pion-pion interactions. We know experimentally that there is a P-wave resonance at 750 MeV with a width of 120 MeV (the ρ meson) and a D-wave resonance in isospin $I=0$ (the f^0) at 1250 MeV. It is reasonable to assume that the pion-pion phase shifts are known with certainty in these regions, namely they have to pass through 90° and their energy dependence is given by the width of the resonances. The P-wave phase shift used in the analysis of X^0 decay can be regarded as trustworthy since the total energy of the two pions lies entirely within the range of the resonance. This is not the case for η^0 decay which has a much smaller Q value. We must use some theoretical method to estimate the pion-pion phase shift away from the resonance region. This method may not be reliable, but for our purpose, this is the best we can do. In the next section we shall discuss the implication of large deviation from the phase shift given below.

For the P-wave resonance, we use the effective-range formula of Frazer and Fulco[11]:

$$\left[\frac{(s-4)^3}{s}\right]^{1/2}\cot\delta_1=\frac{1}{\gamma_\rho}(s_\rho-s)+\frac{8}{\pi}\left(\frac{s-4}{s}\right)^{1/2} \times\ln\left[\left(\frac{s}{4}-1\right)^{1/2}+\left(\frac{s}{4}\right)^{1/2}\right]. \quad (10a)$$

[9] R. Omnes, Nuovo Cimento **8**, 316 (1958); N. I. Muskhelishvili, *Singular Integral Equations* (P. Noordhoff Ltd., Groningen, The Netherlands, 1953).

[10] G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960).

[11] W. R. Frazer and J. R. Fulco, Phys. Rev. Letters **2**, 365 (1959).

It is a good approximation to replace (10a) by

$$[(s-4)^3/s]^{1/2}\cot\delta_1=(1/\gamma_\rho)(s_\rho-s), \quad (10b)$$

which yields the following approximate expression for the D_1 function

$$\frac{1}{D_1}=\frac{s_\rho-4}{s_\rho-s-i\gamma_\rho[(s-4)^3/s]^{1/2}}. \quad (11)$$

Using the experimental mass and width of ρ we obtain $s_\rho=30$ and $\gamma_\rho=0.15$. Likewise we shall characterize the D-wave phase shift by

$$[(s-4)^5/s]^{1/2}\cot\delta_2=(1/\gamma_f)(s_f-s), \quad (12)$$

which gives

$$\frac{1}{D_2}=\frac{s_f-4}{s_f-s-i\gamma_f[(s-4)^5/s]^{1/2}}, \quad (13)$$

where $s_f=81$ and $\gamma_f=0.001$ for the observed mass of f^0 of 1250 MeV and width of 100 MeV. In the range of two-pion energy available in η^0 and X^0 decay it is a good approximation to put

$$1/D_2 \simeq s_f-4/s_f-s. \quad (14)$$

4. NUMERICAL RESULTS

Substituting Eqs. (11) and (14) into Eq. (5) we have

$$\Gamma(s,\cos\theta)=(\text{const})(m^2-s)^3\left(\frac{(s-4)^3}{s}\right)^{1/2}\sin^2\theta\left[\frac{1}{|D_1(s)|^2} -\frac{c}{D_2(s)}\,\text{Im}\frac{1}{D_1(s)}\left(\frac{s-4}{s}\right)^{1/2}\left(\frac{m^2-s}{M^2}\right)\cos\theta +\frac{c^2}{4|D_2(s)|^2}\left(\frac{s-4}{s}\right)\left(\frac{m^2-s}{M^2}\right)^2\cos^2\theta\right], \quad (15)$$

where $c=g_2/f_1$ and we have used the approximation that $1/D_2(s)$ is real.

In order to compare η^0 and X^0 decays, we assume that they have the same characteristic decay radius, i.e., that the ratio of the C-violating amplitude to the C conserving one is the same for both decays. This may not be a bad assumption since, assuming that the width of $\eta^0\to\pi^+\pi^-\gamma$ is 30 eV it predicts a width for $X^0\to\pi^+\pi^-\gamma$ of 40 keV which competes favorably with the strong decay mode $X^0\to\pi^+\pi^-\eta^0$ as observed experimentally.[12] We arbitrarily set $M=m_X$ in both decays and study the asymmetry as a function of c.

[12] R. H. Dalitz and D. G. Sutherland, Nuovo Cimento **37**, 1777 (1965); L. M. Brown and H. Faier, Phys. Rev. Letters **13**, 73 (1964).

TABLE I. Calculated values of the asymmetry parameter α and branching ratio R for η^0 and X^0 decays [Eqs. (18), (20)].

$\sqrt{c}$	α_{η^0} (%)	R_{η^0}	α_{X^0} (%)	R_{X^0}
1	0.04	0.0003	1.4	0.001
2	0.18	0.003	5.4	0.01
3	0.39	0.015	11.1	0.05
4	0.65	0.046	16.3	0.13
5	0.90	0.100	18.4	0.23
6	1.06	0.170	17.5	0.32
7	1.13	0.220	15.6	0.38
8	1.09	0.28	13.2	0.42

Upon integrating over the angular dependence we obtain the two-pion energy distribution

$$\frac{d\Gamma}{ds}\propto (m^2-s)^3\left[\frac{(s-4)^3}{s}\right]^{1/2}\left[\frac{4}{3}\left|\frac{1}{D_1(s)}\right|^2+\frac{1}{15}c^2\left|\frac{1}{D_2(s)}\right|^2\left(\frac{s-4}{s}\right)\left(\frac{m^2-s}{m_X^2}\right)^2\right]. \quad (16)$$

The average angular distribution is given by

$$\frac{d\Gamma(\cos\theta)}{d\cos\theta}=\int_4^{m^2}\Gamma(s,\cos\theta)ds.$$

It has the following numerical value for η^0 decay;

$$d\Gamma_\eta/d\cos\theta \propto (1+0.0011c\cos\theta+0.002c^2\cos^2\theta)\sin^2\theta, \quad (17a)$$

and, for X^0 decay,

$$d\Gamma_{X^0}/d\cos\theta\propto(1+0.037c+0.007c^2\cos^2\theta)\sin^2\theta. \quad (17b)$$

The asymmetry parameter for η^0 decay is given by

$$\alpha_\eta=0.044c/(1+4\times10^{-4}c^2)\%, \quad (18a)$$

and, for X^0 decay,

$$\alpha_X=1.4c/(1+1.4\times10^{-3}c^2)\%. \quad (18b)$$

It is also convenient to define the branching ratio R:

$$R_{\eta,X}=\frac{\Gamma(\eta^0, X^0\to\pi^0\pi^0\gamma)}{\Gamma(\eta^0X^0\to\pi^+\pi^-\gamma)}. \quad (19)$$

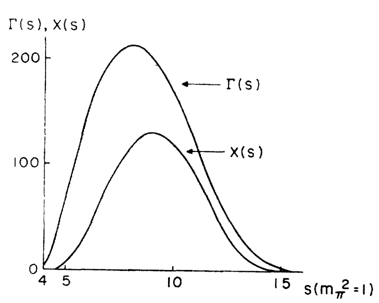

FIG. 1. $\eta^0\to\pi^+\pi^-\gamma$ decay. The two-pion energy distribution $\Gamma(s)$ and the P- and D-wave interference $X(s)$ as a function of the two-pion energy squared.

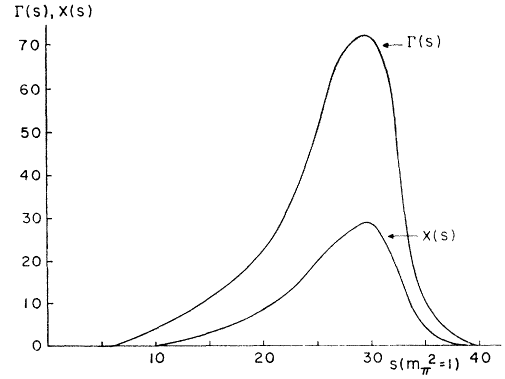

FIG. 2. $X^0\to\pi^+\pi^-\gamma$ decay. The two-pion energy distribution $\Gamma(s)$ and the P- and D-wave interference $X(s)$ as a function of two-pion energy squared.

They have the following numerical values:

$$R_\eta=2\times10^{-4}c^2/(1+4\times10^{-4}c^2), \quad (20a)$$

$$R_X=7\times10^{-4}c^2/(1+1.4\times10^{-3}c^2), \quad (20b)$$

which have the limiting value of $\frac{1}{2}$ when the D amplitude dominates completely the P-wave amplitude.

For values of $\alpha_X<15\%$, Eqs. (18b) and (20b) give the following simple relation:

$$R_X=3.6\alpha_X^2, \quad (21)$$

where α is the percentage asymmetry. For example an asymmetry of 5% corresponds to $R_X=0.01$. This clearly indicates that measuring the asymmetry in X^0 decay is a much more sensitive experiment than detecting the existence of the decay $X^0\to\pi^0\pi^0\gamma$ mode.

The values of R_X, R_η, α_X, and α_η as a function of c are tabulated in Table I. In Fig. 1 we plot $X(s)$ and $\Gamma(s)$ as functions of s for η^0 decay and in Fig. 2 for X^0 decay.

It is seen from Eqs. (18a) and (18b) and Table I that, for a given value of c, the asymmetry parameter is much larger in X^0 decay than in η^0 decay. This is due to the fact that in X^0 decay the two pion p-wave phase shift can go through 90°, while in η^0 decay the maximum phase, as given by Eq. (10), is about 5° at the highest available energy. As can be seen in Fig. 1, $\Gamma(s)$ and $X(s)$ reach their maximum value at $s=7$–10. At these energies the phase shift as given by Eq. (10) is of the order of a few degrees. In Fig. 2, $X(s)$ and $\Gamma(s)$ for X^0 decay reach their maximum value at $s\approx30$ which is at the ρ resonance. Since most of the events are in the region $15<s<35$, the asymmetry calculated with the P-wave phase shift as given by Eq. (10) can be taken with good confidence. This is not, however, the case for the η^0 asymmetry parameter.

In X^0 decay the asymmetry parameter reaches a maximum value of 18% at $\sqrt{c}=5$, while in η^0 decay its maximum value is 1.1% at $\sqrt{c}=7$. The maximum asymmetry parameter is reached at the point where both P and D amplitudes contribute equally to the rate of η^0, $X^0 \to \pi^+\pi^-\gamma$ decay. It is unlikely that the experimental data on X^0 decay can exceed the upper limit of 18% given here since the P-wave pion-pion phase shift as explained above can be used with confidence for this decay. It is possible that the upper limit of the asymmetry in η^0 decay can be exceeded in reality because of the uncertainty in using formula (10) for values of s far from resonance. In this case one should try a more complicated expression for the phase shift and also for the D functions.

The ratio $X(s)/\Gamma(s)$ which measures the asymmetry in X^0 decay remains more or less constant in the energy range, $15 \lesssim s \lesssim 35$. The reason for this is that the asymmetry depends on the product of $\sin\delta_1$, δ_1 being the P-wave phase shift, and the D- to P-wave ratio; at an energy where $\sin\delta_1$ is large, the P-wave amplitude is also large, the D- to P-wave ratio is thus small and their combined effect remains essentially constant over the energy range of X^0 decay. Had the width of ρ^0 been much narrower, this would not have been the case and the expected asymmetry would have been much smaller.

Existing experimental data[13] indicate a possible upper limit of 8% for the asymmetry in X^0 decay which is consistent with the upper limit given here. This value of the asymmetry parameter corresponds to a value $\sqrt{c}=3$. The corresponding asymmetry in η^0 decay is 0.3%. Supposing that the P-wave phase shift was underestimated by one order of magnitude, the asymmetry in η^0 would only be 3% which is still smaller than that in X^0 decay.

From our analysis, it is evident that the absence of a detectable asymmetry in $\eta^0 \to \pi^+\pi^-\gamma$ would not rule out the possibility of a C violation in electromagnetic processes. On the other hand such a lack of asymmetry in $X^0 \to \pi^+\pi^-\gamma$ decay would likely cast doubts on the notion of "*maximally*" C-violating in electromagnetic interactions of strongly interacting particles.

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APPENDIX: EFFECT OF A C VIOLATION IN THE STRONG INTERACTION ON η, $X^0 \to \pi^+\pi^-\gamma$ DECAY

Following the notation of Ref. 5 we introduce a C-violating interaction $ig_{\eta\rho\pi}\rho^\mu\cdot(\eta\partial_\mu\pi - \pi\partial_\mu\eta)$. This I-conserving coupling gives rise to the following C-violation amplitude $\mathfrak{M}_2$:

$$\mathfrak{M}_2 = 2ig_{\eta\rho\pi}g_{\rho\pi\gamma}\epsilon_{\mu\nu\sigma\tau}e_\gamma{}^\mu p_\gamma{}^\nu p_+{}^\sigma p_-{}^\tau \times\left[\frac{1}{(p_-+p_\gamma)^2-m_\rho{}^2}-\frac{1}{(p_++p_\gamma)^2-m_\rho{}^2}\right], \quad \text{(A1)}$$

where the $\rho\pi\gamma$ vertex is written as $g_{\rho\pi\gamma}\epsilon_{\alpha\beta\gamma\delta}p^\alpha(\rho)e^\beta(\rho)\times p^\gamma(\gamma)e^\delta(\gamma)$. In this approximation the C-conserving amplitude $\mathfrak{M}_1$ can also be written as

$$\mathfrak{M}_1 = 2g_{\rho\eta\gamma}g_{\rho\pi\pi}\epsilon_{\mu\nu\sigma\tau}e_\gamma{}^\mu p_\gamma{}^\nu p_+{}^\sigma p_-{}^\tau \times[1/(p_++p_-)^2-m_\rho{}^2]. \quad \text{(A2)}$$

For the two-pion energy available in η^0 decay, we approximate Eq. (A1) by:

$$\mathfrak{M}_2 \approx 4ig_{\rho\eta\pi}g_{\rho\pi\gamma}\epsilon_{\mu\nu\sigma\tau}e_\gamma{}^\mu p_\gamma{}^\nu p_+{}^\sigma p_-{}^\tau p_\gamma\cdot(p_+-p_-). \quad \text{(A3)}$$

Comparing Eqs. (A2) and (A3) with Eqs. (1), (2), and (15) with $M=m_X$ we find

$$c = 2\left(\frac{g_{\rho\eta\pi}}{g_{\rho\pi\pi}}\right)\left(\frac{g_{\rho\pi\gamma}}{g_{\rho\eta\gamma}}\right)\frac{m_X{}^2(m_\rho{}^2-4)}{m_\rho{}^4}. \quad \text{(A4)}$$

Inserting the SU_3 relation $\sqrt{3}g_{\rho\pi\gamma}=g_{\rho\eta\gamma}$ and the present upper limit[5]

$$g_{\rho\eta\pi}{}^2/4\pi \lesssim 4\times10^{-2},$$

i.e.,

$$g_{\rho\eta\pi}/g_{\rho\pi\pi} \lesssim 0.14,$$

we obtain

$$c \lesssim 0.22, \quad \text{(A5)}$$

which is much smaller than any values of c considered in Table I for η^0 decay. From Eq. (20a) we have

$$R_\eta \lesssim 10^{-5}. \quad \text{(A6)}$$

Hence a detection of R_η larger than this value would indicate a C violation in electromagnetic interactions. This is however a very difficult experiment.

Similarly, we can calculate the C-violating amplitude for X^0 decay. Because of a much larger energy available, we must treat the ρ as unstable particle. For a rough approximation, which is needed here, we neglect the momentum dependence of the width, and we obtain a similar equation to (A1):

$$\mathfrak{M}_2 = 4ig_{\rho X\pi}g_{\rho\pi\gamma}\epsilon_{\mu\nu\sigma\tau}e_\gamma{}^\mu p_\gamma{}^\nu p_+{}^\sigma p_-{}^\tau p_\gamma \cdot(p_+-p_-)[1/(m_\rho{}^2-u)(m_\rho{}^2-t)], \quad \text{(A7)}$$

where $u=(p_\gamma+p_+)^2$, $t=(p_\gamma+p_-)^2$, and m_ρ is complex. The C-conserving amplitude $\mathfrak{M}_1$ is given by (A2) with $g_{\rho\eta\gamma}$ replaced by $g_{\rho X^0\gamma}$. There is no simple approximation for (A7) in X^0 decay. The asymmetry can be computed numerically and is found to be less than $\frac{1}{2}$%, if we assume $g_{\rho X\gamma}\approx g_{\rho\eta\gamma}$ and use the upper limit[5] $g_{\rho X\pi}{}^2/4\pi \lesssim 2\times10^{-4}$.

[13] A. Rittenberg and G. Kalbfleisch, Phys. Rev. Letters 15, 556 (1965).