be quite relevant to consider the "extended" algebra which contains operators representing the inclusion of that breakage. Thus it is important to specify commutation relations for the physical operators

$$\dot{A}_i = \int d^3x\, D_i(x_0,\mathbf{x}).$$

Suppose the densities $D_i(x)$ commute with one another as they could because of our discussion. Then V_i and D_i, $i=1,\cdots 8$, generate a noncompact subalgebra within the extended algebraic system with representations perfectly analogous to those of $SL(3,C)$. This assertion follows from the argument that the $D_i(x)$ form an octet of operators as do the noncompact operators of $SL(3,C)$ and, furthermore, since they commute like translations with infinite degrees of freedom, they too can construct similar unitary, infinite-dimensional representations. In particular, one may form[10] unit vectors $\hat{D}_i = D_i/(\mathbf{D}^2)^{\frac{1}{2}}$ which in turn can be used to construct Hermitian, noncompact operators $N_i = i[C,\hat{D}_i]$, where C is the bilinear Casimir operator of $SU(3)$. Then V_i and N_i generate the classical group $SL(3,C)$. An example of one of the unitary, infinite-dimensional representations involving a sum over compact $SU(3)$ representations is (8, 27, 64, 125, $\cdots$).

Should the quantities $D_i(x)$ prove useful for multiplet building, the *origin* of possible manifestations in a particle spectrum of a noncompact group could lie in the brokenness of the compact $[SU(3)\times SU(3)]$ symmetry. The fact that $SU(3)$ symmetry itself is broken perhaps partially accounts for the approximate influence or "traces" of noncompactness in a physical particle spectrum. Also, we noted previously that the bootstrap approach could support the hypothesis of no subtractions in our discussion of the commuting property for the $D_i(x)$. Thus a connection could be established between the bootstrap and approximate noncompactness of a particle spectrum.

Finally we mention that the notions of this paper can perhaps be applied to particle algebraic systems (where breakage is important) other than the particular chiral $SU(3)\times SU(3)$ system of operators and densities used here. For example, one may view our discussion as an attempt to identify physical operators—broken densities—with at least some noncompact operators of the "gigantic" symmetries such as $SL(6,C)$ (actually, their mathematically "contracted" or translation versions as described above).

ACKNOWLEDGMENTS

The author especially wishes to thank Professor Murray Gell-Mann for useful criticism. He also wishes to acknowledge discussions with Professor Fredrik Zachariasen and Dr. Frederick Gilman.

PHYSICAL REVIEW VOLUME 147, NUMBER 4 29 JULY 1966

High-Energy πp Scattering and the π-π $J=0$, $T=0$ Antibound State*

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(Received 26 October 1965; revised manuscript received 4 April 1966)

A model for high-energy πp scattering is presented. Partial waves with $l\leq l_a$ are taken as completely imaginary and those with $l_c\geq l>l_a$ are given by their Born terms. Values of the partial-wave inelasticity $\eta_{l\pm}{}^T$ are calculated dynamically. A force similar to the exchange of a scalar particle of mass $<2\mu$ but with residue of opposite sign is found. This force gives a repulsive real part of the forward scattering amplitude. The origin of this force is explained as due to the exchange of the π-π $J=0$, $T=0$ antibound state or ABC particle.

I. INTRODUCTION

In a recent model calculation for high-energy p-p scattering,[1] it was found that a simple approximation of the force yielded (i) physically acceptable values of $\eta_l = e^{-2\mathrm{Im}\delta_l}$ dynamically, and (ii) a repulsive real part of the forward scattering amplitude, in agreement with experimental results. The approximate force was similar to the force due to the exchange of a scalar particle with mass $<2\mu$, but with a residue which was opposite in sign. The opposite sign of the residue indicated that this force did not correspond to the exchange of a physical scalar particle. One wonders whether this result is a reflection of the model, or has some deeper physical significance. With this in view, we have investigated the high-energy πp scattering. This problem is of further interest, since a negative real part of the forward scattering amplitude has also been found for πp scattering.[2] As in the p-p case, we approximate the force as that due to the exchange of a scalar system whose mass and residue are taken as

* Supported, in part, by the U. S. Atomic Energy Commission.

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parameters of the theory. Further, to make the model more realistic, the spin of the nucleon is taken into account and a force corresponding to the exchange of the ρ meson is included.

A formulation of the model for the equal-mass spinless case has been given in I. Here, in Sec. II the formulation for πN scattering is given and in Sec. III the results of the calculation are presented. In Sec. IV the physical interpretation of the results is discussed.

II. FORMULATION OF THE MODEL FOR πN SCATTERING

In the model we are discussing, the partial-wave inelasticity $\eta_{l\pm}{}^T$ is calculated from the force.[3] The physical partial-wave amplitude

$$f_{l\pm}{}^T=(\eta_{l\pm}{}^T e^{2i\mathrm{Re}\delta_{l\pm}{}^T}-1)/2ik$$

is related to an amplitude $\mathfrak{F}_{l\pm}{}^T$, which obeys elastic unitarity throughout the physical region, by the equation[4]

$$1+2ikf_l(s)=e^{2i\theta_l(s)}[1+2ik\mathfrak{F}_l(s)]\,, \tag{1}$$

where

$$\theta_l(s)=\frac{k^{2l+1}}{\pi}\int_{s_i}^{\infty}\frac{\mathrm{Im}\delta_l(s')ds'}{k'^{2l+1}(s'-s)}\,, \tag{2}$$

k = c.m. momentum, and s_i = inelastic threshold. The amplitude $\mathfrak{F}_l(s)$ can be written as

$$\mathfrak{F}_l(s)=(e^{2i\alpha_l(s)}-1)/2ik\,, \tag{3}$$

where $\alpha_l(s)$ is given by $\alpha_l(s)=\delta_l(s)-\theta_l(s)$ and is real for $s>(m+\mu)^2$. Writing $\theta_l(s)=\Delta_l(s)+i\,\mathrm{Im}\delta_l(s)$, one can express $\theta_l(s)$ as an integral over $\Delta_l(s)^3$, namely,

$$\theta_l(s)=\frac{k^{2l+1}(s-s_i)^{1/2}}{\pi i}\int_{s_i}^{\infty}\frac{\Delta_l(s')ds'}{k'^{2l+1}(s'-s_i)^{1/2}(s'-s)}\,. \tag{4}$$

As discussed in I, we impose the following asymptotic behavior:

$$\frac{\theta_l(s)}{k}=O(|s|^{-\epsilon})\quad\text{for}\quad |s|\to\infty\,(\epsilon>0)\,. \tag{5}$$

Then,

$$\frac{\theta_l(s)}{k^{2l+1}(s-s_i)^{1/2}}=O(|s|^{-l-\frac{1}{2}-\epsilon})\qquad\text{for}\qquad |s|\to\infty\,. \tag{6}$$

Using the identity relation

$$\frac{1}{s'-s}=-\sum_{n=0}^{l-1}\frac{[s'-(m+\mu)^2]^n}{[s-(m+\mu)^2]^{n+1}}+\frac{[s'-(m+\mu)^2]^l}{[s-(m+\mu)^2]^l(s'-s)}\,, \tag{7}$$

we have from (4),[4a]

$$\theta_l(s)=\frac{k^{2l+1}(s-s_i)^{1/2}}{\pi i[s-(m+\mu)^2]^l}\int_{s_i}^{\infty}\frac{\Delta_l(s')[s'-(m+\mu)^2]^l ds'}{k'^{2l+1}(s'-s_i)^{1/2}(s'-s)}\,. \tag{8}$$

Now $\Delta_l(s)=\mathrm{Re}\delta_l(s)-\alpha_l(s)$, and if we take $\mathrm{Re}\delta_l(s)\approx 0$ for $s>s_i$, then from (8) we obtain

$$\mathrm{Im}\delta_l(s)\approx\frac{k^{2l+1}(s-s_i)^{1/2}}{[s-(m+\mu)^2]^l}P\frac{1}{\pi}\times\int_{s_i}^{\infty}\frac{\alpha_l(s')[s'-(m+\mu)^2]^l ds'}{k'^{2l+1}(s'-s_i)^{1/2}(s'-s)}\,. \tag{9}$$

As in I, for $\alpha_l(s')$ in Eq. (9) we make the approximation[5]

$$\alpha_l(s')\approx k'f_l{}^B(s')\,, \tag{10}$$

where $f_l{}^B(s)$ is the Born approximation of the amplitude $f_l(s)$. Since the Born amplitudes are completely determined by the input force, so Eqs. (9) and (10) provide a dynamical method for calculating $\eta_l=e^{-2\mathrm{Im}\delta_l}$.

To obtain the Born terms, we consider that the invariant functions $A^{(\pm)}(s,t)$ and $B^{(\pm)}(s,t)$ of πN scattering

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[4] For simplicity, we drop the $\pm$ signs and the superscript T. $l\pm$ stands for l, $J=l\pm\frac{1}{2}$, and T stands for the isotopic spin. We use $c=\hbar=1$, m = nucleon mass, μ = pion mass.

[4a] *Note added in proof.* Instead of Eq. (8), a simpler equation can be derived from (4) using the asymptotic behavior (6). For this purpose, let us write

$$\frac{1}{k'^{2l}(s'-s)}=\sum_{m=1}^{l}\frac{A_m(s)}{(s'-s_1)^m}+\sum_{m=1}^{l}\frac{B_m(s)}{(s'-s_2)^m}+\frac{1}{k^{2l}(s'-s)}\,,$$

where $s_1=(m+\mu)^2$, $s_2=(m-\mu)^2$. Examination of the coefficients $A_m(s)$ and $B_m(s)$ shows that

$$|A_m(s)|,\quad |B_m(s)|\geq O\left(\frac{1}{|s|^l}\right)\quad\text{for}\quad s\to\infty\,.$$

This, together with Eq. (6), then leads to

$$\theta_l(s)=\frac{k(s-s_i)^{1/2}}{\pi}\int_{s_i}^{\infty}\frac{\Delta_l(s')ds'}{k'(s'-s_i)^{1/2}(s'-s)}\,,$$

which is the same as in the equal-mass case.

[5] It is interesting to look at the physical meaning of the approximation (10). For this purpose, let us recall the steps which lead to this result:

(1) On the basis of potential scattering $\mathfrak{F}_l(s)$ is considered, for s large, to approach its Born term $\mathfrak{F}_l{}^B(s)$, which is real. Writing

$$\mathfrak{F}_l(s)=(e^{2i\alpha_l}-1)/2ik$$
$$=\alpha_l/k+i\alpha_l{}^2/k\cdots\,, \tag{a}$$

we find that α_l should be small for large s, so that the higher terms in (a) can be neglected and $\mathfrak{F}_l(s)$ can approach a real quantity. This leads to

$$\alpha_l/k\approx\mathfrak{F}_l{}^B. \tag{b}$$

(2) The force for the amplitude $\mathfrak{F}_l(s)$ is approximated by that of the physical amplitude $f_l(s)$, i.e., the left-hand cut contribution is calculated by ignoring the inelastic scattering. This gives

$$\mathfrak{F}_l{}^B\approx f_l{}^B(s). \tag{c}$$

From (b) and (c), we then arrive at the approximation (10), namely,

$$\alpha_l(s)\approx kf_l{}^B(s). \tag{d}$$

The physical meaning of (d) is now clear; it gives the asymptotic phase shift which we would obtain if we had the same input force as that of the physical amplitude and no inelastic scattering.

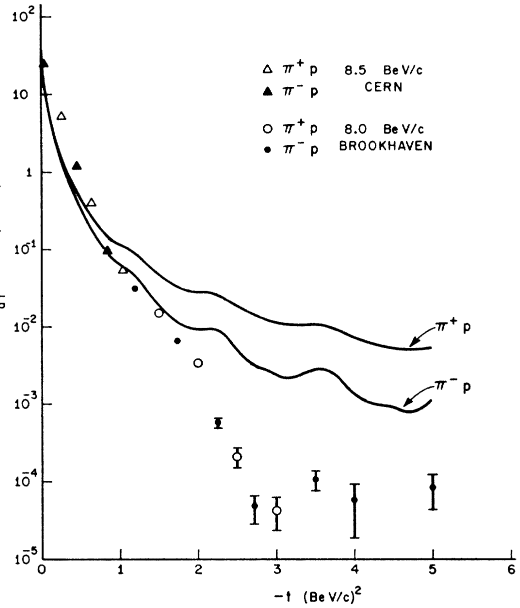

FIG. 1. Calculated angular distribution for pion incident momentum 8 BeV/c. The experimental points are from Ref. 7 (CERN) and from Ref. 8 (Brookhaven).

are given by

$$A^{(+)}(s,t)=\lambda/(m_0^2-t), \tag{11a}$$

$$B^{(+)}(s,t)=0; \tag{11b}$$

$$A^{(-)}(s,t)=\frac{g_m g_{\pi\rho}}{2m}\frac{s-u}{m_\rho^2-t}, \tag{12a}$$

$$B^{(-)}(s,t)=-\frac{2(g_e+g_m)g_{\pi\rho}}{m_\rho^2-t}. \tag{12b}$$

Equations (11) are similar to the contribution due to the exchange of a scalar meson of mass m_0 and isospin zero. Equations (12) correspond to the exchange of the ρ meson. The $NN\rho$ and the $\pi\pi\rho$ interactions have been taken as

$$\begin{aligned} H_{NN\rho}&=ig_e\bar{\psi}\gamma_\mu\tau\psi\rho^\mu+(g_m/2m)\bar{\psi}\sigma_{\mu\nu}\tau\psi\partial_\mu\rho^\nu, \\ H_{\pi\pi\rho}&=\tfrac{1}{2}g_{\pi\rho}\epsilon_{rst}\rho_r^\mu\pi_s\overleftrightarrow{\partial}_\mu\pi_t. \end{aligned} \tag{13}$$

The Born amplitudes can now be calculated using the standard formula for $f_{l\pm}{}^T$.

The πN elastic differential cross section is given by

$$d\sigma_{\mathrm{el}}/d\Omega=|f(\theta)|^2+|g(\theta)|^2, \tag{14}$$

where

$$f(\theta)=\sum_{l=0}^{\infty}[(l+1)f_{l+}+lf_{l-}]P_l(\cos\theta),$$

$$g(\theta)=i\sum_{l=1}^{\infty}[f_{l+}-f_{l-}]P_l{}^1(\cos\theta);$$

$$P_l{}^1(\cos\theta)=\sin\theta(dP_l(\cos\theta)/d\cos\theta).$$

We assume that the partial waves with $l\leq l_a$ are completely imaginary and those with $l>l_a$ are given by the Born amplitudes and therefore, are real. Further, all partial waves with $l>l_c$ are neglected, where l_c is a large number. In this approximation, we have

$$\begin{aligned} f(\theta)=&\frac{i}{2k}\sum_{l=0}^{l_a}[(l+1)(1-\eta_{l+})+l(1-\eta_{l-})]P_l(\cos\theta) \\ &+\sum_{l=l_a+1}^{l_c}[(l+1)f_{l+}{}^B+lf_{l-}{}^B]P_l(\cos\theta), \end{aligned} \tag{15}$$

$$\begin{aligned} g(\theta)=&\frac{1}{2k}\sum_{l=1}^{l_a}[\eta_{l+}-\eta_{l-}]P_l{}^1(\cos\theta) \\ &+i\sum_{l=l_a+1}^{l_c}[f_{l+}{}^B-f_{l-}{}^B]P_l{}^1(\cos\theta). \end{aligned} \tag{16}$$

Equation (15) shows that there will be a real part of the forward scattering amplitude coming from the Born terms.

III. RESULTS OF CALCULATION

There are five main parameters in our calculation, namely, s_i, λ, m_0, g_e, and l_a. As for the other parameters, m_ρ and $g_{\pi\rho}$ are obtained from the experimental mass (750 MeV) and width (100 MeV) of the ρ meson; to obtain g_m we have kept the ratio g_m/g_e fixed at a value -3.7, as indicated by the form-factor data.[6] Regarding the parameter l_c, the calculated results are insensitive to it so long it is sufficiently large. Our method of finding the unknown parameters has been to assume a set of values for them and then to calculate values of $\eta_{l\pm}{}^T$, the differential cross sections and the total cross sections. For an acceptable set of values we require that the calculated η_l's should satisfy the unitarity requirement $1\geq\eta_l>0$ and the calculated total cross sections should be in agreement with experimental values. A set of parameters which we have found satisfying the above criteria for pion incident momentum 8 BeV/c is $s_i=12.0$ (BeV)^2, $\lambda/4\pi=-5.615$ BeV, $m_0=0.25$ BeV, $g_e=-0.8633$, $l_a=25$, and $l_c=60$. For this set the calculated differential cross sections $d\sigma/dt$ are shown in Fig. 1 and the calculated values of $\mathrm{Im}\delta_{l\pm}{}^T$ are given in Fig. 2. Some experimental points[7,8] are shown in Fig. 1 for purpose of comparison. The theoretical angular distribution shows a diffraction peak, a sharp fall with

[6] For a discussion on how the relative signs and magnitudes of g_m, g_e, and $g_{\pi\rho}$ are determined, see M. M. Islam and R. Pinon, (unpublished).

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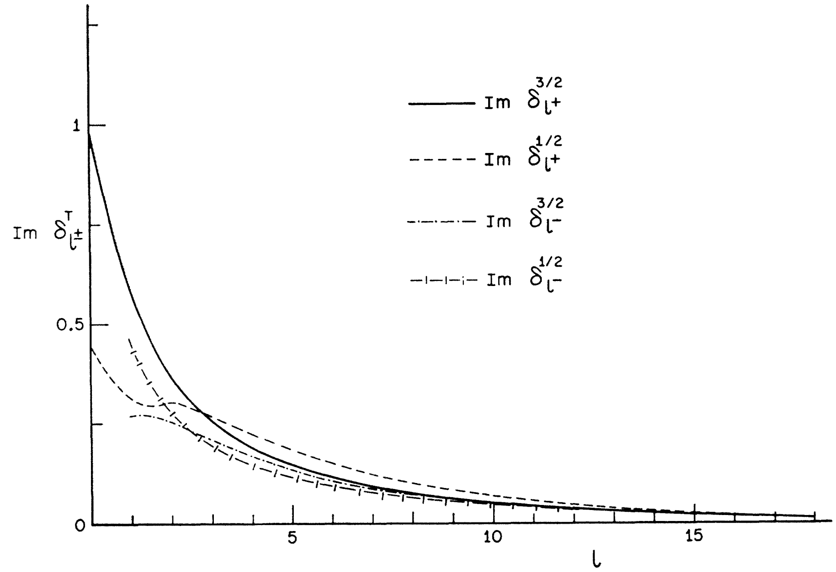

FIG. 2. Im$\delta_{l\pm}^{T}$ as a function of l. The calculated values at integral l are joined by smooth lines.

increasing momentum transfer and a tendency to level off at large angles. For large $-t$ the calculated cross sections are 2 orders of magnitude larger than the experimental values. This possibly indicates that the approximate force we have considered does not adequately represent the short-range interactions. Our calculated total cross sections for π^+p and π^-p scattering are 26.4 mb and 27.1 mb, respectively. These values are in reasonable agreement with the experimental values 25.1 mb and 27.5 mb.[9] We find a repulsive real part of the forward scattering amplitude for π^+p as well as for π^-p scattering. The calculated ratio of the real part to the imaginary part of the forward scattering amplitude is -0.168 for π^+p and -0.163 for π^-p. These values are consistent with the experimental results[2] and forward dispersion-relation calculations.[10,11] Further, the ρ exchange contribution to the real part of $f(\theta=0)$ is found to be negligible. We have also calculated total elastic and charge-exchange cross sections using only the partial waves which are absorbed, i.e., those with $l\leq l_a$. The results are $\sigma_{\mathrm{el}}^{\pi^+p}=2.67$ mb, $\sigma_{\mathrm{el}}^{\pi^-p}=2.60$ mb, and $\sigma_{\mathrm{c.e.}}=0.027$ mb. These values are smaller by about a factor of 2 than the corresponding experimental elastic[7] and charge-exchange[12] cross sections.

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IV. PHYSICAL INTERPRETATION

From the above results, we find that, as in p-p scattering,[1] a force similar to the exchange of a scalar particle, but with opposite sign of the residue, gives a repulsive real part of the forward scattering amplitude and also physically acceptable values of η_{lJ}^{T}. In the following paper,[13] we shall show that a π-π $J=0$, $T=0$ "antibound state" gives a contribution in the crossed channel similar to that due to the exchange of a scalar particle of mass $<2\mu$, but with residue of opposite sign. Thus, our explanation of the negative real part of the forward scattering amplitude for π-p and p-p scattering is that it is due to the exchange of this antibound state, or equivalently, the ABC particle.[14] Physically, we may consider that virtual pions in the cloud of an elementary particle such as nucleon or pion tend to stick together in pairs forming the antibound state. The exchange of this state between two particles produces repulsion. It is worth pointing out that the quantum numbers of this antibound state will be consistent with theorems regarding the exchange of quantum numbers for forward elastic scattering at high energy.[15,16]

ACKNOWLEDGMENT

The author wishes to thank Professor D. Feldman for his interest and encouragement.

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