Infrared Approach to Large-Angle Scattering at Very High Energies*

H. A. KASTRUP†
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received 17 January 1966; revised manuscript received 23 March 1966)

The hypothesis is investigated that the observed exponential decrease of the elastic large-angle high-energy proton-proton cross section is a consequence of soft-meson emission in strong interactions at very high energies. This interpretation is suggested by the observation of leading particles and low c.m. energies of the secondary mesons in cosmic-ray collisions. Neglecting the recoil of the soft mesons implies the factorization of the cross section into a potential part and an infrared part which is given by Poisson's distribution law. These properties alone provide a number of interesting experimental predictions. Adding the assumption of asymptotic dilatation invariance determines the energy dependence of the potential cross section and yields new results: The magnetic form factor $G_M(t)$ of the nucleon becomes a simple function of the multiplicity of soft mesons produced in quasi-elastic electron-nucleon scattering. If this relation can be confirmed experimentally, it will provide a new method of measuring electromagnetic form factors at very high energies. Furthermore, the factor $\exp(-\bar{n})$ of the Poisson distribution can account for the exponential decrease of the elastic proton-proton cross section, and this interpretation gives several new experimental predictions.

I. INTRODUCTION

RECENT experiments[1,2] indicate that the elastic cross section for proton-proton scattering at high energies and for large angles falls off more strongly than any power of the energy. This has stimulated a number of interesting theoretical considerations and speculations.

One explanation is suggested on the basis of Fermi's statistical model.[3–5] This model makes plausible the exponential decrease of the elastic cross section, but, as Bialas and Weisskopf pointed out,[6] it requires an enormous computational effort to calculate the phase-space factors at higher energies. From a more theoretical point of view one may ask whether the notion of a statistical equilibrium is appropriate for such collisions up to the highest energies.

Other interpretations have been given by considering potentials which are highly singular at the origin.[7–9]

Van Hove and collaborators,[10] mainly concerned with forward scattering, have developed an approach which combines phenomenological elements with some general principles such as unitarity, crossing symmetry, etc.

In this paper we want to discuss to what extent the observed behavior of the elastic proton-proton scattering cross section and related phenomena of strongly interacting particles can be understood on the basis of a kind of infrared mechanism in the following sense[11]: In very high-energy collisions the fields of the incoming particles are strongly Lorentz-contracted along the directions of motions of the particles and spread out perpendicular to these directions. The same holds for the highly energetic outgoing particles. If we, therefore, have the elastic scattering of two such particles with high momentum transfer, the long-range parts of the fields have to readjust themselves considerably and there is a certain probability that this readjustment is accompanied by the emission of soft (infrared) secondary particles from the long-range parts of the fields.

It is, for instance, well known in quantum electrodynamics[12] that these infrared contributions form the dominant part of the radiative corrections in large-momentum-transfer reactions.

In strong interactions such an infrared interpretation is suggested by the following observations in cosmic-ray experiments[13]: Most of the nucleon-nucleon reactions are quasi-elastic, i.e. the two incoming nucleons emerge from their collision with a relatively small energy loss (30–40%) over a long range of energies (20–10^6 GeV in the laboratory). The generally soft secondaries are mostly pions (about 80%) and the ratio between pions and kaons is almost independent of the energy of the primaries. Since the pions form the long-range part of the nucleon field these observations fit well into the infrared picture.

The idea of interpreting the meson production in strong interactions in analogy to the bremsstrahlung in electromagnetic interactions is, of course, not new. Heisenberg[14] seems to be the first one who discussed this possibility shortly after the famous paper[15] of Bloch and Nordsieck on the infrared divergencies in electrodynamics. Later Lewis, Oppenheimer and

* Work supported by the U. S. Air Force.
† On leave of absence from the University of Munich, Munich, Germany.
[1] G. Cocconi *et al.*, Phys. Rev. Letters **11**, 499 (1963).
[2] W. Baker *et al.*, Phys. Rev. Letters **12**, 132 (1964).
[3] R. Hagedorn, Nuovo Cimento Suppl. **3**, 147 (1965).
[4] G. Cocconi, Nuovo Cimento **33**, 643 (1964).
[5] H. A. Bethe, Nuovo Cimento **33**, 1167 (1964).
[6] A. Bialas and V. F. Weisskopf, Nuovo Cimento **35**, 1211 (1965).
[7] R. Serber, Rev. Mod. Phys. **36**, 649 (1964).
[8] G. Tiktopoulos, Phys. Rev. **138**, B1550 (1965).
[9] W. N. Cottingham and N. Dombey, Rutherford Laboratory Report (unpublished).
[10] A. Bialas and L. Van Hove, Nuovo Cimento **38**, 1385 (1965); references to earlier papers can be found here.
[11] We always mean the c.m. system, if not stated otherwise.
[12] D. R. Yennie, S. C. Frautschi, and H. Suura, Ann. Phys. (N.Y.) **13**, 379 (1961); K. E. Eriksson, in *Theoretical Physics* (International Atomic Energy Agency, Vienna, 1963), p. 543; N. Meister and D. R. Yennie, Phys. Rev. **130**, 1210 (1963).
[13] P. H. Fowler and D. H. Perkins, Proc. Roy. Soc. (London) **A278**, 401 (1964).
[14] W. Heisenberg, Z. Physik **113**, 61 (1939).
[15] F. Bloch and A. Nordsieck, Phys. Rev. **52**, 54 (1937).

Wouthuysen[16] considered special field-theoretical models for strong interactions, to which they applied the Bloch-Nordsieck method. Something similar had been done before by Tomonaga and collaborators,[17] whose papers became known later. More recent work along these lines has been done by Eriksson,[18] Chilinski[19] and Bialas and Ruijkrok.[20]

All these approaches rely more or less on field-theoretical models. Since there is no satisfactory field theoretical description of strong interactions it is difficult to see which results are a consequence of the basic picture and which depend on the rather crude field-theoretical approximations.

We want to point out in this paper that it is possible to test the basic idea experimentally without any specific field theoretical model. The reason for this is the following: The main assumption of the infrared approach is that the recoil of the secondary soft particles is negligible. This implies:

1. The cross section for the quasi-elastic scattering of the two primary particles accompanied by the emission of soft secondaries is the product of two factors, the first one of which describes the noninfrared scattering of the two primary particles, and the second, the emission of soft secondaries. This factorization has been found in many examples,[12,21,22] and it is nothing other than the multiplication law of probability theory,[23] which says that the probability for the sequence of two events, the second one of which is a consequence of the first one, is the product of the two probabilities which are attributed to each of the two events.

2. Neglecting the recoil of the soft secondaries means that they are emitted independently and that we can treat their source as a classical quantity. From this one can show that the probability distribution of the secondaries is of Poisson's type.[24]

These two properties—factorization of the cross section and Poisson distribution of the secondaries—imply a number of consequences, which can be tested experimentally. This is discussed in detail in Sec. II.

We obtain more special predictions if we assume the noninfrared part of the cross section to be determined by the hypothesis of asymptotic dilatation invariance.[25,26] The considerations of this paper are therefore also a first attempt of comparing some consequences of this symmetry with experiments. In Sec. III we apply the general results to electron-nucleon scattering and obtain an interesting relationship between the magnetic form factors and the multiplicity of soft mesons. If this relation can be confirmed experimentally, it will provide a new method of determining form factors by inelastic processes. Comparison with available data seems to give at least qualitatively reasonable results. Further experiments at higher momentum transfer are very desirable. The same holds for proton-proton scattering, which we discuss in Sec. IV.

Pending further experimental confirmation, one of the general conclusions of our analysis is the same as that of Wu and Yang[27]: The behavior of cross sections in very high-energy and high-momentum-transfer reactions involving hadrons is dominated by the accompanying statistical long-range effects which obscure the short-range potential scattering. It is, therefore, very hard to extract information about the latter from those experiments.

The assumptions, discussed in this paper, do not provide any interesting information about the angular distribution of the secondaries. From our qualitative picture of the infrared mechanism we expect a strong correlation between the direction of the soft secondaries and those of the primary particles. Such correlations seem to exist experimentally, for instance in the approximate equality of the almost constant average transverse momentum of primaries and secondaries and in other anisotropies.[13,28,29] We shall not discuss these features in this paper.

II. THE GENERAL CASE

According to our previous discussion the cross section $d\sigma_n$ for the high-energy quasi-elastic scattering of two primary particles with c.m. energies E and accompanied by the emission of n secondary mesons is given by the expression

$$d\sigma_n(E,\theta)=\bar{\sigma}^0(E,\theta)w_n(E,\theta)d\Omega\,,\quad d\Omega=2\pi\sin\theta d\theta\,,\quad (1)$$

where θ is the c.m. scattering angle of the primary particles and

$$w_n(E,\theta)=e^{-\bar{n}(E,\theta)}[\bar{n}(E,\theta)]^n/n! \quad (2)$$

the Poisson probability for the emission of n soft mesons. $\bar{n}(E,\theta)$ is the average number of the secondaries. Since it depends not only on the energy E of the primary particles but also on their scattering angle θ, we shall call it the *differential multiplicity*.

The factorization (1) is exactly true only in the limit of vanishing mass and vanishing energy of the soft

[16] H. W. Lewis, R. Oppenheimer, and S. A. Wouthuysen, Phys. Rev. **73**, 127 (1948).
[17] T. Miyazima and S. Tomonaga, Sci. Papers Inst. Phys. Chem. Res. (Tokyo) **40**, 21 (1942), reprinted in Progr. Theoret. Phys. Suppl. **2**, 21 (1955).
[18] K. E. Eriksson, Phys. Letters **1**, 291 (1962); K. E. Eriksson and S. A. Yngstrom, Phys. Rev. **131**, 1805 (1963).
[19] Z. Chylinski, Nucl. Phys. **44**, 58 (1963).
[20] A. Bialas and T. Ruijkrok, Nuovo Cimento **39**, 1061 (1965).
[21] Y. Nambu and D. Lurié, Phys. Rev. **125**, 1429 (1962); Y. Nambu and E. Shrauner, *ibid.* **128**, 862 (1962).
[22] S. L. Adler, Phys. Rev. **139**, B1638 (1965).
[23] See, for instance, Marek Fisz, *Probability Theory and Mathematical Statistics* (John Wiley and Sons, Inc., New York, 1963), 3rd ed., p. 20.
[24] See, for example, J. D. Bjorken and D. S. Drell, *Relativistic Quantum Fields* (McGraw-Hill Book Company, Inc., New York, 1965), p. 202.
[25] H. A. Kastrup, Nucl. Phys. **58**, 561 (1964).
[26] H. A. Kastrup, Phys. Rev. **142**, 1060 (1966).
[27] Tai Tsun Wu and C. N. Yang, Phys. Rev. **137**, B708 (1965).
[28] N. A. Dobrotin and E. L. Feinberg, Proc. Roy. Soc. (London) **A278**, 391 (1964).
[29] J. Orear, Phys. Letters **13**, 190 (1964).

secondaries, but we assume it to be a reasonable approximation also in the case of finite masses and energies as long as the total c.m. energy of the secondaries is small compared to E. We also neglect the spin and isospin (unitary spin) dependence of the cross section, and consider only one kind of mesons, assuming that these simplifications give already the essential features of our approach.

We see from Eqs. (1) and (2) that the emission of the secondaries dampens the purely elastic cross section $d\sigma_0$ by a factor $\exp(-\bar{n})$. In the language of S-matrix theory this dampening is a consequence of the unitarity of the S matrix which relates the elastic amplitudes to the inelastic ones. In a field-theoretical description this factor is due to the virtual soft particles which are almost on the mass shell.[12] We see that we can determine this factor experimentally by measuring the multiplicity of soft mesons for given E and θ. Knowing $\bar{n}(E,\theta)$ and using the Eqs. (1) and (2) we can predict all the cross sestions $d\sigma_n$, $n=0,1,\cdots$, if we know one of them.

Since the assumption of negligible recoil is no longer justified if n exceeds a certain value for a given energy E, this limitation holds also for the factorization (1). The number of secondaries is limited anyhow by energy conservation. Thus one has to keep in mind in the following discussion:

Since

$$\sum_{n=0}^{\infty} w_n = 1\,,$$

we obtain

$$d\bar{\sigma}_{\text{tot}}(E,\theta)=\bar{\sigma}(E,\theta)d\Omega\,, \tag{3}$$

if we sum Eq. (1) over all n. $d\sigma_{\text{tot}}$ is the total cross section for the quasi-elastic scattering of the primary particles, summed over all secondaries. The sum rule (3) might be an interesting additional test for the consistency of our picture, if the contributions for $n\gg\bar{n}(E,\theta)$ become negligible. An important test of the quasi-elastic scattering hypothesis would be to see, whether the outgoing primary particles have approximately opposite directions and momenta in the c.m. system, particularly for large angles.

Another experimental possibility to test the above ideas, is the measurement of the rms fluctuation which is

$$[\bar{n}(E,\theta)]^{1/2}$$

for the Poisson distribution.

The above discussions show that the differential multiplicity is the most interesting physical quantity in our approach and its experimental determination is of considerable interest.

We have also seen that the hypothesis of a kind of infrared mechanism in high-momentum-transfer reactions which involve strongly interacting particles can be tested without any assumption about the potential cross section $\bar{\sigma}$, which is supposed to describe the short-range properties of the collision. It has been suggested[25,26] that at very high energies the usual space-time symmetries of the Poincaré group are enlarged by the addition of the dilatations and the special conformal group. This has been discussed in detail in Refs. 25 and 26. Those deliberations are concerned with the short-range behavior of the interactions and apply therefore only[30] to the potential cross section $\bar{\sigma}$. It follows from the discussions in Ref. 25 that the invariance under dilatations restricts $\bar{\sigma}(E,\theta)$ for $\theta\neq 0,\pi$ to the form[31] (c.m. system):

$$\bar{\sigma}(E,\theta)=E^{-2}|B(\theta)|^2\,, \tag{4}$$

where $B(\theta)$ is a function of θ alone. Equation (3) indicates how the assumption (4) can be tested directly, at least approximately. It is easy to see that in any common relativistic field theory with a dimensionless coupling constant the Born approximation has the form (4) in the limit of negligible rest mass. But we do not maintain that $\bar{\sigma}(E,\theta)$ is the Born approximation. This approximation might be a good starting point for theories with small coupling constants such as electrodynamics and the intermediate-boson theory for weak interactions. The Møller cross section times the infrared corrections indeed seem to give a reasonable description for high-energy and high-momentum-transfer scattering.[12,32]

Since $\bar{n}$ is proportional to the square of the coupling constant,[24] the cross section (1) involves an infinite number of Feynman graphs. A discussion in terms of these graphs will be given elsewhere.[33] As the electromagnetic coupling constant is small, the infrared factor gives only a small correction to the potential cross section,[12] at least for present accelerator energies, but we expect the second factor to be dominant in strong interactions, because the strong-coupling constant is so large. This will be confirmed by our discussion of the following examples.

III. ELECTROMAGNETIC FORM FACTORS FOR LARGE MOMENTUM TRANSFER

Under the assumption of one-photon-exchange the theoretical differential cross section for electron-proton

[30] Equation (9) of Ref. 25 might be true generally, but the arguments leading to Eqs. (12) and (28) apply only to the short-range part of the scattering.

[31] If we write the potential cross section in the form

$$d\bar{\sigma}=s^{-1}|F(s,t)|^2 d\Omega,$$

where $F(s,t)$ is an invariant scattering amplitude, then invariance under dilatations implies that F is a function of the scattering angle alone (see Ref. 25). The special conformal group (see Ref. 26) can be generated by the transformation which represents the reflection by reciprocal radii and since this transformation commutes with the homogeneous Lorentz group, it leaves any function of an angle invariant. In this sense the special conformal group does not impose new restrictions on the scattering amplitude which go beyond those obtained by dilatation invariance.

[32] Y. S. Tsai, Phys. Rev. **120**, 269 (1960).

[33] G. Mack, Electromagnetic Form Factor of the Nucleon for High Momentum Transfer (to be published).

scattering is given by[34]

$$\frac{d\sigma_0}{dt}=\frac{4\pi\alpha^2}{t^2(s-M^2)^2}[st+(s-M^2)^2]\times\left[\frac{t}{t-4M^2}G_M{}^2(t)+\frac{t^2}{2[st+(s-M^2)^2]}G_M{}^2(t)-\frac{4M^2}{t-4M^2}G_E{}^2(t)\right].$$

s is the squared total c.m. energy of the two particles, and $t\leqslant 0$ is the squared invariant momentum transfer, M is the proton mass, the electron mass is neglected, and $G_E(t)$ and $G_M(t)$ are the proton form factors with the normalization $G_E(0)=1$, $G_M(0)=1+k$, where k is the anomalous magnetic moment of the proton. We neglect the electromagnetic radiative corrections, which seem to be of the order of 10–20% in the GeV region.[35]

If we assume $G_E(t)\leqslant G_M(t)$, which is indicated experimentally, and if $s\gg 4M^2$, $-t\gg 4M^2$, we can neglect the term with the electric form factor and get

$$\frac{d\sigma_0}{dt}\approx\sigma_B(s,t)G_M{}^2(t)\,,\quad \sigma_B(s,t)=\frac{2\pi\alpha^2}{t^2s^2}(t^2+2st+2s^2)\,.$$

σ_B is Born approximation. Comparison with Eqs. (1) and (2) for $n=0$ gives $\bar{n}(s,t)=-\ln G_M{}^2(t)+\ln(\bar{\sigma}/\sigma_B)$. According to our assumption (4), the quantity $\bar{\sigma}/\sigma_B$ depends only on the ratio s/t or only on θ. One might therefore suspect that $\ln(\bar{\sigma}/\sigma_B)$ is negligible in comparison to $-\ln G_M{}^2(t)$ for large $-t$, i.e. we assume that the strong decrease of $G_M(t)$ for large $-t$ is dominated by soft-meson emission. If we do this we get the following expression for $G_M(t)$:

$$\bar{n}(t)=-\ln G_M{}^2(t)\,,\quad -t\gg 4M^2. \tag{5}$$

This is an interesting relationship between the differential multiplicity of the soft secondary hadrons, mostly pions, in quasi-elastic electron-proton scattering and the magnetic form factor of the proton. If it can be confirmed experimentally, at least approximately, it will provide a new method of determining such form factors by inelastic processes. Equation (5) says that the differential multiplicity should depend only on the invariant momentum transfer. This provides a simple test for the validity of the approximation (5).

The condition $-t\gg 4M^2$ is hardly fulfilled in present experiments. Chen *et al.*[36] give for their highest value $-t=6.8(\text{GeV}/c)^2$ the upper limit $G_M\approx 0.05$. This gives $\bar{n}\approx 6.0$ and for the rms fluctuation the value ≈ 2.5. These are not unreasonable numbers, but further experiments are certainly necessary to test the relation (5).

[34] See, for instance, G. Källén, *Elementary Particle Physics* (Addison-Wesley Publishing Co., Inc., Reading, Massachusetts, 1964), p. 224.
[35] Y. S. Tsai, Phys. Rev. **122**, 1898 (1961).
[36] K. W. Chen *et al.*, Phys. Rev. **141**, 1267 (1966).

In our special example the sum rule (3) has the form

$$d\sigma_{\text{tot}}/dt=\bar{\sigma}(s,t)\,,$$

where (s,t) is a homogeneous function of degree -2 in s and t, if we assume dilatation invariance. This is in agreement with an expression obtained by Bjorken[37] from a chiral $U_6\otimes U_6$ algebra of current densities.

An interesting question is, whether $\bar{\sigma}=\sigma_B$ is a reasonable approximation.

The usual dispersion-theoretical analysis[38] of the form factors relate their structure for negative values of t to the singularities in the crossed t channel. One obtains an approximation for the form factors by taking into account only the nearest singularities in that channel. But our approach deals with very large $-t$ and that means that other singularities become equally or more important. It is an interesting problem how one can justify the relation (5) by dispersion-theoretical methods. We shall not pursue this question here.

Cosmic-ray experiments[13] suggest that the multiplicity of secondaries created in reactions which involve baryons, increases more strongly than logarithmically with increasing energy. Although that multiplicity is not the differential multiplicity discussed here, it seems not unlikely that the energy dependencies are similar (see Sec. IV). If this is true, then the relation (5) implies an exponential decrease of G_M with increasing energy. This was already suggested by Wu and Yang[27] on the basis of slightly different statistical arguments. Since this increase of the multiplicity seem to persist up to the highest cosmic-ray energies, it indicates that G_M goes to zero for $-t\to\infty$. This behavior has been anticipated theoretically.[39]

The same arguments as above can be applied to the magnetic form factor $G_{Mn}(t)$ of the neutron. We obtain

$$\bar{n}_n(t)=-\ln G_{Mn}{}^2(t)\,. \tag{6}$$

Since the meson fields of the proton and the neutron are almost the same, we expect the differential multiplicities $\bar{n}_p$ and $\bar{n}_n$ to be almost equal for equal t. Experiments[40] and symmetry arguments[41] suggest the relation

$$G_{Mn}{}^2(t)=\left[\frac{\mu_n}{1+k}\right]^2G_{Mp}{}^2(t)\,,$$

where μ_n is the magnetic moment of the neutron. From

[37] J. D. Bjorken, Phys. Rev. Letters **16**, 408 (1966), and Phys. Rev. (to be published).
[38] For a review with many references see S. D. Drell, in *Proceedings of the International School of Physics "Enrico Fermi", Course 26 (1962)* (Academic Press, New York, 1963), p. 184.
[39] S. D. Drell and F. Zachariasen, Phys. Rev. **119**, 463 (1960); L. E. Evans, Nucl. Phys. **17**, 163 (1960); see however R. G. Sachs, Phys. Rev. **126**, 2256 (1962) and Phys. Rev. Letters **12**, 231 (1964); asymptotic properties of form factors are also discussed by M. Gell-Mann and F. Zachariasen, Phys. Rev. **123**, 1065 (1961).
[40] J. R. Dunning, Jr. *et al.*, Phys. Rev. **141**, 1286 (1966).
[41] R. Delbourgo and R. White, Nuovo Cimento **40A**, 1228 (1965); this paper contains references to earlier related work.

this it follows that

$$\bar{n}_n(t) = \bar{n}_p(t) + 0.17 .$$

Thus the difference is indeed very small.

Equation (6) might be of considerable experimental interest if our approach turns out to be a reasonable approximation for very high momentum transfers. According to this equation we can determine $G_{Mn}(t)$ if we know the momentum transfer of the electron and the number of soft secondaries produced in the quasi-elastic collisions. It is not necessary to measure the cross section itself, a task which is particularly difficult for neutrons. It might therefore perhaps be possible to determine G_{Mn} by shooting a highly energetic electron beam across a dense beam of free (reactor) neutrons, measuring the electron momentum transfer and counting the number of soft mesons.

Furthermore, since the number of soft mesons produced in electron-proton scattering and nucleon-nucleon scattering should be related for given momentum transfers, one can determine the electromagnetic form factors from cosmic-ray experiments, if one knows this relationship or makes some plausible assumption about it.[27,33]

The above discussion can be generalized to muon-nucleon scattering and lepton-baryon scattering. If we adopt the conserved-vector-current hypothesis,[42] we can relate the form factors of the weak vector current of the hadrons to their electromagnetic form factors. The infrared behavior of the axial-vector-current form factors has to be discussed independently. We shall not do this here.

IV. HIGH-ENERGY LARGE-ANGLE PROTON-PROTON SCATTERING

Orear fitted[29] the experimental data for elastic large-angle proton-proton scattering up to 30-GeV accelerator energies by the expression

$$\sigma_0(E,\theta) = AE^{-2}e^{-ap \sin\theta} , \tag{7}$$

where p is the c.m. momentum of the protons, $a \approx 6(\text{GeV}/c)^{-1}$ and $A \approx 1.5 \times 10^2 (\text{GeV})^2 \text{mb/sr}$. We see that this expression indeed has the form suggested by Eqs. (1), (2) and (4), and one is tempted to put

$$\bar{n}(E,\theta) \approx 6p \sin\theta . \tag{8}$$

Equation (8) implies that the differential multiplicity has its maximum for $\theta = \frac{1}{2}\pi$, if p is given. This one would expect, for the elastic proton-proton cross section, averaged over spins, has to be symmetric with respect to the angle $\theta = \frac{1}{2}\pi$ (exclusion principle), and in this sense both protons are deflected maximally at this angle. According to our infrared picture the average number of secondaries should also be maximal in this case.

By means of the expression (8) for $\bar{n}(E,\theta)$ we can now predict the cross sections $d\sigma_1$, $d\sigma_2$, etc.

The differential multiplicity in Eq. (8) is linear in the c.m. energy (if we neglect the rest mass), whereas the experimental integral multiplicity $\bar{n}_{\text{tot}}(E)$, observed over a long range of cosmic-ray energies, seems to be proprtional to the square root of the c.m. energy.[13] $\bar{n}_{\text{tot}}(E)$ is the total number of secondaries at a given energy of the primary particles, irrespective of their scattering angle. In order to relate $\bar{n}(E,\theta)$ approximately to $\bar{n}_{\text{tot}}(E)$, we have to introduce an—unfortunately unknown—weight factor $\rho(E,\theta)$ which takes into account that the scattering angles around $\theta = \frac{1}{2}\pi$ are relatively rare. We then have

$$\bar{n}_{\text{tot}}(E) = \left[\int d\Omega\, \rho(E,\theta) \bar{n}(E,\theta) \right] \Big/ \int d\Omega\, \rho(E,\theta) . \tag{9}$$

Fowler and Perkins give[13] for the multiplicity of charged secondaries a formula which reads in the c.m. system:

$$\bar{n}_{\text{tot}}^{c}(E) \approx 2(2/M)^{1/4} E^{1/2} , \quad E \text{ in GeV}. \tag{10}$$

Although $\bar{n}(E,\theta)$ and $\bar{n}_{\text{tot}}(E)$ are different quantities, they differ essentially by an integration over the angle. Therefore one might expect that their energy dependence is similar. This is not so for the expressions (8) and (10), but Fig. 79 of Ref. 13 shows that $\bar{n}_{\text{tot}}^{c}(E)$ increases at accelerator energies stronger than the formula (10) indicates, in qualitative agreement with the expression (8), which has been deduced from accelerator experiments. From Eq. (10) we expect a slower decrease of the elastic cross section at energies higher than the present accelerator energies, if $\bar{n}(E,\theta)$ and $\bar{n}_{\text{tot}}^{c}(E)$ have the same energy dependence.

It is further interesting to compare the numbers of secondaries given by the Eqs. (8) and (10) at, for instance, $E \approx cp \approx 10$ GeV. This numerical value of E is far above the experimentally verified realm of the expression (8), but this unjustified extrapolation is nevertheless instructive. We have $\bar{n}(10,\theta) \approx 60 \sin\theta$ and $\bar{n}^{c}(10) \approx 8$. If we assume the secondaries to be pions then charge independence tells us that $\bar{n}_{\text{tot}}(10) \approx 12$. This shows that our interpretation (8) of the exponent in Eq. (7) is at least qualitatively reasonable, for the weight of $\bar{n}(E,\theta)$ in Eq. (9) is very small around $\theta = \frac{1}{2}\pi$. If we put $\bar{n}_{\text{tot}}(10) = \bar{n}(10,\bar{\theta})$, we get $\bar{\theta} \approx 12^0$. Nevertheless the multiplicities predicted by Eq. (8) may be too large, even if they are allowed energetically. This can be an indication that the term $\ln(E^2\bar{\sigma}/A)$ is not negligible in comparison to $6p \sin\theta$ and that we are not in the asymptotic region at 30 GeV, or that our whole approach is a rather rough approximation for nucleon-nucleon collisions.

A final remark on forward scattering: Since the long-range parts do not have to readjust themselves in this case, we expect the number of soft secondaries to be

[42] S. S. Gershtein and I. B. Zeldovich, Zh. Eksperim. i Teor. Fiz. **29**, 698 (1955) [English transl.: Soviet Phys.—JETP **2**, 576 (1956)]; R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); Nicola Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

small. Furthermore, the arguments leading to the form (4) of the potential cross section are no longer applicable for forward scattering.[25,43] The optical-shadow picture seems to be more appropriate for this case. Similar arguments hold for backward scattering, when the Mandelstam variable u is small.

Note added in proof. After the revised version of this paper was submitted for publication, Stephen Weinberg has published a note [Phys. Rev. Letters **16**, 879 (1966)], which deals with the problem of multipion production within the framework of the conserved or partially conserved axial vector current hypothesis (PCAC). Weinberg concludes that from this point of view the soft pions are not emitted independently. I should like to make two remarks:

1. A typical contribution in the PCAC approach is

$$\cdots[(\not{p}+m)/k\cdot p]\tau_\alpha\gamma_5\not{k}u(p,\zeta),$$

where $u(p,\zeta)$ is a spinor with the property $(\not{p}-m)u=0$. The physical way to go to the limit $k=0$ seems to be the sequence[21] $\mathbf{k}\to 0$, $k_0=\mu\to 0$. It then follows that the above expression is proportional to

$$\cdots\tau_\alpha(\boldsymbol{\sigma}\cdot\mathbf{p}/p_0)u(p,\zeta).$$

This term does not contribute to the scattering amplitude if we are in the rest system ($\mathbf{p}=0$) of the primary particle with 4-momentum p.

On the other hand, if the energies p_0 of the primary particles are very high, so that we can neglect their rest masses, then these particles are practically in a state of definite helicity before and after the scattering, and we have

$$(\boldsymbol{\sigma}\cdot\mathbf{p}/p_0)u(p)=\pm u(p).$$

If several spurions are emitted, we see that at very high energies they are emitted independently as far as the spin of the nucleons is concerned, because the helicity "recoil" becomes negligible in this case.

The isospin parts τ_α, τ_β etc. of the different spurion vertices do not commute, but one expects that the isospin properties can be treated statistically at very high energies when many channels are open.[27] If, for instance, a large number of pions is created in very high energy nucleon-nucleon collisions, one might expect that in the average one third of them is π^+, one third π^- and the rest π^0.

The assumption of negligible correlations between the secondaries has been discussed and employed in detail by Van Hove and collaborators[10,20] and they find reasonable agreement with experiments, particularly with the features of forward scattering. But direct experimental data concerning the assumption of negligible correlation between the soft mesons are rare and appropriate experiments are very desirable.

2. Since the mass of the real pion does not vanish, the notion of soft pions depends on the frame of reference.[21] Experimentally the pions seem to be soft in the c.m. system.[13] Furthermore, at very high energies we are away from the pole $p\cdot k=0$ in the c.m. system and at the moment there seems to be no argument why the realistic collision amplitude should be dominated at very high energies by the pole terms in the PCAC approach, even when the physical mesons are soft. The other terms, which vanish in the limit $\mu=0$, may contribute as well or even more for $p\cdot k\gg 0$.

I am indebted to Professor Y. Nambu for stimulating discussions concerning these remarks.

ACKNOWLEDGMENTS

I am very much indebted to Professor E. P. Wigner, Professor M. L. Goldberger, Professor M. Froissart, Professor S. B. Treiman, Professor G. F. Chew, Dr. D. Hestenes and Dipl.-Physiker G. Mack for stimulating discussions and critical remarks at the various states in the development of this paper.

---

[43] The deliberations in Ref. 25 give a generalized Regge-like behavior for forward scattering. If we adopt this form for the potential part in Eq. (1), then the supposedly small infrared factor might be large enough to obscure the Regge-behavior, in agreement with many experiments. For the time being this is a mere speculation, extrapolated from the large-angle properties discussed above. But since the soft-meson emission is a purely relativistic effect, which does not occur in potential scattering, from which the Regge behavior has been inferred, this speculation might not be completely unreasonable.