the Σ_{AB} given by Galbraith *et al.*[5] Our solution yields $\chi^2=22$ indicating an adequate fit to the data. The fitted parameters α_T, F, and δ were determined to be

$$\alpha_T=0.39\pm0.24\,,\quad F=2.0\pm0.6\,,\quad \delta=2.3\pm1.1\,. \tag{13}$$

Consequently the ansatz condition $\delta=2F-1$ is to be compared with the fitted values $\delta=2.3\pm1.1$ and $2F-1=3.0\pm1.2$. Hence the total-cross-section data indicate approximate validity of the ansatz for the tensor nonet couplings. If we constrain $\delta=2F-1$ for the fit (which decouples S from the nucleons), then we obtain

$$\alpha_T=0.57\pm0.22\,,\quad F=2.2\pm1.0\,, \tag{14}$$

and thus

$$\delta=2F-1=3.4\pm2.0\,.$$

However, since this value of α_T is somewhat larger than the value $\alpha_R\approx0.4$ determined[9] from the $\pi^-p\rightarrow\eta n$ differential-cross-section data, it appears that S is approximately but perhaps not entirely decoupled from $\bar{N}N$.[10] More precise measurements on the Σ_{AB} will permit considerable refinement of this analysis (such as removal of the constraint $\alpha_R=\alpha_{P'}=\alpha_S$).

ACKNOWLEDGMENTS

We thank Professor D. Cline, Professor C. Goebel, and Professor M. Ross for their interest in this work.

[9] R. J. N. Phillips and W. Rarita, Phys. Letters 19, 598 (1965).

[10] With $\delta=2F-1$ we have not been able to find a χ^2 minimum for $\alpha_R<\alpha_S-0.1$. However, this does not exclude the possible existence of such solutions with an acceptable χ^2. The decoupling of the physical particle s depends somewhat on the precise value of the mixing angle (Ref. 7). An independent test of the $s\bar{N}N$ decoupling hypothesis would be experimental observation of suppression of $K^-p\rightarrow\Lambda s$ relative to $K^-p\rightarrow\Lambda f$ at backward angles.

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Separable Two-Body Potentials for Multiparticle Scattering*

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Separable potential forms which lead to on-energy-shell partial-wave transition amplitudes for two-body scattering which have analyticity properties very similar to those proved in local potential theory are studied. An explicitly energy-dependent potential form is shown numerically to give a better approximation to the amplitude from a local Yukawa potential than the best previous form, for $l>0$ in the scattering region. Some remarks are made about the applicability of these potential forms to multiparticle scattering calculations.

I. INTRODUCTION

RECENT papers on multiparticle scattering[1] have stressed the need for a good separable approximation to the two-particle transition amplitude. In particular, the introduction of a separable approximation for the off-energy-shell two-body amplitude in the integral equations for the three-body transition amplitude reduces the dimensionality of these integral equations and makes it possible to solve them on a computer.

Not long ago, Noyes[2] and Kowakski[3] gave a separable approximation to the off-shell two-body amplitude derived from local potential theory. However, Basdevant[4] points out that their approximation has cuts for $k^2<0$ (we take $\hbar=2m=1$, so that k^2 is the energy variable) that are not present in the full off-energy-shell two-particle transition amplitude defined by the Lippmann-Schwinger equation. He further points out that these cuts will be in regions of k^2 over which one must integrate in the three-body problem and that they can lead to complex eigenvalues in the three-body bound-state region. Finally, numerical solutions of the equations of Noyes and Kowalski for a Yukawa potential have shown that the term neglected in their approximation to $t_l(p,p',k)$ is comparable in size to the term retained when p and p' differ from k by an amount comparable to k.

In the light of these developments, the spirit of the present note is as follows:

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[1] C. Lovelace, Phys. Rev. 135, B1225 (1964); B. S. Bhakar and A. N. Mitra, Phys. Rev. Letters 14, 143 (1965); R. Aaron, R. D. Amado, and Y. Y. Yam, *ibid.* 13, 574 (1964); M. Bander, Phys. Rev. 138, B322 (1965); J. L. Basdevant, *ibid.* 138, B892 (1965); J. H. Hetherington and L. H. Schick, *ibid.* 139, B1164 (1965).

[2] H. Pierre Noyes, Phys. Rev. Letters 15, 538 (1965).

[3] K. L. Kowalski, Phys. Rev. Letters 15, 798 (1965).

[4] J. L. Basdevant (Centre de Recherches Nucléaires, Strasbourg), private communication to H. Pierre Noyes.

(a) A good separable approximation to the off-shell two-body transition amplitude is still needed.

(b) Since the on-shell amplitudes derived from local potential theory are known to be of some use in describing the interaction of subatomic particles, we shall demand that our approximation to the off-energy-shell two-body amplitude have the same analyticity properties *on the energy shell* as those derived from local potential theory. We will see that our separable potentials will not have the higher left-hand cuts arising from higher Born approximations to the local potential amplitude. However, we will say that amplitudes differing only in these higher left-hand cuts have the "same" analyticity properties.

(c) Although the off-energy-shell properties of two-body amplitudes required to give a useful description of physical phenomena are not nearly as well known as the corresponding on-shell properties, we shall use the analyticity properties of the off-shell two-body amplitudes known from local potential theory[5] as a guide, at least to avoid introducing troublesome singularities.

II. SEPARABLE POTENTIALS

It has long been known that one way of obtaining a separable two-particle transition amplitude in each partial wave is to use a single separable potential in each partial wave, and that the choice of a separable potential determines the off-energy-shell behavior of the two-particle amplitude, which is of fundamental importance in the calculation of multiparticle amplitudes. Mitra[6] has given criteria which a separable potential must satisfy in order that the on-energy-shell partial-wave transition amplitude will have the analyticity structure which it is known to possess in local potential theory. He points out that almost all of the separable potentials commonly used do *not* lead to the correct analyticity structure for the on-shell two-body amplitude. However, the commonly used separable potentials are at least capable of reproducing the correct strength and range of the interaction in each partial wave, and since the present multiparticle scattering calculations are still too crude to be sensitive to any finer details of the interactions, we may understand the relative success of recent calculations.

Mitra proposes a separable potential of the form

$$V_l(p,p')=\lambda g_l(p)g_l(p')\,,$$

where

$$g_l(p)=\left[\frac{|G|}{4\pi^2}\frac{1}{p^2}Q_l\left(1+\frac{\mu^2}{2p^2}\right)\right]^{1/2}.$$

G is the coupling constant, μ is the inverse range, and λ is $+1$ if G is positive, -1 if G is negative. He shows that this leads to the proper analytic structure for the on-shell two-body amplitude and is "equivalent" to a local Yukawa potential,

$$V(r)=G(e^{-\mu r}/r)\,,$$

in the loose sense that they both have the same Born approximation in a given partial wave, and lead to amplitudes with the same analytic structure.

We feel that this "equivalence" is interesting because a local Yukawa potential is one of the very few simple potentials known to have any value in describing the interaction of elementary particles.

We would like to extend the work of Mitra and propose separable potentials of the form

$$V_l(p,p')=\lambda g_l(p)g_l(p')\,,$$

where

$$g_l(p)=\left[\sum_i\frac{G_i}{4\pi^2}\frac{1}{p^2}Q_l\left(1+\frac{\mu_i^2}{2p^2}\right)\right]^{1/2},$$

G_i, μ_i, and λ are analogous to the similar quantities appearing in the previous potential, and the summation must have definite sign for $p>0$. The advantage of this form of potential is that, subject to the restriction mentioned above, we obtain a transition amplitude

$$t_l(p,p';k)=\lambda\left[\sum_i\frac{G_i}{4\pi^2}\frac{1}{p^2}Q_l\left(1+\frac{\mu_i^2}{2p^2}\right)\right]^{1/2}\left[\sum_i\frac{G_i}{4\pi^2}\frac{1}{p'^2}Q_l\left(1+\frac{\mu_i^2}{2p'^2}\right)\right]^{1/2}\Bigg/\left\{1+4\pi\lambda\int_0^\infty dq\frac{q^2}{q^2-k^2-i\epsilon}\left[\sum_i\frac{G_i}{4\pi^2}\frac{1}{q^2}Q_l\left(1+\frac{\mu_i^2}{2q^2}\right)\right]\right\},$$

which has the same Born approximation and the same analyticity structure as that arising from a *superposition* of Yukawa potentials in local potential theory, even though we have used only *one* separable term. The requirement that the summation entering into $g_l(p)$ be of definite sign is necessary to prevent the appearance of spurious cuts in the numerator function of $t_l(p,p';k)$. A separable potential cannot change sign in momentum space without introducing spurious cuts into the amplitude, which is a defect common to this and all earlier separable potential approaches. Finally, this potential is Hermitian and leads to an off-energy-shell amplitude with the same analytic structure as that obtained from the Lippmann-Schwinger equation in local potential theory. The same trick could be used with other po-

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[6] A. N. Mitra, Phys. Rev. **123**, 1892 (1961). A. N. Mitra and J. D. Anand, *ibid.* **130**, 2117 (1963).

tential shapes, but then our analyticity arguments might not hold.

III. SEPARABLE POTENTIALS WITH EXPLICIT ENERGY DEPENDENCE

Let us now consider the introduction of an explicit energy dependence into our separable potentials. This is done in an attempt to obtain a form for $t_l(k)$ in the scattering region similar to that obtained in local potential theory by Noyes and Kowalski and displayed in Sec. IV, Eq. (3).

We choose a Hermitian energy-dependent separable potential of the form

$$V_l(p,p';k)=\lambda g_l(p,k)g_l(p',k),$$

where

$$g_l(p,k)=\left[\sum_i \frac{G_i}{4\pi^2}\frac{1}{pk}Q_l\left(\frac{p^2+k^2+\mu_i^2}{2pk}\right)\right]^{1/2}$$

for $k^2>0$ (scattering region),

$$g_l(p,k)=\left[\sum_i \frac{G_i}{4\pi^2}\frac{1}{p^2}Q_l\left(1+\frac{\mu_i^2}{2p^2}\right)\right]^{1/2}$$

for $k^2<0$ (bound-state region). G_i, μ_i, and λ have the same meaning as before,

$$\sum_i \frac{G_i}{4\pi^2}\frac{1}{pk}Q_l\left(\frac{p^2+k^2+\mu_i^2}{2pk}\right)$$

must be of definite sign for $p>0$ and $k^2>0$, and

$$\sum_i \frac{G_i}{4\pi^2}\frac{1}{p^2}Q_l\left(1+\frac{\mu_i^2}{2p^2}\right)$$

must be of definite sign for $p>0$. The k independent form was chosen in the bound-state region to avoid the introduction of cuts similar to those which appear in the Noyes and Kowalski approximation and to insure Hermiticity. That is, if the form used for $k^2>0$ were used for $k^2<0$, we would encounter two problems, cuts and non-Hermiticity. The form chosen in the scattering region, the square root of a superposition of Yukawa potentials in momentum space, was picked because numerical analysis has shown that it represents the on-shell scattering amplitude from a superposition of local Yukawa potentials substantially better than the generalization of Mitra's form does for $l>0$ and in the scattering region. Note that when $p=k$ (on the energy shell) this new separable form reduces to the earlier form

$$g_l(k,k)\equiv g_l(k)=\left[\sum_i \frac{G_i}{4\pi^2}\frac{1}{k^2}Q_l\left(1+\frac{\mu_i^2}{2k^2}\right)\right]^{1/2}$$

for all values of k^2. Considering the *on-shell* properties of this potential further, we see that it satisfies Mitra's criteria and thereby leads to an on-shell two-particle

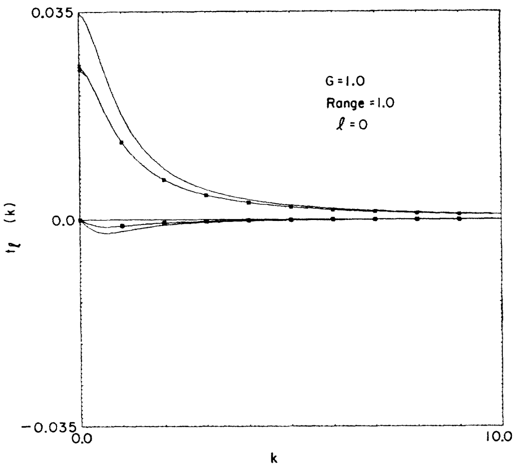

FIG. 1. $t_l(k)$ versus k for $l=0$. The upper curves are the real parts of $t_l(k)$ from the various approaches; the lower curves are the imaginary parts. The curves without symbols are the curves of $t_l(k)$ from a local Yukawa potential; the marked curves represent $t_l(k)$ obtained from the two separable potentials, which give identical results in this case. In all figures, k has units $1/R$ where R is the range of the potential.

amplitude which has the same analyticity properties as those proved for a superposition of Yukawa potentials in local potential theory and that it is "equivalent" to a local Yukawa potential in the same sense as the earlier form.

When we turn to the more general properties of the potential and the resulting amplitude (i.e., $p\neq k$) we see that, although the introduction of an explicit energy dependence in the potential does not impair its usefulness for multiparticle scattering calculations, we can no longer speak of analyticity properties of the resulting amplitude in the whole k^2 plane; instead, we may speak only of the analyticity properties in the half-planes $k^2>0$ (scattering region) and $k^2<0$ (bound-state region). Of course, within these regions, separately, the amplitude $t_l(p,p';k)$ from our energy-dependent separable potential has the same analytic structure as the amplitudes from both the earlier energy-independent separable potential and the local potential theory approach based on Yukawa potentials and the Lippmann-Schwinger equation. To sum up, this potential leads to an off-shell scattering amplitude with a discontinuity between the bound-state and scattering regions, while the on-shell scattering amplitude that one obtains from this potential has no such discontinuities and has analyticity properties which are the same as those obtained in local potential theory from a superposition of Yukawa potentials.

We shall want to see some benefits to this energy-dependent separable potential before we accept the accompanying loss of analyticity properties. It will develop that the energy-dependent separable form represents the on-shell scattering amplitude obtained from a local Yukawa potential for $l>0$ in the scattering region ($k^2>0$) substantially better than the corre-

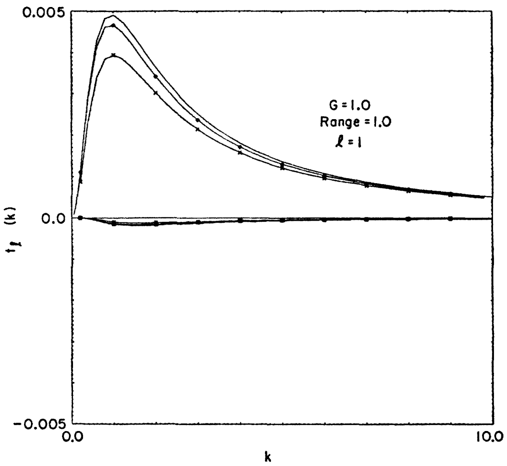

FIG. 2. $t_l(k)$ versus k for $l=1$. The upper curves are the real parts of $t_l(k)$; the lower curves are the imaginary parts. The unmarked curves represent $t_l(k)$ from a local Yukawa potential, the curves marked with x represent $t_l(k)$ from Mitra's separable potential, and the curves between them represent $t_l(k)$ from our separable potential.

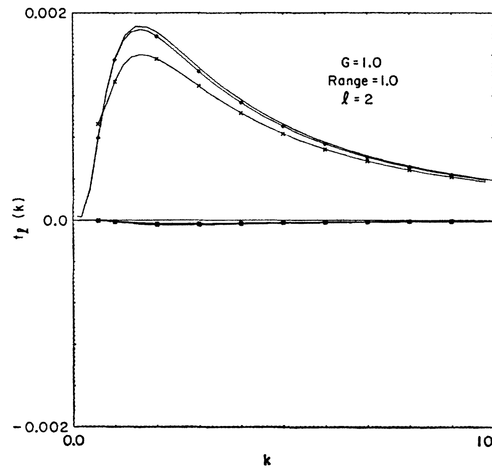

FIG. 3. $t_l(k)$ versus k for $l=2$. The upper curves are the real parts of $t_l(k)$; the lower curves are the imaginary parts. The unmarked curves represent $t_l(k)$ from a local Yukawa potential, the curves marked with x represent $t_l(k)$ from Mitra's separable potential, and the curves between them represent $t_l(k)$ from our separable potential.

sponding energy-independent form. This will be seen in Eqs. (1) and (2) below and in Figs. 1, 2, and 3.

IV. RESULTS

We have studied numerically the real and imaginary parts of the on-shell two-body transition amplitude $t_l(k)$ in the scattering region ($k^2>0$) for amplitudes obtained from

(a) solution of the Lippmann-Schwinger equation with a local Yukawa potential, $V(r)=Ge^{-\mu r}/r$ with $G=\mu=1$,

(b) Mitra's potential,

$$V_l(p,p')=\lambda g_l(p)g_l(p'),$$

with

$$g_l(p)=\left[\frac{G}{4\pi^2p^2}Q_l\left(1+\frac{\mu^2}{2p^2}\right)\right]^{1/2}$$

and $G=\mu=\lambda=1$, and

(c) the potential proposed in this note,

$$V_l(p,p';k)=\lambda g_l(p,k)g_l(p',k),$$

with $k^2>0$, so

$$g_l(p,k)=\left[\frac{G}{4\pi^2pk}Q_l\left(\frac{p^2+k^2+\mu^2}{2pk}\right)\right]^{1/2}$$

and $G=\mu=\lambda=1$.

We use a system of units in which $\hbar=2m=1$ and the range R of the potential ($R=1/\mu$) is taken as the unit of length, so that the unit of energy is $\hbar^2/2mR^2$. The Lippmann-Schwinger equation for the transition amplitude is

$$t_l(p,p';k)=V_l(p,p')-4\pi\int_0^\infty dq\frac{q^2}{q^2-k^2-i\epsilon}V_l(p,q)t_l(q,p';k).$$

(a) For a Yukawa potential, $V(r)=G(e^{-\mu r}/r)$ and

$$V_l(p,q)=\frac{G}{4\pi^2pq}Q_l\left(\frac{p^2+q^2+\mu^2}{2pq}\right);$$

the equation was solved numerically, by matrix inversion techniques, for various l and k values.

(b) For Mitra's separable potential, the solution can be given explicitly

$$t_l(p,p';k)=\left[\frac{G}{4\pi^2p^2}Q_l\left(1+\frac{\mu^2}{2p^2}\right)\right]^{1/2}\left\{1+4\pi\int_0^\infty dq\frac{q^2}{q^2-k^2-i\epsilon}\left[\frac{G}{4\pi^2q^2}Q_l\left(1+\frac{\mu^2}{2q^2}\right)\right]\right\}^{-1}\left[\frac{G}{4\pi^2p'^2}Q_l\left(1+\frac{\mu^2}{2p'^2}\right)\right]^{1/2}$$

and

$$t_l(k)=\frac{G}{4\pi^2k^2}Q_l\left(1+\frac{\mu^2}{2k^2}\right)\Big/\left\{1+4\pi\int_0^\infty dq\frac{q^2}{q^2-k^2-i\epsilon}\left[\frac{G}{4\pi^2q^2}Q_l\left(1+\frac{\mu^2}{2q^2}\right)\right]\right\}. \quad (1)$$

(c) For the potential proposed in this note, one finds, analogously,

$$t_l(p,p';k)=\left[\frac{G}{4\pi^2pk}Q_l\left(\frac{p^2+k^2+\mu^2}{2pk}\right)\right]^{1/2}\left\{1+4\pi\int_0^\infty dq\frac{q^2}{q^2-k^2-i\epsilon}\left[\frac{G}{4\pi^2qk}Q_l\left(\frac{q^2+k^2+\mu^2}{2qk}\right)\right]\right\}^{-1}\left[\frac{G}{4\pi^2p'k}Q_l\left(\frac{p'^2+k^2+\mu^2}{2p'k}\right)\right]^{1/2}$$

and

$$t_l(k)=\frac{G}{4\pi^2k^2}Q_l\left(1+\frac{\mu^2}{2k^2}\right)\Big/\left\{1+4\pi\int_0^\infty dq\frac{q^2}{q^2-k^2-i\epsilon}\left[\frac{G}{4\pi qk}Q_l\left(\frac{q^2+k^2+\mu^2}{2qk}\right)\right]\right\} \tag{2}$$

for $k^2>0$.

Note that even though the two separable potentials and their Born terms are equal when $p=p'=k$ (on the energy shell), the expressions (1) and (2) for the full on-shell amplitude are different because of the explicit energy dependence of our potential. In spite of this, expressions (1) and (2) seem to give nearly identical results (to within 1%) for $l=0$, although the difference is rather pronounced for $l>0$ and $G>0.5$.

Noyes[2] and Kowalski[3] have derived for the on-shell transition amplitude from local potential theory the expression

$$t_l(k)=V_l(k,k)\Big/\left\{1+4\pi\int_0^\infty dq\frac{q^2}{q^2-k^2-i\epsilon}V_l(k,q)f_l(q,k)\right\},$$

where $f_l(q,k)$ satisfies a nonsingular integral equation, and $f_l(k,k)=1$. When $V_l(k,q)$ is a Yukawa potential, this becomes

$$t_l(k)=\frac{G}{4\pi^2k^2}Q_l\left(1+\frac{\mu^2}{2k^2}\right)\Big/\left\{1+4\pi\int_0^\infty dq\frac{q^2}{q^2-k^2-i\epsilon}\left[\frac{G}{4\pi^2kq}Q_l\left(\frac{q^2+k^2+\mu^2}{2qk}\right)\right]f_l(q,k)\right\}. \tag{3}$$

Thus, the numerator and the imaginary part of the denominator of our expression (2) and this expression are identical, and they differ only in the factor $f_l(q,k)$ in the principal value integral. Finally, $f_l(q,k)$ is close to unity in the vicinity of the singularity at $q=k$. One may also note that the transition amplitudes obtained from the expression above and the expressions (1) and (2) approach each other at high k as the contribution from the integral term in the denominator becomes less important.

We have investigated numerically the behavior of $t_l(k)$, obtained from the three approaches mentioned above, as a function of k. We have held μ fixed ($\mu=1.0$), as $R=1/\mu$ has been taken as the unit of length, and varied G, the coupling constant. The results for $G=1.0$ and $l=0$, 1, 2 are shown in Figs. 1, 2, and 3. The agreement of the separable potentials with the Yukawa potential is poorest for low k in the s wave, where both separable potentials give the same result. When $G=1.0$ and $k=0$, we find that $t_l(k)$ from the separable potentials is 73% of $t_l(k)$ obtained from a Yukawa potential.

We have investigated coupling constants in the range from $G=0.01$ to $G=15.0$ and found that the results have the same qualitative behavior as those for $G=1.0$. For the *real part* of $t_l(k)$, we find that:

(a) The agreement of the separable potentials with the Yukawa potential is poorest at $k=0$ in the s wave.

(b) Both separable potentials lead to the same results for $l=0$.

(c) For $l>0$, the agreement of the separable potentials with the Yukawa potential is poorest at the peak in the curve of $t_l(k)$ versus k.

(d) For $l>0$, in *all* cases the curves from the potential proposed in this note lie higher (and thus closer to the curve from the local Yukawa potential) than the curves from Mitra's potential, even though the difference may be negligible for small G.

(e) In *no* case did the curves from the separable potentials lie higher than those from the local Yukawa potential. In other words, the real part of the on-shell scattering amplitude from a local Yukawa potential is always greater than the real part of the on-shell amplitude obtained from either of the separable potentials considered.

(f) The agreement of the separable forms with the Yukawa potential (and with each other for $l>0$) rapidly improves as G goes below 1.0 and deteriorates as G becomes greater than 1.0.

Let us look at the results in more detail. For $G=0.01$, we get 97% of the Yukawa amplitude from the separable potentials at $k=0$ in the s wave. In addition, both separable potentials give essentially the same result in all partial waves and these results never differ from the results for a Yukawa potential by more than 3%. For $G=0.2$, we obtain 91% of the Yukawa amplitude from the separable potentials at $k=0$ in the s wave; the amplitudes from the two separable potentials never differ from the Yukawa amplitude by more than 10%; and (for $l>0$) they never differ from each other by more than 5%.

Therefore, for G less than about 0.5 one can use the Mitra separable form as a good approximation to a Yukawa potential in all partial waves. This is because both separable potentials lead to amplitudes which are approximately equal to the amplitude from a Yukawa potential, and the Mitra separable potential leads to an amplitude with the correct analyticity properties for the full off-shell scattering amplitude. For G greater than about 0.5 and $l>0$, the curves of $t_l(k)$ versus k from the potential proposed in this note agree with the curves from a Yukawa potential substantially better than the curves from Mitra's potential (see Figs. 2 and 3). As a further example, consider $G=5.0$, where we obtain 49% of the Yukawa amplitude from the separable potentials at $k=0$ in the s wave. For $l=1$, at the peak of the curves of $t_l(k)$ versus k, where the agreement with the amplitude from a Yukawa potential is poorest, the amplitude from the potential in this note is 79% of the amplitude from a Yukawa potential, while the amplitude from Mitra's potential is only 47.5% of the Yukawa amplitude. For $l=2$, at the peak of the curves, the numbers are 89% for our potential and 56% for Mitra's potential. For G greater than about 5.0 both separable potentials give a very poor representation of the on-shell scattering amplitude from a local Yukawa potential. For example, for $G=15.0$, we obtain only 35% of the Yukawa amplitude from the separable potentials at $k=0$ in the s wave. Thus, even though our potential enjoys an ever-widening advantage over Mitra's potential for $l>0$ and G greater than about 5.0, they are both so poor that the advantage seems unimportant.

The same kinds of remarks could be made about the imaginary part of the on-shell amplitude $t_l(k)$ which shows the same general behavior as the real part. However, for G less than about 5.0, the imaginary parts are small and the imaginary parts of the amplitudes from both separable potentials are quite similar.

V. SIGN CHANGE AND ADDITIONAL BOUND STATES

All separable potential approaches which have been studied, including those in this note, suffer from two major defects:

(a) The potentials are not allowed to change sign in momentum space.

(b) The potentials can support only one bound state. We have noted that an explicit energy dependence in a separable potential does not impair its (practical) usefulness for multiparticle scattering calculations. Thus one might take a potential of the form

$$V_l(p,p';k)=f(k^2)g_l(p,k)g_l(p',k)$$

and attempt to circumvent the limitations mentioned above by proper choice of the function $f(k^2)$ [we choose $f(k^2)$ to avoid making the potential non-Hermitian if $g_l(p,k)g_l(p',k)$ is already Hermitian]. However, this will not work because any singularities in $f(k^2)$ will appear in *both* the numerator and denominator functions of the resulting two-body scattering amplitude. However, the numerator and denominator functions are supposed to have no singularities in common. Thus $f(k^2)$ must be analytic for all k^2, and since it must be bounded to keep the potential finite, $f(k^2)$ is a constant by Liouville's theorem of analysis.

VI. CONCLUSION

We have shown that one may obtain a two-particle partial-wave scattering amplitude from a *single* separable potential which has the same on-shell Born approximation and the same analyticity properties as the amplitude obtained in local potential theory from a *superposition* of Yukawa potentials. This is subject to two restrictions;

(1) The separable potential can support, at most, one bound state.

(2)
$$\sum_i \frac{G_i}{4\pi^2}\frac{1}{k^2}Q_l\left(1+\frac{\mu_i^2}{2k^2}\right),$$

the on-shell Born approximation of the superposition of Yukawa potentials, must be of definite sign for $k>0$.

We may obtain an on-shell amplitude which represents the on-shell amplitude obtained from a local Yukawa potential substantially better (for $l>0$ and $G>0.5$) than the method mentioned above, at the cost of introducing a discontinuity between the bound-state ($k^2<0$) and scattering ($k^2>0$) regions in the off-shell amplitude (the on-shell amplitude is *not* affected). That is, we obtain a two-particle partial-wave scattering amplitude from a *single* separable potential which has the same on-shell Born approximation, the same on-shell analyticity properties and, in the regions $k^2>0$ and $k^2<0$ separately, the same analyticity properties as the full off-shell scattering amplitude obtained from local potential theory with a superposition of Yukawa potentials. This is subject to one further restriction in addition to those mentioned above. That is,

$$V_l(p,k)=\sum_i \frac{G_i}{4\pi^2}\frac{1}{pk}Q_l\left(\frac{p^2+k^2+\mu_i^2}{2pk}\right),$$

the superposition of Yukawa potentials in momentum space, must be of definite sign for $p>0$ and $k^2>0$.

Finally, for $l=0$ and G less than about 5.0, both of the separable potential forms in this note lead to the same result, which represents the on-shell scattering amplitude from a local Yukawa potential quite well. In addition, Mitra's form has the advantage that it leads to the correct analyticity properties for the full off-shell amplitude. For $l>0$ and G less than about 0.5 both separable potential forms lead to amplitudes which represent a Yukawa potential quite well, but the superior analyticity properties of Mitra's form lead us to suggest its use in this case. For $l>0$ and G less than about 5.0 and greater than about 0.5, our separable potential leads to an amplitude which represents the on-shell scattering amplitude from a Yukawa potential (in the scattering region $k^2>0$) substantially better than the corresponding amplitude using Mitra's form. The improvement is such that we might suggest its use despite the attendant introduction of a discontinuity in the full off-shell amplitude.

For G greater than about 5.0, both of the separable potential forms in this note lead to poor representations of the scattering amplitude from a local Yukawa potential.