Study of Possible Practical Relativistic Generalizations of the Faddeev Equations in an Actual Model*

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The relativistic Faddeev equations derived from the Bethe-Salpeter equation are considered with the alternative rules of Blankenbecler and Sugar which put the intermediate particles on the mass shell. The above rules give rise to three alternative sets of equations. Another set of equations is obtained by introducing relativistic kinematics and phase-space factors in the (nonrelativistic) Faddeev equations. Each alternative set of equations is applied to the problem of the possible existence of the pion as a bound state of three pions, considered previously. Although qualitative results are similar, quantitative results from the various alternatives differ considerably. Because of the phase-space factors, no cutoff is needed in these calculations. It is also found that the three pions can form a bound state at the pion mass only if the scattering length in the two-body amplitude is negative.

I. INTRODUCTION

It is by now well known that a straightforward solution of the Lippmann-Schwinger equation for a system of three particles is in general not possible.[1,2] The main difficulty comes from the possibility that two of the particles can interact while at the same time the third one remains free. This, among other things, gives rise to the so-called dangerous δ function in the kernel. The difficulties were removed by Faddeev[1] and in a slightly different way by Weinberg.[2] However, the resulting equations are still too complicated for practical calculations—even after the partial-wave decomposition. A possible simplification occurs in some cases where the two-body amplitudes which enter in the equations can be approximated by a separable form. This may be expected to be the case when these two-body amplitudes are determined by resonances or bound states.[3] In this separable approximation one then obtains a coupled set of one-dimensional equations which are amenable in actual numerical calculations.[4]

Having found a somewhat practical three-particle theory in the nonrelativistic case, one is then tempted to make a relativistic generalization with possible applications in the case of strongly interacting particles. A natural generalization of the above procedure is to use the Bethe-Salpeter (B-S) equation, rather than the Lippmann-Schwinger equation, as a starting point. Obviously, the three particle B-S equation can be written in Faddeev form. In contrast to the nonrelativistic case, however, there is the extra complication of the intermediate particles being off the mass shell. This gives rise to additional integration variables (i.e. the fourth components of the momenta) in the equations. A procedure of putting the intermediate particles on the mass shell, based on the Cutkosky rules is given by Blankenbecler and Sugar.[5] This procedure is based on the requirement that the right-hand discontinuity be correctly given. This method, however, yields no unique result and three natural alternatives arise.

In the present paper we compare the results using the different alternatives in an actual numerical calculation. We also find that the resulting equations from one of the three alternatives are closely related to those obtained from the Faddeev equations by introducing relativistic kinematics and also relativistic phase-space factors to make the volume elements of the integration invariant. We expect the difference between the various alternatives to be more pronounced for bound-state problems than for the case of resonances. We have thus considered for definiteness the problem of the possible description of a pion as a bound state of three pions.[4] In this model the two-body amplitude is approximated by the (isospin-zero S wave) scattering-length formula. We find as a general result that the phase-space factors suppress the effective force considerably. Thus no cutoffs in the integration are required. We also find that, in all the various alternatives, unless the scattering length is negative, the force is not sufficient to produce the pion as a three-pion bound state. This result is in agreement with the arguments given by Chew[6] in favor of a negative scattering length.

In the next section we write down the B-S equations in the Faddeev form. We also give the free resolvents obtained from the Blankenbecler and Sugar alternatives. In Sec. III we then proceed to give the resulting formulas in our model for the pion. Finally in Sec. IV the numerical results are presented and discussed.

II. THE RESOLVENTS

Let us first write down the B-S equation in the Faddeev form. For simplicity we consider spinless equal-

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[1] L. D. Faddeev, Zh. Eksperim. i Teor. Fiz. **39**, 1459 (1960) [English transl.: Soviet Physics—JETP **12**, 1014 (1961)].
[2] S. Weinberg, Phys. Rev. **133**, B232 (1964).
[3] C. Lovelace, Phys. Rev. **135**, B1225 (1964).
[4] A. Ahmadzadeh and J. Tjon, Phys. Rev. **139**, B1085 (1965).
[5] R. Blankenbecler and R. Sugar, Phys. Rev. **142**, 1051 (1966).
[6] G. F. Chew, Phys. Rev. Letters **16**, 60 (1966).

mass particles. Furthermore, the interactions between the particles are assumed to be only due to two-body forces. The three-particle Bethe-Salpeter equation for the T matrix can be written as

$$T=\sum_i I_i-\sum_i I_i D_i^{(0)}T, \qquad (2.1)$$

where I_i represents all the two-particle irreducible diagrams in which particles j and k are interacting while the particle i remains free (i, j, k=1, 2, 3; $i\neq j\neq k\neq i$). In the ladder approximation I_i is given by the one-particle exchange process between particles j and k. Furthermore, $D_i^{(0)}$ denotes the free Green's function of the particles j and k,

$$D_i^{(0)}=-(2i/\pi)d_j d_k, \qquad (2.2)$$

where

$$d_j=[k_j^2-1]^{-1},$$

k_j being the four-momentum of particle j. Equation (2.1) can now be rewritten in the Faddeev form (see also Ref. 7). Defining

$$T^i=I_i-I_iD_i^{(0)}T, \qquad (2.3)$$

we have for the relativistic generalization of the Faddeev equations

$$\begin{pmatrix}T^1\\T^2\\T^3\end{pmatrix}=\begin{pmatrix}T_1\\T_2\\T_3\end{pmatrix}-\begin{pmatrix}0 & T_1D_1^{(0)} & T_1D_1^{(0)}\\ T_2D_2^{(0)} & 0 & T_2D_2^{(0)}\\ T_3D_3^{(0)} & T_3D_3^{(0)} & 0\end{pmatrix}\begin{pmatrix}T^1\\T^2\\T^3\end{pmatrix} \qquad (2.4)$$

with $T=T^1+T^2+T^3$ and where T_i are the two-body T matrices satisfying the equation

$$T_i=I_i-I_iD_i^{(0)}T_i. \qquad (2.5)$$

Therefore, a procedure to solve the three-particle Bethe-Salpeter equation (in the absence of the three-body force) would be to find the two body amplitudes T_i from Eq. (2.5) and then solve Eq. (2.4) for the three-body amplitude. In practice one would like to use the two-body amplitudes from experiments which are given for on the mass shell and on the energy shell only. The off-energy-shell effects are approximated by introduction of appropriate form factors.[3,4] In view of the conservation of total four momentum P, Eq. (2.4) is a coupled set of integral equations with eight integration variables. To reduce the number of variables in the integration and to be able to utilize the phenomenological two-body amplitudes, one would like to find an approximation to put the intermediate particles in Eq. (2.4) on their mass shell. A procedure for doing this, based on the Cutkosky rules, is given by Blankenbecler and Sugar.[5] This prescription is not unique, however, and various natural alternatives arise as we now see. Let us write instead of the Green's function $D_i^{(0)}$ a dispersion integral in the three-particle invariant-energy parameter s. We have[8]

$$A_i=4\int_9^\infty \frac{ds'}{s'-s}\delta[\{\tfrac{1}{2}(P'-k_i)+k_{jk}\}^2-1]\times\delta[\{\tfrac{1}{2}(P'-k_i)-k_{jk}\}^2-1], \qquad (2.6)$$

with

$$s=P^2,\quad P'=(s'/s)^{1/2}P,\quad k_{jk}=\tfrac{1}{2}(k_j-k_k);\quad (i\neq j\neq k).$$

Note that the definition of A_i is relativistically invariant. Without any loss of generality we may evaluate equation (2.6) in the over-all c.m. system. Equation (2.6) becomes

$$A_i=\frac{2}{\omega_j\omega_k}\frac{\sum_i\omega_i}{(\sum_i\omega_i)^2-s}\delta(k_{j0}-k_{k0}-\omega_j+\omega_k), \qquad (2.7)$$

where $\omega_i=(\mathbf{k}_i^2+1)^{1/2}$. Now from Eq. (2.4) we see that this propagator is multiplied by the two-body amplitude T_i for which we have

$$\langle k_1k_2k_3|T_i|k_1''k_2''k_3''\rangle=\delta^4(k_i-k_i'')\langle k_jk_k|t_i(\Omega_i)|k_j''k_k''\rangle. \qquad (2.8)$$

It can easily be seen from Eqs. (2.4), (2.7), and (2.8) that this procedure amounts to nothing more than putting the intermediate particles on the mass shell and writing for the "on-mass-shell" propagators

$$A_i=\frac{2}{\omega_j\omega_k}\frac{\sum_i\omega_i}{(\sum_i\omega_i)^2-z^2}, \qquad (2.9)$$

where $z=\sqrt{s}$. Similarly one can write the dispersion integral in the two-particle invariant energy,

$$B_i=4\int_4^\infty\frac{d\sigma_i'}{\sigma_i'-\sigma_i}\delta[(\tfrac{1}{2}P_{jk}'+k_{jk})^2-1]\times\delta[(\tfrac{1}{2}P_{jk}'-k_{jk})^2-1] \qquad (2.10)$$

with

$$\sigma_i=(P-k_i)^2=P_{jk}^2,\quad P_{jk}=k_j+k_k,\quad P_{jk}'=(\sigma_i'/\sigma_i)^{1/2}P_{jk}.$$

Explicit evaluation of Eq. (2.10) yields

$$B_i=\frac{2}{\bar\omega_j\bar\omega_k}\frac{\bar\omega_j+\bar\omega_k}{(\bar\omega_j+\bar\omega_k)^2+\mathbf{k}_i^2-(z-\omega_i)^2}\delta(l_{i0}), \qquad (2.11)$$

where $\bar\omega_i=(\mathbf{l}_i^2+1)^{1/2}$ and l_i is the same as the four-vector k_j-k_k, but its value should always be computed in the c.m. system of the j-k pair. Furthermore, $\mathbf{k}_i^2$ and ω_i are computed in the over-all c.m. system. This alternative corresponds again to putting the intermediate particles on the mass shell but with now as propagator

$$B_i=\frac{2}{\bar\omega_j\bar\omega_k}\frac{\bar\omega_j+\bar\omega_k}{(\bar\omega_j+\bar\omega_k)^2+\mathbf{k}_i^2-(z-\omega_i)^2}. \qquad (2.12)$$

[7] D. Stojanov and A. N. Tavkhelidze, Phys. Letters 13, 76 (1964); V. P. Shelest and D. Stojanov, *ibid.* 13, 253 (1964).

[8] V. A. Alessandrini and R. L. Omnes, Phys. Rev. 139, B167 (1965).

We can also write a similar dispersion integral in the one-particle invariant energy. Instead of the Green's function $D_i^{(0)}$ in the expression $T_iD_i^{(0)}T^j$ of equation (2.4) we can use

$$C_{ij}=4\int_1^\infty \frac{d\eta'}{\eta'-\eta}\delta[K_k'^2-1]\delta[(K_k'+2k_{jk})^2-1] \quad (2.13)$$

with

$$\eta=(P-k_i-k_j)^2=k_k^2,\quad K_k'=(\eta'/\eta)^{1/2}k_k.$$

Thus

$$C_{ij}=\frac{2}{\bar{\omega}_j}\frac{1}{\omega_k^2-(z-\omega_i-\omega_j)^2}\delta(l_{i0}). \quad (2.14)$$

In this case the propagator becomes

$$C_{ij}=\frac{2}{\bar{\omega}_j}\frac{1}{\omega_k^2-(z-\omega_i-\omega_j)^2}. \quad (2.15)$$

Finally let us consider the nonrelativistic free resolvent

$$G_0=(\textstyle\sum_i \mathbf{k}_i^2/2-\xi)^{-1}. \quad (2.16)$$

With relativistic kinematics this becomes

$$G_0=(\textstyle\sum_i \omega_i-z)^{-1},$$

where $z=\xi+3$. Now, if we symmetrize this expression with respect to z we find

$$G_0'=\frac{1}{\sum_i\omega_i-z}+\frac{1}{\sum_i\omega_i+z}=\frac{2\sum_i\omega_i}{(\sum_i\omega_i)^2-z^2}. \quad (2.17)$$

The only difference between Eqs. (2.17) and (2.9) is the phase space factors $1/\omega_j\omega_k$. Thus as a fourth alternative we may use the unsymmetrized form with the phase-space factors, namely

$$E_i=\frac{1}{\omega_j\omega_k}\frac{1}{\sum_i\omega_i-z} \quad (2.18)$$

Thus, if we use E_i for D_i in equation (2.4), the resulting equation is indentical to the one obtained from the nonrelativistic Faddeev equations with relativistic kinematics[4]—provided that in the latter case one also includes the phase space factors which make the volume element of the integration relativistically invariant. It is easily seen that in the nonrelativistic limit all the four alternatives—Eqs. (2.9), (2.12), (2.15), and (2.18)—become equal.

The numerical calculations consist of using the above alternatives in our model of the pion and comparing the results. Let us make the remark that all alternatives give rise to the same right-hand cut in z. They have different extra cuts, however, For example, the propagator given by Eq. (2.9) gives a left-hand cut in z starting at $z=-3$. Also, the propagator in Eq. (2.15) which corresponds to that given by Freedman, Lovelace and Namyslowski[9] gives an additional right-hand cut starting at $z=0$. For this reason we have not used it here for the bound-state problem although it produces the on-shell three-particle unitarity correctly.

III. THE PION PROBLEM

Our starting point here is Eqs. (3.1) and (3.2) of Ref. 4 which, after some manipulation, become

$$\Psi(k_1)=\int dk_2K(k_1,k_2)\Psi(k_2) \quad (3.1)$$

with

$$K(k_1,k_2)=\frac{-4\pi}{3}\int_{-1}^1 d\cos\theta_{k_1k_2}\hat{t}_0(p_1,p_2,\xi-q_1^2)\times\frac{k_2^2}{p_2^2+q_2^2-\xi}, \quad (3.2)$$

where p_i and q_i are the same as in Ref. 4, namely,

$$\mathbf{p}_1=\tfrac{1}{2}(\mathbf{k}_2-\mathbf{k}_3),\quad \mathbf{q}_1=-\tfrac{1}{2}\sqrt{3}\mathbf{k}_1,\quad \mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3=0,$$

with cyclic permutation of (1,2,3) for $\mathbf{p}_2$, $\mathbf{q}_2$, etc. Equation (3.2) as it stands is of course purely nonrelativistic with the free resolvent

$$G_0(\xi)=(p_2^2+q_2^2-\xi)^{-1}. \quad (3.3)$$

In this nonrelativistic version we use for the two-body amplitude

$$t_0(p_1,p_2,\xi-q_1^2)=\frac{-1}{2\pi^2}\frac{g(p_1)g(p_2)}{1/a_0-i(\xi-q_1^2)^{1/2}}. \quad (3.4)$$

The form factor $g(p)$ is assumed to be given by a constant with a cutoff.

Although the separation of angular momentum as carried out in Ref. 4 is nonrelativistic, we shall use it here too. As we are only considering zero angular momenta, this should make little or no difference. For a fully relativistic partial wave decomposition, one may apply the method given by Wick.[10]

Furthermore, the quantity Ω_i in Eq. (2.8) is taken to be the relativistic generalization of $s-q^2$. It is given by

$$\Omega_i=(z-\omega_i)^2-\mathbf{k}_i^2.$$

The two-body amplitude used here for the relativistic case is given by the scattering-length formula of Chew and Mandelstam.[11] As in Ref. 4 we have

$$t_0(p_1,p_2,\xi-q_1^2)=-[g(p_1)g(p_2)/2\pi^2]A_0(\nu), \quad (3.5)$$

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[9] D. Freedman, C. Lovelace, and J. Namyslowski, CERN report, 1965 (unpublished).

[10] G. C. Wick, Ann. Phys. (N. Y.) 18, 65 (1962).

[11] G. F. Chew and S. Mandelstam, Phys. Rev. 119, 476 (1960).

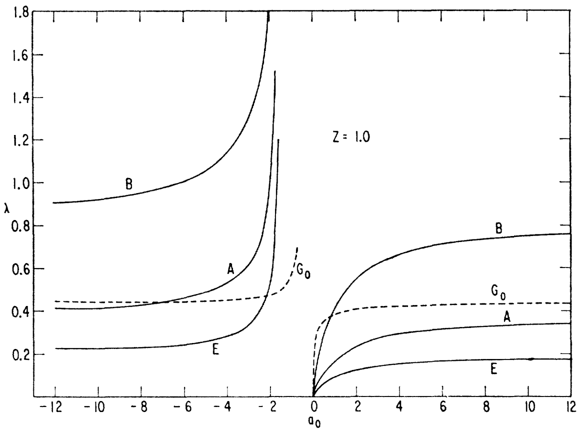

FIG. 1. The dependence of the largest eigenvalue λ on the scattering-length parameter a_0 for the various alternatives.

where we have[11]

$$A_0(\nu)=N(\nu)/D(\nu) \tag{3.6}$$

with

$$N(\nu)=a_I=a_0(1+2a_0/\pi)$$

and

$$D(\nu)=1-\frac{a_I(\nu+1)}{\pi}\int_0^\infty\left(\frac{\nu'}{\nu'+1}\right)^{1/2}\frac{d\nu'}{(\nu'-\nu)(\nu'+1)} \tag{3.7}$$

and, where

$$\nu=\tfrac{1}{4}[(z-(\tfrac{4}{3}q^2+1)^{1/2})^2-\tfrac{4}{3}q^2-4]=\tfrac{1}{4}[(z-\omega_1)^2-k_1^2-4].$$

In Eq. (3.5) we set $g(p)=1$, and in all the relativistic alternatives used here no cutoff is necessary.

The numerical calculations consist of using (3.5) with the alternative propagators of the previous section [instead of (3.3)] in Eq. (3.2). We then compute the eigenvalues of the matrix K in the same way as in Ref. 4. We want a situation in which, for a given $z<3$ and a given a_0, the kernel $K(k_1,k_2)$ has a unit eigenvalue. This then corresponds to the result that there is a three-particle bound state at that energy z; and the binding is provided by the two-body amplitude with the scattering length a_0. If all the eigenvalues are smaller than unity, it means that the force is not strong enough to bind the three particles.

IV. NUMERICAL RESULTS AND DISCUSSION

Here we present our numerical results in the form of figures. In Fig. 1 we give the dependence of the largest eigenvalue on the scattering-length parameter a_0 for various alternative forms of the propagator, for $z=1$. Notice that for $a_0<0$, small values of $|a_0|$ have not been considered. We have discarded these values from our calculations since the kernel becomes singular for these values of a_0. This is due to the occurence of a bound-state pole in the two-body amplitude in the region of integration. Each alternative is identified by symbol designating the propagator, namely:

Case A corresponds to solution with propagator A_i of Eq. (2.7).

Case B corresponds to solution with propagator B_i of Eq. (2.11).

Case E corresponds to solution with propagator E_i of Eq. (2.18).

Case G_0 corresponds to solution with propagator G_0 of Eq. (3.3).

Figure 2 shows the variation of the eigenvalues with the three-particle energy parameter z for $a_0=2$ and for $a_0=-2$. Because the results for separate alternative propagators have the same qualitative features, only a typical case (case A) is given. For the sake of comparison we have also given in this figure the results of the purely nonrelativistic case with propagator G_0. It is interesting and somewhat curious that in the relativistic case for a certain range of z the eigenvalue is a decreasing function of z. The reason for this behavior might be that a three-particle bound state in this region would correspond to a ghost solution. A check for this would be the determination of the sign of the residue of the three-particle bound-state pole.

As can be seen from Fig. 1, with the exception of the nonrelativistic case which is cutoff-dependent, for a positive scattering length the largest eigenvalue is always smaller than unity. We interpret this to mean that for a positive scattering length the force is not large enough to bind the three particles. It is the relativistic phase-space factors which keep the eigenvalues small. However, with negative scattering length it is possible to bind the three particles. In a recent calculation by Rothe[12] based on Regge poles the scattering length has been estimated to be $a_0=-1.7$. Also, Chew has recently given arguments in favor of negative scattering length.[6] Furthermore, it seems that nearly all the phenomenological fitting which has been made with a positive scattering length could equally be done with a negative scattering length.[13] It would be interest-

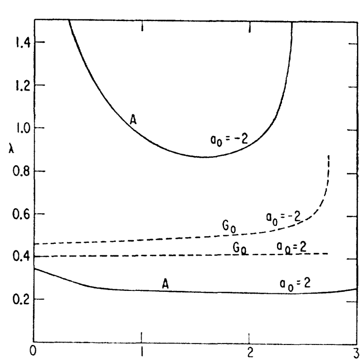

FIG. 2. The dependence of the largest eigenvalue λ on the energy parameter z for $a_0=+2$ and $a_0=-2$ in case A. For comparison the nonrelativistic case is also shown.

[12] H. Rothe, Phys. Rev. 140, B1421 (1965).

[13] We are indebted to Professor D. Y. Wong for discussion on this point.

ing to have an experimental determination of the sign of this quantity. One such possible experiment has already been suggested by Chew.[6] We should remark that we have presented the nonrelativistic case only for the sake of comparison. However, even for the nonrelativistic case the eigenvalues are very small for $a_0>0$ unless unreasonably large cutoffs are introduced. Although there is a qualitative agreement between the various alternatives, quantitative results differ rather substantially. For this reason it would be interesting to consider the problem without putting the intermediate particles on the mass shell and compare the results with the above alternatives. On the other hand we have seen that the qualitative results of the Blankenbecler-Sugar alternatives applied to the Bethe-Salpeter equation are also obtained by introducing relativistic kinematics and phase-space factors in the Faddeev equation. A similar situation holds in the two particle case. There the Blankenbecler-Sugar rule applied to the two-particle Bethe-Salpeter equation gives the same result as the Lippmann-Schwinger equation with relativistic kinematics and phase-space factors.

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Nonet Meson Couplings to Baryons*

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A nonet coupling ansatz which requires zero couplings for the nonstrange baryons to the $\phi(1^{--}$, 1020 MeV) and $s(2^{++}$, 1525 MeV) $I=0$ mesons is investigated in the framework of a Regge-pole model for forward elastic scattering amplitudes at high energy. The analysis indicates that total cross sections are roughly consistent with the ansatz.

INTRODUCTION

An extension of the vector-meson nonet scheme relevant to the couplings of the vector mesons to baryons has recently been proposed[1] to account for (i) the relative suppression of backward ϕ/ω production of K^-p collisions, (ii) the proportionality of electric and magnetic form factors, and (iii) the isospin independence of the hard core in nucleon-nucleon scattering. The explanation of the above phenomena follows from a postulated $SU(3)$-invariant interaction Lagrangian in which all couplings of the nonstrange baryons to the physical ϕ meson are zero. Such an ansatz is made in the framework of a $SU(3)$ quark model in which the quark indices of the vector-meson nonet wave function are only allowed to couple *directly* to the quark indices of the baryon wave function. This anatz is independent of the f/d ratios of the $\bar{B}BV$ vertex and is therefore less restrictive than Lagrangians derived from higher symmetry schemes.[2]

In this article we investigate the validity of such a coupling ansatz for both the vector-meson and the tensor-meson nonets in the framework of a Regge-pole model[3] for forward elastic scattering amplitudes at high energy. Our analysis indicates that the total cross sections are roughly consistent with the ansatz.

VECTOR-MESON NONET

On the basis of mass formula and decay rates, the vector mesons [$\rho(760)$, $K^*(890)$, $\phi(1020)$, $\omega(783)$] are assigned to a $SU(3)$ nonet[4] $(V)_\alpha{}^\beta=(V_8)_\alpha{}^\beta+(1/\sqrt{3})V_1\delta_\alpha{}^\beta$ with the $\omega-\phi$ mixing specified by $\phi=(\sqrt{2}\phi_8-\omega_1)/\sqrt{3}$ and $\omega=(\phi_8+\sqrt{2}\omega_1)/\sqrt{3}$. The 3×3 matrix form of V may be written as

$$V=\begin{bmatrix}(\rho^0+\omega)/\sqrt{2} & \rho^+ & K^{*+}\\ \rho^- & (-\rho^0+\omega)/\sqrt{2} & K^{*0}\\ K^{*-} & \bar{K}^{*0} & -\phi\end{bmatrix}. \quad (1)$$

Then the most general $SU(3)$-invariant interaction Lagrangians for the vector-meson Regge-pole residues at $t=0$ are

$$L_{VMM}=\sqrt{2}\gamma_{MV}\langle M[V,M]\rangle,$$
$$L_{V\bar{B}B}=\sqrt{2}\gamma_{NV}(f\langle\bar{B}[V,B]\rangle+(1-f)\langle\bar{B}\{V,B\}\rangle+\beta\langle V\rangle\langle\bar{B}B\rangle), \quad (2)$$

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[1] H. Sugawara and F. von Hippel, Phys. Rev. **141**, 1331 (1966).

[2] In this paper we shall specifically deal with the $\bar{B}BV$ vertex which enters in the Reggeized vector-meson-exchange contribution to the *forward* elastic-scattering amplitude. In general both the conventional γ_μ and $\sigma_{\mu\nu}$ vector-meson nucleon couplings can contribute to the forward s-channel helicity nonflip Regge amplitude. Consequently, the f/d ratio for the $\bar{B}BV$ vertex at $t=0$ will represent a combination of both the electric- and magnetic-coupling contributions.

[3] V. Barger and M. Olsson, Phys. Rev. Letters **15**, 930 (1965); **16**, 545 (1966); Phys. Rev. (to be published).

[4] S. Okubo, Phys. Letters **5**, 165 (1963).