flection r_0 for $V(r)$ is greater than the inflection r_0' for $V'(r)$. Consider a third potential

$$V''(r) = -G[\exp(mG/B)]r^{-1}e^{-mr}$$

which has the same range m^{-1} as $V'(r)$, but a greater strength so that it passes through the point $(r_0, -B)$ where the binding energy plotted negatively intersects $V(r)$. For $r<r_0$, $V''(r)<V(r)$, so that the binding energy B'' of the ground state in $V''(r)$ is greater than B. Therefore the strength G' of $V'(r)$ necessary to give binding energy B is less than $G[\exp(mG/B)]$ and $V'(r_0') = -B$ occurs for $r_0'<r_0$ as was to be shown. This same argument shows that for given binding energy the size of the ground state in a Yukawa potential of range m^{-1} is larger than that in a Yukawa potential of range $(m')^{-1}$ for $m'>m$. A similar statement holds for any family of monotonically increasing (attractive) potentials characterized by a single range parameter.

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Meson Masses and Decays*

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We consider here the masses, decay rates, and decay spectra of the octet of 0^- mesons, the nonet of 1^- mesons, the $X(960)$ resonance, and a proposed $\eta^*(1620)$ resonance having the quantum numbers of $\eta(550)$. We use spin–unitary-spin symmetry and nonet symmetry, so that the mesons fall into 36 states, combining an $SU(6)\mathbf{1}$, with an $SU(6)\mathbf{35}$. Further, we fix the strength of the couplings of the $SU(3)$ octets in the **36** with respect to the $SU(3)$ singlets by assuming that the form of the coupling remains unchanged when we include the singlet states. We are then able—after making a particular choice of Hamiltonion—to predict the masses $(M_{\eta^*}, M_{\rho^\pm}, M_{\rho^0}, M_{K^{*\pm}}, M_{K^{*0}})$ and the decay rates $\Gamma_{\omega\to\pi^+\pi^-}$, $\Gamma_{\phi\to\pi^+\pi^-}$, $\Gamma_{\phi\to\pi^+\pi^-\pi^0}$, using as input the masses of the octet of 0^- mesons and (M_ϕ, M_ω). These predictions are in reasonable agreement with presently existing experimental data. We further discuss related meson decays using the above scheme to eliminate the arbitrariness in the relative coupling strength of the singlet states. Specifically we consider $V\to P+P'$, $V\to 3P$, $V\to P+\gamma$, $V\to P+P'+\gamma$ [discussing C noninvariance and S-wave pion-pion resonances and their effect in $(\omega,\phi)\to\pi^+\pi^-\gamma$], $V\to l^+ + l^-$, $P\to 2\gamma$, $P\to P'+P''+\gamma$, and $P\to l^+ + l^- + \gamma$. We use the model of Gell-Mann, Sharp, and Wagner and give invariant-mass spectra for the three-body decays.

INTRODUCTION

In the present work, we consider a theory in which the eight 0^- mesons, the nine 1^- mesons, and a possible ninth 0^- meson, are all equivalent in the absence of symmetry-breaking forces. We thus begin with 36 equivalent states. The particular form of the symmetry breaking and the symmetry itself, are made plausible by the assumption that the known particles are built up out of three very heavy fractionally charged objects,[1–4] schematically named (n_0, p_0, Λ_0). Following Zweig,[1] we call these objects "aces." It is to be noted that the symmetry and the form of its breaking are in no sense rigorously derivable from the ace assumption. In fact, the aces could be considered merely a mathematical convenience, and thus at this level are a purely phenomenological construction. In order to decide why nature seems to reflect certain properties of our symmetry a more extensive theoretical investigation is required, dealing with the underlying dynamics.[5]

In Sec. I we discuss the consequences of the assumption that the mesons are bound states of ace-antiace pairs. We assume that in the limit of perfect symmetry, the ace-ace forces are unitary-spin, and spin, independent. This will lead us to a symmetry between 36 meson states: $3\times 9=27$ vector-meson states, and 9 pseudoscalar-meson states. It should be emphasized that we assume a coupling of the form $\mathrm{Tr}(VVP)$ and thus fix the couplings of the $SU(3)$ singlets with respect to the $SU(3)$ octets. There are not then two arbitrary amplitudes for the VVP couplings,[6,7] and we shall see the consequences of this assumption in detail

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[1] G. Zweig, CERN Report TH. 412, 1964 (unpublished); lectures given at the Majorana Summer School, Erice, Sicily, August 1964 (unpublished).

[2] M. Gell-Mann, Phys. Letters **8**, 214 (1964).

[3] T. D. Lee, M. Nauenberg, and F. Gürsey, Phys. Rev. **135**, B467 (1964).

[4] J. Schwinger, Phys. Rev. **135**, B816 (1964).

[5] Experimentally, there are lower limits to the ace mass of a few tens of BeV. For a detailed discussion of the implications of the existence of real aces, fundamental triplets, etc., see: Y. Nambu, University of Chicago report, 1965 (unpublished); in *Proceedings of the 1965 Coral Gables Conference* (W. H. Freeman and Company, San Francisco, California, 1965), p. 274; Y. Nambu and S. Fenster, Progr. Theoret. Phys. (Kyoto) (to be published).

[6] S. Okubo, Phys. Letters **5**, 165 (1965).

[7] S. Glashow, Phys. Rev. Letters **11**, 48 (1963).

below. We emphasize that the part of our considerations dealing with the combination of spin and unitary spin symmetries is used to calculate static properties only. The extension to the relativistic case is beyond the scope of this work.

In Sec. II, we use the theory of Sec. I, plus the dynamical considerations of Refs. 8, 9, and 10 to discuss certain meson decays. The theory of Sec. I provides transition masses for the calculation of such decay amplitudes as that for the (assumed) process $\omega^0 \to \rho^0 \to \pi^+ + \pi^-$. We consider the vector-meson decays:

(1) $V \to P + P'$
(2) $\to 3P$
(3) $\to P + \gamma$
(4) $\to P + P' + \gamma$
(5) $\to l^+ + l^-$

and the pseudoscalar-meson decays:

(6) $P \to \gamma + \gamma$
(7) $\to P' + P'' + \gamma$
(8) $\to l^- + l^+ + \gamma$,

where V means 1^- meson, P means 0^- meson, and l means lepton. We note here that all these decays have been considered by other authors in different contexts and/or with the different input data.[11] We also consider in Sec. II the energy spectra to be expected from the 3-body decays (4,7,8). In the case of (7) we consider the possibility of C violation through the process $V \to (\gamma V') \to \gamma + P + P'$ and also the possible contribution of an s-wave PP' resonance.

I. SYMMETRY CONSIDERATIONS

In this section, we discuss mass formulas and physical states for a meson **36**, using $SU(6)$ and ace symmetry and including electromagnetic effects. The particular Hamiltonian that we choose contains seven arbitrary parameters, enabling us to fix 5 of the 12 meson masses in question, using as input the experimental values of the other 7. The eigenvectors of the Hamiltonian lead to a definite prediction for the amount of $\omega\rho^0$ and $\phi\rho^0$ mixing and thus to a prediction of the decay rates $\Gamma_{\omega\to\pi^+\pi^-}$, $\Gamma_{\phi\to\pi^+\pi^-}$, processes forbidden by isospin conservation. We also obtain a prediction of the amount of mixing of the "pure nonet" $\omega'\phi'$ states,[12] and thus are able to calculate $\Gamma_{\phi\to\pi^+\pi^-\pi^0}$, on the basis of this model.[13]

We follow Zweig[1] and construct 36 meson states by using as building blocks the six "ace" states

$$A_i = (n_0\uparrow, n_0\downarrow, p_0\uparrow, p_0\downarrow, \Lambda_0\uparrow, \Lambda_0\downarrow)\,, \tag{1.1}$$

where the arrows represent spin states. The physical meson states are linear combinations of ace-antiace states called deuces. We deal with an $SU(6)$ **1**, and an $SU(6)$ **35**. We now make up a meson matrix K for the **36**, following Zweig.[1]

$$K = (\mathbf{V}\cdot\boldsymbol{\sigma} + P\sigma_0)/\sqrt{2} = \bar{A}A\,. \tag{1.2}$$

We have written K as a product of spin and unitary spin pieces. $\boldsymbol{\sigma}$ and σ_0 are the Pauli matrices and the 2×2 identity matrix. K is formed symmetrically out of the ace sextuplets A defined above. The 0^- piece, P, is defined as[12]

$$P = G_P \otimes (\bar{A}_\uparrow A_\uparrow + \bar{A}_\downarrow A_\downarrow)/\sqrt{2}\,, \tag{1.3}$$

where G_P is the matrix for the 0^- nonet. In parentheses is the antiace-ace spin-0 state, whose direct product with G_P is taken to form P. Similarly $\mathbf{V}$[12] is the direct product of a spin vector times the matrix for the vector-meson nonet.

$$\mathbf{V} = G_V \otimes \mathbf{S}\,, \tag{1.4}$$

where $\mathbf{S}$ has the components

$$S_x = \frac{1}{\sqrt{2}}(\bar{A}_\uparrow A_\downarrow + \bar{A}_\downarrow A_\uparrow)\,;\quad S_y = \frac{-1}{\sqrt{2}}(\bar{A}_\uparrow A_\downarrow - \bar{A}_\downarrow A_\uparrow)\,;$$
$$S_z = \frac{1}{\sqrt{2}}(\bar{A}_\uparrow A_\uparrow - \bar{A}_\downarrow A_\downarrow)\,. \tag{1.5}$$

We choose the Hamiltonian for the **36**, including symmetry-breaking terms

$$\begin{aligned} H = {} & M_0\,\mathrm{Tr}\bar{K}K + M_1(\mathrm{Tr}\bar{K}\,\mathrm{Tr}K + \mathrm{Tr}K\,\mathrm{Tr}\bar{K}) \\ & + M_2\,\mathrm{Tr}(\bar{K}\lambda_0 \otimes \hat{S}^2 K) \\ & + M_3\,\mathrm{Tr}(K\lambda^2 \otimes \sigma_0 \bar{K} + K\lambda^2 \otimes \sigma_0 \bar{K}) \\ & + M_4\,\mathrm{Tr}(\bar{K}\beta_3 K + K\beta_3 \bar{K}) \\ & + M_5\,\mathrm{Tr}(\bar{K}\beta_1 K + K\beta_1 \bar{K}) + M_6\,\mathrm{Tr}(\bar{K}\beta_1 K\beta_1)\,, \end{aligned} \tag{1.6}$$

where

$$\beta_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};\quad \beta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix};\quad \beta_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

[8] J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960).

[9] M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961).

[10] M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Letters **8**, 261 (1962). Details of the calculations are given in W. G. Wagner, Ph.D. thesis, California Institute of Technology, 1962 (unpublished).

[11] For references and a review see J. J. Sakurai, in *Proceedings of the International School of Physics, Enrico Fermi, Course XXVI* (Academic Press Inc., New York, 1963), p. 41.

[12] We have

$$G_V = \begin{array}{c} \\ p_0 \\ n_0 \\ \Lambda_0 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} \bar{p}_0 & \bar{n}_0 & \bar{\Lambda}_0 \end{array} \\ \begin{pmatrix} 2^{-1/2}(\omega' + \rho_0^0) & \rho^+ & K^{*+} \\ \rho^- & 2^{-1/2}(\omega' - \rho_0^0) & K^{*0} \\ K^{*-} & K^{*0} & \phi' \end{pmatrix} \end{array}$$

and similarly for G_P. The states $\phi'(\eta^{*\prime})$, $\omega'(\eta')$ are defined by

$$\phi' = \bar{\Lambda}_0\Lambda_0 = -(\tfrac{2}{3})^{1/2}Y + (\tfrac{1}{3})^{1/2}B \equiv -aY + bB,$$
$$\omega' = (\bar{n}_0 n_0 + \bar{p}_0 p_0)/\sqrt{2} = (\tfrac{1}{3})^{1/2}Y + (\tfrac{2}{3})^{1/2}B \equiv bY + aB,$$

where Y and B are the pure octet and singlet states, respectively.

[13] For a more detailed discussion of nonet symmetry see J. Yellin, Enrico Fermi Institute, University of Chicago Report C00-264-278, 1965 (unpublished).

The M_0 term is totally **36**-symmetric. The M_1 term splits the **1** from the **35**. (At this point we have omitted a term like $\mathrm{Tr}K\ \mathrm{Tr}K\beta_3$. Note that it would contribute to the 0^- part only. We could include it but it would have negligible effect on our predictions for the 1^- meson masses.) The term in M_2 splits spin 1 from spin 0. $I\otimes S^2$ is just the identity in $SU(3)$ space times the total (spin)^2. This picks out the **V** piece in (1.2). The M_3 term is just the analog of the M_2 term, $\lambda^2=\sum_i\lambda_i^2$. It splits the two $SU(3)$ singlets from the two octets in the **36**. The M_4 term distinguishes the strange ace from the nonstrange aces. The M_5 term does the same thing, this time distinguishing p_0 from the neutral aces. The M_6 term is a second-order electromagnetic term contributing to the self-energy for which, of course, there is no necessary analog like $\mathrm{Tr}\bar{K}\beta_3K\beta_3$. We have included such a term because the first-order term produces no π^+-π^0 mass difference. Note that we have included in H only the simplest terms and have neglected interference terms like $\mathrm{Tr}K\hat{S}^2\beta_3\bar{K}$.

The splitting of Λ_0 from n_0p_0 and p_0 from $n_0\Lambda_0$, caused by the M_4 and M_5 pieces, is the same for the 0^- and 1^- parts. This gives rise to the two relations[14]: (In the following the symbol for a meson will indicate its mass squared.)

$$(M_4)\qquad K^{*\pm}-\rho^\pm=K^\pm-\pi^\pm,\tag{1.7}$$

$$(M_5)\qquad K^{*\pm}-K^{*0}=K^\pm-K^0.\tag{1.8}$$

The first four terms in (1.6) give

$$\begin{aligned}M_0\,\mathrm{Tr}\bar{K}K&+M_1(\mathrm{Tr}\bar{K}\ \mathrm{Tr}K+\mathrm{Tr}K\ \mathrm{Tr}\bar{K})\\&+M_2\,\mathrm{Tr}(\bar{K}\lambda_0\otimes S^2K)\\&+M_3\,\mathrm{Tr}(\bar{K}\lambda^2\otimes\sigma_0K+K\lambda^2\otimes\sigma_0\bar{K})\\&=(M_0+M_2+M_3')\,\mathrm{Tr}\bar{G}_VG_V+(M_0+M_3')\,\mathrm{Tr}\bar{G}_PG_P\\&\quad+M_3''(\mathrm{Tr}\bar{G}_V\ \mathrm{Tr}G_V+\mathrm{Tr}G_V\ \mathrm{Tr}\bar{G}_V)\\&\quad+(M_1+M_3'')(\mathrm{Tr}\bar{G}_P\ \mathrm{Tr}G_P+\mathrm{Tr}G_P\ \mathrm{Tr}\bar{G}_P),\end{aligned}\tag{1.9}$$

where we have split the M_3 term into separate pieces multiplying the singlet and the octet parts. Note that all four terms are independent. Defining

$$M_0{}^V=M_0+M_2+M_3';\quad M_0{}^P=M_0+M_3';$$
$$M_S{}^V=M_3'';\qquad M_S{}^P=M_1+M_3'',\tag{1.10}$$

we can write out the Hamiltonian (1.6) in matrix form $H=H_V+H_P$. In terms of $\phi'(\eta^{*\prime})$ and $\omega'(\eta')$, H_V and H_P have the same form. We have

$$H_V=\begin{matrix}\phi'\\\omega'\\\rho'^0\\K^{*\pm}\\K^{*0}\\\rho^\pm\end{matrix}\begin{bmatrix}M_0{}^V+M_S{}^V+2M_4&(2)^{1/2}M_S{}^V&0&0&0&0\\(2)^{1/2}M_S{}^V&M_0{}^V+M_5+M_6+2M_S{}^V&M_5+M_6&0&0&0\\0&M_5+M_6&M_0{}^V+M_5+M_6&0&0&0\\0&0&0&M_0{}^V+M_4+M_5&0&0\\0&0&0&0&M_0{}^V+M_4&0\\0&0&0&0&0&M_0{}^V+M_5\end{bmatrix},\tag{1.11}$$

and to get H_P, substitute P for V. For the mass parameters we get

$$M_0{}^P=K^0-K^\pm+\pi^+=0.0194\ \mathrm{BeV}^2,\tag{1.12}$$

$$M_5=K^0-K^\pm=K^{*0}-K^{*\pm}=(4.17\pm0.49)\times10^{-3}\ \mathrm{BeV}^2,\tag{1.13}$$

$$M_4=K^{*\pm}-\rho^\pm=K^+=-\pi^\pm=0.225\pm0.001\ \mathrm{BeV}^2,\tag{1.14}$$

where we have used the 0^- masses[15] to calculate the numbers above.[14]

We now need to fix the values of the four remaining mass parameters $(M_6,M_S{}^V,M_S{}^P,M_0{}^V)$. We will use as input the masses of the 0^- octet and (M_ϕ,M_ω).[15] This means we know two of the eigenvalues of H_P': (M_{η^0},M_{π^0}) and two of the eigenvalues of H_V': (M_ϕ,M_ω). The remaining eigenvalues (M_{η^*},M_{ρ^0}) will be determined from the values of the four remaining mass parameters. (By H_P', H_V' we mean the nondiagonal 3×3 parts of H_V and H_P.) We fix the mass parameters by using the secular equations for H_P' and H_V'. They can be written in the form

$$(0^-)\quad-\lambda_i{}^3+P(M_5+M_6,M_S{}^P)\lambda_i{}^2+Q(M_5+M_6,M_S{}^P)\lambda_i+R(M_5+M_6,M_S{}^P)=0,\tag{1.15}$$

$$(1^-)\quad-\lambda_i{}^3+P'(M_0{}^V,M_S{}^V)\lambda_i{}^2+Q'(M_0{}^V,M_S{}^V)\lambda_i+R'(M_0{}^V,M_S{}^V)=0,\tag{1.16}$$

where the λ_i are the eigenvalues, and P, Q, R, P', Q', R' are functions of the four unknown variables, as shown. Expanding $\det(H'-\lambda I)$ we have

$$\begin{aligned}P(x,y)&=\mathrm{Tr}H_P'=3M_0{}^P\\&=3M_0{}^P+2x+3y+2M_4=\eta^*+\eta+\pi^0,\\Q(x,y)&=-(y+M_0{}^P+2M_4)(2M_0{}^P+2x+2y)\\&\quad-(M_0{}^P+x)(x+2y+M_0{}^P)+x^2+2y^2,\\R(x,y)&=-x^2(M_0{}^P+y+2M_4)-2y^2(x+M_0{}^P)\\&\quad+(M_0{}^P+x)(2y+x+M_0{}^P)(M_0{}^P+2M_4+y),\end{aligned}\tag{1.17}$$

[14] V. Kadiyshevskii, R. Muradyan, and Y. A. Smorodinski, Dubna Report P-2061, 1965 (unpublished). Additional references are listed there. $K^*-\rho=K-\pi$ has been discussed by several authors.

[15] A. Rosenfeld *et al.*, Rev. Mod. Phys. **36**, 977 (1964).

and P', Q', R' can be obtained from (1.17) by substituting M_6+M_5 for x, and x for $M_0{}^P$. Using the first two equations of (1.15), ($i=1$, 2; $\eta=\lambda_2$, $\pi^0=\lambda_1$), we can solve for M_5+M_6 and $M_S{}^P$. Inserting these values into $P(x,y)$ we then get M_{η^*}. The answers are, using (1.13)

$$M_6=(-1.2\pm0.5)\times10^{-3}\ \mathrm{BeV}^2, \qquad (1.18)$$

$$M_{\eta^*}=1617\pm30\ \mathrm{MeV}, \qquad (1.19)$$

$$M_S{}^P=M_{\eta^{*\prime}\eta^{\prime}}{}^2/(2)^{1/2}\cong0.793\ \mathrm{BeV}^2. \qquad (1.20)$$

Inserting the above value for M_6 into $H_V{}'$ and using the last two equations of (1.16), ($i=2$, 3; $\lambda_2=\omega$, $\lambda_3=\phi$) we can similarly solve for $M_0{}^V$ and $M_S{}^V$. We get

$$M_0{}^V=K^{*0}-K^{*\pm}+\rho^{\pm}=0.580\pm0.003\ \mathrm{BeV}^2, \quad (1.21)$$

$$M_{\omega'\phi'}{}^2=(2)^{1/2}M_S{}^V=(2.0\pm0.5)\times10^{-2}\ \mathrm{BeV}^2, \quad (1.22)$$

while the trace again yields the third eigenvalue

$$\rho^0=0.568\pm0.009\ \mathrm{BeV}^2 \quad \text{or } M_{\rho^0}=754\pm6\ \mathrm{MeV}. \quad (1.23)$$

In Fig. 1 we have shown the energy-level diagram for the **36**. The eigenvectors of H' yield the physical states. Defining the mixing parameters by

$$
\begin{aligned}
|\omega'\rangle&=A_{\omega'\phi}|\phi\rangle+A_{\omega'\omega}|\omega\rangle+A_{\omega'\rho}|\rho\rangle,\\
|\phi'\rangle&=A_{\phi'\phi}|\phi\rangle+A_{\phi'\omega}|\omega\rangle+A_{\phi'\rho}|\rho\rangle,\\
|\rho'\rangle&=A_{\rho'\phi}|\phi\rangle+A_{\rho'\omega}|\omega\rangle+A_{\rho'\rho}|\rho\rangle,\\
|\eta^*\rangle&\cong A_1|\eta_1\rangle-A_2|\eta_2\rangle,\\
|\eta\rangle&\cong A_1|\eta_2\rangle+A_2|\eta_1\rangle, \qquad (1.24)
\end{aligned}
$$

we get

$$A_{\omega'\phi}{}^2=(4.0\pm1.0)\times10^{-3}, \qquad (1.25)$$

$$A_{\rho'\phi}{}^2\cong5.9\times10^{-7}, \qquad (1.26)$$

$$A_2{}^2=0.09. \qquad (1.27)$$

The transition mass we need to calculate the $\omega^0\to\pi^+\pi^-$ decay rate is given by

$$M_{\omega'\rho'}{}^2=M_5+M_6=-(5.4\pm1.0)\times10^{-3}\ \mathrm{BeV}^2. \quad (1.28)$$

Comparing this with $M_{YB}{}^2=M_4=0.22$ BeV^2 we see that $M_{\omega'\rho'}{}^2\simeq\alpha M_{YB}{}^2$, as one would expect. This result may be compared to that of Picasso *et al.*,[16] who used the observed baryon masses to fix the ratio of electromagnetic to strong contributions to the meson masses. They give

$$M_{\omega\rho}{}^2\cong M_{B\rho}{}^2\cong-3.0\times10^{-3}\ \mathrm{BeV}^2. \qquad (1.29)$$

The $M_{Y\rho}{}^2$ contribution is not included in this estimate.[16]

Summarizing our results, using the Hamiltonian (1.6) for the meson **36** we are able to predict five of the twelve meson masses involved. Essentially the introduction of the electromagnetic terms leads us to five

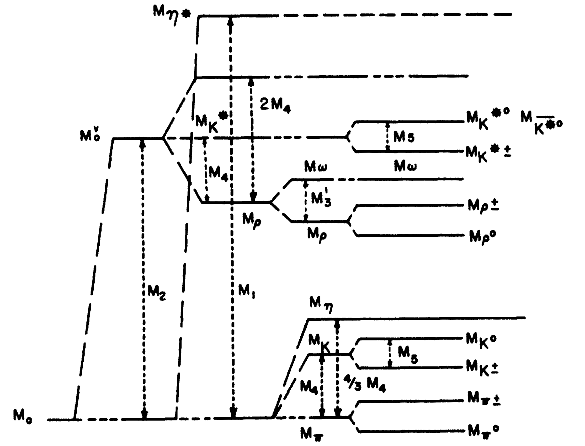

FIG. 1. The energy-level diagram for the meson **36** is shown. M_2 is the spin splitting; M_1 is the splitting of the ninth 0^- state from the octet; M_4 is the strangeness or Gell-Mann–Okubo splitting, M_3' splits the singlet and octet pieces of the vector-meson nonet; M_5 is the electromagnetic splitting.

relations: two slightly perturbed quadratic mass formulas for each nonet; $K^{*0}-K^{*+}=K^0-K^+$; $K^{*+}-\rho^+=K^+-\pi^+$; and a relation between the electromagnetic mass splittings of the π's and ρ's which involves the amount of $\omega\rho$ mixing. Using as input[15] (M_ϕ,M_ω,M_η, $M_{K^\pm}$,M_{K^0},$M_{\pi^\pm}$,M_{π^0}) the predicted masses are

$$
\begin{aligned}
M_{K^{*\pm}}&=895\pm3\ \mathrm{MeV}; \quad M_{K^{*0}}=897\pm2\ \mathrm{MeV},\\
M_{\rho^\pm}&=759\pm3\ \mathrm{MeV}; \quad M_{\rho^0}=754\pm6\ \mathrm{MeV},
\end{aligned} \qquad (1.30)
$$

where we have used (1.12–1.14) and (1.10). The K^* electromagnetic splitting, from (1.8) is

$$M_{K^{*0}}-M_{K^{*\pm}}=2.1\pm0.1\ \mathrm{MeV}. \qquad (1.31)$$

The vector-meson transition masses are

$$
\begin{aligned}
M_{\omega'\rho'}{}^2&=-(5.4\pm1.0)\times10^{-3}\ \mathrm{BeV}^2,\\
M_{\omega'\phi'}{}^2&=(2.0\pm1.0)\times10^{-2}\ \mathrm{BeV}^2.
\end{aligned}
$$

The mixing parameters of interest for the calculation of relative couplings are

$$
\begin{aligned}
A_{\omega'\phi}{}^2&=(4.0\pm1.0)\times10^{-3},\\
A_{\rho'\phi}{}^2&\cong5.9\times10^{-7},\\
A_2{}^2&\cong0.09,\\
A_1{}^2&\cong1.0.
\end{aligned}
$$

Comparable experimental results for the masses are[17]

$$M_\rho=763\pm4\ \mathrm{MeV},$$

$$M_{K^{*\pm}}=891\pm1\ \mathrm{MeV}.$$

In Fig. 1 we have drawn the energy level diagram arising from (1.6).

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[16] L. Picasso, L. Radicati, J. J. Sakurai, and D. Zanello, Nuovo Cimento **37**, 187 (1965); L. A. Radicati, L. E. Picasso, D. P. Zanello, and J. J. Sakurai, Phys. Rev. Letters **14**, 160 (1965). I am now informed by Professor Sakurai that taking into account the $M_{Y\rho}{}^2$ contribution to $M_{\omega\rho}{}^2$ via the methods of Picasso *et al.*, gives a total effect quite comparable to (1.28).

[17] For the mass of charged K^* we use the experimental result of Wojcicki, Phys. Rev. **135**, B484 (1964). The number for M_ρ comes from Ref. 15. For a theoretical discussion of the 0^- meson electromagnetic mass differences see R. Socolow, Phys. Rev. **137**, B1221 (1965) and S. Bose and R. Marshak, Nuovo Cimento **25**, 529 (1962).

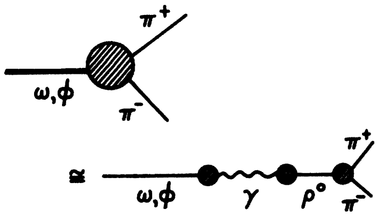

FIG. 2. One-photon-exchange model for $(\omega^0,\phi^0) \to \pi^+ + \pi^-$.

TABLE I. Experimental and theoretical decay rates for strongly allowed $V \to 2P$ decays. Theoretical rates are computed including exact 9 symmetry for the V's.

Decay	Theory	Experiment
$\rho \to 2\pi$	(input)	106 ±5 MeV
$K^* \to K\pi$	31.7 ±1.8 MeV	50 ±2 MeV
$\phi \to \bar{K}K$	2.73±0.16 MeV	2.5±0.8 MeV

II. MESON DECAYS

In this section we discuss decay rates and spectra. Unless otherwise noted we have used the masses and widths of Rosenfeld *et al.*[15]

In the Appendix we discuss the coupling of the photon to vector mesons. We list below the content of the various parts of Sec. II:

(II-1) $V \to P + P'$

We consider the strongly allowed decays of this type and then use the model of Sec. I to predict the rates $\Gamma_{\omega,\phi\to\pi^+\pi^-}$. The results for the strongly allowed modes are not new and we include them for completeness.

(II-2) $V \to 3P$

We consider: $\omega^0 \to \pi^+ + \pi^- + \pi^0$; $\phi^0 \to \pi^+ + \pi^- + \pi^0$; $K^* \to K + \pi + \pi$. The first of these was discussed originally by Gell-Mann, Sharp, and Wagner (GMSW),[10] and we include the $\pi^\pm\pi^0$ electromagnetic mass difference and new values for the masses and for the ρ width, and perform the numerical integration. Similarly, we calculate the rates for the other two decays, including both ρK and $K^*\pi$ intermediate states for the K^* decay.[18]

(II-3) $V \to P + \gamma$

For completeness we calculate these rates from 9 symmetry, using as input $\Gamma_{\omega\to\pi\gamma}$. Then we repeat the GMSW[10] calculation of $\Gamma_{\omega\to\pi\gamma}/\Gamma_{\omega\to 3\pi}$ with our present input, obtaining essentially the same number. We then estimate the effect of the finite width of the ω^0 on this branching ratio and find a relatively weak dependence.

(II-4) $V \to PP\gamma$

We consider $(\phi,\omega) \to \pi^+\pi^-\gamma$ and estimate the possible effects of s-wave pion-pion resonances and C violation.

(II-5) $V \to l^+ + l^-$

We review the results of previous authors for these rates and then comment on some recent experimental results.

(II-6) $P \to 2\gamma$

We use our present input to calculate these decay rates. We then get, because of different input, an even larger discrepancy between theory and experiment for $\Gamma_{\pi^0\to 2\gamma}$ than that given by Dashen and Sharp.[19]

(II-7) $P \to PP\gamma$

We use the GMSW model to calculate the rates and spectra for these decays, specifically considering $(\eta,\eta^*,X) \to \gamma + \pi^+ + \pi^-$. We compare the photon spectra expected from the decay $X(960) \to \pi^+\pi^-\gamma$ with experiment and find fair agreement.

(II-8) $P \to \gamma + l^+ + l^-$

We calculate the effect of intermediate vector-meson states on the rates and invariant-mass spectra for these decays.

(II-9) $\eta^*(1600) \to V + V'$

We consider here $\eta^*(1600) \to (2\rho) \to 4\pi$; $\eta^*(1600) \to (2\omega) \to 6\pi$. Using the GMSW model these rates come out so large due to the high mass that this state will be essentially unobservable.

(II-1) $V(1^-) \to P + P'$

We have, for the widths

$$\Gamma_{V\to PP'} = \left(\frac{f_{VPP'}^2}{4\pi}\right)\frac{M_V}{12} \times\left[1 - \frac{2(M_P^2 + M_{P'}^2)}{M_V^2} + \frac{(M_P^2 - M_{P'}^2)^2}{M_V^4}\right]^{3/2}. \quad (2.1)$$

This implies

$$f_{\rho\pi\pi}^2/4\pi = 2.07 \pm 0.12. \quad (2.2)$$

Taking the VVP couplings from 9 symmetry, we get the rates in Table I, where we have taken all charge modes into account. These decays have been discussed by

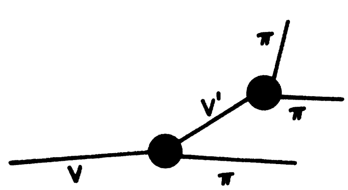

FIG. 3. Gell-Mann, Sharp, Wagner model for $V \to 3\pi$ via $(V'\pi)$.

[18] See M. Sweig, Phys. Rev. **131**, 860 (1963) for a calculation of $K^* \to K\pi\pi$ in the nonrelativistic approximation and including only $K\rho$ intermediate states.

[19] R. Dashen and D. Sharp, Phys. Rev. **133**, B158 (1964).

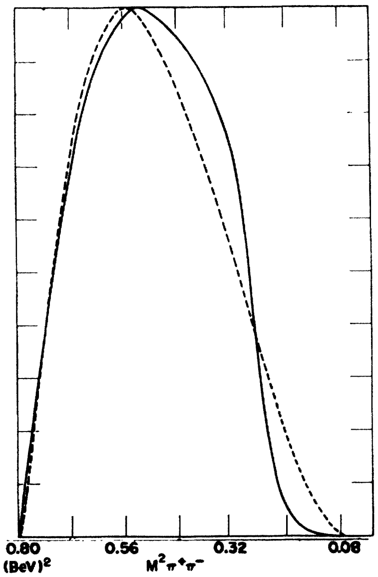

FIG. 4. Invariant-mass spectrum for $\pi^+\pi^-$ pair from $\phi \to \pi^+ + \pi^- + \pi^0$ on the Gell-Mann, Sharp, Wagner model. The dashed line shows the predictions for the simplest matrix element.

Sakurai[20] and by Gell-Mann,[21] in an $SU(3)$ invariant framework, rather than including the effects of YB mixing. Our width $\Gamma_{\phi\to K\bar{K}}$ differs somewhat from that of Dashen and Sharp,[19] who included YB mixing, because of different input data.

Now we pass to the isospin-violating decays of the type $(\omega^0,\phi^0) \to \pi^+ + \pi^-$. First, we try the model of one-photon exchange, combining the results of Dashen and Sharp[19] and of Sakurai and Nambu.[22] This model is pictured in Fig. 2. We have

$$\Gamma_{\omega\to\pi\pi} \cong \frac{\alpha^2 M\omega}{108(f_{\rho\pi\pi}^2/4\pi)}\left[1-\frac{4M_\pi^2}{M_\omega^2}\right]^{3/2} \times \frac{1}{(1-M_\omega^2/M_\rho^2)^2+\Gamma_\rho^2/M_\rho^2}, \quad (2.3)$$

$$\Gamma_{\phi\to\pi\pi} \cong \frac{\alpha^2 M\phi}{54(f_{\rho\pi\pi}^2/4\pi)}\left[1-\frac{4M_\pi^2}{M_\phi^2}\right]^{3/2} \times \frac{1}{(1-M_\phi^2/M_\rho^2)^2+\Gamma_\rho^2/M_\rho^2}. \quad (2.4)$$

These give

$$\Gamma_{\omega\to\pi\pi} \cong 6.9 \text{ keV}; \quad \Gamma_{\omega\to\pi\pi}/\Gamma_\omega^{\text{tot}} \cong 5.3\times10^{-4}, \quad (2.5)$$

$$\Gamma_{\phi\to\pi\pi} \cong 1.0 \text{ keV}; \quad \Gamma_{\phi\to\pi\pi}/\Gamma_\phi^{\text{tot}} \cong 3.2\times10^{-4}. \quad (2.6)$$

Using the model of Sec. I we have,

$$\Gamma_{\omega\to\pi\pi} \cong \frac{M_{\omega\rho'}^4}{[(M_\omega^2-M_\rho^2)^2+\Gamma_\rho^2 M_\rho^2]}\Gamma_{\rho\to\pi\pi}, \quad (2.7)$$

$$\Gamma_{\phi\to\pi\pi} \cong A_{\rho'\phi}^2 \frac{M_\phi}{12}\frac{f_{\rho\pi\pi}^2}{4\pi}, \quad (2.8)$$

where for the ϕ decay we use the mixing (1.24) directly since the ϕ and ρ masses are so far away from each other.

Then we get,[23]

$$\Gamma_{\omega\to\pi\pi}/\Gamma_\phi^{\text{tot}} \cong (3.0\pm1.0)\times10^{-2}, \quad (2.9)$$

$$\Gamma_{\phi\to\pi\pi}/\Gamma_\phi^{\text{tot}} \cong (3.2\pm0.6)\times10^{-5}. \quad (2.10)$$

The experimental results are

$\Gamma_{\omega\to\pi\pi}/\Gamma_\omega^{\text{tot}}$
$= (2.9\pm1.0)\times10^{-2}$, or $(8.2\pm3.0)\times10^{-2}$ (Flattè *et al.*)[24]
$= <0.8\times10^{-2}$ (Steinberger-Lüjtens)[25] (2.11)
$= (1.8_{-0.6}^{+1.2})\times10^{-2}$ (Walker *et al.*)[26]
$= <2\times10^{-2}$ (Alff *et al.*)[27]

(II-2) $V \to 3P$

We consider here the decays

$$\omega^0 \to \pi^+ + \pi^- + \pi^0,$$
$$\phi^0 \to \pi^+ + \pi^- + \pi^0,$$
$$K^* \to K + \pi + \pi.$$

We use the model of Gell-Mann, Sharp, and Wagner[10] as shown in Fig. 3.

The pion energy spectrum for the ω^0 decay differs little from the spectrum calculated with propagators deleted. The spectrum for the ϕ decay is shown in Fig. 4. Note that we have included the $\pi^\pm$-π^0 mass difference, and that our input is somewhat different from the original GMSW calculation.[10] Performing the integration numerically and defining

$$\frac{g_{\omega\rho\pi}^2}{(138 \text{ MeV})^2} \equiv \frac{f_{\omega\rho\pi}^2}{4\pi}, \quad \frac{g_{\phi\rho\pi}^2}{(138 \text{ MeV})^2} \equiv \frac{f_{\phi\rho\pi}^2}{4\pi}, \quad (2.12)$$

[20] J. J. Sakurai, Phys. Rev. 132, 434 (1963). See also Ref. 19.
[21] M. Gell-Mann, CTSL-20 (1961) published in *The Eightfold Way* (W. A. Benjamin, Inc., New York, 1964); Y. Ne'eman, Nucl. Phys. 26, 222 (1961); S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962).
[22] Y. Nambu and J. J. Sakurai, Phys. Rev. Letters 8, 49 (1962).
[23] See also the discussion of P. Singer, Phys. Rev. Letters 12, 524 (1964) who uses nonet symmetry to calculate the electromagnetic self-energies of the vector mesons. He gives $\Gamma_{\phi\to\pi^++\pi^-}/\Gamma_{\phi\to\bar{K}+K} \cong 5\times10^{-3}$, a number radically different from our estimate: 3.2×10^{-5}. Singer uses as input in his calculation M_ω, M_ρ, Γ_ρ, and $\Gamma_{\omega\to2\pi}/\Gamma_\omega^{\text{tot}} = 1.8\times10^{-2}$. He uses one-photon exchange and fixes $f_\rho^2/4\pi \cong 0.41$ instead of $f_{\rho\pi\pi}^2/4\pi \cong 2$ by requiring the branching ratio $\Gamma_{\omega\to2\pi}/\Gamma_\omega^{\text{tot}}$ to be 1.8%.
[24] S. Flattè *et al.*, Phys. Rev. Letters 14, 1095 (1965).
[25] J. Steinberger and G. Lütjens, Phys. Rev. Letters 12, 517 (1964).
[26] W. D. Walker *et al.*, Phys. Letters 8, 208 (1964).
[27] C. Alff *et al.*, Phys. Rev. Letters 8, 325 (1962).

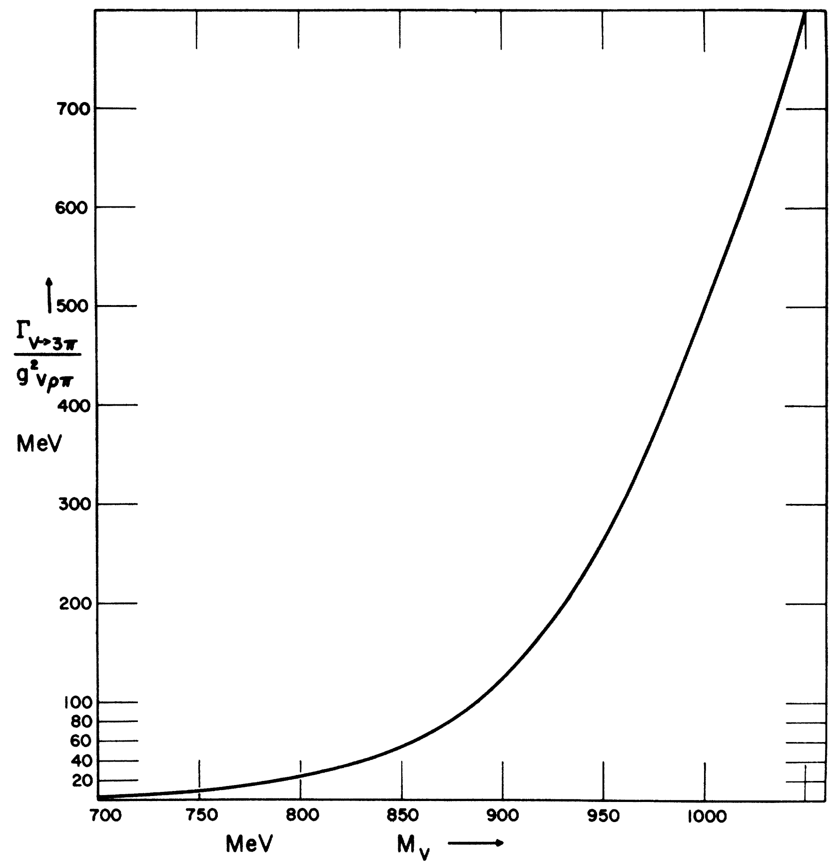

FIG. 5. Decay rate $V \to (\rho\pi) \to 3\pi$ as a function of the mass M_V of the decaying state.

we get

$$\Gamma_{\omega\to3\pi} = (17.6\pm1.0) g_{\rho\omega\pi}{}^2 \times \tfrac{1}{2}(f_{\rho\pi\pi}{}^2/4\pi)\ \text{MeV}, \quad (2.13)$$

$$\Gamma_{\phi\to3\pi} = (600\pm10) g_{\phi\rho\pi}{}^2 \times \tfrac{1}{2}(f_{\rho\pi\pi}{}^2/4\pi)\ \text{MeV}. \quad (2.14)$$

It is interesting to note that the stable ρ approximation gives

$$\Gamma_{\phi\to\rho\pi} = 384 g_{\phi\rho\pi}{}^2 \times \tfrac{1}{2}(f_{\rho\pi\pi}{}^2/4\pi)\ \text{MeV}.$$
(all charge modes). (2.15)

The errors in (2.11) and (2.12) come from the uncertainty in the ρ mass.

In Fig. 5 we show

$$2\Gamma_{V\to3\pi}/g_{V\rho\pi}{}^2(f_{\rho\pi\pi/4\pi}{}^2) \quad (2.16)$$

as a function of M_V. Using $\Gamma_\omega{}^{\text{tot}} = 13.4\pm2$ MeV,[28] and[29]

$$\left(\frac{\Gamma_{\omega\to3\pi}}{\Gamma_\omega{}^{\text{tot}}}\right)_{\text{expt}} \cong 1 - \left(\frac{\Gamma_{\omega\to\text{neutrals}}}{\Gamma_\omega{}^{\text{tot}}}\right)_{\text{expt}}$$
$$\cong 0.90\pm0.04, \quad (2.17)$$

[28] D. Miller, Ph.D. thesis, Columbia University, 1965 (unpublished).

[29] N. Gelfand (private communication). We acknowledge many interesting discussions on the properties of ω mesons with Professor Gelfand. The corresponding number of Ref. 24 is $(9.7\pm1.6)\times10^{-2} = \Gamma_{\omega\to\text{neutrals}}/\Gamma_\omega{}^{\text{tot}}$, as compared to Gelfand's $(9.9\pm0.4)\times10^{-2}$, which we use in the text.

we get from (2.13)[30]

$$g_{\rho\omega\pi}{}^2 = 0.68\pm0.07, \quad (2.18)$$

and using (1.24) and (2.14) we get

$$\Gamma_{\phi\to(\rho\pi)\to3\pi} \cong A_{\phi'\omega}{}^2 g_{\rho\omega\pi}{}^2(600\pm10)\ \text{MeV}$$
$$= 1.6\pm0.5\ \text{MeV}; \quad (2.19)$$

$$\Gamma_{\phi\to3\pi}/\Gamma_\phi{}^{\text{tot}} = 0.51\pm0.21. \quad (2.20)$$

The experimental results are

$$\Gamma_{\phi\to3\pi}/\Gamma_\phi{}^{\text{tot}} = 0.18\pm0.08 \quad \text{(Lindsey } et\ al.)^{31} \quad (2.21)$$

$$= 0.4_{-0.2}{}^{+0.4} \quad \text{(Samios } et\ al.)^{32}, \quad (2.22)$$

$$\Gamma_{\phi\to3\pi}/(\Gamma_{\phi\to3\pi}+\Gamma_{\phi\to\bar{K}K})$$
$$= 0.51\pm0.09 \quad \text{(Badier } et\ al.)^{33}. \quad (2.23)$$

We get, using $f_{K^*K\pi}{}^2/4\pi = 1.73$, from Table I and performing the numerical integration

$$\Gamma_{K^{*+}\to K^0\pi^0\pi^+} = 5.73\ \text{keV},$$
$$\Gamma_{K^{*+}\to K^+\pi^+\pi^-} = 2.73\ \text{keV}, \quad (2.24)$$
$$\Gamma_{K^{*+}\to K^+\pi^0\pi^0} = 0.24\ \text{keV},$$

[30] Dashen and Sharp (Ref. 19) give $g_{\omega\rho\pi}{}^2 = 0.41\pm0.09$ rather than 0.68 ± 0.07 due to a different $\Gamma_\omega{}^{\text{tot}}$ and $\Gamma_{\omega\to\text{neutrals}}/\Gamma_\omega{}^{\text{tot}}$.

[31] J. Lindsey *et al.*, Bull. Am. Phys. Soc. **10**, 502 (1965).

[32] N. Samios *et al.*, Bull. Am. Phys. Soc. **10**, 66 (1965).

[33] J. Badier *et al.*, Phys. Letters **17**, 337 (1965).

giving a branching ratio and total width over all charge modes of

$$\Gamma_{K^*\to K\pi\pi}/\Gamma_{K^*}{}^{\text{tot}}\cong 3.5\times 10^{-4};\ \Gamma_{K^*\to K\pi\pi}\cong 8.7 \text{ keV}, \quad (2.25)$$

where from experiment[34]

$$\Gamma_{K^*\to K\pi\pi}/\Gamma_{K^*\to K\pi}\lesssim 2\times 10^{-3}. \quad (2.26)$$

(II-3) $V\to P\gamma$

From experiment[29]

$$\Gamma_{\omega\to \text{neutrals}}/\Gamma_{\omega}{}^{\text{tot}}=(9.9\pm 0.4)\times 10^{-2}. \quad (2.27)$$

Assuming that the neutral decay products are all $\pi^0\gamma$, we have

$$\Gamma_{\omega\to\pi^0\gamma}=1.30\pm 0.15 \text{ MeV}. \quad (2.28)$$

We then have the results shown in Table II. Using 9 symmetry we can also estimate the ratio $\Gamma_{\omega\to\pi\gamma}/\Gamma_{\omega}{}^{\text{tot}}$ directly, repeating the original GMSW[10] calculation. We get

$$\Gamma_{\omega\to\pi\gamma}/\Gamma_{\omega\to 3\pi}\cong 0.20. \quad (2.29)$$

TABLE II. Experimental and theoretical decay rates for $V\to P+\gamma$.

Decay	Theory	Experiment
$\omega\to\pi\gamma$	input	1.30±0.15 MeV
$\omega\to\eta\gamma$	6.95 keV	···
$\phi\to\eta\gamma$	0.335 MeV	20% of $\phi\to\bar{K}K$[a] 15% of $\phi\to\bar{K}K$[b]
$\phi\to\pi\gamma$	12 keV	···
$\rho\to\pi\gamma$	0.133 MeV	···
$\rho\to\eta\gamma$	50 keV	···
$K^{*+}\to K^+\gamma$	7 keV	···
$K^{*0}\to K^0\gamma$	28 keV	

[a] See Ref. 33.
[b] J. Lindsey and G. Smith, Phys. Rev. Letters 15, 221 (1965).

The original Gell-Mann, Sharp, and Wagner paper gave 0.17, close to our result (2.29) but a factor of 2 greater than the experimental value. We can try to estimate the effect of finite ω^0 width and see if this gives better agreement. We write

$$\frac{d\sigma_{\pi\gamma}}{d\sigma_{3\pi}}=\frac{\int d\omega^2 d\sigma_0(\omega)\Gamma^{\omega\to\pi\gamma}(\omega)\{(\omega^2-M_0{}^2)^2+M_0{}^2[\Gamma^{\omega\to\pi\gamma}(\omega)+\Gamma^{\omega\to 3\pi}(\omega)]^2\}^{-1}}{\int d\omega^2 d\sigma_0(\omega)\Gamma^{\omega\to 3\pi}(\omega)\{(\omega^2-M_0{}^2)^2+M_0{}^2[\Gamma^{\omega\to\pi\gamma}(\omega)+\Gamma^{\omega\to 3\pi}(\omega)]^2\}^{-1}}, \quad (2.30)$$

where $d\sigma_0(\omega)$ is the cross section for making a stable ω^0 of mass ω and M_0 is the measured ω^0 mass.[35] Assuming that $d\sigma_0(\omega)$ is constant over the relevant range of ω^2, we can take out the dependence on the production mechanism, obtaining

$$d\sigma_{\pi\gamma}/d\sigma_{3\pi}\cong 0.18, \quad (2.31)$$

so that the branching ratio in question is rather insensitive to the ω^0 width.

(II-4) $V\to PP\gamma$

We consider here the process $V\to PP\gamma$, with the view of obtaining information about the pion-pion interaction. Note that $G_\omega=-G_{2\pi}$. Therefore, it is the isoscalar piece of the electromagnetic current which contributes in lowest order, and the pions appear in a state with $T=0$.[36] C invariance gives $C_{\omega^0}=-1=C_\gamma C_{2\pi}=-(-1)^{2\pi}$ implying $L_{2\pi}=0, 2, 4, \cdots$. We will assume only s-wave is present.

We now examine the consequences of the assumption that the pions resonate in the final state to form a 0^+ object ξ^0, mass M_{ξ^0}, width Γ_{ξ^0}.[37] The process is pictured in Fig. 6. Assuming no momentum dependence at the vertices, the amplitude is

$$A^\xi=\frac{f_{\xi\omega\gamma}f_{\xi\pi\pi}}{p_{\pi\pi}{}^2-M_\xi{}^2}\omega_\nu A_\mu\left(\delta_{\mu\nu}-\frac{p_\nu q_\mu}{pq}\right). \quad (2.32)$$

The rate is, from (2.32)

$$\Gamma_{\omega\to\pi^+\pi^-\gamma}=\frac{f_{\xi\gamma\omega}{}^2 f_{\xi\pi\pi}{}^2}{12(2\pi)^3 M_\omega{}^3}\int_0^{x_{\max}}dx\, I^\xi(x)$$

$$\equiv\frac{f_{\xi\gamma\omega}{}^2 f_{\xi\pi\pi}{}^2}{12(2\pi)^3 M_\omega{}^3}J^\xi, \quad (2.33)$$

where $x_{\max}=E_\gamma{}^{\max}/M_\omega=\frac{1}{2}(1-4M^2{}_\pi/M_\omega{}^2)$. The photon spectrum is given by

$$I^\xi(x)=\frac{x}{[1-2x-M_\xi{}^2/M_\omega{}^2]^2+K_\xi{}^2}\times\left[1-\frac{1-2x_{\max}}{1-2x}\right]^{1/2}, \quad (2.34)$$

with $K_\xi{}^2=\Gamma_\xi{}^2 M_\xi{}^2/M_\omega{}^4$.

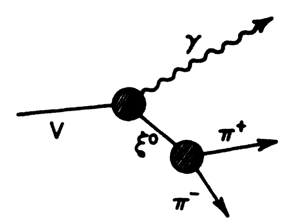

FIG. 6. Model for decay $V\to\pi^+\pi^-\gamma$ with intermediate s-wave resonance.

[34] S. Wojcicki, M. Alston, and G. R. Kalbfleisch, Phys. Rev. 135, B495 (1964).
[35] J. D. Jackson, Nuovo Cimento 34, 1644 (1965).
[36] G. Feinberg and A. Pais, Phys. Rev. Letters 9, 45 (1962).
[37] Scalar resonances have been proposed in the 400-MeV region by Brown and Singer [see L. M. Brown, in *Proceedings of the 2nd Coral Gables Conference* (Freeman and Company, San Francisco, California, 1965), p. 219; a complete discussion of the σ^0 meson is given there] and in the region of the ρ by Durand and Chiu, Phys. Rev. Letters 14, 329 (1965); 14, 680 (E) (1965); 14, 1029 (1965).

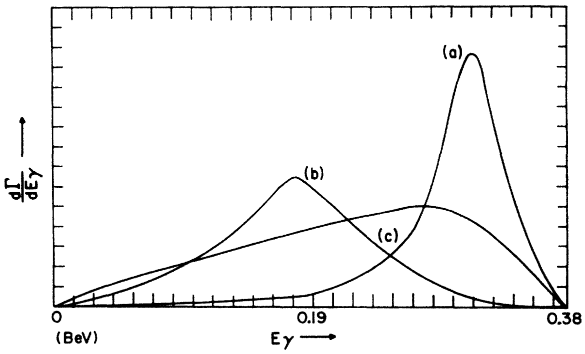

FIG. 7. Photon spectra for $\omega^0 \to \pi^+ + \pi^- + \gamma$ with intermediate s-wave resonance. (a) $M_{\xi^0}=400$ MeV; (b) $M_{\xi^0}=730$ MeV; (c) simplest matrix element.

In Figs. 7 and 8 are shown the photon spectra for $M_{\xi^0}=300$, $\cdots$, 1000 MeV, $\Gamma_{\xi\pi\pi}=100$ MeV. These results are to be compared with those of Singer[38] who used a model with an intermediate $\rho\pi$ state as in Fig. 9. To get the rate on the ξ^0 model, we need to calculate $f_{V\xi\gamma}$. We can do that in two orthogonal ways. In the first method, we use the universal coupling hypothesis of Coleman and Glashow.[39] In the second we apply the tadpole model of Sakurai.[20]

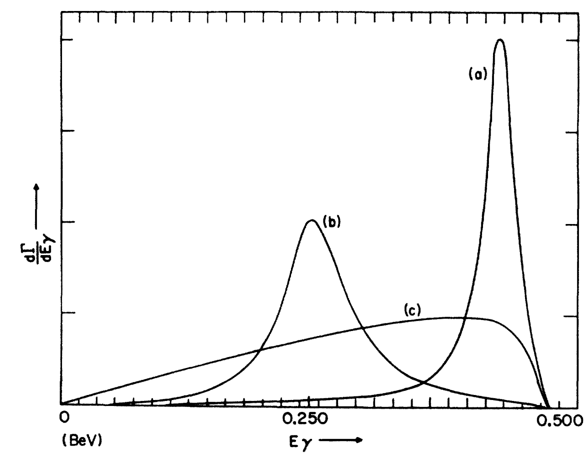

FIG. 8. Photon spectra for $\phi^0 \to \pi^+\pi^-\gamma$; shown are effects of an s-wave $\pi\pi$ resonance ξ^0. (a) $M_\xi=400$ MeV; (b) $M_\xi=730$ MeV; (c) simplest matrix element.

In the universal coupling theory, the piece of the interaction Lagrangian that is relevant is

$$g(\mathrm{Tr}PP\phi+\mathrm{Tr}VV\phi)=\mathcal{L}\,, \tag{2.35}$$

where ϕ is the matrix for the scalar octet, and we have suppressed the spatial part of the interaction.

Assuming ξ^0 is the $T=0$, $Y=0$ member of the scalar octet we have

$$\Gamma_{\omega\to\pi^+\pi^-\gamma}\cong\frac{8\Gamma_{\xi\pi\pi}{}^2M_\xi{}^2\alpha b^2J_\omega{}^\xi}{9\pi M_\omega{}^3(f_{\rho\pi\pi}{}^2/4\pi)(1-4M_\pi{}^2/M_\xi{}^2)}\,. \tag{2.36}$$

We then get the rates shown in Table III.

The branching ratio $\Gamma_{\omega^0\to\pi^+\pi^-\gamma}/\Gamma_\omega{}^{\mathrm{tot}}=3\%$ reported by Rosenfeld *et al.* is incorrect. There is no present experimental evidence for this decay.[40] Our branching ratio for $\omega^0\to\pi^+\pi^-\gamma$ should be compared with that of Singer,[38] who used $\omega^0\to\rho^\pm\pi^\mp\to\pi^+\pi^-\gamma$. Singer gets 2×10^{-5}, about an order of magnitude less than the above.

We can also estimate the rate from the tadpole model.[20] As in Fig. 10, we write

$$\delta m_Y{}^2=f_{\xi YY}\langle\mathrm{vac}|\,\xi\,|\mathrm{vac}\rangle\,, \tag{2.37}$$

$$\delta m_\pi{}^2=f_{\xi\pi\pi}\langle\mathrm{vac}|\,\xi\,|\mathrm{vac}\rangle\,, \tag{2.38}$$

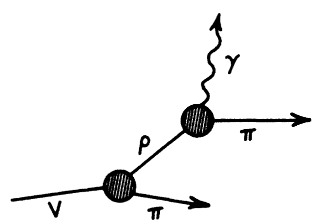

FIG. 9. Model of Singer for $\omega^0\to\pi^+\pi^-\gamma$ via $(\rho\pi)$ intermediate state.

where $\delta m_Y{}^2$, $\delta m_\pi{}^2$ are the deviations of $m_Y{}^2$ and $m_\pi{}^2$ from the values predicted by the Gell-Mann–Okubo formula.[21]

We then get

$$\delta m_Y{}^2=m_Y{}^2-\tfrac{1}{2}(m_\rho{}^2+m_Y{}^2)=\tfrac{1}{2}(m_Y+m_\rho)(m_Y-m_\rho)$$
$$\cong\tfrac{1}{2}(0.273)\ \mathrm{BeV}^2\,, \tag{2.39}$$

$$\delta m_\pi{}^2=\tfrac{1}{2}(m_\pi+m_\eta)(m_\eta-m_\pi)\cong\tfrac{1}{2}(0.327)\ \mathrm{BeV}^2\,. \tag{2.40}$$

These give

$$f_{\xi YY}=f_{\xi\pi\pi}(\delta m_Y{}^2/\delta m_\pi{}^2)\,. \tag{2.41}$$

Using (2.36) and (2.41) we have

$$\frac{\Gamma^{\mathrm{Tadpole}}}{\Gamma^{\mathrm{Universal\ cplg}}}\cong\frac{\delta m_Y{}^2}{\delta m_\pi{}^2}\frac{1}{3b^2}\cong\frac{\delta m_Y{}^2}{\delta m_\pi{}^2}=0.83\,. \tag{2.42}$$

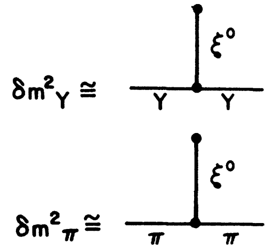

FIG. 10. Deviation from Gell-Mann–Okubo formula from tadpole mechanism of Sakurai.

[38] P. Singer, Phys. Rev. **128**, 2789 (1962). His branching ratio $\Gamma_{\omega\to\pi\pi\gamma}/\Gamma_\omega{}^{\mathrm{tot}}$ depends critically on the size of $\Gamma_{\rho\to\pi\gamma}$. If one takes the experimental result of H. R. Couch *et al.*, Phys. Rev. Letters **13**, 640 (1964), $\Gamma_{\rho\to\pi\gamma}\cong1.65$ MeV, then $\Gamma_{\omega\to\pi\pi\gamma}/\Gamma_\omega{}^{\mathrm{tot}}=2\times10^{-3}$, 100 times larger than the number in the above reference. We thank Dr. Singer for a private communication on this point.

[39] S. Coleman and S. Glashow, Phys. Rev. **134**, B671 (1964).

[40] J. Shafer (private communication). In Ref. 24 is given an experimental upper limit $\Gamma_{\omega\to\pi\pi\gamma}/\Gamma_\omega{}^{\mathrm{tot}}<5\%$. Further experimental results are given by J. Lindsey and G. Smith, Phys. Rev. Letters **15**, 221 (1965).

TABLE III. Decay rates for $(\phi^0,\omega^0)\rightarrow\pi^+ +\pi^- +\gamma$ including the effect of an s-wave pion-pion resonance ξ^0.

Decay	Γ_ξ(MeV)	M_ξ(MeV)	$100(\Gamma_{V\rightarrow\pi^+\pi^-\gamma}/\Gamma_V^{\mathrm{tot}})$
$\omega^0\rightarrow\pi^+\pi^-\gamma$	100	300	4.7×10^{-2}
		400	7.2×10^{-2}
		500	8.0×10^{-2}
		600	6.8×10^{-2}
		700	4.3×10^{-2}
		800	1.5×10^{-2}
$\phi^0\rightarrow\pi^+\pi^-\gamma$	100	300	1.1
		400	0.62
		500	0.62
		600	0.63
		700	0.58
		800	0.48

Similarly for $\phi^0\rightarrow\pi^+\pi^-\gamma$ we have

$$\Gamma_{\phi\rightarrow\pi^+\pi^-\gamma}\cong\frac{8J_\phi{}^\xi\alpha b^2M_\xi{}^2}{9\pi(1-4M_\pi{}^2/M_\xi{}^2)M_\phi{}^3(f_{\rho\pi\pi}{}^2/4\pi)}. \quad (2.43)$$

If we include the possibility of C noninvariance the pion pair can be in a p state.[41] Assuming the diagram of Fig. 11 dominates, we have the amplitude

$$\frac{\lambda[(\phi\cdot A)(p\cdot s)-(\phi\cdot q)(s\cdot A)]}{p_{\pi\pi}{}^2-M_\rho{}^2}\equiv A^\rho, \quad (2.44)$$

where $q=p_\gamma$, $s=p_+-p_-$, and $p=p_\phi$. Combining A^ρ and A^ξ we have, for the total amplitude squared,

$$|M_{\omega\rightarrow\pi^+\pi^-\gamma}|^2=B_1+\lambda B_2+\lambda^2B_3, \quad (2.45)$$

where

$$B_1=2f_{\xi\gamma V}{}^2f_{\xi\pi\pi}{}^2|D_\xi|^2, \quad (2.46)$$

$$B_2=2(p\cdot s)f_{\xi\gamma V}f_{\xi\pi\pi}[D_\xi D_\rho{}^*+D_\rho D_\xi{}^*], \quad (2.47)$$

$$B_3=2|D_\rho|^2\left\{\frac{(p\cdot s)^2}{2}-\frac{s^2}{2p^2}(pq)^2+\frac{(pq)(ps)}{p^2}\right\}, \quad (2.48)$$

B_2 and B_3 contain the C violation.

For the γ spectrum, B_2 does not contribute, and we have

$$I^{\mathrm{tot}}(x)=I^\xi(x)+I^\rho(x)\lambda^2M_V{}^4/3f_{\xi\gamma V}{}^2f_{\xi\pi\pi}{}^2, \quad (2.49)$$

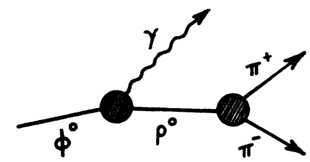

FIG. 11. Model for C-violating piece of decay $\phi^0\rightarrow\pi^+ +\pi^- +\gamma$.

where

$$I^\rho(x)=x(1-x)R^{3/2}(x)N^{-1}(x)+3x^2R^{5/2}(x)N^{-1}(x),$$

$$R(x)\equiv1-\frac{1-2x_{\max}}{1-2x};$$

$$N(x)=\left[1-2x\frac{M_\rho{}^2}{M_V{}^2}\right]-\frac{\Gamma_\rho{}^2M_\rho{}^2}{M_V{}^4}. \quad (2.50)$$

If we take $\lambda/f_{\xi\gamma V}f_{\xi\pi\pi}\cong1$, we get

$$\Gamma_{\phi\rightarrow\pi^+\pi^-\gamma}/\Gamma_\phi{}^{\mathrm{tot}}\cong6.5\times10^{-3}$$

for $M_\xi=730$ MeV, near the ϵ^0 mass.[37] Unless $M_\xi\gtrsim800$ MeV, the photon spectra do not show the ρ effect very well.

For the $\pi^\pm$ energy spectra, we have

$$d\Gamma/dE_+=I_0(E_+)+K_+(E_+), \quad (2.51)$$

$$d\Gamma/dE_-=I_0(E_-)+K_-(E_-), \quad (2.52)$$

where

$$I_0(E_+)=\int B_1(E_+,E_-)dE_- +\lambda^2\int B_3(E_+,E_-)dE_-\ K_+(E_+),$$

$$K_+(E_+)=\lambda\int dE_-\ B_2(E_+,E_-). \quad (2.53)$$

$$K_-(E_-)=\lambda\int dE_+B_2)E_+,E_-).$$

Using $p\cdot s=M_V(E_+-E_-)$ and

$$p_{\pi\pi}{}^2=-M_V{}^2+2M_V(E_++E_-)$$

and defining $x^\pm=E_\pm/M_V$, $\mu=M_\pi/M_V$, $\xi=M_\xi/M_V$, $\rho=M_\rho/M_V$, $f=f_{\xi\gamma V}f_{\xi\pi\pi}$, we have

$$B_1(x_+,x_-)=\frac{f^2}{C^2(x_+,x_-,\xi)+\Gamma_\xi{}^2\xi^2/M_V{}^2},$$

$$B_2(x_+,x_-)=\frac{f(x_+,x_-)[C(x_+,x_-\xi)C(x_+,x_-\rho)+\rho\xi\Gamma_\xi\Gamma_\rho/M_V{}^2]}{[C^2(x_+,x_-,\xi)+\Gamma_\xi{}^2\xi^2/M_V{}^2][C^2(x_+,x_-,\rho)\Gamma_\rho{}^2\rho^2/M_V{}^2]}, \quad (2.54)$$

$$B_3(x_+,x_-)=\frac{\{(x_+-x_-)^2+x_++x_-+(1-x_+-x_-)(x_+-x_-)-\frac{1}{2}-2\mu^2\}}{C^2(x_+,x_-,\rho)+\Gamma_\rho{}^2\rho^2/M_V{}^2},$$

[41] After this section was written, our attention was called to a report by J. Bernstein, G. Feinberg, and T. D. Lee, now published in Phys. Rev. 139, B1650 (1965), in which $(\phi^0,\omega^0)\rightarrow\pi^+\pi^-\gamma$ is also discussed.

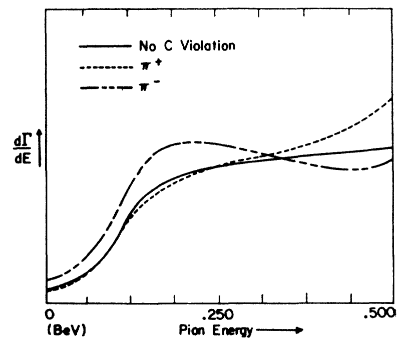

FIG. 12. $\pi^{\pm}$ energy spectra from C-violating $\phi^0 \to \pi^+\pi^-\gamma$ decay with $(\rho\gamma)$ and $(\xi\gamma)$ intermediate states.

where

$$C(x_+\cdot x_-,\beta)=2(x_+ + x_-)-1-\beta. \qquad (2.55)$$

From (2.53) and (2.54), $K_+(x)=-K_-(x)=K(x)$. The π^+ and π^- spectra for maximal violation are compared in Fig. 12.[42] Experimental upper limits to the relevant rates are as follows:

$$\Gamma_{\omega\to\pi^+\pi^-\gamma}\lesssim 0.05\Gamma_\omega^{\text{tot}}, \qquad (2.56)$$

$$\Gamma_{\phi\to\pi^+\pi^-\gamma}\lesssim 0.05\Gamma_{\phi\to\bar{K}K}, \qquad (2.57)$$

$$\Gamma_{\phi\to\rho\gamma}\lesssim 0.03\Gamma_{\phi\to\bar{K}K}, \qquad (2.58)$$

$$\Gamma_{\phi\to\omega\gamma}\lesssim 0.09\Gamma_{\phi\to\bar{K}K}. \qquad (2.59)$$

(II-5) $B\to l^+ + l^-$

We have,[43] for the widths,

$$\Gamma_{V\to l^+l^-}=\frac{G_{V\gamma}^2}{M_V^4}\frac{e^2M_V}{12\pi}\left(1+\frac{2M_l^2}{M_V^2}\right)\left(1-\frac{4M_l^2}{M_V^2}\right)^{1/2}. \qquad (2.60)$$

This gives

$$\frac{\Gamma_{\phi\to l^+l^-}}{\Gamma_{\omega\to l^+l^-}}=\left(\frac{a}{b}\right)^2\frac{M_\phi}{M_\omega}\cong\frac{2M_\phi}{M_\omega}, \qquad (2.61)$$

$$\frac{\Gamma_{\rho\to l^+l^-}}{M_\rho}=9\frac{\Gamma_{\omega^0\to l^+l^-}}{M_\omega}, \qquad (2.62)$$

yielding the rates of Table IV. If one looks for e^+e^- from the reaction

$$\pi^- + p \to (\omega^0,\phi^0,\rho^0)+n, \qquad (2.63)$$

followed by $(\omega^0,\phi^0,\rho^0)\to e^+ + e^-$, one predicts

$$\frac{\sigma_{\pi^-p\to\omega^0n}R_\omega+\sigma_{\pi^-p\to\rho n}R_\rho}{\sigma_{\pi^-p\to\phi n}R_\phi}\equiv Z, \qquad (2.64)$$

the ratio of the number of pairs with invariant $(\text{mass})^2 \approx M_\rho^2\approx M_\omega^2$ to the number of pairs with invariant $(\text{mass})^2\approx M_\phi^2$.

Experimentally,[15]

$$\sigma_{\pi^-p\to\omega^0n}\cong 0.5\ \text{mb}, \quad \sigma_{\pi^-p\to\rho^0n}\cong 1.6\ \text{mb}. \qquad (2.65)$$

According to Cohn *et al.*,[44] vector-meson exchange plus absorption agrees well with experimental data for $\pi^+ + n\to\omega^0 + p$. Assuming that this is true for $\pi + N\to(\omega,\phi)+N$ at incident pion energies of several BeV, and neglecting the mass difference $M_\omega - M_\phi$, we can estimate

$$\frac{\sigma_{\pi^-p\to\phi^0n}}{\sigma_{\pi^-p\to\omega^0n}}\cong\frac{f_{\phi\rho\pi}^2}{f_{\omega\rho\pi}^2}\cong A_{\omega'\phi}^2=4\times10^{-3}, \qquad (2.66)$$

where we have used (1.25). This gives, using (2.65),

$$\sigma_{\pi^-p\to\phi n}\cong 2\mu\text{b}. \qquad (2.67)$$

Using (2.64) and Table IV, we have $Z\cong 100$. Recently published experimental data give[45]

$$R_\rho=(0.5_{-0.3}^{+0.6})\times10^{-4};\ R_\omega=(1.0_{-0.8}^{+1.2})\times10^{-4}; \quad R_\phi\times\sigma_{\pi^-p\to\phi n}=(0.29\pm0.15)\mu\text{b}. \qquad (2.68)$$

Using (2.67), which should be good within an order of magnitude, (2.68) gives

$$R_\phi=0.15\pm0.08, \quad Z=0.6_{-0.5}^{+2.3} \qquad (2.69)$$

in gross disagreement with the theoretical branching ratio $R_\phi=6.5\times10^{-4}$, and with our estimate for Z above. In order for (2.69) to be consistent with our R_ϕ, $\sigma_{\pi^-p\to\phi^0n}$ must be of the same order of magnitude as $\sigma_{\pi p\to\omega^0n}$, and the peripheral model must break down.

(II-6) $P\to 2\gamma$

For the widths we have

$$\Gamma_{\eta\to2\gamma}=\Gamma_{\pi\to2\gamma}(M_\eta/M_\pi)^3\tfrac{1}{3}A_1^2=21.4\,\Gamma_{\pi\to2\gamma}, \qquad (2.70)$$

$$\Gamma_{\eta^*\to2\gamma}=\Gamma_{\pi\to2\gamma}(M_{\eta^*}/M_\pi)^3\tfrac{1}{3}A_2^2\cong 3\ \text{MeV}. \qquad (2.71)$$

TABLE IV. Theoretical branching ratios and rates for the decays $V\to l^+ + l^-$.

Decay	Rate (keV)	$10^4\times$branching ratio
$\omega\to l^+ + l^-$	0.8	$0.6\ =R_\omega$
$\rho\to l^+ + l^-$	6.7	$0.65=R_\rho$
$\phi\to l^+ + l^-$	2.0	$6.5\ =R_\phi$

[42] The branching ratios given here are much smaller than those of J. Prentki and M. Veltman, Phys. Letters **17**, 77 (1965), who were unaware that the 3% branching ratio for $\omega\to\pi\pi\gamma$ quoted in Ref. 15 was incorrect. We thank Dr. Veltman for a private communication on this point.

[43] For a recent discussion of these decays see D. Beder, California Institute of Technology CTSL Internal Report No. 15, 1965 (unpublished). These decays were originally discussed by Nambu and Sakurai (Ref. 22). We obtain the results below by combining their work with that of Dashen and Sharp (Ref. 19).

[44] Cohn *et al.*, Phys. Letters **15**, 344 (1965). The actual experimental process is $\pi^+ + d\to p+(p)+\pi^+ + \pi^- + \pi^0$.

[45] R. Zdanis *et al.*, Phys. Rev. Letters **14**, 721 (1965). Zdanis' results indicate $\sigma_{\pi^-p\to\phi^0n}<10\ \mu$b at the relevant π^- momentum. [R. Hess *et al.*, Bull. Am. Phys. Soc. **10**, 1196 (1965)]. We thank R. Hess for reporting this result to us.

Using[46] $\Gamma_{\pi\to 2\gamma}=6.3\pm 1.0$ eV, we have

$$\Gamma_{\eta\to 2\gamma}=135\pm 22 \text{ eV}, \qquad (2.72)$$

$$\Gamma_{\eta^*\to 2\gamma}\cong 0.3 \text{ keV}. \qquad (2.73)$$

We can estimate $\Gamma_{\pi\to 2\gamma}$ directly, writing

$$\Gamma_{\pi\to 2\gamma}=(f^2_{\pi\gamma\gamma}/64\pi)M_\pi^3 \qquad (2.74)$$

and

$$f_{\pi\gamma\gamma}=-\frac{2f_{Y\rho\pi}G_{Y\gamma}G_{\rho\gamma}}{M_Y^2M_\rho^2}\cong-\frac{\alpha f_{Y\rho\pi}}{(3)^{1/2}}. \qquad (2.75)$$

This gives

$$\Gamma_{\pi\to 2\gamma}=\frac{\alpha^2b^2}{48\pi}M_\pi^3\frac{f_{\omega\rho\pi}^2}{4\pi}\left(1-\frac{af_{\phi\rho\pi}}{bf_{\omega\rho\pi}}\right)^2, \qquad (2.76)$$

and with $f_{\omega\rho\pi}^2/4\pi\cong 2/3M_\pi^2$, $b^2\cong 2a^2\cong\frac{2}{3}$

$$\Gamma_{\pi\to 2\gamma}\cong 35\pm 3 \text{ eV}, \qquad (2.77)$$

which compares poorly with experiment.[46]

These decays were also discussed recently by Sharp and Dashen[19] who got a value of

$$\Gamma_{\pi\to 2\gamma}\approx(24.8\pm 5.3)\left[1-\frac{af\phi\rho\pi}{bf\omega\rho\pi}\right]^2 \text{eV}, \qquad (2.78)$$

the difference being due to their $g_{\omega\rho\pi}^2=0.41\pm 0.09$, compared to our $g_{\omega\rho\pi}^2=0.68\pm 0.08$.

(II-7) $P\to\gamma+P'+P''$

We consider the decays[47]

$$\left\{\begin{array}{l}\eta(550)\\ \eta^*(1620)\\ X(960)\end{array}\right\}\to(\gamma+\rho^0)\to\gamma+\pi^++\pi^-. \qquad (2.79)$$

In this model, we have the rate

$$\Gamma_{\pi^+\pi^-\gamma}{}^P=\frac{f_{P\gamma\rho}^2f_{\rho\pi\pi}^2M_P^3}{12(2\pi)^3}\int_0^{\frac{1}{2}(1-4M_\pi^2/M_P^2)}I^P(x)dx, \qquad (2.80)$$

where the photon spectrum is given by

$$I^P(x)=x^3\left[1-\frac{4M_\pi^2/M_P^2}{1-2x}\right]^{3/2}\times\left[\left(1-2x-\frac{M_V^2}{M_P^2}\right)^2+\frac{\Gamma_V^2M_V^2}{M_P^4}\right]^{-1}. \qquad (2.81)$$

Using $f_{P\gamma\rho}\cong ef_{P\rho\rho}/f_\rho$ we have

$$\Gamma_{\pi^+\pi^-\gamma}{}^P=\frac{\alpha M_P^3}{96\pi M_\pi^2}\left(\frac{f_{P\rho\rho}^2}{4\pi}\right)M_\pi^2\int I^P(x)dx. \qquad (2.82)$$

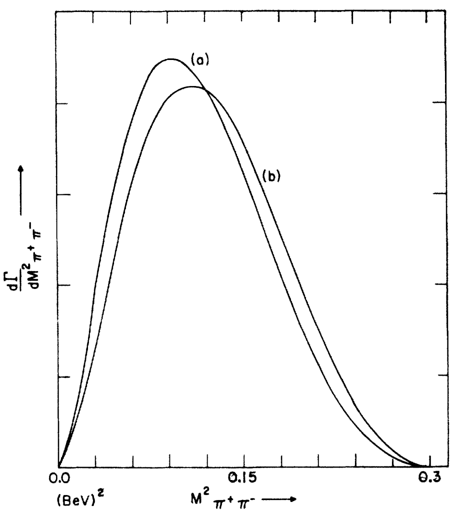

FIG. 13. $\pi^+\pi^-$ invariant-mass distribution for $\eta^0\to\pi^+\pi^-+\gamma$. (a) Simplest matrix element; (b) $\rho^0\gamma$ intermediate state. Curves are normalized to equal area.

The rates are, integrating numerically,

$$\begin{aligned}\Gamma&=789\ \ g_{\eta\rho\rho}^2 \text{ eV}\\ &=\ 40.5g_{\eta^*\rho\rho}^2 \text{ MeV}\\ &=960\ \ g_{X\rho\rho}^2 \text{ keV},\end{aligned} \qquad (2.83)$$

where we have defined

$$g^2\equiv f^2M_\pi^2/4\pi. \qquad (2.84)$$

Using the theory of Sec. I,

$$\Gamma_{\eta\to\gamma\pi^+\pi^-}\cong 175 \text{ eV},\quad \Gamma_{\eta^*\to\gamma\pi^+\pi^-}\cong 3 \text{ MeV}. \qquad (2.85)$$

Experimentally[15,48]

$$\Gamma_{\eta\to\gamma\pi^+\pi^-}/\Gamma_\eta^{\text{tot}}=(5.5\pm 1.3)\times 10^{-2}, \qquad (2.86)$$

$$\Gamma_{X\to\gamma\pi^+\pi^-}/\Gamma_X^{\text{tot}}=0.25\pm 0.14. \qquad (2.87)$$

In Figs. 13 and 14 are shown spectra from these decays. Using

$$\Gamma_{\eta\to 2\gamma}/\Gamma_\eta^{\text{tot}}=35.3\pm 3.0 \qquad (2.88)$$

and our estimate $\Gamma_{\eta\to 2\gamma}=135$ eV, we see (2.85) agrees rather poorly with (2.86). This is just the same discrepancy as appeared for the estimate of $\Gamma_{\pi^0\to 2\gamma}$. Taking $\Gamma_{\pi\to 2\gamma}\cong 35$ eV and using (2.70), we get a number 7.9%, to compare to (2.86).

(II-8) $P(0^-)\to\gamma+l^++l^-$

We are interested here in the process[49] $P\to 2\gamma$, with one photon converting internally into a pair of leptons.

[46] G. Von Dardel *et al.*, Phys. Rev. Letters 4, 51 (1963).

[47] These decays have also been considered by L. Brown and P. Singer, Phys. Rev. Letters 8, 460 (1962); S. Okubo and B. Sakita, *ibid.* 11, 50 (1963); L. Brown and H. Faier, *ibid.* 13, 73 (1964); and by W. Wagner (Ref. 10).

[48] P. Dauber *et al.*, Phys. Rev. Letters 13, 449 (1964); G. R. Kalbfleisch *et al.*, *ibid.* 13, 349a (1964). The curves of Fig. 14 differ somewhat from those shown in the latter reference. This is due to the fact that the exact k dependence was not inserted in the preliminary experimental analysis. More data are now available and are being compared to the curves computed from the same matrix elements used here. We thank Dr. G. Kalbfleisch for a discussion of this point.

[49] The decays $(\eta^0,\pi^0)\to(\gamma e\bar{e},\gamma\mu\bar{\mu})$ have been considered by Celeghini and Gatto, Nuovo Cimento 28, 1496 (1963), independently of the present model.

TABLE V. Branching ratios $P\to\gamma+l^++l^-/P\to\gamma+\gamma$; shown are the ratios for $F_P(x)=1$, and including form-factor variation.

Mode	$100\Gamma_{\gamma ll}/\Gamma_{\gamma\gamma}$ $F_P(x)=1$	Vector mesons included	Experiment[a]
$\pi^0\to\gamma e^+e^-$	1.1855	1.1879	1.166±0.047
$\eta^0\to\gamma e^+e^-$	1.6200	1.6454	
$\eta^{0*}(1620)\to\gamma e^+e^-$	1.96	7.16	
$\eta^{0*}(1620)\to\gamma\mu^+\mu^-$	0.35	5.48	
$X^0(960)\to\gamma e^+e^-$	1.79	2.07	
$X^0(960)\to\gamma\mu^+\mu^-$	0.173	0.422	

[a] See Ref. 51.

We have

$$\frac{\Gamma_{P\to\gamma l^+l^-}}{\Gamma_{P\to\gamma\gamma}}=\frac{2\alpha}{3\pi}\int_{4M_l^2/M_P^2}^{1}\frac{dx}{x}(1-x)^3\times\left(1+\frac{2M_l^2}{xM_P^2}\right)\left(1-\frac{4M_l^2}{xM_P^2}\right)^{1/2}|F_P(x)|^2, \quad (2.89)$$

where we have written the integrand in terms of the invariant (mass)^2 of the lepton pair.[50] The form factor $F_P(x)$ is normalized so that $F_P(0)=1$. Including the (ρ^0,ω,ϕ) states, we can calculate $F_P(x)$ and thus obtain the branching ratios for the various decay modes, as in Table V.[51]

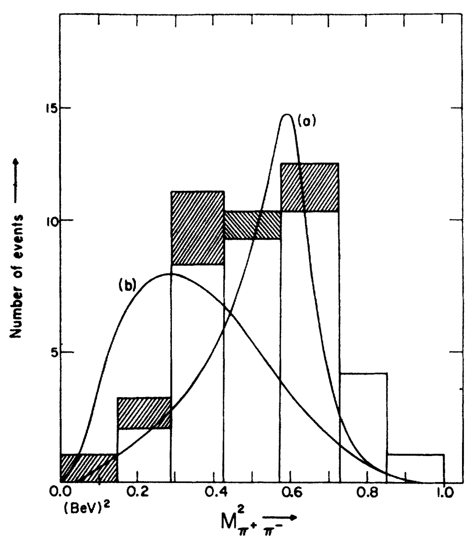

FIG. 14. Invariant-mass distribution for $\pi^+\pi^-$ from $X^0(960)\to\pi^++\pi^-+\gamma$. The histogram is taken from the data of Kalbfleisch *et al.*, Phys. Rev. Letters **13**, 349 (1964). (a) $\rho^0\gamma$ intermediate state; (b) simplest matrix element. Curves are normalized to equal areas.

[50] R. H. Dalitz, Proc. Roy. Soc. (London) **A64**, 667 (1951); S. Berman and D. Geffen, Nuovo Cimento **18**, 1192 (1960).

[51] H. Samios, Phys. Rev. **121**, 275 (1961). As pointed out in Ref. 19, the 3% effect to be expected at the high-energy end of the photon spectrum is not inconsistent with the data of Samios and of H. Kobrak, Nuovo Cimento **20**, 1115 (1961). We thank Dr. Kobrak for several helpful discussions.

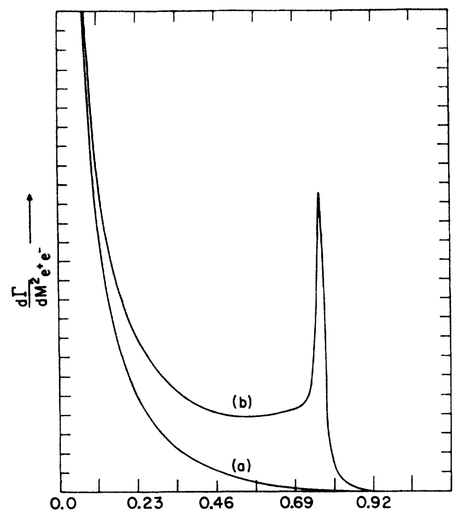

FIG. 15. Invariant-mass distribution for e^+e^- pair from $X^0(960)\to\gamma+e^+e^-$. (a) no vector-meson effects, $F_P(x)=1$; (b) vector-meson effects included. Curves are unnormalized.

We estimate that with present techniques, to measure the form-factor variation for $\eta^0\to\gamma+e^++e^-$ would take about 2×10^6 bubble chamber pictures.[52]

In Fig. 15 we show the invariant mass distribution for $X^0(960)\to\gamma+e^++e^-$. Compared there are the distributions for $F_X(x)=1$ and for $F_X(x)$ including vector-meson effects and assuming X is the ninth 0^- meson.

(II-9) *Decay of the $\eta^*(1620)\to V+V'$*

We can get a rough estimate of the width of this high-mass object by assuming the 4π and 6π decays go through the $\rho\rho$ and $\omega\omega$ intermediate states. We have

$$\Gamma_{\eta^*\to\rho\rho}\approx\Gamma_{\eta^*\to\omega\omega}\cong 400\text{ MeV}.$$

This large width, which should be an underestimate, implies that our $\eta^*(1620)$ will be essentially unobservable.

Note added in proof. The considerations of Sec. II can be extended to the case $(\eta^0, X, \eta^*)\to\pi^+\pi^-\pi^0\gamma$ via $\omega^0\gamma$. *A priori*, since $\eta^0\to3\pi$ and $\eta^0\to2\gamma$ are of order α^2, one might expect the lower order decay into $\omega^0\gamma$ to be large. However, we find that this branching ratio is quite small for the $\eta(550)$, though the X and η^* cases are more favorable. For details see M. Veltman and J. Yellin (to be published).

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[52] We thank Professor N. Gelfand and Professor S. C. Wright for discussing the feasibility of the above experiment and providing the estimate of the number of bubble chamber pictures required which appear in the text.

related topics, at CERN and at Cal Tech. I would also like to acknowledge the kind help of Dr. Yasuo Hara.

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APPENDIX

We derive in this Appendix the formulas we need for inserting the free photon into the theory of Sec. I.

Just as Telegdi and Gell-Mann,[53] and Radicati,[54] introduced the electromagnetic symmetry breaking into the $SU(2)$ symmetry of nuclear physics, with the photon transforming like a combination of $T=1$ and $T=0$ states; in the $SU(3)$ theory, the photon transforms like a particular linear combination of members of the unitary octet, namely the U-spin singlet.[55] [The reader can check this by writing down the $U=0$, $U_z=0$ deuce state $(\bar{\Lambda}_0\Lambda_0+\bar{n}_0n_0-2\bar{p}_0p_0)/\sqrt{6}$ and expressing it as a linear combination of the ρ^0, Y^0 states.]

Symbolically

$$\gamma\sim\tfrac{1}{2}(\sqrt{3}\rho^0-Y^0). \tag{A1}$$

If we further make the dynamical assumption that the vector-meson contributions dominate the coupling of a photon to an arbitrary strongly interacting system X, we have

$$f_{X\gamma}=\sum_i \frac{f_{XV_i}G_{V_i\gamma}}{M_{V_i}^2-q^2}, \tag{A2}$$

where V is a vector meson and we sum over all possible intermediate states. We have suppressed the spatial part; q is the γ 4-momentum; and we have neglected the momentum dependence of the XV and $V\gamma$ vertices, assuming the q^2 dependence to come from the propagator of V alone.

[53] V. L. Telegdi and M. Gell-Mann, Phys. Rev. **91**, 169 (1953).
[54] L. Radicati, Proc. Phys. Soc. (London) **A66**, 139 (1953).
[55] S. Meshkov, C. Levinson, and H. Lipkin, Phys. Rev. Letters **10**, 361 (1963).

For the $V\gamma$ couplings[19] we have

$$\begin{aligned}
G_{\rho\gamma}&=eM_\rho^2/f_\rho,\\
G_{Y\gamma}&=eM_Y^2/2f_Y=-G_{\rho\gamma}/\sqrt{3},\\
G_{\omega\gamma}&=-bG_YM_\omega^2/M_Y^2,\\
G_{\phi\gamma}&=+aG_{Y\gamma}M_\phi^2/M_Y^2,
\end{aligned} \tag{A3}$$

where f_ρ and f_Y are the couplings of ρ^0 and Y^0 to the isospin and hypercharge currents respectively.[8] $f_Y=-\frac{1}{2}\sqrt{3}f_\rho$ from (A1). Note that $G_{\omega\gamma}$, $G_{\phi\gamma}$ depend only on the *physical* masses M_ω, M_ϕ.

The isoscalar part of (A2) is independent of the mixing parameters (a,b) by virtue of the assumption (A1), provided $q_\gamma^2=0$. We can see this by calculating

$$f_{X\gamma}{}^{I=0}=\frac{f_{X\omega}{}^{I=0}G_{\omega\gamma}}{M_\omega^2}+\frac{f_{X\phi}{}^{I=0}G_{\phi\gamma}}{M_\phi^2}. \tag{A4}$$

Using (A3)

$$f_{X\gamma}{}^{I=0}=\frac{G_{Y\gamma}}{M_Y^2}(af_{X\phi}-bf_{X\omega})=\frac{G_{Y\gamma}f_{X\gamma}}{M_Y^2} \tag{A5}$$

independent of (a,b); Q.E.D.

Sakurai has remarked[56] that the baryon magnetic moments, in this model, are independent of a and b; this is just a special case of (A5). Making X a $pp\gamma$ vertex, and considering the magnetic piece only, we get

$$\mu_p=(e/f_\rho)[f_{\rho pp}{}^{\text{Mag}}+f_{Ypp}{}^{\text{Mag}}], \tag{A6}$$

or, in terms of D and F couplings:

$$\mu_p=(e/f_\rho)[G_D{}^{\text{Mag}}+\tfrac{1}{3}G_F{}^{\text{Mag}}] \tag{A7}$$

which reproduces Sakurai's result.[56] Going through the same procedure for all the baryons, one gets the well-known Coleman-Glashow[57] relations for the baryon magnetic moments. Thus the dynamical assumption of vector-meson dominance is still consistent with the symmetry assumption (A1).

[56] J. J. Sakurai, Nuovo Cimento **34**, 1582 (1964). See also S. Coleman and H. Schnitzer, Phys. Rev. **134**, B863 (1964) who also note $\omega\phi$ mixing does not affect the Coleman-Glashow relations (Ref. 57).
[57] S. Coleman and S. Glashow, Phys. Rev. Letters **6**, 423 (1961).