Algebra of Current Components and Decay Widths of ρ and K^* Mesons

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By using the equal-time commutation relations of the chiral $SU(3)\otimes SU(3)$ current algebra and the notion of partially conserved axial-vector current, decay widths of the ρ and K^* mesons are calculated and compared with experiment.

A POWERFUL method has been derived by Fubini and Furlan[1] to extract physical information from the current algebra proposed by Gell-Mann.[2] This method together with the notion of a partially conserved axial-vector current[3] (PCAC) has been used by Adler[4] and Weisberger[5] to obtain the renormalization of the β-decay axial-vector coupling constant in impressive agreement with experiment. The method has since then been extended to various other decays with encouraging results.[6] In this paper we apply this approach to a calculation of the decay widths of $\rho\rightarrow 2\pi$ and $K^*\rightarrow K\pi$. In particular we obtain the following relations:

$$\frac{\gamma^2}{4\pi}=\frac{1}{4\pi}\frac{m_\rho^2}{4f_\pi^2},\tag{1}$$

$$\frac{\gamma_K^2}{\gamma^2}=\frac{m_{K^*}^2}{m_\rho^2},\tag{2}$$

where f_π is related to the decay rate of $\pi\rightarrow\mu\nu$. The quantity $\gamma^2/4\pi$ determines the width of $\rho\rightarrow 2\pi$ decay and $\gamma_K^2/4\pi$ that of $K^*\rightarrow K\pi$. As is discussed below, the relation (1) gives the decay width of the ρ in fair agreement with experiment. The relation (2) then determines the width of $K^*\rightarrow K\pi$. In the exact unitary symmetry limit $\gamma_K=\gamma$. The relation (2) gives the width of K^* in better agreement with experiment than that obtained on the exact unitary limit. The implication of this relation with respect to broken $SU(3)$ is discussed below.

We start with the identity[7]

$$iq_\mu M_{\mu\nu}=i\int d^4x\, e^{-iq\cdot x}\theta(x_0)\langle 0|[\partial_\mu\mathfrak{F}_{\mu,3}{}^5(x),J_\nu{}^{A,0}]|\rho^-\rangle$$
$$+i\int d^4x\, e^{-iq\cdot x}\delta(x_0)\langle 0|[\mathfrak{F}_{0,3}{}^5(x),J_\nu{}^{A,0}(0)]|\rho^-\rangle,\tag{3}$$

where

$$M_{\mu\nu}=i\int d^4x\, e^{-iq\cdot x}\theta(x_0)\langle 0|[\mathfrak{F}_{\mu,3}{}^5(x),J_\nu{}^{A,0}(0)]|\rho^-\rangle\tag{4}$$

and

$$J_\nu{}^{A,0}=\mathfrak{F}_{\nu,1}{}^5+i\mathfrak{F}_{\nu,2}{}^5\tag{5}$$

$\mathfrak{F}_{\nu,\alpha}{}^5(\alpha=1,2,3)$ denote the first three components of the octet of axial-vector currents. The matrix elements in the above equations are taken between the vacuum and a ρ^- meson state of momentum p. According to the partially conserved axial-vector current (PCAC) hypothesis

$$\partial_\mu\mathfrak{F}_{\mu,\alpha}{}^5=-(f_\pi/\sqrt{2})\mu^2\phi_\alpha,\tag{6}$$

where μ denotes the pion mass and ϕ_α denotes the pion field. By the Goldberger-Treiman relation[8]

$$(f_\pi/\sqrt{2})=-(G_A/G)(m/g_r),\tag{7}$$

where m is the nucleon mass, G_A is the axial-vector coupling constant, $G_A\approx -1.18$ G, and g_r is the strong

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pion-nucleon coupling constant, $g_r^2/4\pi \approx 14.5$. With the use of relation (6), Eq. (3) can be written as

$$i\int d^4x\, e^{-iq\cdot x}\delta(x_0)\langle 0|[\mathfrak{F}_{0,3}{}^5(x),J_\nu{}^{A,0}(0)]|\rho^-\rangle$$
$$=iq_\mu M_{\mu\nu}+(f_\pi/\sqrt{2})(\mu^2/(q^2+\mu^2))R_\nu(\nu,q^2)\,, \quad (8)$$

where

$$R_\nu(\nu,q^2)=i\int d^4x\, e^{-iq\cdot x}(\mu^2-\Box^2)$$
$$\times\langle 0|\theta(x_0)[\phi_3(x),J_\nu{}^{A,0}(0)]|\rho^-\rangle\,, \quad (9)$$

and

$$\nu=-(p\cdot q)/m_\rho\,.$$

Consider now the limit in which q_μ vanishes. We now assume that near $q_\mu\to 0$, $R_\nu(\nu,q^2)$ is dominated by the pole at $-q^2=\mu^2$ and thus obtain from (8)

$$i\langle 0|[I_3{}^5(0),J_\nu{}^{A,0}(0)]|\rho^-\rangle$$
$$=\lim_{q_\mu\to 0} iq_\mu M_{\mu\nu}+(f_\pi/\sqrt{2})\lim_{\nu\to 0}R_\nu(\nu,-\mu^2)\,, \quad (10)$$

where

$$R(\nu,-\mu^2)=\langle\pi^0|J_\nu{}^{A,0}(0)|\rho^-\rangle(2q_0)^{1/2}\,,$$
$$I_3{}^5(t)=\int d^3x\,\mathfrak{F}_{0,3}{}^5(\mathbf{x},t)\,, \quad (11)$$

and the chiral $SU(3)\otimes SU(3)$ algebra of current components gives

$$[I_3{}^5,J_\nu{}^{A,0}]=\mathfrak{F}_{1,\nu}+i\mathfrak{F}_{2,\nu}$$
$$=J_\nu{}^{V,0}. \quad (12)$$

Thus in the limit $q_\mu\to 0$, we obtain

$$i\langle 0|J_\nu{}^{V,0}|\rho^-\rangle=\lim_{q_\mu\to 0} iq_\mu M_{\mu\nu}$$
$$+(f_\pi/\sqrt{2})\lim_{\nu\to 0}R_\nu(\nu,-\mu^2)\,. \quad (13)$$

The first term on the right-hand side of Eq. (13) vanishes[7] unless an intermediate state contributes which is degenerate in mass with the ρ meson. This is not the case here. Thus this term does not contribute and we obtain

$$i\langle 0|J_\mu{}^{V,0}|\rho^-\rangle=(f_\pi/\sqrt{2})\lim_{\nu\to 0}\langle\pi^0|J_\mu{}^{A,0}|\rho^-\rangle(2q_0)^{1/2}. \quad (14)$$

Now the matrix element relevant to $\rho^-\to l^-+\bar\nu$ decay is

$$(2p_0)^{1/2}\langle 0|J_\mu{}^{V,0}|\rho^-\rangle=f_\rho\epsilon_\mu\,, \quad (15)$$

where ϵ_μ is the polarization vector of ρ. Also for the ρ^- β-decay, the matrix element, which survives when q_μ or $\nu\to 0$, can be written as

$$(2q_0)^{1/2}(2p_0)^{1/2}\langle\pi^0|J_\mu{}^{A,0}|\rho^-\rangle=i(G_A{}^{\rho\pi}/G)\epsilon_\mu\,. \quad (16)$$

Equation (14) then gives

$$f_\rho=(G_A{}^{\rho\pi}/G)(f_\pi/\sqrt{2})\,. \quad (17)$$

This relation is not useful since neither f_ρ nor $G_A{}^{\rho\pi}$ is measureable. However, by a Goldberger-Treiman type relation applied to $\rho^-\to\pi^0+l^-+\nu$ decay, one can relate $G_A{}^{\rho\pi}$ to f_π and $\gamma_{\rho\pi\pi}$:

$$f_\pi=\frac{|G_A{}^{\rho\pi}/G|}{2\gamma_{\rho\pi\pi}}\,, \quad (18)$$

where $\gamma_{\rho\pi\pi}$ is defined by the Lagrangian density

$$L_{\rm int}=2\gamma(\boldsymbol{\pi}\times(\partial\boldsymbol{\pi}/\partial x_\mu))\rho_\mu$$

with

$$\gamma_{\rho\pi\pi}=2\gamma\,. \quad (19)$$

On the other hand if we assume that the $\pi^-\to\pi^0+l+\nu$ decay is dominated by the ρ-meson pole[2,9] and use the conserved vector current (CVC) hypothesis, we obtain

$$\sqrt{2}=\gamma_{\rho\pi\pi}|f_\rho|/m_\rho{}^2. \quad (20)$$

Equations (17), (18), and (20) then give

$$\gamma_{\rho\pi\pi}{}^2=m_\rho{}^2/f_\pi{}^2$$

or

$$\gamma^2/4\pi=(1/4\pi)m_\rho{}^2/4f_\pi{}^2. \quad (21)$$

By considering the $\Delta S=1$ axial-vector current $J_\mu{}^{A,1}$ and by taking the matrix elements between the vacuum and K^* meson in Eqs. (3), (4), etc., we obtain in exactly the same manner as above by employing the commutation relation

$$[I_3{}^5,J_\mu{}^{A,1}]=\tfrac{1}{2}J_\mu{}^{V,1}\,,$$

the following relation:

$$\tfrac{1}{2}|f_{K^*}|=|G_A{}^{K^*\pi^0}/G|f_\pi/\sqrt{2}\,, \quad (22)$$

where the Goldberger-Treiman relation in this case gives

$$|f_K|=|G_A{}^{K^*\pi}/G|/2|\gamma_K| \quad (23)$$

f_{K^*} and $G_A{}^{K^*\pi}$ are defined in the same manner as f_ρ and $G_A{}^{\rho\pi}$ in Eqs. (15) and (16). Here f_K is the decay constant for $K_{\mu l}$ decay and γ_K is defined by the Lagrangian density

$$i\gamma_K K^{*-}(K^+\partial_\alpha\pi^0-\pi^0\partial_\alpha K^+)\,. \quad (24)$$

Let us now consider the decay $K^-\to\pi^0+e^-+\bar\nu$, whose matrix element in the limit of zero momentum transfer, one defines as

$$(2p_{K0}2p_{\pi 0})^{1/2}G\langle\pi^0|J_\mu{}^{V,1}|K^-\rangle$$
$$=G[(1/\sqrt{2})f_+(0)(p_K+p_\pi)_\mu]. \quad (25)$$

Assuming as before that this decay is dominated[2,9] by the K^* meson pole for low momentum transfer, we obtain

$$(1/\sqrt{2})|f_+(0)|=|\gamma_K||f_{K^*}|/m_{K^*}{}^2. \quad (26)$$

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Equations (22), (23), and (26) then give

$$\frac{\gamma_K^2}{4\pi}=\frac{1}{4\pi}\frac{m_{K^*}^2}{4f_\pi}\left|\frac{f_+(0)}{f_K}\right|. \tag{27}$$

In Cabibbo's theory,[10] where $\theta_A=\theta_V$, we have $f_+(0)=\sin\theta$, $f_K=f_\pi\sin\theta$, and relation (27) reduces to

$$\frac{\gamma_K^2}{4\pi}=\frac{1}{4\pi}\frac{m_{K^*}^2}{4f_\pi^2}. \tag{28}$$

On using Eq. (21), we can express this result as

$$\gamma_K^2/\gamma^2=m_{K^*}^2/m_\rho^2. \tag{29}$$

In the exact $SU(3)$ limit, $m_{K^*}=m_\rho$, $\gamma_K=\gamma$.

Let us now confront the relations (21) and (28) to experimental check. In Cabibbo's theory the $\pi_{\mu 2}$ decay is determined by $f_\pi\cos\theta$ and the experimental decay rate[11] for $\pi_{\mu 2}$ gives

$$|f_\pi\cos\theta|=0.94m_\pi. \tag{30}$$

Now[10] $\theta\approx0.25$ and we obtain

$$f_\pi^2=0.94m_\pi^2.$$

With this value of f_π^2 and using[11] $m_\rho=763$ MeV, Eq. (21) gives

$$\gamma^2/4\pi\approx0.6,\quad \Gamma_\rho\approx122\text{ MeV}. \tag{31}$$

The experimental ρ width[11] is

$$\Gamma_\rho^{\text{expt}}=106\pm5\text{ MeV}. \tag{32}$$

The agreement is fair in view of the fact that the usual Goldberger-Treiman relation (7) is also satisfied experimentally to within 20% or so. In fact, with the above value of f_π^2, relation (7) gives $g_r^2/4\pi$ to be about 11, whereas its experimental value is 14.5. Relation (29) gives

$$\frac{\Gamma(K^{*-}\to K\pi)}{\Gamma_\rho}=\frac{3}{4}\frac{\gamma_K^2}{\gamma^2}\frac{p_{K\pi}^3/m_{K^*}^2}{p_{\pi\pi}^3/m_\rho^2}=\frac{3}{4}\frac{m_{K^*}^2}{m_\rho^2}\frac{p_{K\pi}^3/m_{K^*}^2}{p_{\pi\pi}^3/m_\rho^2}. \tag{33}$$

Using (32), one obtains

$$\Gamma(K^{*-}\to K\pi)=(44.7\pm2.2)\text{ MeV} \tag{34}$$

to be compared with[11]

$$\Gamma^{\text{expt}}(K^{*-}\to K\pi)=(50\pm2)\text{ MeV}. \tag{35}$$

In the exact $SU(3)$ limit, with Γ_ρ given in (32), one gets

$$\Gamma_{SU(3)}(K^{*-}\to K\pi)=(32.9\pm1.6)\text{ MeV}. \tag{36}$$

The value (34) obtained in our approach agrees fairly well with the experimental value.

If one does not use Cabibbo's theory, then

$$f_\pi=0.94m_\pi, \tag{37}$$

and the experimental values[12] of $f_+(0)$ and f_K are

$$|f_+(0)|=0.226\pm0.14,\quad |f_K|=(0.247\pm0.003)m_\pi. \tag{38}$$

With these values,

$$\Gamma_\rho=137.7\text{ MeV} \tag{39}$$

and the relations (21) and (27), if one uses the experimental value (32) of Γ_ρ, give

$$(36\pm2)\leqslant\Gamma_{K^*}\leqslant(41.4\pm2). \tag{40}$$

The agreement is somewhat better on Cabibbo's theory than obtained here.

The small discrepancies between our results and experiments may be due to approximating the form factors $K_{\pi NN}(0)$, $K_{\rho\pi\pi}(0)$ and $K_{K^*K\pi}(0)$, which in general appear in relations (7), (18), and (21) and (23) and (27), respectively, by unity whereas the first two form factors are normalized to unity at $k^2=-m_\pi^2$ and the last one at $k^2=-m_K^2$, k being the momentum transfer. The small variation of the above form factors from the value at $k^2=-m_\pi^2$ or $k^2=-m_K^2$ to that at zero may account for the above discrepancies.

Relation (29) is very interesting because it shows that the departure for the strong coupling constant γ_K of $K^*\to K\pi$ decay from its exact $SU(3)$ value, namely, γ is completely determined by the ratio of K^* and ρ meson masses. A similar conclusion was already drawn by one of the authors[13] and by Freund and Nambu[14] who showed, by the use of generalized Goldberger-Treiman relations, that the departures for the strong meson-baryon coupling constants from their exact $SU(3)$ value are completely determined by the baryon mass spectrum, e.g., $g_{K\Lambda N}/g_{\pi NN}=(g_{K\Lambda N}/g_{\pi NN})_{SU(3)}\times(m_\Lambda+m)/2m$.

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