Association between the Dip in the $\pi^- p \to \pi^0 n$ High-Energy Angular Distribution and the Zero of the ρ Trajectory*

FARZAM ARBAB AND CHARLES B. CHIU
Lawrence Radiation Laboratory, University of California, Berkeley, California
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The angular distributions of $\pi^- p \to \pi^0 n$ near the forward direction from 3 to 18 GeV/c measured by the Saclay-Orsay group displayed a dip near $t=-0.6$ (GeV/c)^2 independent of energy. To analyze the data we assumed that the amplitude is dominated by the ρ-trajectory exchange, and we parametrized it with six parameters (two for the ρ trajectory, two for the non-spin-flip residue, and two for the spin-flip residue). The data above 5 GeV/c with $|t| \leqslant 1.4$ are included in this analysis. We found a solution with $\chi^2=98$ for a total of 62 points. The result for the ρ trajectory is $\alpha_\rho=0.56(\pm0.01)+1.08(\pm0.03)\,t$. Our result shows that the observed dip can be explained by the necessary vanishing of the spin-flip amplitude when the exchange angular momentum passes through 0.

THE angular distribution of $\pi^- p \to \pi^0 n$ near the forward direction have been measured by the Saclay-Orsay group at CERN at several momenta between 3 and 18 GeV/c.[1] The observed energy dependence gives important support to the hypothesis of Regge behavior controlled by the ρ trajectory. A dip at about $t=-0.6(\text{GeV}/c)^2$ together with a secondary maximum is a general feature of these distributions.

Figure 1 shows the angular distributions at 5.85, 9.8, 13.3, and 18.2 GeV/c. Phillips and Rarita fitted these distributions,[2] assuming that the amplitudes are dominated by a ρ Regge pole in the crossed channel, with the differential cross sections given by the expression

$$\frac{d\sigma}{dt}(s,t)=\frac{1}{\pi s}\left(\frac{M}{4k}\right)^2\left[\left(1-\frac{t}{4M^2}\right)|A|^2+\frac{t}{4M^2}\left(s-\frac{s+p^2}{1-t/4M^2}\right)|B|^2\right], \quad (1)$$

where

$$A=C(t)\frac{1-\exp(-i\pi\alpha)}{\sin\pi\alpha}\left(\frac{E}{E_0}\right)^\alpha$$

is the nonhelicity-flip amplitude and

$$B=D(t)\frac{1-\exp(-i\pi\alpha)}{\sin\pi\alpha}\left(\frac{E}{E_0}\right)^{\alpha-1}$$

is the helicity-flip amplitude. The symbols s and t are the invariant squares of energy and momentum transfer, respectively, p and E are the incident-pion momentum and total energy in the laboratory system, k is the center-of-mass momentum, M is the nucleon mass, and E_0 is a scale factor arbitrarily taken to be 1 GeV. The ρ trajectory is designated by $\alpha(t)$.

Phillips and Rarita parametrized $C(t)$ as $(2\alpha+1)$ times the difference of two decreasing exponentials, while $D(t)$ was represented as α times such a difference. The trajectory $\alpha(t)$ was assigned the Pignotti form. They obtained a solution with $\alpha(0)=0.540\pm0.002$ and $\alpha'(0)=0.65\pm0.02$ from 75 data points with $\chi^2=144$. Assuming a linear trajectory, they found a less satisfactory fit with $X^2=175$ for $\alpha(0)=0.530\pm0.003$ and $\alpha'=0.47\pm0.02$. In both fits the value of $|t|$ where $\alpha(t)$ crosses zero is much larger than 0.6. In their solutions, the dip in the cross sections is explained by the change in sign of the difference of the two exponentials in the B amplitude (which is much larger than A), the position of the dip being near the position where the difference of the exponentials vanishes. At the same time, they pointed out the possibility that the dip might be associated with the vanishing of the factor $\alpha(t)$ in the B amplitude. In fact, they noticed that if one assumes a linear trajectory that goes through the position of the ρ resonance and through $\alpha\approx\frac{1}{2}$ at $t=0$, this linear trajectory should go through $\alpha(t)=0$ near $t=-0.6$. We proceeded to study this possibility using this idea as the ingredient, and have found a fit to the data, which is actually slightly better than the preferred fit of Phillips and Rarita.

As a preliminary to our analysis, the trajectory function $\alpha(t)$ was first studied by the so-called model-independent method, which has been used, for example, by Logan,[3] Höhler,[4] and others. The value of α at each t can be determined from the dependence of $d\sigma/dt$ on E, the incident-pion lab energy, since we have

$$\frac{d\sigma}{dt}=\left(\frac{E}{E_0}\right)^{2\alpha-2}F(t).$$

A linear form for the trajectory gave a statistically adequate fit, as shown in Fig. 2 (curve I) leading to $\alpha(0)=0.56\pm0.03$ and $\alpha'=0.81\pm0.08$. We thus chose a linear form to parametrize the trajectory in the following analysis.

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[1] Saclay-Orsay Collaboration: (a) Phys. Rev. Letters **14**, 763 (1965); (b) Phys. Letters **20**, 75 (1966).

[2] R. J. N. Phillips and W. Rarita, Phys. Rev. **139**, B1336 (1965).

[3] R. K. Logan, Phys. Rev. Letters **14**, 414 (1965).

[4] G. Höhler, J. Baacke, H. Schlaile, and P. Sonderegger, Phys. Letters **20**, 79 (1966).

For the residue functions $C(t)$ and $D(t)$ we chose forms based on Wang's analysis[5] of the poles and zeros of the helicity amplitudes A and B, showing that there should be no poles beyond those at $\alpha=1, 3, 5, \cdots$ in both amplitudes, while the kinematically required zeros occur at $\alpha=-1, -2, \cdots$, in both $C(t)$ and $D(t)$, and at $\alpha=0$ in $D(t)$. The sequence of zeros at negative odd integers cancel out the spurious poles at these points in the function $[1-\exp(-i\pi\alpha)]/\sin\pi\alpha$. The data in question will carry us near the point[6] $\alpha=-1$ (see Fig. 2) but not near $\alpha=-2, -3, \cdots$, so in our parametrization we have included only the first zero of this sequence. In addition $D(t)$ must have a zero at $\alpha=0$. It is possible that further (dynamical) zeros occur in the residue functions, but we have sought a "simple" fit where such complications are absent. Accordingly we chose the expressions

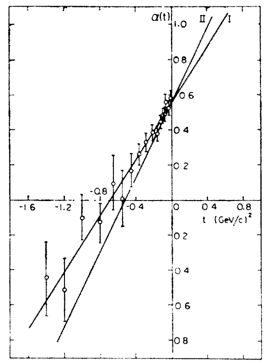

FIG. 2. The ρ trajectory plotted as a function of t. Curve I = best linear fit to $\alpha(t)$ values determined by the model-independent method from data of Ref. 1(a); $\alpha(t)=0.56+0.81t$. Curve II: $\alpha(t)=0.56+1.08t$.

$$C(t)=(\alpha+1)C_0\exp(C_1t)\,,$$

and

$$D(t)=\alpha(\alpha+1)D_0\exp(D_1t)\,,$$

where C_0, C_1, D_0, and D_1 are adjustable constants. Note that these forms are somewhat different from those of Phillips and Rarita. In their parametrization there is an undesired zero at $\alpha=-\frac{1}{2}$ in A and poles at $\alpha=-1$ in both A and B amplitudes.

The data points for $|t|\leqslant 1.4(\text{GeV}/c)^2$ at $p_\pi=5.85$, 13.3, and 18.2 GeV/c and for $|t|\leqslant 0.8(\text{GeV}/c)^2$ at $p_\pi=9.8$ GeV/c were included. With a total of 62 points,[7] the best solution we found had $\chi^2=98$. We did not include the normalization uncertainty, which would result in a lower χ^2 value. The values of the six adjustable parameters for our best solution are given in Table I, the corresponding fit being shown in Fig. 1.

Since both C_1 and D_1 are essentially consistent with zero, we made a search demanding that $C_1=D_1=0$, and obtained a solution with the same χ^2 value. We feel this four-parameter fit is just an accident, however, since in the expression for the cross section an arbitrary exponential dependence on t has already been introduced through the choice of the value E_0. That is, if we were to choose a different value for E_0, the values of C_1 and D_1 would then be different from zero. Furthermore, there is no *a priori* reason to assume that the scale factor in the A amplitude should be the same as that in the B amplitude. Thus we feel six parameters are still needed.

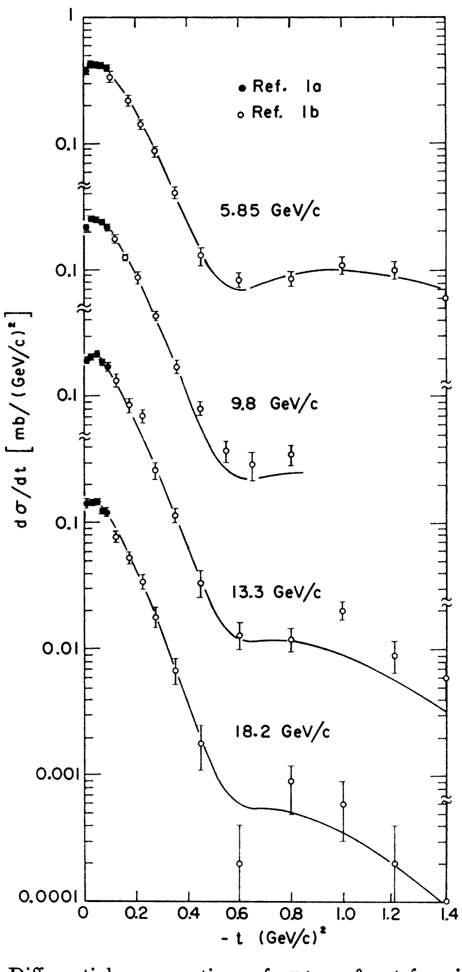

FIG. 1. Differential cross sections of $\pi^-p\to\pi^0n$ at four incident-pion momenta. The smooth curves are our best statistical fits.

TABLE I. Parameters for πN charge-exchange amplitudes.

$\alpha(0)$	0.56 ± 0.01	C_0	2.3 mb GeV
		C_1	0.01 GeV^{-2}
α'	1.08 ± 0.03	D_0	38.9 mb
		D_1	0.01 GeV^{-2}

[5] L. L. Wang, Lawrence Radiation Laboratory, private communication, 1966.

[6] Although at one point Gribov and Pomeranchuk argued that the Regge-pole formalism should never carry below the angular momentum $l=-1$, it can be shown that their argument is not valid. We thank Professor G. F. Chew and Professor Stanley Mendelstam for explaining this point to us.

[7] The Saclay-Orsay 75 preliminary data points used in Ref. 2 have $|t|\leqslant 0.85$ (GeV/c)^2. These data were regrouped and published as 56 points in Ref. 1(a). Notice the data included in this analysis cover a t range larger than the corresponding range included in Ref. 2.

Note that the trajectory parameters given in Table I are essentially compatible with values that we determined from the model-independent method. Fig. 2 shows that $\alpha(t)$ should have a slight curvature, so that if we consider only small-momentum-transfer data points, we will get a higher value than 0.81 for the slope of the trajectory. In fact, the trajectory $\alpha(t) = 0.56 + 1.08t$ gives a satisfactory fit with $\chi^2 = 14$ for 14 points, i.e., $|t| < 0.8$. Incidentally, the trajectory given in Table I predicts $M_\rho \approx 640$ MeV, while the trajectory obtained by the model-independent method leads to $M_\rho \approx 740$ MeV.

To summarize, we find we can explain the dip at $t \approx -0.6(\text{GeV}/c)^2$ in terms of the necessary vanishing of the helicity-flip amplitude when the exchanged angular momentum passes through zero. If such is in fact the origin of this minimum in the angular distribution, one should expect to observe similar minima *at the same value of momentum transfer* in other reactions where the ρ trajectory plays a prominent role.

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Dynamics and Symmetries of Nonleptonic Hyperon Decays; Scalar Meson Couplings*

DAVID LOEBBAKA† AND JOGESH C. PATI
Department of Physics and Astronomy, University of Maryland, College Park, Maryland

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We investigate the dynamics and symmetries of nonleptonic hyperon decays under the following assumptions: (i) the parity-violating (p.v.) and parity-conserving (p.c.) amplitudes are dominated by the K_1 and κ_1 tadpoles, respectively; (ii) the corresponding inverse associated production amplitudes $K_1(\kappa_1)+Y \to \pi+N$ are dominated by the baryon octet pole terms in s and u channels and by κ (p.v. case) and K (p.c. case) pole terms in the t channel; and (iii) the strong vertices satisfy $SU(3)$ symmetry, although the particles are allowed to have their observed masses. [The reason for assuming the dominance of the p.v. amplitudes by the K_1 tadpole, even though the latter is forbidden in the limit of $SU(3)$, has been discussed in a previous work by Pati and Oneda. It is essentially based upon the *observed* rate of the $K_1 \to 2\pi$ decay, which is also forbidden in the limit of $SU(3)$.] The model, thus described, involves six unknowns. Having solved for these in terms of a convenient set of input experimental parameters, the model *predicts* the γ parameter of $\Lambda \to p + \pi^-$ decay, as well as the rate, the asymmetry parameter, and the γ parameter of $\Xi^- \to \Lambda + \pi^-$ decay. All of these four predictions, except for the magnitude of $\gamma(\Xi_-^-)$, are in good agreement with experiments [this includes the sign of $\gamma(\Xi_-^-)$ as well]. Furthermore, we predict that $\Sigma^+ \to n+\pi^+$ decay must proceed via *P wave* (S-wave assumption is shown to be inconsistent with the model). By using the observed values of (1) the rates of Λ_-, Σ_+^+, and Σ_-^- decay, (2) the asymmetry parameter of Σ_+^+ decay, and (3) the signs of $\alpha(\Lambda_-)$, $\alpha(\Xi_-^-)$, and $\gamma(\Xi_-^-)$, we obtain the *unique* prediction that $\Sigma^- \to n+\pi^-$ decay must proceed via pure *S wave*. In obtaining this last prediction we do not utilize the observed values of the Σ_0^+-decay parameters, but instead obtain them as predictions as well. In addition we arrive at a host of other interesting conclusions regarding (a) the d/f ratio in baryon-pseudoscalar meson coupling (this is predicted to be nearly 1.8), (b) the over-all strength and the d'/f' ratio in baryon-scalar meson coupling, (c) the strengths of scalar and pseudoscalar tadpoles, and (d) the degree of validity of Lee's sum rule for p.v. and p.c. decays. Comment is also made on the radiative $\Sigma^+ \to p+\gamma$ decay in an analogous model.

I. INTRODUCTION, MODEL

It has been suggested[1–4] that from the dynamical point of view, the parity-violating (p.v.) and the parity-conserving (p.c.) hyperon decay amplitudes may be well represented by appropriate extrapolation of the amplitudes of associated productions[5] $K+Y \to \pi+N$ and $\kappa+Y \to \pi+N$, respectively. (κ stands for the normal scalar meson with $I=\frac{1}{2}$ and $|Y|=1$, assuming that it exists.) This is based on writing a dispersion integral in the variable $x=(p_Y-p_N-p_\pi)^2$ for the amplitude of $Y \to N+\pi$ decay and assuming that the said integral is well approximated by low-mass pole terms. This leads to the tadpole model of Salam and Ward[1] and Coleman and Glashow,[3] in which the p.v.

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[1] A. Salam and J. C. Ward, Phys. Rev. Letters 5, 390 (1960); A. Salam, Phys. Letters 8, 217 (1964).

[2] M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

[3] S. Coleman and S. L. Glashow, Phys. Rev. 134, B671 (1964).

[4] J. C. Pati and S. Oneda, Phys. Rev. 140, B1351 (1965).

[5] For Ξ decays, Y (i.e., Σ or Λ) and N should be replaced by Ξ and Y, respectively.