intermediate boson. In this case, the effect of (1) on $\nu_e+e^- \to \nu_e+e^-$ depends on the ratio of the mass of W_l^0 to that of the charged intermediate boson.

The original motivation for raising the present question is as follows: Since the neutral lepton currents are not coupled to the schizons of Lee and Yang,[1] is it possible that these currents are coupled to some other intermediate boson? It must be emphasized that we have no cogent reason at all to believe in the existence of the interaction (1); we are rather asking how we can find out experimentally about the existence of this interaction. If (1) is found to be present, then it becomes tempting to speculate in numerous directions. For example, if we think that all weak interactions are mediated by bosons, then we may ask whether there is a strong trilinear coupling involving W_l^0 and the charged intermediate bosons. Such a coupling may be used to account for the large masses of the bosons, and may have other desirable properties. In particular, this can be fitted into the schizon scheme of Lee and Yang[1] if we assign W_l^0 to be a singlet under isotopic spin rotation and require this trilinear coupling to conserve isotopic spin with the schizons treated as two doublets.

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Sum Rules in Nuclear Beta Decay*

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The previously developed "elementary-particle" treatment of nuclear beta decay, $N_i \to N_f+e^{\mp}+\nu_e$, is combined with the Adler-Weisberger procedure, to obtain sum rules relating axial-vector coupling constants for *nuclear* beta decays, $G_A(N_i \to N_f)$, to integrals over pion energy of pion–final-nucleus total cross sections, $\mathscr{s}(\pi^\pm,N_f)$. It is shown that the ratio $[\mathscr{s}(\pi^\pm,N_f)/\mathscr{s}(\pi^\pm,p)]=-[\mathscr{s}(\pi^\pm,N_f)/\mathscr{s}(\pi^\pm,n)]$ is just equal to the difference between the number of protons and the number of neutrons in N_f, $[Z-(A\text{-}Z)]$, multiplied by a factor which becomes unity in the approximation of neglect of meson-exchange corrections to $G_A(N_i \to N_f)$.

I. INTRODUCTION

In the present work the previously developed "elementary-particle" ("E-P") treatment of nuclear beta decay, $N_i \to N_f+e^{\mp}+\nu_e$,[1] is combined with the Adler-Weisberger (A-W) procedure,[2,3] to obtain sum rules relating axial-vector coupling constants for *nuclear* beta decays, $G_A(N_i \to N_f)$, to integrals over pion energy of pion-final nucleus total cross sections, $\mathscr{s}(\pi^\pm,N_f)$. It is shown that the ratio

$$[\mathscr{s}(\pi^\pm,N_f)/\mathscr{s}(\pi^\pm,p)]=-[\mathscr{s}(\pi^\pm,N_f)/\mathscr{s}(\pi^\pm,n)]$$

is just equal to the difference between the number of protons and the number of neutrons in N_f, $[Z-(A-Z)]$, multiplied by a factor which becomes unity in the approximation of neglect of meson-exchange corrections to $G_A(N_i \to N_f)$. Experimental verification of this relationship between "purely strong-interaction type" nuclear properties will provide further confirmation of the basic assumptions underlying the "E-P" treatment and the A-W procedure.

II. CALCULATIONS

We start by writing down the equal-time commutation (ETC) relation[4]

$$Q_A^{(+)}(t)Q_A^{(-)}(t)-Q_A^{(-)}(t)Q_A^{(+)}(t)=2I^{(3)},$$
$$Q_A^{(\pm)}(t)\equiv -i\int \{J_{A;4}^{(1)}(\mathbf{x},t)\pm iJ_{A;4}^{(2)}(\mathbf{x},t)\}d\mathbf{x}, \quad (1)$$

where $J_{A;\mu}^{(i)}$ and $I^{(3)}$ are, respectively, the axial-vector strangeness-preserving weak current and the third component of isospin; the assumption of the validity of this ETC relation and also of the partially conserved-axial-vector-current (PCAC) relation,[5] i.e. of the pion-pole-dominated unsubtracted dispersion relation for the form factor associated with any hadron $\to$ hadron matrix element of $(\partial/\partial x_\mu)\{J_{A;\mu}^{(1)}(x)\pm iJ_{A;\mu}^{(2)}(x)\}$, constitutes the basis of the A-W procedure.[6] Equation (1) immedi-

* Supported in part by the National Science Foundation.
[1] C. W. Kim and H. Primakoff, Phys. Rev. **139**, B1447 (1965); **140**, B566 (1965).
[2] S. L. Adler, Phys. Rev. Letters **14**, 1051 (1965); Phys. Rev. **140**, B736 (1965).
[3] W. Weisberger, Phys. Rev. Letters **14**, 1047 (1965); Phys. Rev. **143**, 1302 (1966).
[4] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).
[5] See J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo Cimento **17**, 757 (1960) and also Ref. 4.
[6] See Refs. 2 and 3 and also B. Renner, Phys. Letters **20**, 72 (1965).

ately gives

$$(2J_f+1)^{-1}\sum_{M_f=-J_f}^{J_f}\sum_k\sum_{M_k=-J_k}^{J_k}\{\langle N_f;\cdots J_f,M_f|Q_A^{(+)}|H_k;\cdots J_k,M_k\rangle\langle H_k;\cdots J_k,M_k|Q_A^{(-)}|N_f;\cdots J_f,M_f\rangle$$

$$-\langle N_f;\cdots J_f,M_f|Q_A^{(-)}|H_k;\cdots J_k,M_k\rangle\langle H_k;\cdots J_k,M_k|Q_A^{(+)}|N_f;\cdots J_f,M_f\rangle\}$$

$$=(2J_f+1)^{-1}\sum_{M_f=-J_f}^{J_f}\langle N_f;\cdots J_f,M_f|2I^{(3)}|N_f;\cdots J_f,M_f\rangle$$

$$=Z-(A-Z), \tag{2}$$

where $|N_f;\cdots J_f,M_f\rangle$ is the final nuclear state in the beta decay, $|H_k;\cdots J_k,M_k\rangle$ is a member of a complete set of hadron states, and where, because of selection rules on $Q_A^{(\pm)}$, only the states $|H_k;\cdots J_kM_k\rangle$ with $B(H_k)=N(N_f)=A$, $Q(H_k)=Q(N_f)\mp1=Z\mp1$, $I(H_k)=I(N_f)\pm1$ or $I(N_f)$, $S(H_k)=S(N_f)=0$ contribute to the $\sum_k\cdots$ (B, Q, I, S are baryon, charge, isospin, and strangeness quantum numbers). Then, for the case of neutron beta decay: $|N_f;\cdots J_fM_f\rangle=|p;\cdots\frac{1}{2},M_p\rangle$; contributing states $|H_k;\cdots J_k,M_k\rangle=|n;\cdots\frac{1}{2},M_n\rangle$, all $|\{\pi^\pm,p\};\cdots J_k,M_k\rangle$ states, $\cdots$; $A=1$, $Z=1$, and the A-W procedure applied to Eq. (2) yields the sum rule[2,3,6]

$$[G_A(n\to p)]^2+[G_A(\pi\to\text{vacuum})]^2\mathcal{G}(\pi^\pm,p)=1; \tag{3a}$$

$$\mathcal{G}(\pi^\pm,p)\equiv\int_{m_\pi}^{\infty}\frac{dE_\pi(E_\pi^2-m_\pi^2)^{1/2}}{E_\pi^2}\left[\frac{\sigma(\pi^-,p;E_\pi)-\sigma(\pi^+,p;E_\pi)}{\pi(1/m_\pi)^2}\right], \tag{3b}$$

where $\sigma(\pi^\pm,p;E_\pi)$ is the $\pi^\pm$-p total cross section for $\pi^\pm$ of laboratory energy E_π. The A-W sum rule in Eqs. (3a), (3b) is in good agreement with experiment since with $G_A(n\to p)=1.18\pm0.02$ (from measured rate of $n\to p+e^-+\bar{\nu}_e$), $G_A(\pi\to\text{vacuum})=0.95\pm0.01$ (from measured rate of $\pi^+\to\mu^++\nu_\mu$), and $\mathcal{G}(\pi^\pm,p)=-0.50\pm0.01$ [from measured values of $\sigma(\pi^\pm,p;E_\pi)$], the left side of Eq. (3a) is 0.94 ± 0.04.

We next discuss the A-W-type sum rule deduced from Eq. (2) for the case of nuclear beta decay. Here the contributing states $|H_k;\cdots J_k,M_k\rangle=$ all nuclear ground and excited states $|N_i;\cdots J_i,M_i\rangle$ with $B(N_i)=B(N_f)=A$, $Q(N_i)=Q(N_f)\mp1=Z\mp1$, $I(N_i)=I(N_f)\pm1$ or $I(N_f)$, $S(N_i)=S(N_f)=0$, all $|\{\pi^\pm,N_f\};\cdots J_k,M_k\rangle$ states, $\cdots$; $A\geqq2$, $Z\geqq1$, and the A-W procedure gives

$$\sum_i[Q(N_f)-Q(N_i)]\tfrac{1}{3}\eta(J_i,J_f)[G_A(N_i\to N_f)]^2+[G_A(\pi\to\text{vacuum})]^2\mathcal{G}(\pi^\pm,N_f)=Z-(A-Z); \tag{4a}$$

$$\mathcal{G}(\pi^\pm,N_f)\equiv\int_{m_\pi}^{\infty}\frac{dE_\pi(E_\pi^2-m_\pi^2)^{1/2}}{E_\pi^2}\left[\frac{\sigma(\pi^-,N_f;E_\pi)-\sigma(\pi^+,N_f;E_\pi)}{\pi(1/m_\pi)^2}\right], \tag{4b}$$

where $\sigma(\pi^\pm,N_f;E_\pi)$ is the $\pi^\pm-N_f$ total cross section for $\pi^\pm$ of laboratory energy E_π and $\eta(J_i,J_f)$ is a numerical coefficient which depends on J_i and J_f and on the parities of $|N_i;\cdots J_i,M_i\rangle$ and $|N_f;\cdots J_i,M_i\rangle$. We list the values of the $\eta(J_i,J_f)$ appropriate to allowed beta decays and for which, as a consequence, the corresponding $\eta(J_i,J_f)$ $G_A(N_i\to N_f)$ make a dominant contribution to Eq. (4a), viz.,

$$\eta(J,J)=(J+1)/J\,,(J>0)\,,$$

$$\eta(J,J-1)=\frac{2J+1}{2(J-1)+1}\eta(J-1,J)\,, \tag{5}$$

$$\eta(0,1)=\tfrac{1}{2}\,,\quad\eta(\tfrac{1}{2},\tfrac{3}{2})=1\,,\;\cdots$$

$$\vdots$$

We further note that in the evaluation of $G_A(N_i\to N_f)$ in beta-decay-allowed approximation with the states $|N_f;\cdots J_f,M_f\rangle$ and $|N_i;\cdots J_i,M_i\rangle$ described by the nuclear wave functions $\Psi(N_f;\cdots J_f,M_f|\cdots\mathbf{r}_a,\sigma_a^{(3)},\tau_a^{(3)}\cdots)$

$\equiv\Psi(f)$ and $\Psi(N_i;\cdots J_i,M_i|\cdots\mathbf{r}_a,\sigma_a{}^{(3)},\tau_a{}^{(3)}\cdots)\equiv\Psi(i)$, we have[1]

$$[G_A(N_i\to N_f)]^2=[G_A(n\to p)]^2\left(\frac{1}{2J_i+1}\right)\sum_{M_i=-J_i}^{J_i}\sum_{M_f=-J_f}^{J_f}|\langle\Psi(f)|(\boldsymbol{\sigma}^{(\pm)}+\boldsymbol{\sigma}^{(\pm)}\xi)|\Psi(i)\rangle|^2\Big/\left(\frac{2J_f+1}{2J_i+1}\right)\eta(J_i,J_f)$$

$$=[G_A(n\to p)]^2\sum_{M_i=-J_i}^{J_i}|\langle\Psi(f)|(\boldsymbol{\sigma}^{(\pm)}+\boldsymbol{\sigma}^{(\pm)}\xi)|\Psi(i)\rangle|^2/\eta(J_i,J_f);\tag{6}$$

$$\boldsymbol{\sigma}^{(\pm)}\equiv\sum_{a=1}^{A}\tau_a{}^{(\pm)}\boldsymbol{\sigma}_a,\quad \boldsymbol{\sigma}^{(\pm)}\xi\equiv\sum_{a=1}^{A}\tau_a{}^{(\pm)}\boldsymbol{\sigma}_a\sum_{b=1}^{A}\xi(\boldsymbol{\sigma}_a,\boldsymbol{\sigma}_b,\mathbf{r}_a-\mathbf{r}_b),$$

where the $\xi(\boldsymbol{\sigma}_a,\boldsymbol{\sigma}_b,\mathbf{r}_a-\mathbf{r}_b)$ describe the correction to $G_A(N_i\to N_f)$ associated with meson $(\pi,\cdots)$ exchange. As a rough approximation we assume[7]

$$\langle\Psi(f)|\boldsymbol{\sigma}^{(\pm)}\xi|\Psi(i)\rangle\approx\bar{\xi}\langle\Psi(f)|\boldsymbol{\sigma}^{(\pm)}|\Psi(i)\rangle\tag{7}$$

with $\bar{\xi}$ (1) more or less independent of J_i and J_f, (2) varying only slowly with A and Z, and (3) of order 0.05. Then, using Eqs. (6), (7) and closure over the complete set of nuclear wave functions $\Psi(i)$, we get

$$\sum_i[Q(N_f)-Q(N_i)]\tfrac{1}{3}\eta(J_i,J_f)[G_A(N_i\to N_f)]^2$$

$$=\tfrac{1}{3}[G_A(n\to p)]^2(1+\bar{\xi})^2\sum_i\sum_{M_i=-J_i}^{J_i}\{|\langle\Psi(f)|\boldsymbol{\sigma}^{(+)}|\Psi(i)\rangle|^2-|\langle\Psi(f)|\boldsymbol{\sigma}^{(-)}|\Psi(i)\rangle|^2\}$$

$$=\tfrac{1}{3}[G_A(n\to p)]^2(1+\bar{\xi})^2\langle\Psi(f)|(\boldsymbol{\sigma}^{(+)}\cdot\boldsymbol{\sigma}^{(-)}-\boldsymbol{\sigma}^{(-)}\cdot\boldsymbol{\sigma}^{(+)})|\Psi(f)\rangle$$

$$=\tfrac{1}{3}[G_A(n\to p)]^2(1+\bar{\xi})^2\langle\Psi(f)|3\sum_{a=1}^{A}\tau_a{}^{(3)}|\Psi(f)\rangle$$

$$=[G_A(n\to p)]^2(1+\bar{\xi})^2\{Z-(A-Z)\}\tag{8}$$

so that, combining Eq. (8) and Eq. (4a), we obtain the sum rule

$$[G_A(n\to p)]^2(1+\bar{\xi})^2\{Z-(A-Z)\}+[G_A(\pi\to\text{vacuum})]^2\mathcal{G}(\pi^\pm,N_f)=Z-(A-Z).\tag{9}$$

Equations (9) and (3a) yield

$$\mathcal{G}(\pi^\pm,N_f)=\{Z-(A-Z)\}\mathcal{G}(\pi^\pm,p)\left\{\frac{1-[G_A(n\to p)]^2(1+\bar{\xi})^2}{1-[G_A(n\to p)]^2}\right\}\tag{10}$$

or

$$\mathcal{G}(\pi^\pm,N_f)=\{Z\mathcal{G}(\pi^\pm,p)+(A-Z)\mathcal{G}(\pi^\pm,n)\}\left\{\frac{1-[G_A(n\to p)]^2(1+\bar{\xi})^2}{1-[G_A(n\to p)]^2}\right\},\tag{11}$$

whence, with neglect of the meson-exchange beta-decay correction factors $\bar{\xi}$ relative to 1, i.e. in the impulse approximation for the calculation of $G_A(N_i\to N_f)$ in Eq. (6), we have

$$\mathcal{G}(\pi^\pm,N_f)\cong\{Z-(A-Z)\}\mathcal{G}(\pi^\pm,p)\tag{12}$$

or

$$\mathcal{G}(\pi^\pm,N_f)\cong Z\mathcal{G}(\pi^\pm,p)+(A-Z)\mathcal{G}(\pi^\pm,n).\tag{13}$$

III. DISCUSSION

The results in Eqs. (10)–(13) demonstrate that, as a consequence of aspects of hadron dynamics expressed by the PCAC hypothesis and the ETC relation, $\mathcal{G}(\pi^\pm,N_f)$, $\mathcal{G}(\pi^\pm,p)$, and $\mathcal{G}(\pi^\pm,n)$ behave as additive-type quantum numbers in the approximation $\bar{\xi}\ll1$; thus, from the point of view of the calculation of $\mathcal{G}(\pi^\pm,N_f)$ in this approximation, the nucleus N_f can be considered as a collection of noninteracting nucleons each on its own mass shell. We emphasize that the impulse approximation for the calculation of $G_A(N_i\to N_f)$, i.e. $\bar{\xi}\ll1$, is *not* in general

[7] J. S. Bell and R. J. Blin-Stoyle, Nucl. Phys. **6**, 87 (1958); R. J. Blin-Stoyle, V. Gupta, and H. Primakoff, *ibid.* **11**, 444 (1959); R. J. Blin-Stoyle and S. Papageorgiou, *ibid.* **64**, 1 (1965); Phys. Letters **14**, 343 (1965).

equivalent to the impulse approximation for the calculation of $\pi^{\pm}-N_f$ forward elastic scattering, i.e., $\alpha(\pi^{\pm},N_f;E_\pi)\ll 1$ [see Eq. (15) below]. In fact

$$g(\pi^{\pm},N_f)=Zg(\pi^{\pm},p)+(A-Z)g(\pi^{\pm},n)+\int_{m_\pi}^{\infty}dE_\pi\frac{4m_\pi^2}{E_\pi^2}[\mathrm{Im}\alpha(\pi^-,N_f;E_\pi)-\mathrm{Im}\alpha(\pi^+,N_f;E_\pi)], \tag{14}$$

where

$$\begin{aligned}\alpha(\pi^{\pm},N_f;E_\pi)\equiv\ &\mathcal{A}(\pi^{\pm}+N_f\to\pi^{\pm}+N_f;E_\pi,\theta=0)\\ &-[Z\mathcal{A}(\pi^{\pm}+p\to\pi^{\pm}+p;E_\pi,\theta=0)+(A-Z)\mathcal{A}(\pi^{\pm}+n\to\pi^{\pm}+n;E_\pi,\theta=0)],\\ &\alpha(\pi^+,N_f;E_\pi)=\alpha(\pi^-,N_f;E_\pi)\quad\text{for}\quad Z=(A-Z)\end{aligned} \tag{15}$$

and where the optical theorem has been used; comparison of Eq. (14) with Eq. (11) yields

$$\left\{\frac{[G_A(n\to p)]^2\xi(2+\xi)}{[G_A(n\to p)]^2-1}\right\}=\left\{\int_{m_\pi}^{\infty}dE_\pi\frac{4m_\pi^2}{E_\pi^2}[\mathrm{Im}\alpha(\pi^-,N_f;E_\pi)-\mathrm{Im}\alpha(\pi^+,N_f;E_\pi)]\right\}\Big/[Zg(\pi^{\pm},p)+(A-Z)g(\pi^{\pm},n)]. \tag{16}$$

Equation (16) shows that for $Z\neq(A-Z)$ the condition $\xi\cong 0$ implies the condition

$$\int_{m_\pi}^{\infty}dE_\pi\frac{4m_\pi^2}{E_\pi^2}[\mathrm{Im}\alpha(\pi^-,N_f;E_\pi)]\cong\int_{m_\pi}^{\infty}dE_\pi\frac{4m_\pi^2}{E_\pi^2}[\mathrm{Im}\alpha(\pi^+,N_f;E_\pi)] \tag{17}$$

and *not*, in general, the much stronger condition

$$\alpha(\pi^-,N_f;E_\pi)\cong 0,\qquad \alpha(\pi^+,N_f;E_\pi)\cong 0. \tag{18}$$

As a numerical illustration we mention the case of $N_f=C^{12}[Z=(A-Z)]$ where[8]

$$\begin{aligned}\left|\frac{[4\pi/(E_\pi^2-m_\pi^2)^{1/2}]\,\mathrm{Im}\alpha(\pi^+,N_f;E_\pi)}{[Z\sigma(\pi^+,p;E_\pi)+(A-Z)\sigma(\pi^+,n;E_\pi)]}\right|&=\left|\frac{[4\pi/(E_\pi^2-m_\pi^2)^{1/2}]\,\mathrm{Im}\alpha(\pi^-,N_f;E_\pi)}{[Z\sigma(\pi^-,p;E_\pi)+(A-Z)\sigma(\pi^-,n;E_\pi)]}\right|\\ &=\left|\frac{\sigma(\pi^-,N_f;E_\pi)}{[Z\sigma(\pi^-,p;E_\pi)+(A-Z)\sigma(\pi^-,n;E_\pi)]}-1\right|\\ &=0.47\quad\text{for}\quad E_\pi-m_\pi=150\ \text{MeV},\\ &=0.60\quad\text{for}\quad E_\pi-m_\pi=185\ \text{MeV},\\ &=0.16\quad\text{for}\quad E_\pi-m_\pi=300\ \text{MeV},\\ &=0.05\quad\text{for}\quad E_\pi-m_\pi=485\ \text{MeV},\end{aligned} \tag{19}$$

but where ξ may well be quite small.

We feel that experimental tests of Eqs. (10)–(13) would be of considerable interest not only in connection with the confirmation of the general validity of the "E-P" treatment and the A-W procedure but also as a possible source of information about the meson-exchange beta-decay correction factors ξ.[9] Finally, we wish to point out that an "E-P", A-W treatment of hypernuclear beta decay yields relations connecting $g(K^{\pm},N_f)$ with $g(K^{\pm},p)$ and $g(K^{\pm},n)$ analogous to those given in Eqs. (11) and (13), and that, here again, experimental tests would be most welcome.[10]

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[8] A. E. Ignatenko, A. I. Muklin, E. B. Ozerov, and B. M. Pontecorvo, Dokl. Akad. Nauk SSSR **103**, 395 (1957); B. Amblard *et al.*, Phys. Letters **10**, 138 (1964).

[9] It is to be noted that the quantity $\{1-[G_A(n\to p)]^2(1+\bar{\xi})^2\}\{1-[G_A(n\to p)]^2\}^{-1}$ varies rather sensitively with $\bar{\xi}$ having, e.g., the values 1.18, 1.36, and 1.74 for $\bar{\xi}=0.025$, 0.05, and 0.1, respectively. We may also mention that the relation between $g(\pi^{\pm},N_f)$ and $g(\pi^{\pm},p)$ in Eqs. (10)–(13) is invariant to the Goldberger-Treiman replacement in Eqs. (3a) and (9) of $G_A(\pi\to\text{vacuum})=0.95\pm0.01$ by $G_A(n\to p)(2m_p/m_\pi)(\sqrt{2}g_{\pi pp})^{-1}=0.83\pm0.03$, while the value of $G_A(n\to p)$ calculated from Eqs. (3a) and (3b) and measured values of $\sigma(\pi^{\pm},p;E_\pi)$ depends somewhat on whether this replacement is made.

[10] C. W. Kim (to be published). An additional complication in this case arises from the fact that $g(K^-,N_f)$, $g(K^-,p)$, and $g(K^-,n)$ receive contributions from the unphysical region below the K^- elastic-scattering threshold.