The main function of the ether in the present theory is to prove a scale of length and time, through the dependence of particle masses on Λ. If the Weyl theory is true in some sense, then we can even hope to understand changes in the units of length and time as the universe evolves. Mach's principle, as originally stated, is unnecessary in any theory involving an ether. However, this is not the same as saying that distant matter is completely irrelevant. The development of the universe is a most delicate affair, because various self-consistency conditions have to be maintained at every instant. It seems reasonable that one of these should be $\mathcal{G}Nm^2/R \approx m$, where R is the radius of the universe and N the number of particles in it. This relation is true at the present time and is often assumed to have some connection with Mach's principle. We are now trying to find suitable cosmological models which are solutions of the Weyl equation (15); this work may clarify the meaning of the self-consistency conditions.

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Neutron Form Factors from a Study of Inelastic Electron Spectra in the Electrodisintegration of Deuterium*

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Measurements of the ratio of the total inelastic electron-deuteron cross section to the elastic electron-proton cross section have been made to an accuracy of about 3% for values of the square of the four-momentum transfer q^2 in the range 1.5 to 7.5 F^{-2}. These ratios have been analyzed in terms of the form factors of the neutron using the "area method," and it is concluded that the corrections necessary to the simple sum rule given by Jankus are approximately equal in magnitude to those encountered in the use of the more familiar "peak method." A detailed comparison has been made between the shapes of the observed inelastic electron-deuteron cross sections as a function of the scattered electron momentum and the shapes expected according to a theoretical treatment due to Durand.

I. INTRODUCTION

It was first suggested by Hofstadter[1] that experiments on the inelastic scattering of high-energy electrons from the deuteron might provide information on the electromagnetic structure of the neutron. Subsequent experiments by Yearian and Hofstadter[2] confirmed this idea and showed the radius of the magnetic moment distribution in the neutron to be approximately equal to the corresponding radius in the proton. These results were obtained by what is now known as the area method in which the electron-neutron cross section is obtained from the total inelastic electron-deuteron cross section. The area method was quickly superseded, for sound theoretical and experimental reasons, by the so-called peak method in which information about the neutron is obtained from the electron-deuteron cross section at the maximum of the broad inelastic peak (the quasi-elastic region) with the help of a theoretical treatment to allow for the scattering from the proton and the internal motion of the nucleons in the deuteron. The peak method has since been used to measure the variation of both the charge and magnetic form factors of the neutron with the square of the four-momentum transfer, q^2, for values of q^2 up to about 35 F^{-2}.

The most precise experimental information on quasi-elastic electron-deuteron scattering has come from the recent experiments of Hughes *et al.*[3] at Stanford. These data were analyzed by the peak method making use of the most recent theoretical treatment of the inelastic scattering process. The neutron form factors were given for a series of values of q^2 in the range 1.0 to 30.0 F^{-2}. The results suggested that for values of q^2 greater than about 6.0 F^{-2}. the square of the neutron's charge form factor $(G_{En})^2$ was consistent with zero to within an error of the order of 5% in the theoretical cross section. On the other hand for values of q^2 less than 6.0 F^{-2} in

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order to avoid the unattractive result of a negative value of $(G_{En})^2$ a correction to the theory was required which was larger than 5% at some scattering angles. It was concluded that statements regarding the size of the neutron form factors in the low q^2 region could not be made from measurements on quasielastic electron-deuteron scattering until the necessary corrections to the theory were better understood.

In view of this difficulty experienced with the peak method at low values of q^2 it was thought worthwhile to investigate the neutron form factors in this q^2 range by means of the area method. Sum rules for inelastic electron-deuteron scattering, in which the total inelastic corss section is related to the free-proton and electron-neutron cross sections, have been given by Jenkus[4] and Blankenbecler.[5] The sum rule due to Blankenbecler is

$$(d\sigma/d\Omega)_D=(1+\Delta)[(d\sigma/d\Omega)_p+(d\sigma/d\Omega)_n]. \quad (1)$$

The parameter Δ is of the order 0.02 but for the purposes of this paper this correction term to the simple sum rule has been ignored.

One advantage of the sum rule is that it is insensitive to the effects of final-state interactions between the outgoing neutron and proton. On the other hand it is directly sensitive to the effects of meson-exchange currents in the deuteron which could cause Eq. (1) to be seriously in error. Reliable estimates of the meson-exchange contribution to the inelastic cross section are difficult to make, particulary for final electron momenta well below the quasielastic peak, and for this reason the peak method has generally been preferred to the area method as a source of information on the neutron. The peak method, however, is subject to corrections for the final-state interaction and the recent calculations of Nuttall and Whippman[6] have shown that the necessary correction rapidly becomes very large in the low q^2 range. There have been no corresponding calculations of the correction necessary to Eq. (1) for meson-exchange effects and it is the intention of the present experiment to throw light on the size of this and other corrections in the low-q^2 range by investigating the neutron form factors deduced from precise experimental data with the help of Eq. (1).

In the present experiment we have made measurements of the ratio of the total inelastic electron-deuteron cross section to the elastic electron-proton cross section for values of q^2 equal to 1.5 F^{-2}, 2.5 F^{-2}, 4.6 F^{-2}, and 7.5 F^{-2}, where q^2 is defined for electrons scattered at the quasielastic peak. At each value of q^2 this ratio has been measured to an accuracy of about 3% for at least three electron scattering angles in the range 45° to 135°. A single exception to this statement is at $q^2=7.5$ F^{-2} where the measurements are limited to a scattering angle of 120°. Absolute electron-deuteron cross sections are obtained by normalization to the absolute electron-proton cross sections given by Janssens *et al.*[7]

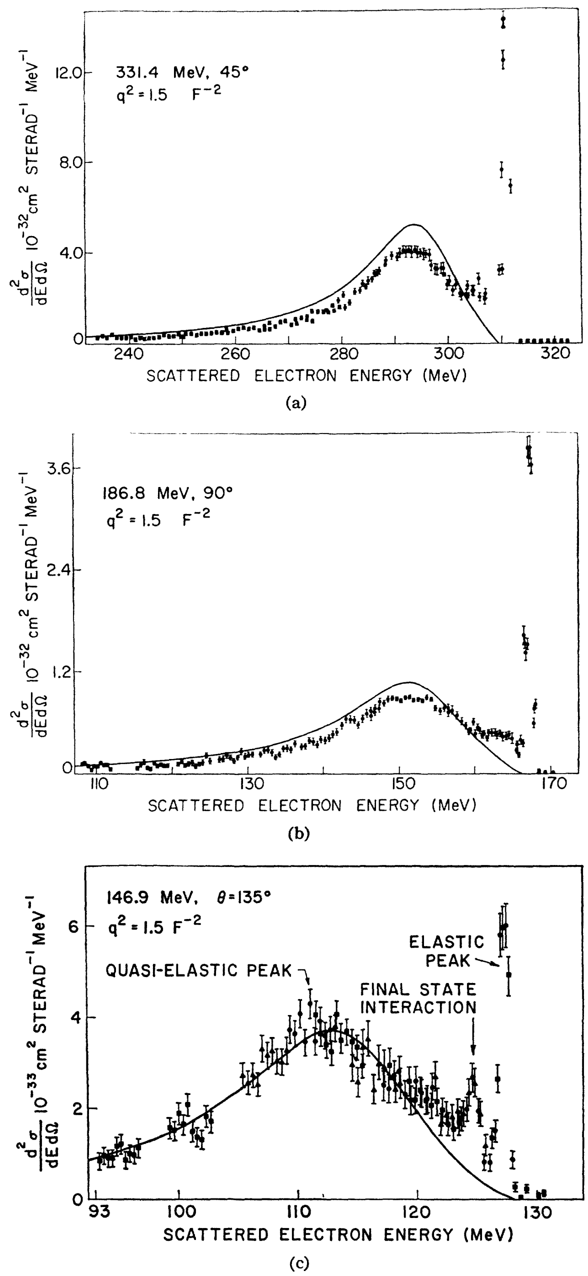

FIG. 1. Examples of the measured momentum distribution of electrons scattered elastically and inelastically from deuterium for scattering angles in the range 45° to 135° and for values of $q^2=1.5$ F^{-2}. The full curves represent the expected variation of the inelastic-cross-section calculated according to the formula given by Durand. No corrections are included in the theory for the effects of the *D*-state component of the deuteron wave function, the final-state interaction, or meson-exchange currents. The experimental resolution function and the radiative corrections are folded into the theoretical curves.

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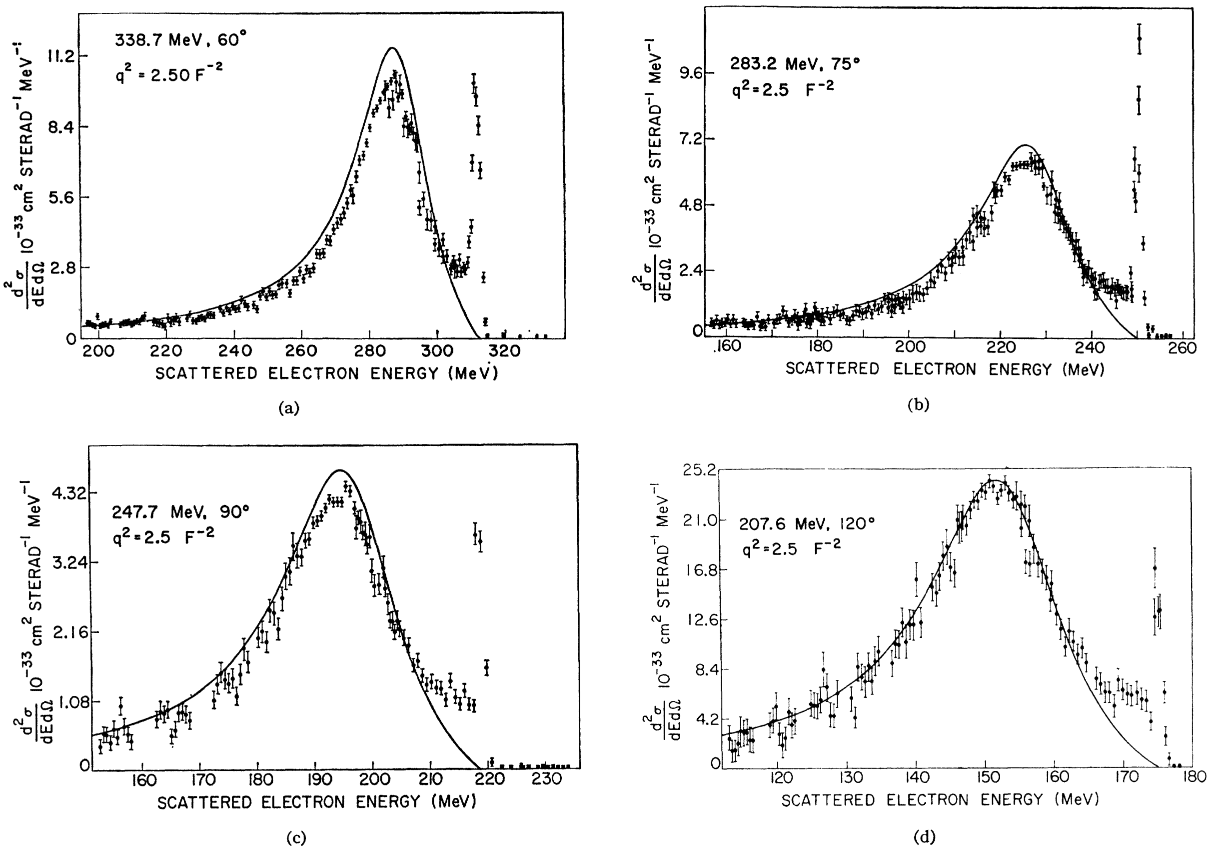

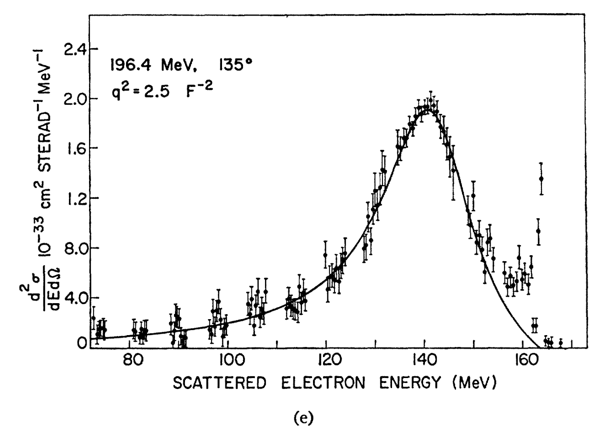

FIG. 2. Examples of the measured momentum distribution of electrons scattered elastically and inelastically from deuterium for scattering angles in the range 45° to 135° and for values of $q^2=2.5$ F^{-2}. The full curves represent the expected variation of the inelastic–cross-section calculated according to the formula given by Durand. No corrections are included in the theory for the effects of the *D*-state component of the deuteron wave function, the final-state interaction, or meson-exchange currents. The experimental resolution functions and the radiative corrections are folded into the theoretical curves.

In the following sections we present the experimental data and discuss its analysis in terms of neutron form factors using the area method.

II. RESULTS AND ANALYSIS

The electron beam for this experiment was supplied by the Stanford Mark III linear accelerator and the experimental technique has been fully described by Hughes *et al.*[3] The momentum distributions of the scattered electrons were obtained by using the 10-channel ladder counter placed in the focal plane of the 72 in. spectrometer. This allowed the simultaneous detection of events in ten closely spaced momentum "bins" with a total acceptance of $\Delta p/p \cong 4\%$.

Figures 1, 2, 3, and 4 show the momentum distributions of the scattered electrons from the deuterium target for a representative series of scattering angles in the low-q^2 range at which measurements were made. The momentum range generally includes both the elastic electron-deuteron peak and the broad quasi-elastic peak and extends to a momentum approximately 35% below the position of the quasielastic peak. At the lowest value of q^2 a second well-defined peak due to the final-state interaction appears at a position about 2

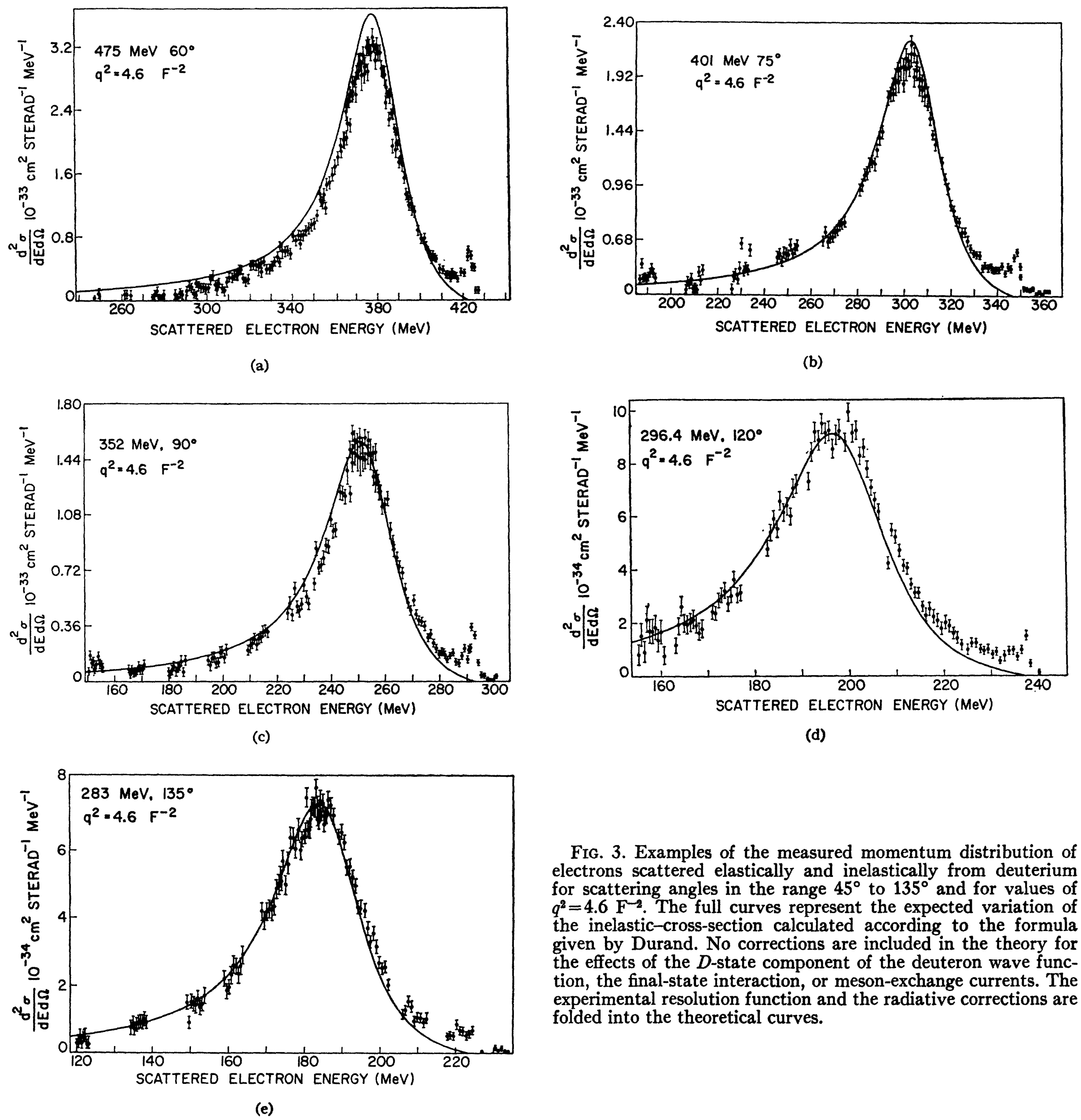

FIG. 3. Examples of the measured momentum distribution of electrons scattered elastically and inelastically from deuterium for scattering angles in the range 45° to 135° and for values of $q^2=4.6$ F^{-2}. The full curves represent the expected variation of the inelastic-cross-section calculated according to the formula given by Durand. No corrections are included in the theory for the effects of the *D*-state component of the deuteron wave function, the final-state interaction, or meson-exchange currents. The experimental resolution function and the radiative corrections are folded into the theoretical curves.

MeV below the elastic peak. The existence of the latter peak, which is attributed to the interaction between the outgoing neutron and proton in the $^1S_0$ state, is well known and has been the subject of discussions by Kendall *et al.*,[8] Barber,[9] and Yearian and Hughes.[10] The theoretical treatment of the final-state-interaction effects is given by Durand.[11]

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The full curves shown in Figs. 1, 2, 3, and 4 represent the expected variation of the inelastic electron-deuteron cross section with scattered electron momentum calculated according to a formula given by Durand[11] which assumes a Hulthén model for the $^3S_1$ component of the deuteron wave function. [The detailed equations used in this calculation are Eqs. (10) through (19) of Ref. 11.] We have inserted in this calculation the neutron and proton form factors given by a dispersion-type model fitted to recent form-factor data.[3] The theoretical cross sections assume the deuteron to be described by a pure $^3S_1$ wave function and do not include corrections for the effects of the D-state component of the deuteron wave function, the final-state interaction and meson-exchange currents. The effects of the finite experimental resolution and of electron radiation both during the interaction and during passage through the target have been folded into the theoretical curves so that they may be directly comparted with the observed cross sections.

The purpose of the comparison shown in Figs. 1, 2, 3, and 4 is to illustrate, using reasonable values for the neutron and proton form factors, the magnitude of the correction that would be needed to the simple Durand theory in order to predict the observed cross section and to show how this correction varies with both q^2 and the scattering angle. For instance, a negative correction is always required at the quasielastic peak. The magnitude of this correction shows little dependence on scattering angle but increases considerably as q^2 decreases. Both these features of the peak correction have been predicted by Durand and by Nuttall and Whippman after allowing for the effects of the D-state component of the deuteron wave function and the final-state interaction. There is no evidence for any marked discrepancy between the simple theory and experiment on the extreme low energy side of the quasielastic peak; in fact, the agreement is surprisingly good since it is in this region that the inelastic cross section should be more sensitive to the structure of deuteron and to meson-exchange effects. On the other hand in the region of the elastic peak the simple theory consistently underestimates the observed cross section by an amount which rapidly increases as q^2 decreases. As mentioned above, this effect is usually attributed to the final-state interaction.

A complete list of the experimental cross sections shown in Figs. 1–4, the associated errors and the predicted theoretical cross sections (both before and after the effects of experimental resolution and radiation have been included) will be tabulated in a Stanford University High Energy Physics Laboratory internal report.

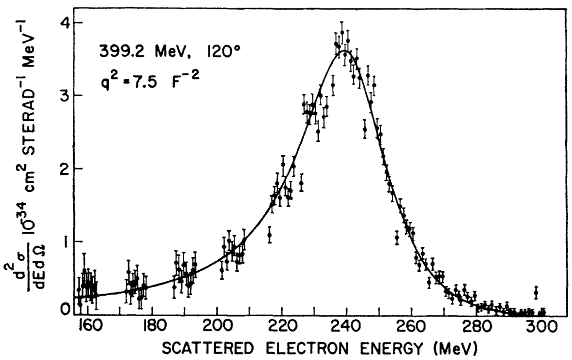


FIG. 4. The measured momentum distribution of electrons scattered elastically and inelastically from deuterium at a scattering angle of 120° and for $q^2=7.5$ F^{-2}. The full curve represents the expected variation of the inelastic-cross-section calculated according to the formula given by Durand. No corrections are included in the theory for the effects of the D-state component of the deuteron-wave function, the final-state interaction, or meson-exchange currents. The experimental resolution function and the radiative corrections are folded into the theoretical curve.

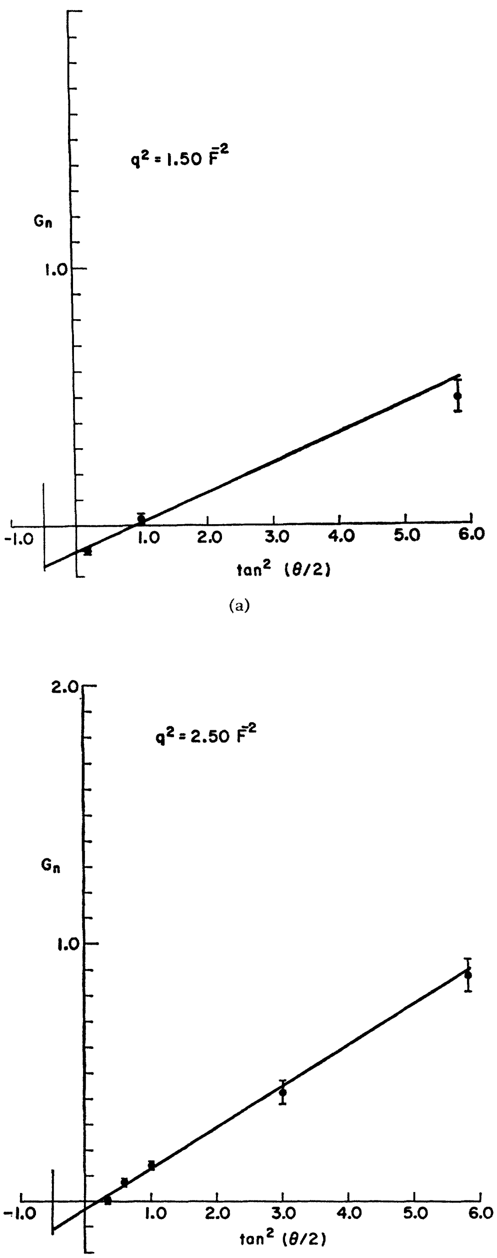
In order to obtain the total inelastic cross section the experimental distributions shown in Figs. 1–4 were integrated numerically using an IBM 7090 computer. The integration was terminated at a momentum ranging from 25% to 35% below the position of the quasielastic peak and did not include the elastic peak. The radiative corrections were obtained to a good approximation from a comparison of the total theoretical cross sections (the smooth curves in Figs. 1, 2, 3, and 4), including and excluding the effects of radiation, integrated to the same momentum cutoff. The theoretical cross section was also used to estimate the correction necessary for that part of the experimental cross section which would fall below the momentum cutoff even in the absence of radiative effects.

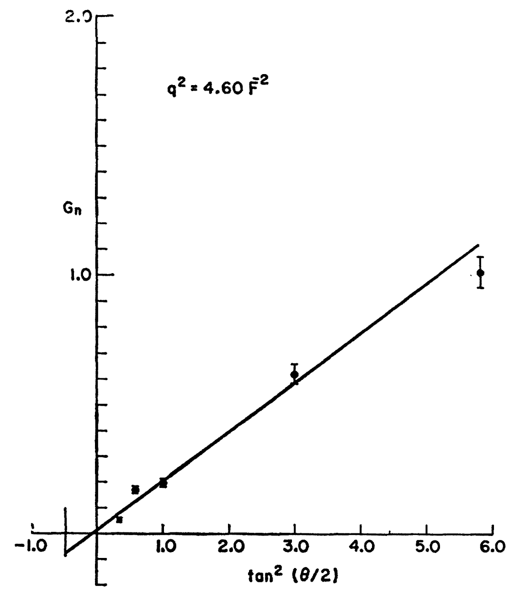
Table I gives the observed ratios of the inelastic electron-deuteron cross section to the elastic electron-proton cross section, together with the corresponding ratios of the elastic electron-neutron and electron-proton cross sections which follow from the inelastic cross sections through Eq. (1). The errors shown in the ratios are due only to counting statistics, which are believed to be the major source of experimental error since most errors of a systematic nature cancel in forming the ratio. Figure 5 shows the corresponding Rosenbluth plots for the neutron for values of q^2 equal to 1.5, 2.5, and 4.6 F^{-2}. In each case the experimental points can be fitted by a straight line from whose slope and intercept the neutron form factors can be determined. The Rosenbluth formulation seems to be well satisfied.

The values we find for the charge and magnetic form factors of the neutron are given in Table II and shown as a function of q^2 in Fig. 6. The error bars include the statistical errors in the ratios and a 4% un-

TABLE I. The experimental ratios σ_D/σ_p as a function of q^2 and scattering angle. Also shown are the ratios σ_n/σ_p which follow from experimental ratios through Eq. (1). The errors shown in the ratios are due only to counting statistics.

q^2 (F^{-2})	E (MeV)	Angle (deg)	σ_D/σ_p	σ_n/σ_p
1.5	331.5	45.0	0.878±0.021	−0.121±0.014
	186.8	90.0	1.032±0.024	0.032±0.014
	146.9	135.0	1.266±0.039	0.266±0.034
2.5	338.7	60.0	1.004±0.025	0.004±0.015
	283.2	75.0	1.091±0.026	0.091±0.016
	247.7	90.0	1.153±0.027	0.153±0.018
	207.6	120.0	1.301±0.038	0.301±0.034
	196.4	135.0	1.419±0.036	0.419±0.034
4.6	473.4	60.0	1.085±0.024	0.085±0.013
	401.0	75.0	1.244±0.026	0.244±0.018
	352.0	90.0	1.243±0.029	0.243±0.021
	296.4	120.0	1.441±0.031	0.441±0.031
	281.4	135.0	1.450±0.032	0.451±0.032
7.5	399.2	120.0	1.387±0.031	0.387±0.036





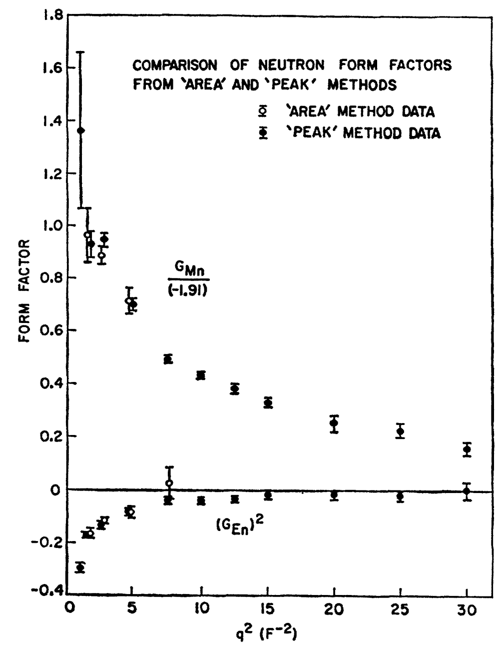


FIG. 5. The Rosenbluth plots for the neutron showing $G_n=\sigma_n/\sigma_{\text{Mott}}$ versus $\tan^2(\theta/2)$ for values of q^2 equal to 1.5, 2.5, and 4.6 F^{-2}. The error bars shown include counting statistics only.

FIG. 6. The square of the neutron's charge form factor $(G_{En})^2$ and magnetic form factor G_{Mn} as a function of q^2 according to the area method. For comparison this figure also shows the neutron form factors found by Hughes *et al.* from the peak method using the simplest form of the Durand theory; namely a pure ${}^3S_1$ deuteron with no account taken of the final-state interaction. At $q^2=7.5$ F^{-2} where the data of the present experiment is restricted to 120° we have shown the value of $(G_{En})^2$ which corresponds to the value of G_{Mn} given by Hughes *et al.* The error bars include the statistical errors and a 4% uncertainty in the absolute electron-proton cross sections.

certainty in the absolute electron-proton cross sections. For comparison, Fig. 6 also shows the neutron form factors found by Hughes *et al.* from the peak method using the simplest form of the Durand theory for the quasielastic cross section; namely a pure ${}^3S_1$ deuteron with no account taken of the D-state component of the deuteron wave function and the final-state interaction. This comparison suggests that in order to obtain positive values of $(G_{En})^2$ the total corrections needed to Eq. (1) and to the simplest form of the Durand theory in the region of the quasielastic peak are approximately equal. It is known, however, that the corrections to the Durand theory are still imperfectly understood in the low q^2 range. On the other hand there has been no attempt to calculate the corrections necessary to the simple sum rule for such values of q^2. Thus from a theoretical point of view the potential of the area method as a means of measuring the neutron form factors in the low q^2 range remains largely unexplored.

IV. CONCLUSIONS

Accurate measurements have been made of the ratio of the total inelastic electron-deuteron cross section to the elastic electron-proton cross section for values of q^2 in the range 1.5 to 7.5 F^{-2}. These ratios have been analyzed in terms of the form factors of the neutron using the area method, and it is concluded that the corrections necessary to the simple sum rule are approximately equal to those encountered in the use of the peak method.

A detailed comparison has been made between the shapes of the observed inelastic electron-deuteron cross sections as a function of the scattered electron momentum and the shapes expected according to a simple form of the Durand theory. Discrepancies between the theory and experiment are observed for momenta close to the elastic peak and near the broad peak of the inelastic spectum, but for momenta less than the quasielastic peak the agreement between theory and experiment is surprisingly good. Up to the present time no reliable estimates have been made of the corrections necessary to the theory for the effects of the final state interaction and meson-exchange currents, but it is expected that there is much to be learned about such effects from attempts to reproduce the cross sections observed in the present experiment.

TABLE II. The neutron form factors $(G_{En})^2$ and G_{Mn} as a function of q^2. The value of G_{Mn} at $q^2=7.5$ F^{-2} is taken from Ref. 7. The quantities G_{En} and G_{Mn} are also frequently called $F_{ch}{}^n$ and $\mu_n F_{mag}{}^n$ respectively, where μ_n is the magnetic moment of the neutron. The errors shown include the statistical errors in σ_n/σ_p and a 4% uncertainty in the absolute electron-proton cross sections.

q^2 (F^{-2})	$(G_{En})^2$	$-(G_{Mn}/\mu_n)$
1.5	-0.163 ± 0.021	0.968 ± 0.102
2.5	-0.114 ± 0.010	0.888 ± 0.032
4.6	-0.084 ± 0.019	0.714 ± 0.051
7.5	$+0.024\pm0.057$	(0.496 ± 0.013)

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