Is the Graviton a Goldstone Boson?*

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A theory is outlined in which Lorentz invariance is spontaneously violated, giving rise to a constant vector field λ^μ. The two quantities which are most simply related to λ^μ are the gravitational constant $\mathcal{G}$ and the decay rate of $K_2^0 \to 2\pi$, assuming that the latter will eventually show an energy dependence. Using a method suggested by Bjorken for electrodynamics, we conclude that the model may support a graviton, and determine the order of magnitude of $\mathcal{G}$. A simple argument relates the K_2^0 decay to $\mathcal{G}$; the agreement with experiment is good.

1. INTRODUCTION

In a recent paper,[1] it was suggested that Lorentz invariance may be violated by the existence throughout space of a constant unit-vector field λ^μ. The experimental limits on the coupling constant g of such a vector were examined, and it was shown that the decay[2] $K_2^0 \to 2\pi$, especially in its lack of energy dependence,[3] ensured that $g\lambda^0 \lesssim 10^{-10}$ eV. Some new experiments to look for λ^μ were suggested, and are now being carried out. Since this vector is strikingly similar to an "ether-drift velocity," we had to ask whether the Michelson-Morley experiment was relevant. In spite of having no very definite model in mind, we could point out that it was difficult to couple λ^μ to A_μ and preserve gauge invariance at the same time. We were left in the uncomfortable position of having introduced an ether, which coupled to massive particles and might even be responsible for their rest mass, but which seemed to demand that the electromagnetic field exist as a separate, independent entity. This is completely contrary to the original aim of ether theories.[4] The idea of an absolute space was first clearly stated by Newton,[5] who invoked it to help explain the difference between inertial and noninertial frames. He felt also that a medium of some sort was necessary to support gravitational interaction. The investigation of electrostatics and magnetism led to the invention of one or more additional "ethers" to carry these forces, though there always remained a hope that there was really only one ether, and that the different forms of action at a distance were merely different distortions of it.

In the present paper we will take this historical idea completely seriously. We will suppose that we have to deal with a theory of the Nambu type,[6] so that ordinary particles are regarded as quasiparticles in a background of massless quanta, which are in a highly correlated state similar to that of a superconductor. This vacuum can then be thought of as a superposition of pair states, and following Ref. 6, Eq. (3.17), it is written as

$$\Omega^{(m)} = \prod_{p,s} \{ [\tfrac{1}{2}(1+\beta_p)]^{1/2} - [\tfrac{1}{2}(1-\beta_p)]^{1/2} \times a^{(0)\dagger}(p,s) b^{(0)\dagger}(-p,s) \} \Omega^{(0)}. \quad (1)$$

The product is continued to infinite energy, to preserve Lorentz invariance. We shall make the simplest modification of this scheme, and cut off the product at a definite energy Λ. This immediately destroys the Lorentz invariance, but to get a vector λ^μ we have to assume that the high-energy states are also unsymmetric under charge conjugation. We will not have to be more specific about the way in which these symmetries are broken. If Λ is very large, then the background may be very nearly Lorentz-invariant, and the effects of λ^μ correspondingly small. This intuitive feeling is strengthened by the detailed considerations which follow.

Our task is to try to explain the gravitational and electromagnetic interactions as collective oscillations of this vacuum state. There is little difficulty with photons, because a theory of this kind has already been worked out by Bjorken.[7] He assumes that the electromagnetic current has a nonzero vacuum expectation value, $Q^\mu \equiv Q\lambda^\mu$, where Q is fixed by a suitable self-consistency relation. The photon then emerges as a kind of Goldstone boson. We shall follow his method closely, and recapitulate it in Sec. 2. However, our emphasis is somewhat different. Bjorken's main objective is to show that his theory is actually compatible with Lorentz invariance, in spite of the existence of Q^μ. To ensure this result, he is obliged to identify $(p^\mu + Q^\mu)$, not simply p^μ, as the momentum operator for charged particles. He also assumes that Q^μ is very large (proportional to the cube of the cutoff energy). The theory does not depend on either of these conjectures, and we will make the opposite assumptions:

* Supported by National Science Foundation Grant NSF-GP4536.

[1] P. R. Phillips, Phys. Rev. **139**, B491 (1965).
[2] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964).
[3] W. Galbraith, G. Manning, A. E. Taylor, B. D. Jones, J. Melos, A. Astbury, N. H. Lipman, and T. G. Walker, Phys. Rev. Letters **14**, 383 (1965); X. de Bouard, D. Dekkers, B. Jordan, R. Mermod, T. R. Willits, K. Winter, P. Scharff, L. Valentin, M. Vivargent, and M. Bott-Bodenhausen, Phys. Letters **15**, 58 (1965).
[4] E. T. Whittaker, *A History of the Theories of Aether and Electricity* (Philosophical Library, Inc., New York, 1951).
[5] I. Newton, *The Mathematical Principles of Natural Philosophy*, edited by F. Cajori (University of California Press, Berkeley, California, 1960), pp. 6–12.
[6] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
[7] J. D. Bjorken, Ann. Phys. (N. Y.) **24**, 174 (1963); G. S. Guralnik, Phys. Rev. **136**, B1404 (1964).

(a) p^μ remains the observable momentum, so that violations of Lorentz invariance can be detected, at least in principle,

(b) Q^μ is a very small vector, so that most experiments are compatible with Lorentz invariance.

Bjorken's theory predicts the existence of photons which travel with velocity c in spite of being collective oscillations in a medium which is not Lorentz invariant. In Secs. 3 and 4 we show that the theory can be adapted with little change to gravitation, with a similar conclusion.

A characteristic of all "superconductor" theories is that physical quantities are intimately connected with the cutoff energy. This is illustrated in Eqs. (3.8) and (3.9) of Ref. 6, for the fermion mass, and Eq. (2.5) of Ref. 7, for the fine-structure constant. Since our aim is to get an estimate of Λ in as many independent ways as possible, it is tempting to invert these equations and find Λ as a function of m and α. Unfortunately, Λ depends exponentially on α, and in an even more delicate way on m, so that neither of these methods can be considered reliable.

There are two quantities which seem likely to be related in a much simpler way to Λ. One is the gravitational coupling constant $\mathcal{G}$, which emerges from our discussion in Secs. 3 and 4. The other is the decay rate[8] for $K_2{}^0 \to 2\pi$, assuming that this will eventually show an energy dependence. This argument is given in Sec. 5. The agreement between the two estimates of Λ is good. In Sec. 6 we summarize our conclusions and try to see some implications of what we have done.

2. PHOTONS IN BJORKEN'S MODEL

The steps taken by Bjorken[7] to derive an expression for the photon propagator are outlined below. I have nothing new to add to the argument, but it seems best to set it out here so that we can see how much of it can be taken over directly to gravitation, and what parts must be changed. We will use λ^μ instead of his η^μ.

(a) A Lagrangian $\mathcal{L}$ is written down[9] for a spin-$\frac{1}{2}$ field; $\mathcal{L}$ contains a vector self-interaction term, with a coupling constant G;

$$\mathcal{L}=\bar{\psi}(x)(i\nabla-m)\psi(x)-\tfrac{1}{8}G[\bar{\psi}(x),\gamma_\mu\psi(x)]\times[\bar{\psi}(x),\gamma^\mu\psi(x)]. \quad (2)$$

(b) It is assumed that the fermion current has a nonzero expectation value Q_μ and the necessary consistency condition is worked out.

$$Q_\mu\equiv\lambda_\mu Q=G\lambda_\mu F(Q^2) \quad (3)$$

with

$$\lambda_\mu F(Q^2)=\mathrm{Tr}\int\frac{d^4k}{(2\pi)^4}\gamma_\mu S_F'(k+\lambda Q). \quad (4)$$

(c) The integral equation is given for the photon self-energy $\Pi_{\mu\nu}(q)$, and it is shown that $\Pi_{\mu\nu}(0)$ can be related to the consistency condition for Q_μ by using Ward's identity.

$$\Pi_{\mu\nu}(q)=i\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\gamma_\mu S_F'(k+Q)\times\Gamma_\nu(k+Q,k+q+Q)S_F'(k+q+Q), \quad (5)$$

$$\Pi_{\mu\nu}(0)=\frac{\partial}{\partial Q^\nu}\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\gamma_\mu S_F'(k+Q)=\frac{\partial}{\partial Q^\nu}\frac{Q_\mu}{(Q^2)^{1/2}}F(Q^2)=\frac{1}{G}(g_{\mu\nu}-\lambda_\mu\lambda_\nu f), \quad (6)$$

where

$$f=1-2Q^2\frac{\partial}{\partial Q^2}\ln F(Q^2).$$

(d) The usual gauge-invariant form is used for

$$\Pi_{\mu\nu}(q)-\Pi_{\mu\nu}(0).$$

(e) The equation is given relating $D_{\mu\nu}(q)$ and $\Pi_{\mu\nu}(q)$:

$$D_{\mu\nu}(q)=-iGg_{\mu\nu}+G\Pi_\mu{}^\lambda(q)D_{\lambda\nu}(q). \quad (7)$$

This is inverted to give an expression for $D_{\mu\nu}(q)$:

$$D_{\mu\nu}(q)=\frac{i}{q^2\Pi(q^2)}\left[g_{\mu\nu}-\frac{(q_\mu\lambda_\nu+q_\nu\lambda_\mu)}{\lambda\cdot q}+\frac{q_\mu q_\nu}{(\lambda\cdot q)^2}\right]-\frac{iGq_\mu q_\nu}{f(\lambda\cdot q)^2}. \quad (8)$$

Only the term proportional to $g_{\mu\nu}$ has physical effects; the other terms vanish when operating between conserved currents.

(f) The theory is extended to all orders, and is shown to be equivalent to electrodynamics.

A few comments on these steps may be helpful:

The form of $\mathcal{L}$ has no deep significance, but is chosen with an eye to simplicity. The constant G which appears in it does not occur in the final expression for $D_{\mu\nu}(q)$, though a divergent integral does. The general form for $\Pi_{\mu\nu}(0)$ is $(ag_{\mu\nu}+b\lambda_\mu\lambda_\nu)$; the chief merit of deducing this by Ward's identity is that we fix the value of a to be $1/G$. If we take b to be zero, we regain ordinary electrodynamics. It is an intriguing feature that we still get acceptable results with a nonzero value of b, but only if a is set equal to $1/G$, as the Ward identity requires. It can also be checked that the gauge-invariant form for $\Pi_{\mu\nu}(q)-\Pi_{\mu\nu}(0)$ is essential for the inversion when $b\neq 0$. If either of these two conditions is violated, we will find terms proportional to $\lambda_\mu\lambda_\nu$ in $D_{\mu\nu}(q)$.

3. THE GRAVITON PROPAGATOR

Bjorken was able to start from a Lagrangian which did not contain a photon field, and derived such a field as a collective excitation. In dealing with gravitation

[8] The theory put forward here bears no relation to previous discussions of $K_2{}^0$ and gravitation, for example, by M. L. Good, Phys. Rev. **121**, 311 (1961).

[9] We set $\hbar=c=1$.

we are not so fortunate, because we cannot begin to work without using a metric and hence introducing the gravitational field. This is not an essential difficulty, and it turns out that a closely analogous deduction is possible. We shall follow the bootstrap philosophy and identify the collective oscillation we arrive at with the metric field introduced at the beginning. This will allow us to connect the gravitational constant with Λ.

The argument given in the previous section falls into two distinct parts. The first is analytic, and takes us up to Eqs. (6) and (7). The second is algebraic, and is concerned with the possibility of finding the inverse of the matrix equation (7). We shall reverse the order here, and tackle the algebraic part in this section. Once we have assured ourselves that we can find a suitable analog of (7), and that it inverts in the expected way, we can start the more delicate task of formulating a self-consistency condition and working from it to an expression for the graviton self-energy.

Equation (7) is the momentum-space version of an integral equation in configuration space. The propagation functions which appear in it depend on the distance between two points, and it becomes hard to see what form the analogous equation might take in a space of arbitrary curvature. We will avoid this difficulty by working entirely in a space which is uniformly curved. The theory should hold to a good approximation in the real world provided that nonlinear effects are small. Propagators in the fictitious space will depend on the "great-circle distance" between two points, and we write the analog of (7) as

$$E_{\mu\nu}{}^{\sigma\rho}(q)=-\frac{i\gamma}{2}(g_\mu{}^\sigma g_\nu{}^\rho+g_\nu{}^\sigma g_\mu{}^\rho)+\gamma\Xi_{\mu\nu}{}^{\alpha\beta}(q)E_{\alpha\beta}{}^{\sigma\rho}(q)\,. \quad (9)$$

Here $E_{\mu\nu}{}^{\sigma\rho}(q)$ is the graviton propagator which we hope to find, and $\Xi_{\mu\nu}{}^{\alpha\beta}(q)$ is the graviton self-energy. γ is a coupling constant, but is not directly related to $\mathcal{G}$. The dimensions of $E_{\mu\nu}{}^{\sigma\rho}$, $\Xi_{\mu\nu}{}^{\alpha\beta}$ and γ are (energy)$^{-4}$, (energy)^4 and (energy)$^{-4}$, respectively. We have written the arguments as q, but we should bear in mind that propagators can be functions of q^2, λ^2 and $(q\cdot\lambda)$. We use $\gamma/2$ rather than γ in the first term to ensure that the equation iterates correctly; $(g_\mu{}^\sigma g_\nu{}^\rho+g_\nu{}^\sigma g_\mu{}^\rho)$ is the analog of $2g_{\mu\nu}$ in electrodynamics. Notice that there is no term involving $g_{\mu\nu}g^{\sigma\rho}$, because this would give an additional contribution depending solely on the scalar part of the gravitational interaction.

In what follows, we will frequently need fourth-order tensors formed from $g_{\mu\nu}$, q_μ, and λ_μ. There are 21 of these which are symmetric in μ, ν and α, β. They will be denoted by $(T_i)_{\mu\nu}{}^{\alpha\beta}$, and are defined in Table I. We will write $\Xi_{\mu\nu}{}^{\alpha\beta}(q)$ as the sum of a self-energy at zero momentum transfer and a term which tends to zero as $q\to 0$; we assume the following form:

$$\Xi_{\mu\nu}{}^{\alpha\beta}(q)=\Xi_{\mu\nu}{}^{\alpha\beta}(0)+(dT_2+eT_6+fT_{13})_{\mu\nu}{}^{\alpha\beta}\Xi(q^2)\,. \quad (10)$$

TABLE I. The 21 tensors $(T_i)_{\mu\nu}{}^{\alpha\beta}$ which can be formed from $g_{\mu\nu}$, q_μ and λ_μ, and are symmetric in μ,ν and in α,β.

Tensor	Definition
T_1	$g_{\mu\nu}g^{\alpha\beta}$
T_2	$g_\mu{}^\alpha g_\nu{}^\beta+g_\mu{}^\beta g_\nu{}^\alpha$
T_3	$g_{\mu\nu}q^\alpha q^\beta$
T_4	$g_{\mu\nu}\lambda^\alpha\lambda^\beta$
T_5	$g_{\mu\nu}(q^\alpha\lambda^\beta+\lambda^\alpha q^\beta)$
T_6	$g_\mu{}^\alpha q_\nu q^\beta+g_\nu{}^\alpha q_\mu q^\beta+g_\mu{}^\beta q_\nu q^\alpha+g_\nu{}^\beta q_\mu q^\alpha$
T_7	$g_\mu{}^\alpha\lambda_\nu\lambda^\beta+g_\nu{}^\alpha\lambda_\mu\lambda^\beta+g_\mu{}^\beta\lambda_\nu\lambda^\alpha+g_\nu{}^\beta\lambda_\mu\lambda^\alpha$
T_8	$g_\mu{}^\alpha q_\nu\lambda^\beta+g_\nu{}^\alpha q_\mu\lambda^\beta+g_\mu{}^\beta q_\nu\lambda^\alpha+g_\nu{}^\beta q_\mu\lambda^\alpha$
T_9	$g_\mu{}^\alpha\lambda_\nu q^\beta+g_\nu{}^\alpha\lambda_\mu q^\beta+g_\mu{}^\beta\lambda_\nu q^\alpha+g_\nu{}^\beta\lambda_\mu q^\alpha$
T_{10}	$q_\mu q_\nu g^{\alpha\beta}$
T_{11}	$\lambda_\mu\lambda_\nu g^{\alpha\beta}$
T_{12}	$(q_\mu\lambda_\nu+\lambda_\mu q_\nu)g^{\alpha\beta}$
T_{13}	$q_\mu q_\nu q^\alpha q^\beta$
T_{14}	$q_\mu q_\nu(q^\alpha\lambda^\beta+\lambda^\alpha q^\beta)$
T_{15}	$(q_\mu\lambda_\nu+\lambda_\mu q_\nu)q^\alpha q^\beta$
T_{16}	$q_\mu q_\nu\lambda^\alpha\lambda^\beta$
T_{17}	$(q_\mu\lambda_\nu+\lambda_\mu q_\nu)(q^\alpha\lambda^\beta+\lambda^\alpha q^\beta)$
T_{18}	$\lambda_\mu\lambda_\nu q^\alpha q^\beta$
T_{19}	$\lambda_\mu\lambda_\nu(\lambda^\alpha q^\beta+q^\alpha\lambda^\beta)$
T_{20}	$(q_\mu\lambda_\nu+\lambda_\mu q_\nu)\lambda^\alpha\lambda^\beta$
T_{21}	$\lambda_\mu\lambda_\nu\lambda^\alpha\lambda^\beta$

Here d, e, and f are functions of q, and Ξ is expected to be a divergent integral. If a "gauge-invariance" condition holds in the form

$$q^\mu[\Xi_{\mu\nu}{}^{\alpha\beta}(q)-\Xi_{\mu\nu}{}^{\alpha\beta}(0)]=0\,,$$

then we must have

$$d=-q^2e\,,\quad\text{and}\quad 2e=-q^2f\,. \quad (11)$$

We cannot yet be sure that such a relation should hold, nor that other tensors, which involve λ^μ, or which pair the indices in an abnormal way, do not appear in the second term of (10). We shall see, however, that the "natural" choice is, in fact, the one which works. Whether this form can be deduced by the canonical methods, using perturbation theory, will be left as an open question.

For $\Xi_{\mu\nu}{}^{\alpha\beta}(0)$, we assume the form

$$\Xi_{\mu\nu}{}^{\alpha\beta}(0)=(AT_2+BT_7+CT_{21})_{\mu\nu}{}^{\alpha\beta}\,. \quad (12)$$

It will be essential in what follows that tensors like T_4 do not appear in (12); such a pairing of the indices is unphysical, since it leads to abnormal scalar contributions, and (9) cannot be suitably inverted if terms of this kind are present. $E_{\alpha\beta}{}^{\sigma\rho}$ can be expressed as a linear combination of the T_i, in which the leading term is proportional to T_2, but in order for it to be acceptable as a propagator, the following terms must not appear in the sum: T_1, T_4, T_7, T_{11}, T_{21}. The first we reject because the indices are incorrectly paired, the rest because they contain no factor of q. The conditions under which an acceptable form for $E_{\alpha\beta}{}^{\sigma\rho}$ may be found (when terms in λ^μ are present) are as follows:

(a) $\gamma A=\frac{1}{2}$. This will be recognized as the analog of the condition derived by Bjorken on the basis of Ward's identity.

(b) $B\neq 0$.

(c) C can have any value except $-4B/\lambda^2$. In particular, it can be zero.

(d) The "gauge-invariance" conditions (11) must hold between d, e, and f.

We now find that

$$E_{\alpha\beta}{}^{\sigma\rho}(q)=\frac{i}{d\Xi(q^2)}\sum_j s_j(T_j)_{\alpha\beta}{}^{\sigma\rho}, \tag{13}$$

where the coefficients s_j are listed in Table II, and we introduce the definitions

$$b\equiv B/\Xi(q^2),\quad\text{and}\quad c\equiv C/\Xi(q^2).$$

All that is needed to obtain this result is patience and a multiplication table of the T_i among themselves. Some equalities hold between the s_j, with the result that $E_{\alpha\beta}{}^{\sigma\rho}$ is symmetric not only under $\mu\leftrightarrow\nu$ and $\alpha\leftrightarrow\beta$, but also under the simultaneous interchange $\mu\leftrightarrow\alpha$, $\nu\leftrightarrow\beta$. This could have been anticipated, and means that we cannot tell one end of the propagator from the other. Notice that some of the s_j are singular as $b\rightarrow 0$. This implies that, as in electrodynamics, the Lorentz-invariant theory is not a uniform limit of the noninvariant one. To make (13) correspond to (8) we must set $d=q^2$ [a multiplicative constant can be absorbed into $\Xi(q^2)$]. This has two immediate consequences:

(a) The dimension of $\Xi(q^2)$ must be (energy)^2, indicating that it probably involves a quadratically-divergent integral. The degree of divergence of the lowest order self-energy integral is actually quartic, and higher order integrals are worse. However, we know that in electrodynamics the worst divergence can be absorbed into $\Pi_{\mu\nu}(0)$, and we will assume that the same can be done here. We will see in Sec. 6 how the gravitational coupling constant $\mathcal{G}$ and the cutoff energy Λ control the magnitude of integrals of arbitrary order. So $\Xi(q^2)\approx\Lambda^2$, and the coupling constant $\mathcal{G}\approx 1/\Lambda^2$. This is a very pleasing result. It has often been conjectured[10] that $\mathcal{G}$ sets a limit on measurements by forcing an effective energy cutoff around $1/\sqrt{\mathcal{G}}\approx 10^{28}$ eV. We have now shown how the converse may occur. A theory with a built-in cutoff can provide a $\mathcal{G}$ which is of just such a magnitude that no greater energies are physically meaningful.

TABLE II. The coefficients s_i used in the expression (18) for $E_{\mu\nu}{}^{\alpha\beta}(q)$.

s_2	$\frac{1}{4}$
s_6	$(b\lambda^2+d)/[4b(q\cdot\lambda)^2]$
s_8	$-1/(4q\cdot\lambda)$
s_9	Equal to s_8
s_{13}	$\lambda^2[2d+4b\lambda^2+c(\lambda^2)^2]/[2(q\cdot\lambda)^4(4b+c\lambda^2)]$
s_{14}	$-[2d+4b\lambda^2+c(\lambda^2)^2]/[2(q\cdot\lambda)^3(4b+c\lambda^2)]$
s_{15}	Equal to s_{14}
s_{16}	$1/[2(q\cdot\lambda)^2]$
s_{17}	$(-dc+4b^2+bc\lambda^2)/[4b(q\cdot\lambda)^2(4b+c\lambda^2)]$
s_{18}	Equal to s_{16}
	All other coefficients are zero.

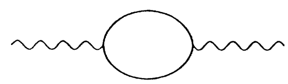

FIG. 1. The lowest order contribution to the graviton self-energy. This is singular at $q^2=0$ if the intermediate particles have zero rest mass.

(b) Setting $d=q^2$, we find from (10) and (11)

$$\Xi_{\mu\nu}{}^{\alpha\beta}(q)-\Xi_{\mu\nu}{}^{\alpha\beta}(0)=(q^2T_2-T_6+2T_{13}/q^2)\Xi(q^2). \tag{14}$$

The term involving T_{13}, which is singular at $q^2=0$, has no analog in electrodynamics. It is associated with diagrams of the type shown in Fig. 1, in which both intermediate particles are massless. They can be simultaneously on the mass shell if the external graviton has $q^2=0$, and this is bound to cause a singularity. This behavior is not, of course, new in our theory, but will appear in a more conventional theory as well.

4. THE GRAVITON SELF-ENERGY

Our aim in this section will be to derive the form (12) (for the self-energy at zero momentum transfer) by starting from a definite equation of motion and formulating an appropriate consistency condition. Since the quantization of the gravitational field is not well understood, there are may ambiguities in what follows, and we will do no more than present the simplest scheme we have come across, without pretending that it is unique or in some way favored among many alternatives. Our starting point will be the Einstein equations, coupled to a scalar matter field. A constant vector has no place in Einstein's theory, which is based on Riemannian geometry. We will at first introduce terms involving such a vector in a tentative way, but it will turn out later that they may have a deeper meaning in the context of a more general geometry originated by Weyl.[11,12]

The vector will be denoted by $S^\mu\equiv S\lambda^\mu$, to differentiate it from the $g\lambda^\mu$ used in Ref. 1, and from the Q^μ of Sec. 2. It is important to make this distinction, because the magnitude and coupling of these vectors need not be the same. If our later geometrical conception is correct, then S^μ is very much smaller than $g\lambda^\mu$. All these vectors

[10] For example, by L. D. Landau, in *Niels Bohr and the Development of Physics*, edited by W. Pauli (McGraw-Hill Book Company, Inc., New York, 1955).

[11] H. Weyl, Berliner Sitzungberichte, 465 (1918); Z. Math. Physik 2, 384 (1918); W. Pauli, *Collected Papers*, edited by R. Kronig and V. Weisskopf (Interscience Publishers, Inc., New York, 1964), Vol. 2, No. 1.

[12] A. S. Eddington, *The Mathematical Theory of Relativity* (Cambridge University Press, New York, 1960), Chap. 7; H. Weyl, *Space-Time-Matter* (Dover Publications, Inc., New York, 1922), Chap. 35.

are regarded as different manifestations of the same ether, and so are proportional to the same unit vector λ^μ. Our vector S^μ is like the vector D^μ introduced in some steady-state cosmologies,[13] and may indeed play a similar role. In Gaussian coordinates it has the form (1,0,0,0), just as in flat-space theories.

We have to be careful in handling covariant and contravariant quantities, and will make the following convention: The independent variables and fields of our theory will be defined as their contravariant components. This means that we take x^μ, λ^μ, $g^{\mu\nu}$ as fundamental, though the momentum $p_\mu \equiv i\partial/\partial x^\mu$, is defined as a covariant operator. There is no physical reason behind this choice; we adopt it simply because it leads directly to the result we are after.

The field equations are taken to be

$$R^{\mu\nu}-\tfrac{1}{2}Rg^{\mu\nu}+\kappa T^{\mu\nu}=\epsilon g^{\mu\nu}-6(S^\mu S^\nu-\eta g^{\mu\nu}S_\sigma S^\sigma)\,. \quad (15)$$

Here ϵ is the cosmological constant, and the constant η is arbitrary. The factor of 6 in front of the terms involving S^μ is included for later convenience. For $T^{\mu\nu}$ we use the expression $T^{\mu\nu}=g^{\mu\alpha}g^{\nu\beta}T_{\alpha\beta}$, with

$$T_{\alpha\beta}=\frac{\partial\phi}{\partial x^\alpha}\frac{\partial\phi}{\partial x^\beta}-\tfrac{1}{2}g_{\alpha\beta}\left(\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x^\nu}g^{\mu\nu}-m^2\phi^2\right). \quad (16)$$

We divide this as

$$T^{\mu\nu}=g^{\mu\alpha}g^{\nu\beta}U_{\alpha\beta}-Vg^{\mu\nu}, \quad (17)$$

where

$$U_{\alpha\beta}\equiv\frac{\partial\phi}{\partial x^\alpha}\frac{\partial\phi}{\partial x^\beta},\quad\text{and}\quad V\equiv\frac{1}{2}\left(\frac{\partial\phi}{\partial x^\sigma}\frac{\partial\phi}{\partial x^\rho}g^{\sigma\rho}-m^2\phi^2\right).$$

We rearrange (15), leaving on the right-hand side only those terms which are scalar multiples of $g^{\mu\nu}$ or $S^\mu S^\nu$:

$$\begin{aligned}(R_{\sigma\rho}+\kappa U_{\sigma\rho})g^{\mu\sigma}g^{\nu\rho}&=(\epsilon+\tfrac{1}{2}R+\kappa V+6\eta S_\sigma S^\sigma)g^{\mu\nu}-6S^\mu S^\nu\\&\equiv\alpha g^{\mu\nu}+\beta\lambda^\mu\lambda^\nu.\end{aligned} \quad (18)$$

Taking the vacuum expectation value of both sides of this equation, we see the right-hand side gives a nonzero result. On the left-hand side we have terms of a much more complicated structure, if the $g^{\mu\nu}$ are quantized. In momentum space we can get closed loops, made up of both gravitons and scalar quanta; we represent all these possibilities in the single diagram, Fig. 2(a). In other words, we regard our version of the Einstein equations as being a direct expression of the Hartree self-consistency condition. I do not know how to decide whether this interpretation is justified, except by trying to understand its consequences. Following the same line of thought as in electrodynamics, we identify diagram 2(a) as a "graviton tadpole." We obtain the self-energy at zero momentum transfer, Fig. 2(b), by differentiating the left-hand side of (18) with respect to $g^{\alpha\beta}$, and dividing by κ, since the addition of a zero-momentum graviton is equivalent to the change $g^{\alpha\beta}\to g^{\alpha\beta}+\kappa h^{\alpha\beta}$. Variation of $R_{\sigma\rho}$ gives a sum of covariant derivatives of Christoffel symbols. These we will discard, according to our assumption of uniform curvature. $U_{\sigma\rho}$ does not change when $g^{\alpha\beta}$ is transformed. So we get for the self-energy:

$$\begin{aligned}\Xi_{\alpha\beta}{}^{\mu\nu}(0)&=(1/2\kappa)(R_{\sigma\rho}+\kappa U_{\sigma\rho})\\&\quad\times[(g_\alpha{}^\mu g_\beta{}^\sigma+g_\alpha{}^\sigma g_\beta{}^\mu)g^{\nu\rho}+g^{\mu\sigma}(g_\alpha{}^\nu g_\beta{}^\rho+g_\alpha{}^\rho g_\beta{}^\nu)]\\&=(1/2\kappa)(\alpha g_\sigma{}^\nu+\beta\lambda_\sigma\lambda^\nu)(T_2)_{\alpha\beta}{}^{\mu\sigma}\\&\qquad+(1/2\kappa)(\alpha g_\rho{}^\mu+\beta\lambda^\mu\lambda_\rho)(T_2)_{\alpha\beta}{}^{\nu\rho}\\&=(\alpha/\kappa)(g_\alpha{}^\mu g_\beta{}^\nu+g_\beta{}^\mu g_\alpha{}^\nu)\\&\qquad+(\beta/2\kappa)(\lambda_\beta\lambda^\nu g_\alpha{}^\mu+\lambda_\alpha\lambda^\nu g_\beta{}^\mu+\lambda^\mu\lambda_\beta g_\alpha{}^\nu+\lambda^\mu\lambda_\alpha g_\beta{}^\nu)\\&=(\alpha/\kappa)(T_2)_{\alpha\beta}{}^{\mu\nu}+(\beta/2\kappa)(T_7)_{\alpha\beta}{}^{\mu\nu},\end{aligned} \quad (19)$$

where we have used (18) and Table I. We now have to estimate the coupling constant γ in (9). Bjorken found it easy to arrive at the analogous Eq. (7) because he started from a Lagrangian which contained a quadratic interaction; the iteration of this gave (7) immediately. Our position is different. We did not arrive at our self-consistency condition by linearizing a quadratic Lagrangian, but took a known set of equations as directly expressing this condition. The next step, therefore, is to reverse Bjorken's procedure, and from the Hartree condition to derive an equivalent quadratic interaction. It may be[11,12] that we ought to have started from a Lagrangian which is quadratic in R and $R_{\mu\nu}$, but we will not go into this here.

Consider the mass of a scalar particle at rest. This is proportional to the trace of the energy-momentum tensor, and we have, using (18):

$$\begin{aligned}T=g^{\mu\nu}T_{\mu\nu}=(1/2\alpha)[&(R^{\mu\nu}+\kappa U^{\mu\nu}+6S^\mu S^\nu)\\&+(\epsilon+\tfrac{1}{2}R+\kappa V+6\eta S_\sigma S^\sigma)g^{\mu\nu}]T_{\mu\nu}.\end{aligned} \quad (20)$$

This gives us the self-energy diagram 2(c); since $U^{\mu\nu}$ and $Vg^{\mu\nu}$ are parts of $T^{\mu\nu}$, the relevant tensor-tensor coupling constant is clearly of order $\kappa/2\alpha$. The derivation of Sec. 3 can now be carried out, making the identifications

$$\gamma=\kappa/2\alpha,\quad A=\alpha/\kappa,\quad B=\beta/2\kappa,\quad C=0. \quad (21)$$

Looking back at the conditions (a)–(d) derived in Sec. 3, we see that we have chosen the right value of γ, and that (b) and (c) are satisfied. So the method works, provided only that we use a gauge-invariant form for $\Xi_{\alpha\beta}{}^{\mu\nu}(q)-\Xi_{\alpha\beta}{}^{\mu\nu}(0)$. The coupling constant κ in Eq. (15)

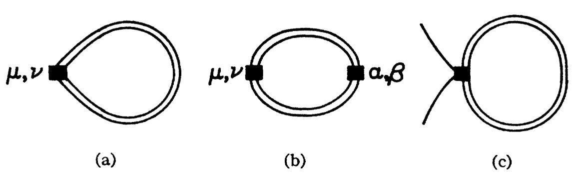

FIG. 2. (a) The "graviton tadpole" of Sec. 4; (b) the self-energy at zero momentum transfer derived from 2(a); (c) the self-energy diagram used in estimating the tensor-tensor coupling constant.

[13] R. Adler, M. Bazin, and M. Schiffer, *Introduction to General Relativity* (McGraw-Hill Book Company, Inc., New York, 1964), Chap. 12.

does not appear explicitly in our estimate of $\mathcal{G}$, which is dominated by the cutoff energy Λ. It is fixed solely by the requirement that perturbation theory agree with experiment, so that, up to factors of 2π, it is identical with $\mathcal{G}$.

There remains the question of what role S^μ can play. The most plausible suggestion I can make is that it is connected with the "gauge vector" introduced by Weyl[11,12] in his conformal geometry. Because this theory, in its original form, was abandoned soon after its invention, and is now not widely known, it seems best to give a brief description of it here. In Einstein's theory, the length l of a measuring rod does not change if it is carried through a small distance dx^μ by parallel displacement, although its direction will, in general, be altered. In Weyl's theory this restriction is lifted, and the length is said to change according to the formula

$$d(\ln l)=S_\mu dx^\mu, \tag{22}$$

where S_μ is a vector field which is a function of position. Weyl pointed out that if S_μ is the gradient of a scalar field, then there is no change in the length of a rod when it is taken round a closed loop, so that S_μ is ambiguous in the same way as the electromagnetic potential. He therefore identified the two, and constructed a theory involving only quantities which are gauge-invariant (which Einstein's equations are not). But it seemed unreasonable to suppose that measuring rods should be permanently altered merely by passing them through an electromagnetic field, nor is this field coupled to matter in a universal way, so the theory was soon rejected. Before its demise, however, Pauli[11] had shown that the Weyl equations, with $S_\mu=0$, could be satisfied by any solution of Einstein's equations, so that they are not in conflict with any of the tests for general relativity.

It may be suggested that if S^μ is not associated with the electromagnetic potential, then it may instead be connected with another long-range field B^μ which is so weakly coupled that it has escaped detection. Such fields have been suggested before,[14] but a classical gauge invariance demands a current which is exactly conserved. We can, for example, use the baryon current, but any such choice destroys the universality of coupling which we need for a geometrical vector.

There is another, quite different, interpretation of S^μ, but to understand it we must recall the type of theory we have in mind here. The basic field of the Nambu theory has spin $\frac{1}{2}$ and is massless, so that the Lagrangian of the field admits a γ_5 transformation. As a result of the peculiar vacuum state, the observable particles become massive, and the Lagrangian, when written in terms of the new fields, possesses no γ_5 transformation, at least of a simple kind. So far we have paid no attention to this feature, because we were concerned with the vectors generated by the vacuum state. But it is very relevant to the Weyl theory, because measuring rods are made of massive particles, while a world containing only massless particles contains no such internal scale. The conjecture about S^μ follows at once: perhaps the basic Lagrangian we should use is conformally invariant, as Weyl would have wished, but the alteration of the vacuum state which leads to massive particles eliminates simultaneously both γ_5 and conformal invariance. The value of S^μ is then fixed, up to an arbitrary choice of reference frame. In a similar way, the fermion mass is fixed, up to an arbitrary choice of "γ_5 angle" [see Ref. 6, Eq. (3.26)]. This idea becomes more plausible when we notice that our equation (15), with $T^{\mu\nu}$ omitted, and η set equal to $\frac{1}{2}$, is the simplest Weyl equation, and is invariant under the transformation: $x^\mu \to x^\mu$, $ds \to \lambda ds$, $g_{\mu\nu} \to \lambda g_{\mu\nu}$, $S_\mu \to S_\mu + \partial(\ln\lambda)/\partial x^\mu$, $\epsilon \to \lambda^{-2}\epsilon$, Here λ is a function of position; the factor of 6 in (15) was included to give this simple form to the gauge transformation.

It is essential for this paper that S^μ becomes fixed at a nonzero value, and we will now try to understand what this means. We assume, as in Ref. 1, that λ^μ is timelike, so that in the "preferred" frames of reference it has the form (1,0,0,0). This implies spatial isotropy; in a given plane, $t=$constant, the clocks and scales will agree. However, in a theory such as Weyl's they will change with time, according to (22). We must interpret this as saying that the vacuum is slowly changing, so that Λ is not constant. Other physical quantities related to Λ will vary as well. We will describe some consequences of this in another paper.

5. CONNECTION OF THE CUTOFF ENERGY WITH OTHER EXPERIMENTAL RESULTS

In Ref. 1 the simplest couplings of λ^μ to matter were considered. The decay $K_2^0 \to 2\pi$ was shown to be one of the strongest limitations on the magnitude of such terms. It seems possible that only by specially designed experiments will we be able to learn more, because few measurements made so far are capable of the energy resolution available in the K^0-$\bar{K}^0$ system. These experiments are in progress, but results are not yet available. Our task is, then, to try to find some connection between the decay rate of $K_2^0 \to 2\pi$ and the cutoff energy Λ, with its resulting constant four-vector. It was the observation of a simple numerical relation which led me to write this paper; however, as the work progressed it became clear that we could say more interesting things about gravitation than about K^0's. Consequently, the argument which follows is scarcely advanced beyond the form in which it first presented itself. There are serious weaknesses in it, which I cannot remedy, but will simply indicate as we come to them.

When λ^μ appears in the Lagrangian in the form $g\lambda^\mu j_\mu$, where g is a coupling constant, and j_μ a current, it results in a splitting of the masses of particle and antiparticle. This statement needs some explanation, since

[14] T. D. Lee and C. N. Yang, Phys. Rev. **98**, 1501 (1955); R. H. Dicke, *ibid.* **126**, 1580 (1962).

λ^μ is assumed in Ref. 7 to have no such effect. We can invoke a gauge principle to get rid of $g\lambda^\mu$ only if j_μ is conserved; this is not necessarily the case, and if the K^0 is involved, then j_μ certainly cannot be the electromagnetic current. Even if j_μ is conserved, we saw in Sec. 1 that we are still not justified in assuming that $p_\mu+g\lambda_\mu$ is the physical momentum, rather than just p_μ. It is solely the requirement of Lorentz invariance which makes us prefer one over the other, and it is just this invariance which is in question here.

The presence of a coupling term $g\lambda^\mu j_\mu$ will result in a γ^2 dependence of the decay rate for $K_2^0 \rightarrow 2\pi$, providing we work at sufficiently high energy. At low energies we may run into trouble because of other, more familiar interactions which cause the decay, and the experimental data[2,3] shows that this does indeed happen. If we assume that $g\lambda^\mu$ causes a mass splitting of $\Delta m/m$ for a K_2^0 at rest, then we can set an upper limit of about 10^{-19} for this ratio. It is quite possible that $\Delta m/m=0$, and the whole theory is wrong, but we will be optimistic and use 10^{-19} in what follows.

The argument would be greatly strengthened if we could show that additional interactions of the correct magnitude could be expected on the basis of our model. It is a serious flaw that I have so far been unable to demonstrate this. There is also the requirement that the phase of this additional term must be almost pure imaginary, and not real, as would be the case for a mass splitting. There is, of course, no reason why extra interactions should not exist, but they do not seem to be demanded.

We must expect marked changes in the mass of a particle, measured in its own rest frame, as we accelerate it towards the limiting energy Λ. Two possibilities immediately suggest themselves: (a) A particle which has energy greater than Λ should behave as if it were free, and hence have $m=0$, *or* (b) energies greater than Λ can never be attained, and particles become very massive as we try to approach this limit. I will not try to decide whether either of these alternatives makes more sense than anything else we might suggest, although I find (b) more satisfying philosophically. All that we need to realize is that we approach Λ, the mass splitting of K^0 and $\bar{K}^0$ should become comparable to the mass of a K^0 at rest. This idea is sketched in Fig. 3. We take Λ as given approximately by the point at which the lower broken line cuts the horizontal axis. This gives

$$2m/\Lambda=\Delta m/m\,, \quad \Lambda=2m^2/\Delta m\approx 10^{28}\text{ eV}.$$

The agreement between this value and the one obtained from the gravitational constant is very striking, considering the crude methods used. If we take it seriously, then we must anticipate that we are on the verge of detecting the energy dependence of the K_2^0 decay rate in the experiments at 10 BeV/c.

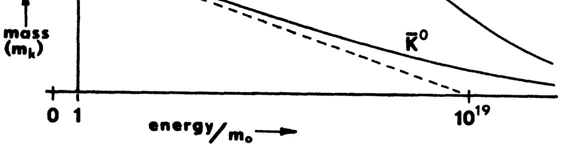

FIG. 3. Possible behavior of the masses of K^0 and $\bar{K}^0$ with increasing energy. Solid curves show the masses tending to zero at energies above a certain upper limit. Broken lines show the extrapolation of the maximum permissible low-energy mass variation. The point at which the lower broken line cuts the horizontal axis gives our estimate of Λ.

6. CONCLUSION

The value of $\mathcal{G}$ which we have obtained has a straightforward connection with the renormalization problem. Consider an arbitrary diagram involving only gravitational interactions; each corner introduces one more power of the momentum than we would have had with a dimensionless coupling constant, and also one power of $\sqrt{\mathcal{G}}$, or equivalently $1/\Lambda$. But integrals over internal momenta are to be cut off at Λ, so the contribution of the diagram should be no larger than we would have obtained using "electrodynamic" coupling. In particular, the higher order integrals in the graviton self-energy (Sec. 3) are of the same general magnitude as the basic lowest order ones. Of course we still have the task of summing families of diagrams, but this too may prove feasible once the divergence problem is seen to be not serious.

The question of the existence of an ether is one of the most protracted that physics has ever faced.[4] After the time of Newton, belief in it persisted for two centuries. The Michelson-Morley experiment and the theory of Maxwell destroyed its relevance for electromagnetic interactions, and with the advent of Einstein's general theory of relativity it seemed that it was not needed for inertial effects either. In the present paper we have reinstated an ether, and have re-examined its connection with electromagnetism and gravitation. It is no longer so obvious that, in their attitudes toward an absolute space, Einstein was right and Newton wrong. Instead, each man appears to have been partly right, Newton in his insistence on the ultimate importance of an ether, and Einstein in seeing that the metric field provides us with an excellent means of avoiding the introduction of an ether in all problems encountered so far. Indeed, since the graviton, like the photon, is a collective oscillation in spite of traveling with velocity c, we will probably have to approach the energy Λ before the graviton concept breaks down.

The main function of the ether in the present theory is to prove a scale of length and time, through the dependence of particle masses on Λ. If the Weyl theory is true in some sense, then we can even hope to understand changes in the units of length and time as the universe evolves. Mach's principle, as originally stated, is unnecessary in any theory involving an ether. However, this is not the same as saying that distant matter is completely irrelevant. The development of the universe is a most delicate affair, because various self-consistency conditions have to be maintained at every instant. It seems reasonable that one of these should be $\mathcal{G}Nm^2/R \approx m$, where R is the radius of the universe and N the number of particles in it. This relation is true at the present time and is often assumed to have some connection with Mach's principle. We are now trying to find suitable cosmological models which are solutions of the Weyl equation (15); this work may clarify the meaning of the self-consistency conditions.

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Neutron Form Factors from a Study of Inelastic Electron Spectra in the Electrodisintegration of Deuterium*

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Measurements of the ratio of the total inelastic electron-deuteron cross section to the elastic electron-proton cross section have been made to an accuracy of about 3% for values of the square of the four-momentum transfer q^2 in the range 1.5 to 7.5 F^{-2}. These ratios have been analyzed in terms of the form factors of the neutron using the "area method," and it is concluded that the corrections necessary to the simple sum rule given by Jankus are approximately equal in magnitude to those encountered in the use of the more familiar "peak method." A detailed comparison has been made between the shapes of the observed inelastic electron-deuteron cross sections as a function of the scattered electron momentum and the shapes expected according to a theoretical treatment due to Durand.

I. INTRODUCTION

IT was first suggested by Hofstadter[1] that experiments on the inelastic scattering of high-energy electrons from the deuteron might provide information on the electromagnetic structure of the neutron. Subsequent experiments by Yearian and Hofstadter[2] confirmed this idea and showed the radius of the magnetic moment distribution in the neutron to be approximately equal to the corresponding radius in the proton. These results were obtained by what is now known as the area method in which the electron-neutron cross section is obtained from the total inelastic electron-deuteron cross section. The area method was quickly superseded, for sound theoretical and experimental reasons, by the so-called peak method in which information about the neutron is obtained from the electron-deuteron cross section at the maximum of the broad inelastic peak (the quasi-elastic region) with the help of a theoretical treatment to allow for the scattering from the proton and the internal motion of the nucleons in the deuteron. The peak method has since been used to measure the variation of both the charge and magnetic form factors of the neutron with the square of the four-momentum transfer, q^2, for values of q^2 up to about 35 F^{-2}.

The most precise experimental information on quasi-elastic electron-deuteron scattering has come from the recent experiments of Hughes *et al.*[3] at Stanford. These data were analyzed by the peak method making use of the most recent theoretical treatment of the inelastic scattering process. The neutron form factors were given for a series of values of q^2 in the range 1.0 to 30.0 F^{-2}. The results suggested that for values of q^2 greater than about 6.0 F^{-2}. the square of the neutron's charge form factor $(G_{En})^2$ was consistent with zero to within an error of the order of 5% in the theoretical cross section. On the other hand for values of q^2 less than 6.0 F^{-2} in

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