Construction of an Equivalent Lagrangian for Interacting Fermion-Boson Fields*

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The principal object of the present work is the derivation of a Lagrangian density $L_\phi(\psi,\bar{\psi})$ which is a function of only the fermion and antifermion fields and contains a boson state at mass μ. We also require $L_\phi(\psi,\bar{\psi})$ to be equivalent to a theory with a Yukawa coupling plus a Matthews term in the limit that the wave-function renormalization constant Z_3 vanishes. We will first consider the case where the Matthews term is neglected and show that $L_\phi(\psi\bar{\psi})$ is just the usual four-fermion theory. In the case where the Matthews term is not neglected, we derive $L_\phi(\psi\bar{\psi})$ in the case of strong coupling and weak coupling. The Lagrangian density is a highly nonlinear function of the fermion fields which, in the weak-coupling limit, reduces to the four-fermion theory as the Matthews term vanishes.

INTRODUCTION

THE understanding of the elementary particles and their properties has been the object of much research. With the development of large accelerators the complexity of this task has multiplied manyfold because of the increasing discoveries of new subatomic particles.

Most of these particles live such a short time that one does not know whether to consider them as new elementary particles or composite states of other constituents. If one considers the particles as composite then one should, in principle at least, be able to calculate the masses and coupling constants of the new particles. On the other hand, if they are considered elementary then one must supply the masses and coupling constants as fundamental parameters which can only be determined by experiments. This reason makes it more appealing to construct a theory where the particles are considered to be composite.

There are several and to some extent equivalent schemes which attempt to realize a self-generating mechanism which will create the composite particles. The bootstrap mechanism of dispersion relations,[1] the vanishing renormalization constants in field theory,[2] the Heisenberg spinor theory,[3] and the superconducting theory of Nambu and Jona-Lasinio[4] are four different theories with a somewhat similar self-generating mechanism. The advantage of these four theories is that the particles can be considered to be composite states of themselves. The forces producing the composite systems are due to the exchange of themselves. Because of this circular self-generating mechanism, it leads to a highly nonlinear theory, namely, a theory similar to a four-fermion interaction. As a consequence of the highly nonlinear character of these theories their utility has been somewhat hampered as far as actually seeing whether or not the theories can achieve their goals. The most extensively studied system of elementary particles has been the pion and nucleon. In particular, Jouvet[5] and later, other authors[6] considered the problem of a fermion field interacting with a boson field via a Yukawa coupling in the limit that the wave-function renormalization constant of the boson field vanished. It was shown that this theory could be considered equivalent to a fermion field interacting with itself via a four-fermion coupling. The particle associated with the boson field appears as a bound state of the four-fermion interaction. In this theory the pion is simply a bound state of a nucleon with an antinucleon. This problem is much too simple to be realistic, since the Yukawa interaction does not take into account the self-interacting effects of the boson field. This effect can be handled by adding, in addition to the Yukawa coupling in the Lagrangian the Matthews term,[7] i.e., a $\lambda\phi^4$ interaction. The presence of the Matthews term in the Lagrangian tremendously increases the complexity of the problem. No longer can this theory be considered equivalent to a four-fermion interaction where the pion appears as a bound state of a nucleon with an antinucleon. This result can be established by observing that the Feynman diagrams of the two theories are no longer in a one-to-one correspondence in the limit of $Z_3=0$ as was the case when the Matthews term was neglected. The question arises as to what fermion-interacting Lagrangian this theory is equivalent to when the Matthews term is not neglected.

The purpose of this paper is to study the properties of a fermion and boson field interacting via a Yukawa coupling plus a Matthews' term in the limit of $Z_3=0$ and to construct an equivalent theory with a self-interacting fermion in which the particle associated with the boson field appears as a bound state. To guess at such a self-interacting-fermion Lagrangian and then try to show a one-to-one correspondence between the Feynman diagrams of the two theories, as was done in the case of the simple Yukawa coupling and four-fermion coupling, is ridiculous because of the many different possibilities available. Therefore, in order to pursue this

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problem one must have a method for calculating such an equivalent field theory. Such a method has been developed by the author[8] and will be used throughout this paper.

The outline of the paper is as follows. In order to illustrate the method developed by the author in Ref. 8 in constructing equivalent field theories we will consider in Sec. II the simple Yukawa coupling in the limit of $Z_3=0$. This is the same problem considered in Refs. 5 and 6 and, as to be expected, we will just reproduce their results. In Sec. III we will consider the case of interest, i.e., the case of a fermion field and a pseudoscalar boson field interacting via a Yukawa coupling plus a Matthews term. An equation for the equivalent Lagrangian will be obtained in terms of a functional integral. The highly nonlinear character of the Matthews term will prohibit one from solving the integral in closed form. The functional integral will be solved by the method of stationary phases where only the lowest order terms are kept. The conclusion will be given in Sec. IV.

II. YUKAWA COUPLING

We shall begin by considering the simplest example of a scalar field coupled to a fermion field. Such a system is described by the Lagrangian density

$$L(\phi,\psi,\bar\psi)=\tfrac{1}{2}\phi(x)[\Box-\mu_0^2]\phi(x) -\bar\psi(x)[\gamma_\mu\partial_\mu+m_0]\psi(x)-g_0\bar\psi(x)\Gamma\psi(x)\phi(x), \quad (1)$$

where Γ is either a scalar or pseudoscalar gamma matrix.

The equations of motion that satisfy Eq. (1) are

$$(\gamma_\mu\partial_\mu+m_0)\psi(x)=-g_0\Gamma\psi(x)\phi(x), \quad (2a)$$

$$(\Box-\mu_0^2)\phi(x)=g_0\bar\psi(x)\Gamma\psi(x). \quad (2b)$$

We now wish to construct a new Lagrangian which will be a function of only the fermion field and whose matrix elements obtained from the new Lagrangian are entirely equivalent to those obtained from Eq. (1). We will also require that $Z_3=0$. Let us denote this new Lagrangian density by $L_\phi(\psi,\bar\psi)$. The notation used here is that of Ref. 8. The subscript refers to the field that has been eliminated and the quantities in the parentheses denote the fields that the Lagrangian density is a function of. As was shown by the author,[8] such a Lagrangian density can be obtained by using Eq. (3) of Ref. 8:

$$\exp\left[i\int L_\phi(\psi,\bar\psi)\,d^4x\right]=\frac{\int \exp[i\int L(\phi,\psi,\bar\psi)\,d^4x]\delta\phi}{\int\exp[i\int L_0(\phi)\,d^4x]\delta\phi}$$
$$=\exp\left[-i\int\bar\psi(\gamma_\mu\partial_\mu+m_0)\psi\,d^4x\right]\frac{\int\exp[\frac{1}{2}i\int\phi(\Box-\mu_0^2)\phi\,d^4x-ig_0\int\bar\psi\Gamma\psi\phi\,d^4x]\delta\phi}{\int\exp[i\int\phi(\Box-\mu_0^2)\phi\,d^4x]\delta\phi}. \quad (3)$$

Solving the functional integral in Eq. (3) will give us a relationship from which we can obtain $L_\phi(\psi,\bar\psi)$. To put the functional integral in more convenient form we will make the change of variable:

$$\phi'(x)=\mu_0\phi(x) \quad (4)$$

and now Eq. (3) becomes

$$\exp\left[i\int L_\phi(\psi,\bar\psi)\,d^4x\right]=\exp\left[-i\int\bar\psi(\gamma_\mu\partial_\mu+m_0)\psi\,d^4x\right]\frac{\int\exp[\frac{1}{2}i\int\phi'(\Box/\mu_0^2-1)\phi'\,d^4x-i(g_0/\mu_0)\int\bar\psi\Gamma\psi\phi'\,d^4x]\delta\phi'}{\int\exp[\frac{1}{2}i\int\phi'(\Box/\mu_0^2-1)\phi'\,d^4x]\delta\phi'}. \quad (5)$$

Now if we place the restraint on the system that $Z_3=0$ and require that the renormalized mass μ of the boson field be finite then as was shown in Ref. 8, this is equivalent to requiring

$$\mu_0\to\infty, \qquad g_0\to\infty, \quad (6)$$

in such a way that

$$g_0^2/\mu_0^2\to g_1<\infty. \quad (7)$$

So in Eq. (5) let us replace g_0/μ_0 by $g_1^{1/2}$ and let $\mu_0\to\infty$. We get

$$\exp\left[i\int L_\phi(\psi,\bar\psi)\,d^4x\right]=\exp\left[-i\int\psi(\gamma_\mu\partial_\mu+m_0)\psi\,d^4x\right]\frac{\int\exp[-\frac{1}{2}i\int\phi'^2\,d^4x-ig_1^{1/2}\int\bar\psi\Gamma\bar\psi\phi'\,d^4x]\delta\phi'}{\int\exp[-\frac{1}{2}i\int\phi'^2\,d^4x]\delta\phi'}. \quad (8)$$

This integral can be solved by making the change of variables

$$\phi=\phi'+g_1^{1/2}\bar\psi\Gamma\psi, \quad (9)$$

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in the numerator of Eq. (8):

$$\exp\left[i\int L_\phi(\psi,\bar{\psi})\,d^4x\right]=\exp\left[-i\int\bar{\psi}(\gamma_\mu\partial_\mu+m_0)\psi\,d^4x\right]\frac{\int\exp[-\frac{1}{2}i\int\phi^2\,d^4x+i(g_1/2)\int\bar{\psi}\Gamma\psi\bar{\psi}\Gamma\psi\,d^4x]\delta\phi}{\int\exp[-\frac{1}{2}i\int\phi'^2\,d^4x]\delta\phi'}. \quad (10)$$

The functional integrals in the numerator and denominator cancel giving us the following equation for $L_\phi(\psi,\bar{\psi})$

$$L_\phi(\psi,\bar{\psi})=-\bar{\psi}(\gamma_\mu\partial_\mu+m_0)\psi+\tfrac{1}{2}g_1(\bar{\psi}\Gamma\psi)(\bar{\psi}\Gamma\psi). \quad (11)$$

Equation (11) now gives us $L_\phi(\psi\bar{\psi})$ whose matrix elements are equivalent to those obtained from Eq. (1). The particle associated with the boson field will appear as a bound state of Eq. (11). This result is equivalent to those obtained in Refs. 5 and 6, however, in this paper it was proved by nonperturbative methods.

In the next section we will do a similar calculation keeping the pion self-interacting effects.

III. YUKAWA PLUS MATTHEWS TERM

The previous method was added to illustrate the method developed by the author in Ref. 8. We will now proceed to apply this procedure in the case of interest.

Let us consider a pseudoscalar boson interacting with a fermion by means of a Yukawa interaction plus a Matthews term. The Lagrangian density for this system is

$$L(\psi,\bar{\psi},\phi)=-\bar{\psi}(\gamma_\mu\partial_\mu+m_0)\psi-ig_0\bar{\psi}\gamma_5\psi\phi+\tfrac{1}{2}\phi(\Box-\mu_0^2)\phi-\lambda\phi^4. \quad (12)$$

As in Sec. II, we will now find a new Lagrangian $L_\phi(\psi,\bar{\psi})$ whose matrix elements are equivalent to those obtained from $L(\psi\bar{\psi}\phi)$ in Eq. (12). We also require the wave-function renormalization constant Z_3 of the boson field to vanish. Such a Lagrangian density can be obtained from Eq. (3) of Ref. 8 which gives

$$\exp\left[i\int L_\phi(\psi,\bar{\psi})\,d^4x\right]=\exp\left[-i\int\bar{\psi}(\gamma_\mu\partial_\mu+m_0)\psi\,d^4x\right]\frac{\int\exp[\frac{1}{2}i\int\phi(\Box-\mu_0^2)\phi\,d^4x+g_0\int\bar{\psi}\gamma_5\psi\phi\,d^4x-i\lambda_0\int\phi^4\,d^4x]\delta\phi}{\int\exp[\frac{1}{2}i\int\phi(\Box-\mu_0^2)\phi\,d^4x]\delta\phi}. \quad (13)$$

Making the change of variables

$$\phi'=\mu_0\phi, \quad (14)$$

Eq. (13) becomes

$$\exp\left[i\int L_\phi(\psi,\bar{\psi})\,d^4x\right]=\exp\left[-i\int\bar{\psi}(\gamma_\mu\partial_\mu+m_0)\psi\,d^4x\right]$$

$$\times\frac{\int\exp[\frac{1}{2}i\int\phi'(\Box/\mu_0^2-1)\phi'\,d^4x+(g_0/\mu_0)\int\bar{\psi}\gamma_5\phi'\,d^4x-(i\lambda_0/\mu_0^4)\int\phi'^4\,d^4x]\delta\phi'}{\int\exp[\frac{1}{2}i\int\phi'(\Box/\mu_0^2-1)\phi'\,d^4x]\delta\phi'}. \quad (15)$$

Requiring $Z_3=0$ and the renormalized boson mass to be finite is equivalent to the restrictions given in Eqs. (6) and (7). Two separate cases appear in satisfying these restrictions:

(i) $\mu_0\rightarrow\infty$ in such a way that $\lambda_0/\mu_0^4\rightarrow 0$,

(ii) $\mu_0,\lambda_0\rightarrow\infty$ in such a way that $\lambda_0/\mu_0^4\rightarrow g_2>0$.

If we consider case (i) then Eq. (15) reduces to

$$\exp\left[i\int L_\phi(\psi,\bar{\psi})\,d^4x\right]=\exp\left[-i\int\bar{\psi}(\gamma_\mu\partial_\mu+m_0)\psi\,d^4x\right]\frac{\int\exp[-\frac{1}{2}i\int\phi'^2\,d^4x-ig_1^{1/2}\int\bar{\psi}\gamma_5\psi\phi'\,d^4x]\delta\phi'}{\int\exp[-\frac{1}{2}i\int\phi'^2\,d^4x]\delta\phi'}. \quad (16)$$

This is exactly equivalent to Eq. (5) so our problem in case (i) has reduced to that of Sec. II. The Matthews term is effectively absent in this case.

If we consider case (ii) Eq. (15) becomes

$$\exp\left[i\int L_\phi(\psi,\bar{\psi})\,d^4x\right]=\exp\left[-i\int\bar{\psi}(\gamma_\mu\partial_\mu+m_0)\psi\,d^4x\right]\frac{\int\exp[-\frac{1}{2}i\int\phi'^2\,d^4x+g_1\int\bar{\psi}\gamma_5\psi\phi'\,d^4x-\frac{1}{4}ig_2\int\phi'^4\,d^4x]\delta\phi'}{\int\exp[-\frac{1}{2}i\int\phi'^2\,d^4x]\delta\phi'}. \quad (17)$$

To obtain an expression for $L_\phi(\psi,\bar\psi)$ defined by Eq. (17) we must solve the functional integral

$$I=\frac{\int \exp[-\int(\frac{1}{2}\phi^2+ig_1\bar\psi\gamma_5\psi\phi+\frac{1}{4}g_2\phi^4)\,d^4x]\delta\phi}{\int \exp[-\int\frac{1}{2}\phi^2\,d^4x]\delta\phi}. \quad (18)$$

The solution to this integral can be obtained by means of a procedure analogous to the method of stationary phases.[9] However, because of the imaginary term $ig_1\bar\psi\gamma_5\psi\phi$ in the exponent of Eq. (18) this method should more appropriately be called the method of steepest descent. The highly nonlinear term $\frac{1}{4}g_2\phi^4$ in Eq. (18) prohibits one from solving this integral in closed form. In the following we will solve the functional integral in Eq. (18) by the method of stationary phases, keeping only the first few terms.

Before proceeding with the solution of Eq. (18), a few remarks are appropriate concerning the meaning of the functional integrals appearing in both Eq. (8) and (18). They are of the form

$$I=\int e^{-iS(\phi)}\delta\phi, \quad (19)$$

where

$$S(\phi)=\int \mathfrak{L}(\phi)\,d^4x.$$

This functional integral is just a generalization of the Feynman path integral,[10] whereas the Feynman path integral is not an integral in the mathematical sense,[11] but can properly be defined through the Wiener integral[12] by a rotation of the time coordinate into the imaginary axis. We find ourselves forced to take a similar approach here in giving the generalized Feynman functional integral meaning. We define the functional integral in Eq. (19) to be related to the generalized Wiener integral defined by Friedrichs and Shapiro[13,14] by a rotation of the time coordinate into the imaginary axis.[15] Upon rotating the time axis, Eq. (19) becomes

$$I=\int e^{-S(\phi)}\delta\phi. \quad (20)$$

Equation (20) yields a well-defined integral and is what will implicitly be implied when we write integrals of the form given by Eq. (19). In solving Eq. (20) by the method of stationary phase, we expand $S(\phi)$ about that field ϕ_c that makes $S(\phi)$ an extremum[16]:

$$S(\phi)=S(\phi_c)+\frac{1}{2}\int\frac{\delta^2S(\phi_c)}{\delta\phi\delta\phi}\phi(x)\phi(y)\,d^4x\,d^4y+\cdots. \quad (21)$$

In general there will be many fields that make $S(\phi)$ an extremum, however, the one that will give the most contribution will be the one that makes $\mathrm{Re}S(\phi_c)$ the smallest.

Keeping only the first two terms in the expansion of $S(\phi)$ and substituting this into Eq. (20), we get

$$I=e^{-S(\phi_c)}\int \exp\left[-\frac{1}{2}\int\frac{\delta^2S(\phi_c)}{\delta\phi\delta\phi}\phi(x)\phi(y)\,d^4x\,d^4y\right]\delta\phi. \quad (22)$$

Applying the method of stationary phase to Eq. (19), we must first find the field $\phi_c(x)$ that makes $S(\phi)$ an extremum. These stationary fields are given by

$$-\delta S/\delta\phi=0=\phi+g_2\phi^3+ig_1\bar\psi\gamma_5\psi. \quad (23)$$

Now this is a cubic equation and has three roots. The three roots are[17]

$$\begin{aligned}\phi_1&=A+B,\\ \phi_2&=-\tfrac{1}{2}(A+B)+i(\tfrac{1}{2}(A+B))\sqrt{3},\\ \phi_3&=-\tfrac{1}{2}(A+B)-i(\tfrac{1}{2}(A-B))\sqrt{3},\end{aligned} \quad (24)$$

where

$$\begin{aligned}A&=\left\{\frac{g_1}{g_2}\bar\psi\gamma_5\psi+\left[\left(\frac{g_1}{2g_2}\bar\psi\gamma_5\psi\right)^2-\left(\frac{1}{3g_2}\right)^3\right]^{1/2}\right\}^{1/3},\\ B&=\left\{\frac{g_1}{g_2}\bar\psi\gamma_5\psi-\left[\left(\frac{g_1}{2g_2}\bar\psi\gamma_5\psi\right)^2-\left(\frac{1}{3g_2}\right)^3\right]^{1/2}\right\}^{1/3}.\end{aligned} \quad (25)$$

Depending on the values of the coupling, two cases of interest exist:

(i) Strong coupling

$$(27/4)g_1^2g_2(\bar\psi\gamma_5\psi)^2>1; \quad (26)$$

(ii) weak coupling

$$(27/4)g_1^2g_2(\bar\psi\gamma_5\psi)^2<1. \quad (27)$$

The ψ and $\bar\psi$ do not represent operators but are variables of integration that appear in the integrand of the functional integrals [c.f. Eq. (2) of Ref. 8]. We see that the inequality in Eq. (26) is satisfied for most values of ψ and $\bar\psi$ if $g_1^2g_2$ is sufficiently large. Likewise the inequality in Eq. (27) is satisfied for most values of $\bar\psi$ and ψ if $g_1^2g_2$ is small.

Let us first discuss the case of weak coupling. The stationary points given by Eq. (24) can be put into the

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form

$$\phi_1 = -2i(1/3g_2)^{1/2}\cos(\tfrac{1}{3}\alpha), \quad (28a)$$

$$\phi_2 = +2i(1/3g_2)^{1/2}\cos(\tfrac{1}{3}\alpha+\tfrac{1}{3}\pi), \quad (28b)$$

$$\phi_3 = +2i(1/3g_2)^{1/2}\cos(\tfrac{1}{3}\alpha-\tfrac{1}{3}\pi), \quad (28c)$$

where

$$\cos\alpha = (27g_1^2g_2/4)^{1/2}\bar{\psi}\gamma_5\psi. \quad (29)$$

Now the stationary point that gives the maximum contribution to the functional integral in Eq. (18) is given by Eq. (28b). Expanding the exponent in the numerator of Eq. (18) about ϕ_2 and keeping only the first two terms of the expansion as done in Eq. (22), cancellation will occur and we get

$$I = \exp\left\{-i\int [-(g_1^2/3g_2)^{1/2}\bar{\psi}\gamma_5\psi\cos(\tfrac{1}{3}\alpha+\tfrac{1}{3}\pi) - (4/9g_2)\cos^4(\tfrac{1}{3}\alpha+\tfrac{1}{3}\pi)]d^4x\right\}. \quad (30)$$

Substituting Eq. (30) into Eq. (17) we obtain the desired Lagrangian density in the weak-coupling limit:

$$L_\phi(\psi,\bar{\psi}) = -\bar{\psi}(\gamma_\mu\partial_\mu+m_0)\psi - (g_1^2/3g_2)^{1/2}\bar{\psi}\gamma_5\psi \times\cos(\tfrac{1}{3}\alpha+\tfrac{1}{3}\pi) - (4/9g_2)\cos^4(\tfrac{1}{3}\alpha+\pi\tfrac{1}{3}), \quad (31)$$

where

$$\alpha = \cos^{-1}[(27g_1^2g_2/4)^{1/2}\bar{\psi}\gamma_5\psi]. \quad (32)$$

In the limit of $g_2 \to 0$, Eq. (31) should reduce to Eq. (11). Indeed, this is true since, as $g_2 \to 0$,

$$\cos(\tfrac{1}{3}\alpha+\tfrac{1}{3}\pi) \to (\tfrac{3}{4}g_1^2g_2)\bar{\psi}\gamma_5\psi, \quad (33)$$

and substituting Eq. (33) into Eq. (31) gives

$$L_\phi(\psi\bar{\psi}) = -\bar{\psi}(\gamma_\mu\partial_\mu+m_0)\psi + \tfrac{1}{2}g_1^2(\bar{\psi}\gamma_5\psi)(\bar{\psi}\gamma_5\psi) \quad (34)$$

which is in agreement with Eq. (11).

Now let us consider the strong-coupling case. In this case the stationary fields can be expressed as

$$\phi_1 = +2i(3g_2)^{-1/2}\csc(2\alpha), \quad (35a)$$

$$\phi_2 = -i(3g_2)^{-1/2}[\csc(2\alpha)+i\sqrt{3}\cot(2\alpha)], \quad (35b)$$

$$\phi_3 = -i(3g_2)^{-1/2}[\csc(2\alpha)-i\sqrt{3}\cot(2\alpha)], \quad (35c)$$

where

$$\tan\alpha = [\tan(\tfrac{1}{2}\beta)]^{1/3},$$

and

$$\sin\beta = \left(\frac{4}{27g_2g_1}\right)^{1/2}\frac{1}{\bar{\psi}\gamma_5\psi}. \quad (36)$$

Equation (35b) gives the stationary field that gives the maximum contribution to the functional integral in Eq. (18). So, as before, we substitute Eq. (35b) into Eq. (18) to give for $L_\phi(\psi,\bar{\psi})$ in the strong-coupling case

$$L_\phi(\psi,\bar{\psi}) = -\bar{\psi}(\gamma_\mu\partial_\mu+m_0)\psi + \left(\frac{g_1^2}{12g_2}\right)^{1/2}\bar{\psi}\gamma_5\psi\{\csc(2\alpha)+i\sqrt{3}\cot(2\alpha)\} - \frac{1}{36g_2}\{\csc(2\alpha)+i\sqrt{3}\cot(2\alpha)\}^4, \quad (37)$$

where α is defined by Eq. (36).

The pion will appear as a bound state of Eqs. (31) and (37).

IV. CONCLUSION

The evasiveness of elementary particles has led to the speculation that no particle is any more elementary than another. The problem then exists of finding the Lagrangian density when various fields do not appear in it but occur as bound states. A method for constructing such Lagrangian densities was developed in Ref. 8. This procedure was applied in Sec. II to the problem of a pion and nucleon interacting by means of a Yukawa coupling. For $Z_3=0$ we established that an equivalent theory can be constructed from $L_\phi(\psi\bar{\psi})$ defined by Eq. (11). The pion appears as a bound state of this Lagrangian density.

This example neglects the self-interaction of the pions. This effect can be properly taken into account by considering Eq. (12). The construction of $L_\phi(\psi\bar{\psi})$ becomes more difficult in this case. The exact solution is given in terms of a functional integral defined by Eq. (17). This functional integral is solved to give an expression for $L_\phi(\psi,\bar{\psi})$ in two cases. The case of strong coupling is given by Eq. (37) and the weak-coupling case by Eq. (31). Both of these Lagrangian densities are highly nonlinear and consequently much of their utility is lost because of the ineffective methods available to solve nonlinear problems.

Consideration of the Lagrangian density defined in Eqs. (31) and (37) certainly imply that the four-fermion interaction is an oversimplification of the pion-nucleon interaction. Also the four-fermion interaction considered in the superconducting theory of Nambu and Jona-Lasinio and the spinor theory of Heisenberg may be much too simple to achieve their aims.

The problem considered here still does not take into account the interactions of the various other particles. Including these interactions in Eq. (17) should certainly give us a more realistic expression for $L_\phi(\psi,\bar{\psi})$ but it has its obvious complications.