from (21) for the case of no collisions (Z=0) and no magnetic field $(Y_L=0)$.

Clemmow¹⁰ has investigated the dispersion relation governing the propagation of plane electromagnetic waves in an infinite, homogeneous medium with a refractive index of fixed value $\sqrt{\epsilon_R}$, traversed by an infinite, uniform, steady, neutral stream of charged particles, where $\epsilon_R = \epsilon/\epsilon_0$. One could use the same method as above by assuming that we have an infinite medium of dielectric constant ϵ instead of ϵ_0 . The corresponding plasma frequency in the moving coordinate system S' will be

$$\omega_{N'^{2}} = \frac{e^{2}N'}{\epsilon m'} = \frac{e^{2}N'}{\epsilon_{R}\epsilon_{0}m'} = \frac{\omega_{N0'^{2}}}{\epsilon_{R}}, \qquad (A4)$$

¹⁰ P. C. Clemmow, Proc. Phys. Soc. (London) 80, 1322 (1962).

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where ω_{N0}' is referred to free space and is invariant. Substituting (A4) into (A2) taking $c/\sqrt{\epsilon_R}$ as the velocity of light everywhere, one has

$$\omega^{\prime 2} - (c^2/\epsilon_R)k^{\prime 2} = \omega^2 - (c^2/\epsilon_R)k^2$$
$$= (\omega_N 0^{\prime 2}/\epsilon_R) = (\omega_N 0^2/\epsilon_R), \quad (A5)$$

which may be rewritten in the form

$$c^2 k^2 = \epsilon_R \omega^2 - \omega_{N0}^2, \qquad (A6)$$

where ω_{N0} is the plasma frequency of the moving plasma in the laboratory system of coordinates S as referred to free space. Equation (A6) can be made identical with the one found by Clemmow¹⁰ by a different method.

3 JUNE 1966

Metastable ${}^{3}P_{2}$ Rare-Gas Polarizabilities*†

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The atomic beam E-H gradient balance method has been used to measure the zz components $\alpha_{zz}(M_J)$ of the diagonalized polarizability tensors in metastable ${}^{3}P_{2}$ neon, krypton, and xenon, in their $M_{J} = +1$ and +2 magnetic substates. These data are sufficient to determine the polarizability tensors in all the substates, as well as the spherically averaged polarizabilities $\bar{\alpha}$. The gross structure of each of the metastable rare gases is similar to that of the ground state of the corresponding alkali, and it is found that the average polarizabilities are comparable. The polarizability tensor components and $\bar{\alpha}$'s are listed in the table below (the values for argon are taken from the earlier measurements of Pollack et al.), together with theoretical values computed using a modification of Sternheimer's method (theory A) and from estimates made of oscillator strengths using the Coulomb approximation of Bates and Damgaard (theory B).

	Polari			
	Neon	Argon	Krypton	Xenon
$\alpha_{zz}(1)$ (Exptl.) $\alpha_{zz}(2)$ (Exptl.) $\overline{\alpha}$ (Exptl.) $\overline{\alpha}$ (Theory A) $\overline{\alpha}$ (Theory B)	$\begin{array}{r} 28.0 \pm 1.4 \\ 26.7 \pm 1.3 \\ 27.6 \pm 1.0 \\ 29.6 \\ 27.8 \end{array}$	$50.5\pm3.544.5\pm3.148.4\pm2.650.548.1$	$53.7 \pm 2.7 \\ 46.7 \pm 2.3 \\ 51.4 \pm 2.0 \\ 59.9 \\ 53.5$	$\begin{array}{c} 68.2{\pm}3.4\\ 56.8{\pm}2.8\\ 64.4{\pm}2.4\\ 78.2\\ 62.5\end{array}$

INTRODUCTION

HE E-H gradient balance method has previously been used to measure alkali polarizabilities¹ and to investigate the polarizability tensor of $3p^{5}4s^{3}P_{2}$

metastable argon.² For metastable argon, the average polarizability was determined to be 48.4×10⁻²⁴ cm³, and the ratio of the diagonalized tensor components $\alpha_{zz}(1)/\alpha_{zz}(2)$ was determined to be $1.13\pm7\%$ (z is the field direction). We report here similar measurements carried out on the metastable ${}^{3}P_{2}$ states of the other rare gases, together with theoretical estimates of the average polarizabilities of the entire set.

The polarizability tensor is defined by its components

$$\alpha_{ij} = 2 \operatorname{Re} \sum_{k \neq 0} \frac{\langle 0|i|k\rangle \langle k|j|0\rangle}{E_k - E_0}; \quad i, j = x, y, z, \quad (1)$$

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[†] Based on a dissertation submitted by E. J. Robinson to the Graduate Faculty of New York University in partial fulfillment of the requirements for the Ph.D. degree, October 1964.

[‡] National Science Foundation Science Faculty Fellow, 1963-1964.

National Science Foundation Graduate Fellow, 1963–1966. A. Salop, E. Pollack, and B. Bederson, Phys. Rev. 124, 1431 (1961).

² E. Pollack, E. J. Robinson, and B. Bederson, Phys. Rev. 134, A1210 (1964).

where

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$$\langle 0|i|k \rangle = \langle 0|\sum_{p=1}^{N} x_p|k \rangle$$
, etc.,

and the summation extends over all the electrons in the atoms (atomic units are used). α_{ij} may be written as the sum of two parts

$$\alpha_{ij} = \bar{\alpha} \delta_{ij} + \alpha_{ij}', \qquad (2)$$

where $\bar{\alpha}$ is the average polarizability, expressible in terms of the oscillator strength summation,

$$\bar{\alpha} = \sum_{k \neq 0} f_{0k} / (E_k - E_0)^2$$
 (3)

and α_{ij}' is a component of a traceless, symmetric second-rank tensor, and is defined according to the prescription

$$\alpha_{ij}' = \frac{1}{3} \left\{ \sum_{k \neq 0} \left(\frac{3\langle 0|i|k \rangle \langle k|j|0 \rangle}{E_k - E_0} + \frac{3\langle 0|j|k \rangle \langle k|i|0 \rangle}{E_k - E_0} - \frac{2\langle 0|\sum_{p=1}^{N} \mathbf{r}_p|k \rangle \langle k|\sum_{p=1}^{N} \mathbf{r}_p|0 \rangle}{E_k - E_0} - \frac{2\langle 0|\sum_{p=1}^{N} \mathbf{r}_p|k \rangle \langle k|\sum_{p=1}^{N} \mathbf{r}_p|0 \rangle}{E_k - E_0} \right\}.$$
 (4)

This tensor possesses the same transformation properties as the quadrupole-moment tensor, so that a single number is sufficient to completely describe it. In analogy with the scalar quadrupole moment, a scalar "polarizability anisotropy" is defined $\alpha' = \alpha_{zz}'|_{M_J=J}$. Thus, two numbers completely specify the polarizability tensors for all the magnetic substates, and a measurement of one pair of independent tensor components is sufficient to determine these. In the argon experiment and the present investigation, the experimental quantities are the zz components of the diagonalized tensors of the $M_J = +1$ and $M_J = +2$ substates. In terms of the measurements, $\bar{\alpha}$, α' , and the other components of $\alpha_{ij}(M_J)$ of the diagonalized tensors are

$$\bar{\alpha} = \frac{1}{3} \left[\alpha_{zz}(2) + 2\alpha_{zz}(1) \right],$$

$$\alpha' = \frac{2}{3} \left[\alpha_{zz}(2) - \alpha_{zz}(1) \right],$$

$$\alpha_{zz}(0) = \frac{1}{3} \left[4\alpha_{zz}(1) - \alpha_{zz}(2) \right],$$

$$\alpha_{xx}(0) = \alpha_{yy}(0) = \frac{1}{3} \left[\alpha_{zz}(1) + 2\alpha_{zz}(2) \right],$$

$$\alpha_{xx}(1) = \alpha_{yy}(1) = \frac{1}{2} \left[\alpha_{zz}(1) + \alpha_{zz}(2) \right],$$

$$\alpha_{xx}(2) = \alpha_{yy}(2) = \alpha_{zz}(1).$$
(5)

(The + and - have been omitted from M_J since the polarizabilities are quadratic in M_J .)

EXPERIMENTAL TECHNIQUE

The apparatus, shown schematically in Fig. 1, is described in detail in Refs. 1 and 2 and will be only briefly discussed here. A beam of metastable atoms is formed when one of the rare gases effuses into a gun where electron bombardment excites a small fraction to the ${}^{3}P_{2}$ and ${}^{3}P_{0}$ states. The beam passes through an exit slit on the gun (designed to eliminate beam broadening due to recoil), enters a region of combined inhomogeneous electric and magnetic fields through a collimating slit, and is detected by the Auger ejection of electrons from the tungsten cathode of a Bendix electron multiplier. The output pulses of the multiplier are fed to a scaler and counted. The half-width of the beam in the detector plane is 0.007 in. (0.002-in. slits are used throughout the apparatus) and the beam height in the field region is limited to 0.218 in. by suitable slits.

The field region consists of a 0.056-in. gap between two soft-iron magnetic pole faces machined to conform to the equipotentials of a two-wire field. The pole faces also serve as electrodes for an inhomogeneous electric field. Since the fields satisfy essentially equivalent boundary conditions if magnetic saturation effects are avoided, E and H are congruent, i.e., they are everywhere proportional and parallel.

The fields are in the z direction and establish the z axis as a symmetry axis for the atomic systems. The polarizability tensors are therefore diagonal with respect to the field direction and a set of arbitrarily chosen x and y axes.

It is then shown in Ref. 1 that the electric and magnetic fields can be adjusted so that for negative effective magnetic moment substates it is possible to balance the induced electric dipole force against the magnetic dipole force. When this E-H balance condition prevails

$$\alpha_{zz}(M_J) = \mu_{\rm eff} H/E^2. \tag{6}$$

In the alkali experiment, H was measured by the zero-moment technique,³ and E calculated from the applied voltage and pole face contour. For the meta-stable rare gases, $\alpha_{zz}(1)$ and $\alpha_{zz}(2)$ were normalized to the polarizability of metastable ${}^{3}S_{1}$ helium, which has been calculated to an accuracy of approximately 1% to be 46.6×10^{-24} cm^{3.4,5}

The polarizabilities were normalized to that of helium by maintaining the magnetic field fixed and obtaining balances on the $M_J = +1$ and +2 substates of the metastable ${}^{8}P_{2}$ rare gas under investigation and the $M_J = +1$ substate of metastable helium. Applying Eq. (6) to the three balanced substates, $\alpha_{zz}(1)$ and $\alpha_{zz}(2)$ are expressible in terms of α (He), the polarizability of metastable helium, and the corresponding balance voltages and magnetic moments, according to the relations

$$\alpha_{zz}(M_J) = \alpha(\text{He}) \frac{\mu_{\text{eff}}(M_J)}{\mu_{\text{eff}}(\text{He})} \left(\frac{V(\text{He})}{V(M_J)} \right)^2.$$
(7a)

³ V. W. Cohen, Phys. Rev. 46, 713 (1934).

⁴ Charles Schwartz (private communication). ⁵ A. Dalgarno and A. E. Kingston, Proc. Phys. Soc. (London)

^{72, 1053 (1958).}



Similarly

per minute.)

$$\alpha_{zz}(1)/\alpha_{zz}(2) = \frac{1}{2} (V(2)/V(1))^2.$$
 (7b)

These measurements are thus independent of the value of the constant magnetic field.

MEASUREMENTS

Data are accumulated in the following manner. A beam of metastable atoms is produced and the magnetic field is set at a convenient value. The field deflects atoms with nonvanishing effective magnetic moments, and the $M_J = 0$ states of ${}^{3}P_2$ and ${}^{3}P_0$ constitute the bulk of the signal. The electric field is turned on, and its value increased upward from zero. This deflects the zero-moment magnetic substates out of the beam, increases the deflection of the negative M_J states, and decreases the deflection of the positive M_J states. As the electric field is further increased, one and then the other of the positive M_J states is brought into balance, and deflected through the undeflected beam position. As an example, a plot of signal against applied voltage is shown in Fig. 2 for krypton at a magnetic field of approximately 314 G. It is seen that the first substate

is in balance at about 12 kV, and the second at about 18 kV. Beyond 18 kV, the beam decreases monotonically with increasing voltage.

After locating the approximate positions of the peaks, these regions were more closely explored in steps of 100 V. Two criteria are used in determining the position of the balance peaks. The counting rate at the peak must be approximately two standard deviations greater than the counting rate at points 100 V above and below the assumed balance voltage, and the plot of signal versus voltage must be a smooth curve. The first requirement reduces the probability that purely statistical errors distort the result, and the second tends to assure that other short-term fluctuations do not introduce appreciable errors into the experiment. In some cases, more than 25 runs were required before a definite maximum could be established. In a few cases it proved impossible to obtain the twostandard-deviations difference and the balance voltage was assigned the value midway between the two highest points, provided each had a counting rate two standard deviations higher than the rest, and they were only 100 V apart. A plot of signal versus voltage



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	Neon	Argon ^a	Krypton	Xenon
$\begin{array}{c} \alpha_{zz}(1) (\text{exptl.}) \\ \alpha_{zz}(2) (\text{exptl.}) \\ \left[\alpha_{zz}(1)/\alpha_{zz}(2)\right] (\text{exptl.}) \\ \alpha_{zz}(0) (\text{exptl.}) \\ \alpha_{xx}(0) (\text{exptl.}) \\ \alpha_{xx}(1) (\text{exptl.}) \\ \alpha_{xx}(2) (\text{exptl.}) \\ \alpha'(\text{exptl.}) \\ \overline{\alpha} (\text{exptl.}) \\ \overline{\alpha} (\text{Theory A})^{\text{b}} \\ \overline{\alpha} (\text{Theory B})^{\circ} \end{array}$	$\begin{array}{c} 28.0 \pm 1.4 \\ 26.7 \pm 1.3 \\ 1.04 \pm 0.05 \\ 28.4 \pm 1.9 \\ 27.1 \pm 1.0 \\ 27.4 \pm 1.0 \\ 28.0 \pm 1.4 \\ -0.9 \pm 1.1 \\ 27.6 \pm 1.0 \\ 29.6 \\ 27.8 \end{array}$	$50.4 \pm 3.5 \\ 44.5 \pm 3.1 \\ 1.13 \pm 0.08 \\ 52.4 \pm 4.8 \\ 46.4 \pm 2.4 \\ 47.25 \pm 2.3 \\ 50.4 \pm 3.5 \\ -3.9 \pm 2.4 \\ 48.4 \pm 2.6 \\ 50.5 \\ 48.1 \\ 1$	$53.7 \pm 2.7 \\ 46.7 \pm 2.3 \\ 1.15 \pm 0.06 \\ 56.0 \pm 3.7 \\ 49.0 \pm 1.8 \\ 50.2 \pm 1.8 \\ 53.7 \pm 2.7 \\ -4.7 \pm 1.9 \\ 51.4 \pm 2.0 \\ 59.9 \\ 53.5 \\ \end{bmatrix}$	$\begin{array}{c} 68.2 \pm 3.4 \\ 56.8 \pm 2.8 \\ 1.20 \pm 0.06 \\ 72.0 \pm 4.6 \\ 60.6 \pm 2.2 \\ 62.5 \pm 2.2 \\ 68.2 \pm 3.4 \\ -7.6 \pm 2.3 \\ 64.4 \pm 2.4 \\ 78.2 \\ 62.5 \end{array}$

TABLE I. Summary of measured and calculated polarizabilities. All polarizability values are in units of 10⁻²⁴ cm³.

See Ref. 2 for argon experimental values

 \overline{a} becomes the second sternheimer method. The effect of the core polarizability on \overline{a} is not included in the quoted values. These corrections are always negative, they tend to reduce \overline{a} . However, accurate values of the ion polarizabilities are not available, and hence quantitive values for these corrections are For the stimate $n p^{s}(n+1)p$ configuration by Bates and Damgaard approximation. difficult

in the region of balance of the $M_J = +2$ substate of krypton at approximately 314 G is shown in Fig. 3.

When data were obtained for both substates of the rare gas being studied, the process was repeated for metastable helium. Equation (6) enables $\alpha_{zz}(1)$, $\alpha_{zz}(2)$, and their ratio to be extracted from the balance voltages. The measurements are summarized in Table I under the headings $\alpha_{zz}(1)$ (exptl.), etc.

The polarizability components in Table I are correct only if the first balance peak is the $M_J = +1$ substate and the second peak the $M_J = +2$ substate. If the reverse were true, then the polarizability ratios $\alpha_{zz}(2)/$ $\alpha_{zz}(1)$ would be 4.2, 4.6, and 4.8 for neon, krypton, and



FIG. 3. Beam intensity as a function of applied voltage for metastable krypton, at a magnetic field of about 314 G, in the vicinity of $M_i = +2$ balance peak.

xenon, respectively. It is very unlikely that this ratio should differ so drastically from unity, since the outer electron which accounts for an overwhelmingly large percentage of the polarizability is in an s-orbital in the zero-order scheme, and would therefore be expected to contribute nearly equal polarizability components. Furthermore, a ratio $\alpha_{zz}(2)/\alpha_{zz}(1) > 4$ would mean, according to Eq. (5), that $\alpha_{zz}(0)$ was negative, which is physically impossible. However, if the error in locating the balance peaks is large enough, it is possible that a ratio $\alpha_{zz}(2)/\alpha_{zz}(1)$ slightly smaller than 4 could be inferred from the data, so that the second argument would not apply. Only for neon does this possibility lie within the estimated experimental error in the balance voltages. These remarks were confirmed by a subsidiary "effective small shift" experiment in which the substates were unambiguously identified (see Ref. 2).

The requirement that the counting rate at each maximum be two standard deviations greater than at points 100 V on either side makes possible estimates of the probability of a given error due to shot noise. It turns out, assuming shot noise alone, that the probability that the counting rate of either side point exceeds that of the assumed peak is only about 20%. (This for an error of ± 100 V.) An assignment of ± 125 V uncertainty to shot noise is thus quite conservative.

Uncertainties of ± 125 V are also assigned to the voltmeter calibration error and to the nonstatistical beam fluctuations. Although the Sensitive Research Company cites $\pm 1\%$ of full scale as the calibration accuracy of the electrostatic voltmeter, calibrations performed by both the manufacturer and the National Bureau of Standards over a period of several years showed the instrument to be correct to within ± 40 V at all cardinal points of the meter range used in the experiment. It therefore seems safe to claim a calibration of $\pm \frac{1}{2}$ % of full scale. There is an apparent shift of the maxima due to contributions to the signal from atoms in unbalanced substates with velocities large enough to prevent them from suffering large deflections.

We estimated that an over-all systematic error no greater than 1% is introduced by this effect.

Multiplier noise was typically only a few counts per minute, small compared to the rms error in the signal integrated over a 30-sec. interval. It is consequently assumed that multiplier noise makes a negligible contribution to experimental error.

Combining the random errors by the rms method, and taking account of the two independent runs one finds an uncertainty in α_{zz} for the $M_J = +1$ substate of neon (the worst case) to be $\pm 5\%$. The other polarizabilities turn out to have somewhat smaller errors but, in view of the fact that some of the main errors could be only rather roughly estimated, no greater claim is made for these. It is thus asserted that the measurements of the polarizability components $\alpha_{zz}(1)$ and $\alpha_{zz}(2)$, as well as their ratios $\alpha_{zz}(1)/\alpha_{zz}(2)$ have been measured to within $\pm 5\%$. The uncertainties in Table I quoted for the other polarizability parameters are computed from the assumed uncertainties in the measured components.

DISCUSSION OF MEASUREMENTS

The polarizability values listed in Table I reveal a number of interesting features. For each of the gases $\bar{\alpha}$ is more than an order of magnitude greater than the polarizability of the ground-state atom, verifying that it is the (n+1)s electron which makes the most significant contribution. As expected, the average polarizabilities increase with atomic number, as does the ratio $\alpha_{zz}(1)/\alpha_{zz}(2).$

Most of the average polarizability of the p^5 core appears to arise from excitations to d orbitals, and with this assumption it can be shown that $|\alpha'|$ of the core cannot exceed about 10% of the core average polarizability. Thus, the core can directly account for a ratio $\alpha'/\bar{\alpha}$ of perhaps 1% or so, much smaller than the observed ratio. Of greater importance than direct core effects should be the fact that the electrostatic and spin-orbit interactions remove the degeneracy of the $np^5n'p$ levels occurring in the zeroth-order scheme, and an anisotropy results from the variation among the energy denominators to a given configuration occurring in the summations of Eq. (1). However, only a fraction of the observed $\alpha_{zz}(1)/\alpha_{zz}(2)$ can be accounted for in this way in all the rare gases. There is no observed sharp increase with atomic number (on the contrary, the fractional effect in neon is greater than that in argon), and there is only a very weak dependence on the type of coupling. Consequently, it is suggested that the anisotropy arises chiefly from deviations from the $np^{5}(n+1)s$ representation. There are two ways in which configuration mixing can give rise to the observed increase of $\alpha_{zz}(1)/\alpha_{zz}(2)$ with atomic number. First, the approximation of a central field should be better in the lighter rare gases. Second, even in the hydrogen atom, there is a rise of the fractional anisotropy of p and delectrons with increasing principal quantum number, so that even if there were the same percentage of mixing of configurations possessing outer electrons with nonvanishing orbital angular momentum, it is reasonable to suppose that these will provide greater fractional anisotropy in the heavier gases, particularly if it should turn out that the admixed configurations in the heavier atom make a generally larger fractional contribution to the average polarizability. This conjecture can be tested only by calculations which employ such configurationsuperposed wave functions.

Calculations of the average polarizabilities can be performed with much simpler wave functions, however. Such estimates are made in a later section.

It was expected that $\bar{\alpha}$ of each of the rare-gas metastables be large, since the gross structure of each is similar to that of the adjacent alkali in the periodic table, that is, a single s electron of a given principal quantum number plus a singly ionized p core of nsmaller by unity. In addition, the electrons in the metastables move in a field which supplies somewhat less binding, so that the rare-gas average polarizability should be greater than that of the corresponding alkali. This is verified by the experiment, with the average polarizability of metastable neon being 38% greater than that of ground-state sodium, and the average polarizability of ${}^{3}P_{2}$ xenon being 23% more than that for cesium.

CALCULATIONS

The strong similarity that exists between the gross structures of the metastable rare gases and the alkali ground states suggests that methods which have proved successful in computing alkali polarizabilities are also appropriate for the calculation of α for the metastable rare gases.

Sternheimer⁶⁻⁸ has developed a procedure for calculating polarizabilities which employs a numerical solution of the first-order Schrödinger equation to implicitly evaluate the summation in Eq. (3). In his alkali calculations, Sternheimer explicitly obtained values for the contribution to the polarizability of the valence electrons, using wave functions devised to approximately reproduce the experimental ionization potentials. The empirical core polarizabilities were added to this, after estimating the effect of the electric field at the position of the core due to the dipole moment induced in the valence-electron charge distribution.8

A variation of Sternheimer's technique is used here to compute the contribution of the valence electrons to the (average) polarizabilities of the ${}^{3}P_{2}$ metastable rare gases, using a single-electron wave function whose energy eigenvalue E_0 is equal to the average ionization potential of the $np^{5}(n+1)s$ configuration. The wave function is obtained by solving the Schrödinger equation

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⁶ R. M. Sternheimer, Phys. Rev. 96, 951 (1954).

 ⁷ R. M. Sternheimer, Phys. Rev. 115, 1198 (1959).
 ⁸ R. M. Sternheimer, Phys. Rev. 127, 1220 (1962).

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in a potential (in rydberg) given by

$$V = V_{\rm HFS} - \alpha_c / (r^2 + r_0^2)^2, \qquad (8)$$

where $V_{\rm HFS}$ is the Hartree-Fock-Slater potential of the rare-gas ground state given by Herman and Skillman,9 and α_c is an estimate of the average core polarizability. The parameter r_0 is adjusted to give the desired energy.

To compute the polarizability of any of the levels in the configuration, it is assumed that the dipole matrix elements do not differ appreciably from those computed from eigenstates of V and that the energy denominators are given to sufficient accuracy by $E_k - E_0 - \Delta$, where E_k, E_0 are the energies of $|k\rangle$ and $|0\rangle$ as computed from Eq. (8), and Δ is the difference between the metastable energy and E_0 . With these simplifications, the valenceelectron polarizability (in a_0^3) is given by

$$\alpha = \frac{4}{3} \int U_0(r) r U_1(r) \, dr \,, \qquad (9a)$$

where U_0 satisfies

$$(-d^2/dr^2 + V - E_0)U_0(r) = 0$$
 (9b)

and U_1 is a solution of

$$(-d^2/dr^2+2/r^2+V-E_0-\Delta)U_1=rU_0.$$
 (9c)

Numerical calculations were performed with a Control Date Corporation Model 1604 computer using a modified version of a program written to compute twophoton transition probabilities.¹⁰

As a check, the polarizabilities of the levels of 1s2shelium were computed. The values obtained for ${}^{3}S_{1}(318 a_{0}{}^{3})$ and ${}^{1}S_{0}(849 a_{0}{}^{3})$ lie within 2% and 8% of earlier calculations. (See Refs. 4 and 5.) The polarizabilities of the ${}^{3}P_{2}$ metastable rare gases found this way are 29.6, 50.5, 59.9, and 78.2, in units of 10⁻²⁴ cm³, for neon, argon, krypton, and xenon. These are listed in Table I under " $\bar{\alpha}$ (Theory A)."

Alkali polarizabilities have been calculated by Dalgarno and Kingston¹¹ using the oscillator strength summation Eq. (3) and a fit of existing oscillator strengths to a set of sum rules. They found that negligible error is introduced by terminating the summation with transitions to the resonance doublet, and that the chief error is the uncertainty in the oscillator strength for this transition. If one assumes that the only

appreciable contributions to $\bar{\alpha}$ for the metastable rare gases arise from transitions to the analogous configuration $np^5(n+1)p$, estimates of these polarizabilities may be easily made. Relative oscillator strengths are obtained by assuming one form of coupling or another (JJ coupling is assumed here, although very small differences result from other choices), and radial integrals obtained from the Coulomb approximation of Bates and Damgaard.¹² The values obtained for ${}^{3}P_{2}$ neon, argon, krypton, and xenon, found in this manner are, in units of 10⁻²⁴ cm³, 27.8, 48.1, 53.5, and 62.5, in remarkable agreement with the experiment. These are included in Table I under the heading $\bar{\alpha}$ (Theory B).

The assertion that transitions other than those to $np^{5}(n+1)p$ are unimportant is supported by the closeness of the values obtained to experiment, by the fact that the oscillator strengths in all four cases are approximately unity, and by a (Bates and Damgaard) calculation of the $4p^{5}5s \rightarrow 4p^{5}5p$ oscillator strength for krypton.

CONCLUSION

The metastable rare gases possess large average polarizabilities due almost entirely to the great spatial extent of their excited s electrons, and much smaller polarizability anisotropies, which appear to result primarily from mixing configurations. The close agreement between the measured average polarizabilities and the calculations by means of Bates and Damgaard oscillator strengths terminating with the $np^5(n+1)p$ configuration, and the somewhat poorer agreement obtained with the numerical solution of Eq. (9c) stress the need for wave functions in polarizability calculations which are truly representative of the atom at large values of the radius.

However, an accurate many-body-type calculation seems to be required to include the effects of electron correlation accurately enough to predict α' .

Since transitions to $np^{5}(n'p)$, $n' \neq (n+1)$ provide a negligible fraction of the polarizability, the ratio $\alpha_{zz}(1)/\alpha_{zz}(2)$ is a function only of the relative oscillator strengths to the $np^5(n+1)p$ levels. A rather rigorous test of these is therefore available, as the polarizability ratio is accurate to about 5%. Only much smaller errors can be tolerated in the eight relative $p^5 s \rightarrow p^5 p$ oscillator strengths if these are to agree with $\alpha_{zz}(1)/\alpha_{zz}(2)$ to within our experimental error.

⁹ F. Herman and S. Skillman, Atomic Structure Calculations (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1963).
¹⁰ E. J. Robinson and S. Geltman (to be published).
¹¹ A. Dalgarno and A. E. Kingston, Proc. Phys. Soc. (London) 72, 455 (1050).

^{73, 455 (1959).}

¹² D. R. Bates and A. Damgaard, Phil. Trans. Roy. Soc. (London) A243, 101 (1949).