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Gravitational Shielding and Absorption*†

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The general theory of relativity enables us to calculate gravitational shielding and absorption. These effects are analogous to the shielding of electromagnetic fields with the exception that in lowest order quadrupoles rather than dipoles are involved. Quasistatic shielding effects occur in the tides, and for some models the shielding effect is several percent of the applied field. The dynamic shielding is much too small to observe at this time.

INTRODUCTION

SHIELDING effects have been observed and well understood in electricity and magnetism for over a century. The distribution of charges and currents is calculated in the presence of a given applied field. If the field of the driven charges results in a reduction of the magnitude of the original driving field, we say there is a shielding effect. There is no intrinsic absorption of the electromagnetic field; all shielding effects are due to charges or multipoles which have been influenced by the applied field. The gravitational case differs in detail, but the same kinds of effects are clearly present.

QUASISTATIC GRAVITATIONAL SHIELDING

The gravitational theories of Newton and Einstein enable us to calculate quasistatic shielding effects. The second derivatives of the gravitational field of one body may induce tidal effects in another. The resulting redistribution of mass gives rise to shielding which is readily observable. Such effects are known to geophysicists under different names and are included here for completeness.

We consider the tidal effects induced in a solid sphere A by another sphere B, using Newton's theory of gravitation. In the region outside of B, the potential of B is given by solutions of the Laplace equation

$$\nabla^2 U_B = 0\,. \qquad (1)$$

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A and B will be in orbital motion. We choose for convenience a spherical coordinate system with the z axis along the line of centers and the center of the coordinate system fixed at the center of mass of A. We thus have cylindrical symmetry and the solution of Eq. (1) is

$$U_B = \sum_n \left[a_n r^n + (b_n / r^{n+1})\right] P_n(\cos\theta)\,. \qquad (2)$$

For values of r smaller than R_{AB} we must have $b_n = 0$. Thus far we have calculated the potential of sphere B as though it were at rest in an inertial frame. In consequence of the orbital motion (free fall) of A the forces at $r=0$ must vanish. This may be accomplished to a good approximation by choosing a new potential

$$U_B' = U_B + L\,. \qquad (3)$$

The radial force at $r=0$ is

$$\partial U_B'/\partial r = \left[\sum n a_n r^{n-1} P_n(\cos\theta) + \partial L/\partial r\right]_{r=0}\,. \qquad (4)$$

Equation (4) will vanish at $r=0$ if we choose $\partial L/\partial r = -a_1 P_1(\cos\theta)$. This then leaves the quadrupole term as the major tide-producing potential. We may determine a_2 by writing, for points on the axis (Fig. 1)

$$U_B = \sum a_n r^n = -G m_B / (R_{AB} \pm r)\,. \qquad (5)$$

Expanding the right side of Eq. (5) enables us to determine a_n, so the quadrupole term is

$$U_2' = -G m_B r^2 P_2(\cos\theta) / R_{AB}^3\,. \qquad (6)$$

The tide-producing potential will distort A and to a first approximation this will result in an induced

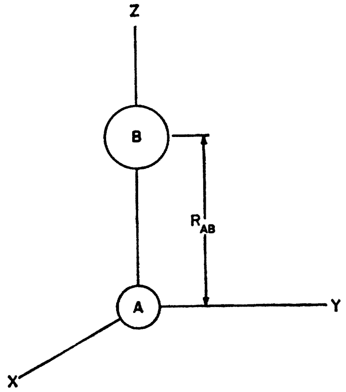

FIG. 1. Gravitational shielding in the tidal interaction of orbiting spheres.

quadrupole, with a change of potential

$$\Delta U = -KGm_B r_s^5 P_2(\cos\theta)/R_{AB}^3 r^3. \quad (7)$$

In Eq. (7), K is a constant which is called a Love number.[1] r_s is the unperturbed radius of sphere A. The form of (7) is chosen because we are "outside" of the induced quadrupole and the coefficient of K must reduce to (6) for $r=r_s$. The acceleration due to gravity at a *fixed radius* r near the surface of A is

$$g = \frac{-Gm_A}{r^2} + \frac{2Gm_B r P_2(\cos\theta)}{R_{AB}^3} - \frac{3KGm_B r_s^5 P_2(\cos\theta)}{R_{AB}^3 r^4} \quad (8)$$

$$= g_0 + [1 - \tfrac{3}{2}K(r_s/r)^5](-\partial U_2'/\partial r). \quad (9)$$

In (9) g_0 is the acceleration due to gravity of the unperturbed sphere A. Calculation of K is a solved problem in the theory of elasticity. For the moon considered as a homogeneous elastic sphere[2] $K \approx 0.03$. Thus at a point near the surface of the moon the shielding of the field of B due to the redistribution of mass of A in consequence of the tidal forces is about $4\frac{1}{2}\%$.

An observer on the surface of sphere A would not necessarily observe a decrease in the part of the acceleration due to gravity of B. This follows because the surface itself moves in response to the tidal force. This displacement is denoted by δ and given approximately by

$$\delta = +hU_B'/g_0. \quad (10)$$

In (10), h is a second kind of Love number. For R_{AB} large compared with R_A it is necessary to correct only the first term of (8) for the displacement to obtain the acceleration due to gravity at a point on the surface of A as

$$g = g_0 + (1 + h - \tfrac{3}{2}K)(-\partial U_2'/\partial r). \quad (11)$$

It is again instructive to consider the moon. Since h has been calculated as ≈ 0.05, it is clear from (11) that an observer in a fixed position on the lunar surface will see, in general, an increase in acceleration due to gravity of sphere B, but the increase is not as great as it would be if it were not for the shielding term involving $-\frac{3}{2}K$. As we noted earlier, for a fixed radius there is a real shielding effect.

DYNAMIC GRAVITATIONAL SHIELDING

A dynamic gravitational field will interact with a mass quadrupole oscillator. The theory of the absorption of energy has been given[3] and the absorption cross section calculated. At resonance the absorption cross section of such an oscillator is given by

$$\sigma = 15\pi G I \omega^2 \tau / 8c^3. \quad (12)$$

In (12), G is the constant of gravitation, I is the quadrupole moment, ω is the angular frequency, c is the speed of light, and τ is the relaxation time. A somewhat more transparent form of (12) is

$$\sigma = (15\pi^3/2)\left(\frac{Gm/c^2}{\lambda}\right)\left(\frac{c\tau}{\lambda}\right)r^2. \quad (12a)$$

(12a) is seen to be the optical cross section r^2, multiplied by the ratio of gravitational length to the wavelength λ, multiplied by the ratio of the integrating length $c\tau$ to the wavelength.

If a gravitational wave is incident in a medium containing mass quadrupoles, a shielding effect may be produced. We proceed to calculate the gravitational absorption coefficient. Dispersion relations may then be employed to calculate the refractive index. Suppose we have an incident plane gravitational wave propagating in the z direction. In a Riemann normal coordinate system, the Riemann tensor is given by

$$\mathcal{B} e^{i\omega z/c - i\omega t - \alpha z}. \quad (13)$$

The time-averaged energy flux t_{0z} has a z dependence

$$t_{0z} = \mathcal{A} e^{-2\alpha z}. \quad (14)$$

The attenuation constant α is given from Eq. (14) as

$$\alpha = -(2t_{0z})^{-1}(dt_{0z}/dz). \quad (15)$$

α may be calculated if the energy absorption dt_{0z}/dz per unit length is known. For simplicity let us assume that we are dealing with uniaxial mass quadrupole oscillators, characterized by a vector r^α which gives the equilibrium position vector of one mass element relative to the other. When driven by the Riemann tensor, the dynamical relative displacement ξ^μ of a given mass quadrupole oscillator is given by the solution of the equation

$$\frac{d^2\xi^\mu}{dt^2} + \frac{D}{m}\frac{d\xi^\mu}{dt} + \frac{k\xi^\mu}{m} = -c^2 R^\mu{}_{0\alpha 0} r^\alpha. \quad (16)$$

[1] A. E. H. Love, *Some Problems of Geodynamics* (Cambridge University Press, London, 1911).

[2] R. Tomaschek, in *Handbuch der Physik* (Springer-Verlag, Berlin, 1957), Vol. 48, pp. 775–845.

[3] J. Weber, *General Relativity and Gravitational Waves* (Interscience Publishers, Inc., New York, 1961), Chap. 8.

Here D is a dissipation factor, k is the force constant. Our Riemann normal coordinate system is assumed to be propagated along the world line of the center of mass of the oscillator. Repeated indices are summed over; $R^{\mu}{}_{\alpha\beta\gamma}$ is the Riemann tensor. Taking the Fourier transform of (16) gives for the Fourier transform $\xi^{\mu}(\omega)$ in terms of the Fourier transform $R^{\mu}{}_{0\alpha 0}(\omega)$:

$$\xi^{\mu}(\omega)=mc^2R^{\mu}{}_{0\alpha 0}(\omega)r^{\alpha}/(\omega^2 m-i\omega D-k). \qquad (17)$$

The absorbed power is given by

$$P_{\text{absorbed}}=\text{Re}[-\tfrac{1}{2}i\omega\xi^{\mu}mc^2(R^{\mu}{}_{0\alpha 0})^*r^{\alpha}]. \qquad (18)$$

In (18), Re stands for the real part and the asterisk indicates a complex conjugate. Making use of (17), we write (18) as

$$P_{\text{absorbed}}=\text{Re}[-i\omega(mc^2)^2(R^{\mu}{}_{0\alpha 0})^*r^{\alpha}R^{\mu}{}_{0\beta 0}r^{\beta}]/2(\omega^2 m-i\omega D-k). \qquad (19)$$

In terms of the energy flux we write[3]

$$\langle(R^{\mu}{}_{0\alpha 0}r^{\alpha})^2\rangle=4\pi^2 IGt_{0z}\omega^2/c^7 m. \qquad (20)$$

In (20), I is the usual quadrupole moment multiplied by a numerical factor of the order of unity. Writing the square of the Riemann tensor in terms of the energy flux has the arbitrariness involved in defining the gravitational energy density and making use of the particular kind of Riemann tensor associated with a linear mass quadrupole oscillator.[3] Equation (20) may be regarded as defining I.

Making use of (20), (19) becomes

$$P_{\text{absorbed}}=\text{Re}(-i\omega^3 m2\pi^2 Gt_{0r}I/(\omega^2 m-i\omega D-k)c^3). \qquad (21)$$

The absorption coefficient (15) for a medium containing n_j quadrupoles per unit volume with quadrupole moment I_j will be

$$\alpha=\text{Re}\sum_j\left(-\frac{i\omega I_j\pi^2 Gn_j}{c^3(1-i/Q_j-\omega^2{}_{0j}/\omega^2)}\right). \qquad (22)$$

In (22), $Q_j=\omega m_j/D_j$, $\omega_{0j}{}^2=k_j/m_j$, and (22) may be written as

$$\alpha=\sum_j\frac{\pi^2\omega I_j Gn_j}{c^3 Q_j\{[1-(\omega_{0j}/\omega)^2]^2+1/Q_j{}^2\}}. \qquad (23)$$

We may obtain the refraction index from the dispersion relation

$$n(\omega)=1+\lim_{\omega_2\to 0}\frac{2c}{\pi}\int_0^{\infty}\frac{\alpha(\omega')d\omega'}{\omega'^2-\omega^2} \qquad (24)$$

with complex $\omega=\omega_1+i\omega_2$.

In most cases (23) gives an incredibly small result. For example suppose we have a case of a medium containing 10^{22} atoms per cubic centimeter, each with a quadrupole moment $\approx 10^{-43}$ g cm^2, $\omega\approx 10^{16}$, $Q\approx 10^8$, $\alpha\approx 10^{-35}$ cm^{-1} at resonance. For the one cycle in a 54-minute quadrupole mode of the earth, about one part in 10^{11} of the incident gravitational wave power is absorbed at resonance.

CONCLUSION

We have noted that static and dynamic gravitational shielding effects exist, in analogy with electrodynamics. For orbiting spheres the quasistatic tidal shielding effects may amount to several percent of the inducing field. The resemblance of (23) to the traditional electromagnetic theory result is evident, with mass quadrupoles taking the usual role assumed by electric dipoles.