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Quantum Kinetic Equations for a Multicomponent System of Charged Particles and Photons*†

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Quantum-mechanical kinetic equations are derived for a homogeneous, isotropic system of charged particles and photons. A hierarchy of equations is introduced by the use of the Wigner distribution operators and quasiphoton creation and annihilation operators. The method of Bogoliubov is used to truncate the hierarchy and to obtain kinetic equations. The resulting kinetic equations contain effects due to the processes of particle-particle scattering, particle-photon scattering, and single and double emission-absorption of photons by particles. In addition, there are terms representing shielding of the particles and photons, as well as other many-body effects. Photon self-energy corrections are also discussed.

INTRODUCTION

THE development of kinetic equations for a plasma including radiation has been a problem of growing interest in recent years. Several authors have developed classical treatments.[1–5] The development of quantum kinetic equations has been generally restricted to the "Golden Rule" approach, i.e., the development of quantum Boltzmann equations via the use of transition rates calculated by the "Golden Rule," or equivalent techniques. In particular, Osborn and Klevans[6] derived master equations by using the repeated random-phase assumption. Dreicer[7] used a strictly Boltzmann approach to derive relativistic, quantum equations. None of these techniques is generally capable of including many-body effects.[8]

In Sec. I of this paper we introduce the Wigner distribution operators,[9,10] and derive the equations of motion satisfied by these operators. The electromagnetic field will be treated as being completely internal (the case of a uniform external magnetic field will be considered in a subsequent paper). The exact equations of motion for the one-particle and one-photon momentum distribution functions for a multicomponent, homogeneous system are presented. The terms in these equations are classified as representing the particle-particle scattering process, the particle-photon scattering process, the single emission-absorption process, and the double emission-absorption process. The various contributions are then treated separately in the subsequent sections. These various terms involve certain correlation functions. Equations of motion are obtained for these correlation functions and a truncation ansatz, which is a simple generalization of that used by Ron,[11] is used to terminate the hierarchy of equations. The Bogoliubov (adiabatic) assumption, that the correlation functions relax in a time short compared to the relaxation time, is used to solve the system of equations and obtain expressions for the correlation functions as functionals of the one-particle and one-photon momentum distribution functions.

The results of Osborn and Klevans[6] and of Dreicer[7] are obtained plus correction terms which represent correlation effects. Among the correlation terms there are some terms which do not contain delta functions that conserve the unperturbed energy. These terms are similar to a term found by Kohn and Luttinger[12] in their work on quantum transport in solids. Similar terms have also been more recently obtained by Mangeney[4] in his treatment of a classical, relativistic plasma with radiation, and by Michel[13] in his treatment of the electron-phonon system. These terms are also

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[8] Osborn and Klevans (Ref. 6) do account for some of the many-body effects by including some of the dispersive nature of the plasma.

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[10] W. E. Brittin and W. R. Chappell, Rev. Mod. Phys. **34**, 620 (1962).

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considered in two recent papers by Chappell and Swenson[14] and Henin *et al.*[15]

I. WIGNER OPERATORS

Let $\Psi_\alpha(\mathbf{r})$ be the quantized wave field for particles of type α. Then, for fermions, Ψ_α and $\Psi_\alpha{}^\dagger$ satisfy the anticommutation relations:

$$\{\Psi_\alpha(\mathbf{r}),\Psi_\beta(\mathbf{r}')\}=\{\Psi_\alpha{}^\dagger(\mathbf{r}),\Psi_\beta{}^\dagger(\mathbf{r}')\}=0,$$

and

$$\{\Psi_\alpha(\mathbf{r}),\Psi_\beta{}^\dagger(\mathbf{r}')\}=\delta_{\alpha,\beta}\delta(\mathbf{r}-\mathbf{r}'). \tag{1}$$

The one-particle Wigner distribution[10,11] operator for α-type particles is given by

$$\hat{f}_\alpha(\mathbf{r},\mathbf{p})=\frac{\hbar^3}{\Omega}\int d\mathbf{l}\, e^{i\mathbf{l}\cdot\mathbf{p}}\Psi_\alpha{}^\dagger(\mathbf{r}+\tfrac{1}{2}\hbar\mathbf{l})\Psi_\alpha(\mathbf{r}-\tfrac{1}{2}\hbar\mathbf{l}), \tag{2}$$

where Ω is the volume of the system and l runs over a region of volume $\Omega/\hbar^3$. The general n-particle operator is

$$\hat{f}_{\alpha_1\alpha_2\cdots\alpha_n}(1,2,\cdots,n)=\frac{\hbar^{3n}}{\Omega^n}\int\cdots\int d\mathbf{l}_1\cdots d\mathbf{l}_n$$
$$\times\exp\left(\sum_{i=1}^{n}\mathbf{l}_i\cdot\mathbf{p}_i\right)\Psi_{\alpha_1}{}^\dagger(\mathbf{r}_1+\tfrac{1}{2}\hbar\mathbf{l}_1)\Psi_{\alpha_2}{}^\dagger(\mathbf{r}_2+\tfrac{1}{2}\hbar\mathbf{l}_2)\cdots$$
$$\times\Psi_{\alpha_n}{}^\dagger(\mathbf{r}_n+\tfrac{1}{2}\hbar\mathbf{l}_n)\Psi_{\alpha_n}(\mathbf{r}_n-\tfrac{1}{2}\hbar\mathbf{l}_n)$$
$$\times\Psi_{\alpha_{n-1}}(\mathbf{r}_{n-1}-\tfrac{1}{2}\hbar\mathbf{l}_{n-1})\cdots\Psi_{\alpha_1}(\mathbf{r}_1-\tfrac{1}{2}\hbar\mathbf{l}_1), \tag{3}$$

where $1=(\mathbf{r}_1,\mathbf{p}_1)$, etc., and the subscripts $\alpha_1\cdots\alpha_n$ denote the particle type. For example, there are two basic types of two-particle distribution functions. These are $f_{\alpha\alpha}(1,2)$ and $f_{\alpha\beta}(1,2)$, with $\alpha\neq\beta$.

Equations (2) and (3) also hold if the particles are bosons. There is no essential difficulty in treating the case where both fermions and bosons are present.

The Wigner operators have the property that

$$f_{\alpha_1\alpha_2\cdots\alpha_n}(1,2,\cdots,n)=\mathrm{Tr}[\rho\hat{f}_{\alpha_1\alpha_2\cdots\alpha_n}(1,2,\cdots,n)]$$

is the Wigner distribution function,[9] where Tr denotes the trace operation and ρ is the density matrix. The properties of these operators are discussed in Ref. 10.

The radiation field will be represented by the vector potential $\mathbf{A}(\mathbf{r})$ (we choose the Coulomb gauge), which can be written in terms of creation and annihilation operators as

$$\mathbf{A}(\mathbf{r})=\sum_{\mathbf{k}\lambda}(2\pi\hbar c/\Omega k)^{1/2}\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}e^{i\mathbf{k}\cdot\mathbf{r}}(a_{\mathbf{k}\lambda}+a_{-\mathbf{k}\lambda}{}^\dagger),$$

where

$$\lambda=1,\,2,\quad \mathbf{k}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}=0, \tag{4}$$
$$[a_{\mathbf{k}\lambda},a_{\mathbf{k}'\lambda'}]=[a_{\mathbf{k}\lambda}{}^\dagger,a_{\mathbf{k}'\lambda'}{}^\dagger]=0,$$

and

$$[a_{\mathbf{k}\lambda},a_{\mathbf{k}'\lambda'}{}^\dagger]=\delta_{\mathbf{k},\mathbf{k}'}\delta_{\lambda,\lambda'}. \tag{5}$$

The Hamiltonian of the system is given by (the coupling of the spins to the radiation field is ignored)

$$H=H_\gamma+\sum\nolimits_\alpha\int d\mathbf{r}\,\Psi_\alpha{}^\dagger(\mathbf{r})\left(-\frac{\hbar^2\nabla^2}{2m_\alpha}\right)\Psi_\alpha(\mathbf{r})+\tfrac{1}{2}\sum_{\alpha,\beta}\int\int d\mathbf{r}\,d\mathbf{r}'\,\Psi_\alpha{}^\dagger(\mathbf{r})\Psi_\beta{}^\dagger(\mathbf{r}')V_{\alpha\beta}(\mathbf{r}-\mathbf{r}')\Psi_\beta(\mathbf{r}')\Psi_\alpha(\mathbf{r})$$
$$-\frac{1}{c}\sum\nolimits_\alpha\int d\mathbf{r}\,\mathbf{J}_\alpha(\mathbf{r})\cdot\mathbf{A}(\mathbf{r})+\sum\nolimits_\alpha\int d\mathbf{r}\,\frac{e_\alpha{}^2}{2m_\alpha c^2}\Psi_\alpha{}^\dagger(\mathbf{r})\Psi_\alpha(\mathbf{r})\mathbf{A}^2(\mathbf{r}), \tag{6}$$

where H_γ is the free-radiation Hamiltonian, $V_{\alpha\beta}(\mathbf{r})$ is the Coulomb potential, and

$$\mathbf{J}_\alpha(\mathbf{r})=\frac{e_\alpha\hbar}{2m_\alpha i}[\Psi_\alpha{}^\dagger(\mathbf{r})\nabla\Psi_\alpha(\mathbf{r})-(\nabla\Psi_\alpha{}^\dagger(\mathbf{r}))\Psi_\alpha(\mathbf{r})]. \tag{7}$$

The equation of motion of the n-particle Wigner operator $\hat{f}_{\alpha_1\alpha_2\cdots\alpha_n}(1,2,\cdots,n)$ is given by

$$\frac{\partial}{\partial t}\hat{f}_{\alpha_1\cdots\alpha_n}(1,2,\cdots,n)+\sum_{j=1}^{n}\frac{\mathbf{p}_j}{m_j}\cdot\nabla_{\mathbf{r}_j}\hat{f}_{\alpha_1\cdots\alpha_n}(1,2,\cdots,n)+\frac{i\hbar^6}{\hbar\Omega^2}\sum_{1\leq j<k\leq n}\sum_{\mathbf{p},\mathbf{p}'}\int\int d\mathbf{l}\,d\mathbf{l}'\,e^{i\mathbf{l}\cdot(\mathbf{p}_j-\mathbf{p})}e^{i\mathbf{l}'\cdot(\mathbf{p}_j-\mathbf{p}')}$$
$$\times[V_{\alpha_j\alpha_k}(|\mathbf{r}_j-\mathbf{r}_k+\tfrac{1}{2}\hbar(\mathbf{l}-\mathbf{l}')|)-V_{\alpha_j\alpha_k}(|\mathbf{r}_j-\mathbf{r}_k-\tfrac{1}{2}\hbar(\mathbf{l}-\mathbf{l}')|)]\hat{f}_{\alpha_1\cdots\alpha_n}(\mathbf{r}_1,\mathbf{p}_1;\cdots;\mathbf{r}_j,\mathbf{p};\cdots;\mathbf{r}_k,\mathbf{p}';\cdots;\mathbf{r}_n,\mathbf{p}_n)$$
$$-\sum_{j=1}^{n}\sum_{\mathbf{p}}\int d\mathbf{l}\,e^{i\mathbf{l}\cdot(\mathbf{p}_j-\mathbf{p})}\frac{e_j\hbar^3}{2m_jc\Omega}[\mathbf{A}(\mathbf{r}_j+\tfrac{1}{2}\hbar\mathbf{l})+\mathbf{A}(\mathbf{r}_j-\tfrac{1}{2}\hbar\mathbf{l})]\cdot\nabla_{\mathbf{r}_j}\hat{f}_{\alpha_1\cdots\alpha_n}(\mathbf{r}_1,\mathbf{p}_1;\cdots;\mathbf{r}_j,\mathbf{p};\cdots;\mathbf{r}_n,\mathbf{p}_n)$$
$$+\frac{i\hbar^3}{\hbar\Omega}\sum_{j=1}^{n}\sum_{\mathbf{p}}\int d\mathbf{l}\,e^{i\mathbf{l}\cdot(\mathbf{p}_j-\mathbf{p})}\frac{e_j}{m_jc}\mathbf{p}\cdot[\mathbf{A}(\mathbf{r}_j+\tfrac{1}{2}\hbar\mathbf{l})-\mathbf{A}(\mathbf{r}_j-\tfrac{1}{2}\hbar\mathbf{l})]\hat{f}_{\alpha_1\cdots\alpha_n}(\mathbf{r}_1,\mathbf{p}_1;\cdots;\mathbf{r}_j,\mathbf{p};\cdots;\mathbf{r}_n,\mathbf{p}_n)$$
$$-\frac{i\hbar^3}{\hbar\Omega}\sum_{j=1}^{n}\sum_{\mathbf{p}}\int d\mathbf{l}\,e^{i\mathbf{l}\cdot(\mathbf{p}_j-\mathbf{p})}\frac{e_j{}^2}{2m_jc^2}[\mathbf{A}^2(\mathbf{r}_j+\tfrac{1}{2}\hbar\mathbf{l})-\mathbf{A}^2(\mathbf{r}_j-\tfrac{1}{2}\hbar\mathbf{l})]\hat{f}_{\alpha_1\cdots\alpha_n}(\mathbf{r}_1,\mathbf{p}_1;\cdots;\mathbf{r}_j,\mathbf{p};\cdots;\mathbf{r}_n,\mathbf{p}_n)$$
$$=\frac{i\hbar^3}{\hbar\Omega}\sum_{j=1}^{n}\sum_{\beta}\sum_{\mathbf{p},\mathbf{p}'}\int d\mathbf{l}\int d\mathbf{r}\,e^{i\mathbf{l}\cdot(\mathbf{p}_j-\mathbf{p}')}$$
$$\times[V_{\alpha_j\beta}(|\mathbf{r}_j-\mathbf{r}+\tfrac{1}{2}\hbar\mathbf{l}|)-V_{\alpha_j\beta}(|\mathbf{r}_j-\mathbf{r}-\tfrac{1}{2}\hbar\mathbf{l}|)]\hat{f}_{\alpha_1\cdots\alpha_n\beta}(\mathbf{r}_1,\mathbf{p}_1;\cdots;\mathbf{r}_j,\mathbf{p}';\cdots;\mathbf{r}_n,\mathbf{p}_n;\mathbf{r},\mathbf{p}). \tag{8}$$

[14] W. R. Chappell and R. J. Swenson, Phys. Fluids **8**, 1195 (1965).
[15] F. Henin, I. Prigogine, P. Résibois, and M. Watabe, Phys. Letters **16**, 253 (1965).

It is convenient to introduce a new set of photon-like or "quasiphoton" operators which are related to the operators $a_{\mathbf{k}\lambda}$ and $a_{\mathbf{k}\lambda}^\dagger$ through a Bogoliubov transformation[16]

$$b_{\mathbf{k}\lambda}=u_{\mathbf{k}\lambda}a_{\mathbf{k}\lambda}+v_{\mathbf{k}\lambda}a_{-\mathbf{k}\lambda}^\dagger,$$

and

$$b_{\mathbf{k}\lambda}^\dagger=u_{\mathbf{k}\lambda}a_{\mathbf{k}\lambda}^\dagger+v_{\mathbf{k}\lambda}a_{-\mathbf{k}\lambda}. \tag{9}$$

The vector potential can then be written as

$$\mathbf{A}(\mathbf{r})=\sum_{\mathbf{k}\lambda}(2\pi\hbar c^2/\Omega\omega_k)^{1/2}\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}e^{i\mathbf{k}\cdot\mathbf{r}}(b_{\mathbf{k}\lambda}+b_{-\mathbf{k}\lambda}^\dagger), \tag{10}$$

and the Hamiltonian is given by

$$H=H_\gamma'+\sum\nolimits_\alpha\int d\mathbf{r}\,\Psi_\alpha^\dagger(\mathbf{r})(-\hbar^2\nabla^2/2m_\alpha)\Psi_\alpha(\mathbf{r})+\frac{1}{2}\int\int d\mathbf{r}\,d\mathbf{r}'\,\Psi_\alpha^\dagger(\mathbf{r})\Psi_\beta^\dagger(\mathbf{r}')V_{\alpha\beta}(\mathbf{r}-\mathbf{r}')\Psi_\beta(\mathbf{r}')\Psi_\alpha(\mathbf{r})$$
$$-\frac{1}{c}\sum\nolimits_\alpha\int d\mathbf{r}\,\mathbf{J}_\alpha(\mathbf{r})\cdot\mathbf{A}(\mathbf{r})+\sum\nolimits_\alpha\int d\mathbf{r}\,(e_\alpha^2/2m_\alpha c^2)\delta n_\alpha(\mathbf{r})\mathbf{A}^2(\mathbf{r}), \tag{11}$$

where

$$\delta n_\alpha(\mathbf{r})=\Psi_\alpha^\dagger(\mathbf{r})\Psi_\alpha(\mathbf{r})-N_\alpha/\Omega,$$
$$H_\gamma=\tfrac{1}{2}\sum_{\mathbf{k}\lambda}\hbar\omega_k(b_{\mathbf{k}\lambda}^\dagger b_{\mathbf{k}\lambda}+b_{\mathbf{k}\lambda}b_{\mathbf{k}\lambda}^\dagger), \tag{12}$$
$$\omega_k=(k^2c^2+\omega_p^2)^{1/2}, \tag{13}$$

and

$$\omega_p^2=\sum\nolimits_\alpha 4\pi N_\alpha e_\alpha^2/m_\alpha\Omega. \tag{14}$$

The quantities N_α, e_α, and m_α are the total number, the charge, and the mass of α-type particles, respectively. These new operators obey the usual boson commutation relations.

The advantage of using these operators is that most of the dispersive effect of the medium is now included. The next term in the dispersion relation is of order $\langle v^2\rangle/c^2$ compared to the plasma frequency ω_p. The dispersion relation used by Osborn[6] is simply the first two terms of the Taylor expansion of Eq. (13) for $kc\gg\omega_p$.

We can rewrite the Hamiltonian as

$$H=H_p+\sum_{\mathbf{k}\lambda}\hbar\omega_k b_{\mathbf{k}\lambda}^\dagger b_{\mathbf{k}\lambda}-\sum_{\alpha,\mathbf{p}}\sum_{\mathbf{k},\lambda}\hbar c_{\alpha k}\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\hat{f}_\alpha(-\mathbf{k},\mathbf{p})(b_{\mathbf{k}\lambda}+b_{-\mathbf{k}\lambda}^\dagger)$$
$$+\sum_{\alpha,\mathbf{p}}\sum_{\mathbf{k},\mathbf{k}'}{}'\sum_{\lambda,\lambda'}\hbar d_{kk'}{}^\alpha\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}\hat{f}_\alpha(\mathbf{k}'-\mathbf{k},\mathbf{p})(b_{\mathbf{k}\lambda}+b_{-\mathbf{k}\lambda}^\dagger)(b_{-\mathbf{k}'\lambda'}+b_{\mathbf{k}'\lambda'}^\dagger), \tag{15}$$

where H_p is that part of the Hamiltonian which contains only particle operators,

$$\hat{f}_\alpha(\mathbf{k},\mathbf{p})=\int d\mathbf{r}\,e^{-i\mathbf{k}\cdot\mathbf{r}}\hat{f}_\alpha(\mathbf{r},\mathbf{p}), \tag{16}$$
$$c_{\alpha k}=(2\pi e_\alpha^2/\hbar m_\alpha^2\omega_k)^{1/2}, \tag{17}$$
$$d_{kk'}{}^\alpha=(\pi e_\alpha^2/m_\alpha\Omega)(\omega_k\omega_{k'})^{-1/2}, \tag{18}$$

and $\sum_{\mathbf{k},\mathbf{k}'}{}'$ means the term $\mathbf{k}'=\mathbf{k}$ is omitted in the summations.

The equations of motion for the operators $b_{\mathbf{k}\lambda}$ and $\hat{f}_\alpha(\mathbf{k},\mathbf{p})$ and the momentum distribution functions are

$$\frac{\partial b_{\mathbf{k}\lambda}}{\partial t}=-i\omega_k b_{\mathbf{k}\lambda}+\sum_{\alpha,\mathbf{p}}c_{\alpha k}\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\hat{f}_\alpha(\mathbf{k},\mathbf{p})+\sum_{\alpha,\mathbf{p}}\sum_{\mathbf{k}',\lambda'}{}'2d_{kk'}{}^\alpha\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}\hat{f}_\alpha(\mathbf{k}-\mathbf{k}',\mathbf{p})(b_{\mathbf{k}'\lambda'}+b_{-\mathbf{k}'\lambda'}^\dagger), \tag{19}$$

$$\left(\frac{\partial}{\partial t}+i\mathbf{k}\cdot\frac{\mathbf{p}}{m_\alpha}\right)\hat{f}_\alpha(\mathbf{k},\mathbf{p})=\tfrac{1}{2}\sum_{\mathbf{l}\lambda}ic_{\alpha l}\hbar\mathbf{k}\cdot\boldsymbol{\varepsilon}_{\mathbf{l}\lambda}[\hat{f}_\alpha(\mathbf{k}-\mathbf{l},\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l})+\hat{f}_\alpha(\mathbf{k}-\mathbf{l},\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{l})](b_{\mathbf{l}\lambda}+b_{-\mathbf{l}\lambda}^\dagger)$$
$$-i\sum_{\mathbf{l}\lambda}c_{\alpha l}\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{l}\lambda}[\hat{f}_\alpha(\mathbf{k}-\mathbf{l},\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l})-\hat{f}_\alpha(\mathbf{k}-\mathbf{l},\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{l})](b_{\mathbf{l}\lambda}+b_{-\mathbf{l}\lambda}^\dagger)$$
$$+\sum_{\mathbf{l}\mathbf{l}'}{}'\sum_{\lambda\lambda'}id_{ll'}{}^\alpha\boldsymbol{\varepsilon}_{\mathbf{l}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{l}'\lambda'}[\hat{f}_\alpha(\mathbf{k}+\mathbf{l}'-\mathbf{l},\mathbf{p}+\tfrac{1}{2}\hbar(\mathbf{l}-\mathbf{l}'))-\hat{f}_\alpha(\mathbf{k}+\mathbf{l}'-\mathbf{l},\mathbf{p}-\tfrac{1}{2}\hbar(\mathbf{l}-\mathbf{l}'))](b_{-\mathbf{l}'\lambda'}+b_{\mathbf{l}'\lambda'}^\dagger)$$
$$+(i/\hbar\Omega)\sum_{\beta,\mathbf{p}'}\sum_{\mathbf{l}}V_{\alpha\beta}(\mathbf{l})[\hat{f}_{\alpha\beta}(\mathbf{k}-\mathbf{l},\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l};\mathbf{l},\mathbf{p}')-\hat{f}_{\alpha\beta}(\mathbf{k}-\mathbf{l},\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{l};\mathbf{l},\mathbf{p}')], \tag{20}$$

[16] W. R. Chappell, S. J. Glass, and W. E. Brittin, Nuovo Cimento 38, 1187 (1965).

$$\frac{\partial\varphi_\alpha(\mathbf{p})}{\partial t}=2\sum_{\mathbf{k}\lambda}c_{\alpha k}\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\,\mathrm{Im}[g_\lambda{}^\alpha(\mathbf{k},\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{k})-g_\lambda{}^\alpha(\mathbf{k},\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{k})]$$
$$-2\sum_{\mathbf{k}\mathbf{k}'}\sum_{\lambda\lambda'}d_{kk'}{}^\alpha\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}\,\mathrm{Im}[h_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p}-\tfrac{1}{2}\hbar(\mathbf{k}'-\mathbf{k}))$$
$$-h_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p}+\tfrac{1}{2}\hbar(\mathbf{k}'-\mathbf{k}))+k_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p}-\tfrac{1}{2}\hbar(\mathbf{k}'-\mathbf{k}))-k_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p}+\tfrac{1}{2}\hbar(\mathbf{k}'-\mathbf{k}))]$$
$$+\frac{2}{\hbar\Omega}\sum_{\beta,\mathbf{p}'}\sum_{\mathbf{l}}V_{\alpha\beta}(-\mathbf{l})\,\mathrm{Im}g_{\alpha\beta}(\mathbf{l},\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l},\mathbf{p}'),\quad(21)$$

and

$$\frac{\partial n_{\mathbf{k}\lambda}}{\partial t}=2\sum_{\alpha,\mathbf{p}}c_{\alpha k}\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\,\mathrm{Im}g_\lambda{}^\alpha(\mathbf{k},\mathbf{p})-4\sum_{\alpha,\mathbf{p}}\sum_{\mathbf{k}'\lambda'}d_{kk'}{}^\alpha\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}\,\mathrm{Im}[h_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p})+k_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p})],\quad(22)$$

where

$$\varphi_\alpha(\mathbf{p})=\int d\mathbf{r}\,\langle\hat{f}_\alpha(\mathbf{r},\mathbf{p})\rangle,\quad(23)$$

$$n_{\mathbf{k}\lambda}=\langle b_{\mathbf{k}\lambda}{}^\dagger b_{\mathbf{k}\lambda}\rangle,\quad(24)$$

$$V_{\alpha\beta}(\mathbf{l})=\int d\mathbf{r}\,e^{-i\mathbf{l}\cdot\mathbf{r}}V_{\alpha\beta}(\mathbf{r}),\quad(25)$$

$$g_\lambda{}^\alpha(\mathbf{k},\mathbf{p})=\langle\hat{f}_\alpha(-\mathbf{k},\mathbf{p})b_{\mathbf{k}\lambda}\rangle,\quad(26)$$

$$h_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p})=\langle\hat{f}_\alpha(\mathbf{k}'-\mathbf{k},\mathbf{p})b_{\mathbf{k}'\lambda'}{}^\dagger b_{\mathbf{k}\lambda}\rangle,\quad(27)$$

$$k_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p})=\langle\hat{f}_\alpha(\mathbf{k}'-\mathbf{k},\mathbf{p})b_{\mathbf{k}\lambda}b_{-\mathbf{k}'\lambda'}\rangle,\quad(28)$$

and

$$g_{\alpha\beta}(\mathbf{k},\mathbf{p}_1,\mathbf{p}_2)=\int\int d\mathbf{r}_1\,d\mathbf{r}_2\,e^{-i\mathbf{k}\cdot(\mathbf{r}_1-\mathbf{r}_2)}g_{\alpha\beta}(1,2).\quad(29)$$

The two-particle correlation function $g_{\alpha\beta}(1,2)$ is defined as

$$g_{\alpha\beta}(1,2)=f_{\alpha\beta}(1,2)-f_\alpha(1)f_\beta(2)-h_{\alpha\beta}(1,2),\quad(30)$$

where

$$h_{\alpha\beta}(1,2)=-\frac{\delta_{\alpha\beta}\hbar^6}{\Omega^2}\sum_{\mathbf{q}_1,\mathbf{q}_2}\sum_{\alpha,s}\int\int d\mathbf{k}\,d\mathbf{l}$$
$$\times\exp[i\mathbf{k}\cdot(\mathbf{p}_1-\tfrac{1}{2}\mathbf{q}_1-\tfrac{1}{2}\mathbf{q}_2)+i\mathbf{l}\cdot(\mathbf{p}_2-\tfrac{1}{2}\mathbf{q}_1-\tfrac{1}{2}\mathbf{q}_2)+(i/\hbar)(\mathbf{q}_2-\mathbf{q}_1)\cdot(\mathbf{r}_1-\mathbf{r}_2)]$$
$$\times f_{\alpha s}(\tfrac{1}{2}[\mathbf{r}_1+\mathbf{r}_2]+\tfrac{1}{4}\hbar(\mathbf{l}-\mathbf{k}),\mathbf{q}_1)f_{\alpha s}(\tfrac{1}{2}[\mathbf{r}_1+\mathbf{r}_2]-\tfrac{1}{4}\hbar(\mathbf{l}-\mathbf{k}),\mathbf{q}_2).\quad(31)$$

The quantity $h_{\alpha\beta}(1,2)$ represents the correlation arising from the Pauli principle. Clearly it is convenient to separate this correlation from the correlation arising from the particle-particle and particle-photon interactions. Except for a change of sign, the expression for $h_{\alpha\beta}(1,2)$ is the same for bosons.

The general structure of the correlation functions gives some insight into their physical nature. Since $\hat{f}_\alpha(\mathbf{k},\mathbf{p})$ contains a particle-creation and a particle-annihilation operator, $g_\lambda{}^\alpha$ is related to single-photon emission-absorption processes, $h_{\lambda\lambda'}{}^\alpha$ to particle-photon scattering, $k_{\lambda\lambda'}{}^\alpha$ to double emission-absorption processes, and $g_{\alpha\beta}$ to particle-particle scattering processes. These relationships are also obvious from Eqs. (21) and (22).

II. PARTICLE-PARTICLE SCATTERING

We begin by considering the contribution to the kinetic equations due to the correlation function $g_{\alpha\beta}$:

$$\left[\frac{\partial\varphi_\alpha(\mathbf{p})}{\partial t}\right]_{\mathrm{p.p.}}=\frac{2}{\hbar\Omega}\sum_{\beta,\mathbf{p}'}\sum_{\mathbf{k}}V_{\alpha\beta}(-\mathbf{k})\,\mathrm{Im}g_{\alpha\beta}(\mathbf{k},\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{k},\mathbf{p}').\quad(32)$$

At this point we will ignore the photon contributions to $g_{\alpha\beta}$. (We shall consider these in Sec. VI.) Thus, the two-particle Wigner distribution function is assumed to obey the following equation of motion:

$$\frac{\partial f_{\alpha\beta}(1,2)}{\partial t}+\frac{\mathbf{p}_1}{m}\cdot\nabla_1 f_{\alpha\beta}(1,2)+\frac{\mathbf{p}_2}{m}\cdot\nabla_2 f_{\alpha\beta}(1,2)+\frac{i\hbar^5}{\Omega^2}\sum_{\mathbf{p},\mathbf{p}'}\int\int d\mathbf{l}\,d\mathbf{k}\,e^{i\mathbf{k}\cdot(\mathbf{p}_1-\mathbf{p})}e^{i\mathbf{l}\cdot(\mathbf{p}_2-\mathbf{p})}$$
$$\times[V_{\alpha\beta}(\mathbf{r}_1-\mathbf{r}_2+\tfrac{1}{2}\hbar(\mathbf{k}-\mathbf{l}))-V_{\alpha\beta}(\mathbf{r}_1-\mathbf{r}_2-\tfrac{1}{2}\hbar(\mathbf{k}-\mathbf{l}))]f_{\alpha\beta}(\mathbf{r}_1,\mathbf{p};\,\mathbf{r}_2,\mathbf{p}')$$
$$=\frac{i\hbar^2}{\Omega}\sum_{\gamma}\sum_{\mathbf{p},\mathbf{p}'}\int\int d\mathbf{l}\,d\mathbf{r}\,e^{i\mathbf{l}\cdot(\mathbf{p}_1-\mathbf{p}')}[V_{\alpha\gamma}(\mathbf{r}_1-\mathbf{r}+\tfrac{1}{2}\hbar\mathbf{l})-V_{\alpha\gamma}(\mathbf{r}_1-\mathbf{r}-\tfrac{1}{2}\hbar\mathbf{l})]$$
$$\times f_{\alpha\beta\gamma}(\mathbf{r}_1,\mathbf{p}';\,\mathbf{r}_2,\mathbf{p}_2;\,\mathbf{r},\mathbf{p})+\frac{i\hbar^2}{\Omega}\sum_{\gamma}\sum_{\mathbf{p},\mathbf{p}'}\int\int d\mathbf{l}\,d\mathbf{r}\,e^{i\mathbf{l}\cdot(\mathbf{p}_2-\mathbf{p}')}$$
$$\times[V_{\beta\gamma}(\mathbf{r}_2-\mathbf{r}+\tfrac{1}{2}\hbar\mathbf{l})-V_{\beta\gamma}(\mathbf{r}_2-\mathbf{r}-\tfrac{1}{2}\hbar\mathbf{l})]f_{\alpha\beta\gamma}(\mathbf{r}_1,\mathbf{p}_1;\,\mathbf{r}_2,\mathbf{p}';\,\mathbf{r},\mathbf{p}).\quad(33)$$

The truncation of this equation involves the use of the following superposition approximation:

$$f_{\alpha\beta\gamma}(1,2,3)\cong f_\alpha(1)f_\beta(2)f_\gamma(3)+f_\alpha(1)[g_{\beta\gamma}(2,3)-h_{\beta\gamma}(2,3)]$$
$$+f_\beta(2)[g_{\alpha\gamma}(1,3)-h_{\alpha\gamma}(1,3)]+f_\gamma(3)[g_{\alpha\beta}(1,2)-h_{\alpha\beta}(1,2)].\quad(34)$$

This is equivalent to the approximation used by Ron. This particular breakup of $f_{\alpha\beta\gamma}$ neglects some higher order exchange contributions.[17] We assume $g_{\alpha\beta}$ and the Coulomb interaction to be small and neglect all terms containing both $g_{\alpha\beta}$ and $V_{\alpha\beta}$ except the screening terms. The screening terms must be retained because of the long-range nature of the Coulomb interaction.

If the average interparticle spacing is assumed to be larger than the thermal wavelength, then terms containing $h_{\alpha\beta}(1,2)$ are small compared to $f_\alpha(1)f_\beta(2)$, and the correlation function $h_{\alpha\beta}$ will only contribute when it has the form $h_{\alpha\gamma}(1,3)$ or $h_{\beta\gamma}(2,3)$. The resulting equation, after a Fourier transformation on the spatial variables, is

$$\left[\frac{\partial}{\partial t}+i\mathbf{k}\cdot\left(\frac{\mathbf{p}_1}{m_\alpha}-\frac{\mathbf{p}_2}{m_\beta}\right)\right]g_{\alpha\beta}(\mathbf{k},\mathbf{p}_1,\mathbf{p}_2)$$
$$-(i/\hbar\Omega)\textstyle\sum_\gamma V_{\alpha\gamma}(\mathbf{k})\Delta_\alpha(\mathbf{k},\mathbf{p}_1)\sum_{\mathbf{p}} g_{\gamma\beta}(\mathbf{k},\mathbf{p},\mathbf{p}_2)+(i/\hbar\Omega)\sum_\gamma V_{\beta\gamma}(\mathbf{k})\Delta_\beta(\mathbf{k},\mathbf{p}_2)\sum_{\mathbf{p}} g_{\alpha\gamma}(\mathbf{k},\mathbf{p}_1,\mathbf{p})$$
$$=(i/\hbar\Omega)\sum_{s_1,s_2} z_\alpha z_\beta V(\mathbf{k})[G_{\alpha s_1}{}^+(\mathbf{k},\mathbf{p}_1)G_{\beta s_2}{}^-(\mathbf{k},\mathbf{p}_2)-G_{\alpha s_1}{}^-(\mathbf{k},\mathbf{p}_1)G_{\beta s_2}{}^+(\mathbf{k},\mathbf{p}_2)],\quad(35)$$

where

$$\Delta_\alpha(\mathbf{k},\mathbf{p})=\varphi_\alpha(\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{k})-\varphi_\alpha(\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{k}),\quad(36)$$

$$G_{\alpha s}{}^\pm(\mathbf{k},\mathbf{p})=\varphi_{\alpha s}(\mathbf{p}\pm\tfrac{1}{2}\hbar\mathbf{k})[1-\varphi_{\alpha s}(\mathbf{p}\mp\tfrac{1}{2}\hbar\mathbf{k})],\quad(37)$$
$$z_\alpha=e_\alpha/e,$$

and

$$V(\mathbf{k})=V_{\alpha\beta}(\mathbf{k})/z_\alpha z_\beta.\quad(38)$$

Equation (35) is the multicomponent generalization of the familiar equation of motion for a quantum electron plasma in the random-phase approximation. The latter has been considered by Guernsey,[18] Ron,[12] and Wyld and Fried.[17] The general technique involves the use of the Bogoliubov assumption. Thus, the one-particle distribution functions are considered to be independent of time and one solves for the asymptotic ($t\to\infty$) limit of the correlation function. The resulting expression is then substituted into the first equation of the hierarchy.

If the asymptotic limit of $g_{\alpha\beta}$ is defined by

$$\tilde{g}_{\alpha\beta}(\mathbf{k},\mathbf{p}_1,\mathbf{p}_2)=\lim_{\epsilon\to0^+}\epsilon\int_0^\infty e^{-\epsilon t}g_{\alpha\beta}(\mathbf{k},\mathbf{p}_1,\mathbf{p}_2)\,dt,\quad(39)$$

and the Bogoliubov assumption is applied to Eq. (35), we obtain

$$[\Phi+\epsilon]\tilde{g}_{\alpha\beta}(\mathbf{k},\mathbf{p}_1,\mathbf{p}_2)=s_{\alpha\beta}(t),\quad(40)$$

[17] H. W. Wyld, Jr., and B. D. Fried, Ann. Phys. (N. Y.) 23, 374 (1963).
[18] R. L. Guernsey, Phys. Rev. **127**, 1446 (1962).

where $s_{\alpha\beta}(t)$ is identical to the right-hand side of Eq. (35) and the term $\Phi\tilde{g}_{\alpha\beta}$ corresponds to all but the first of the terms on the left-hand side of Eq. (35). Equation (40) can then be solved as

$$\tilde{g}_{\alpha\beta}(\mathbf{k},\mathbf{p}_1,\mathbf{p}_2)=\int_0^\infty e^{-[\Phi+\epsilon]\tau}s_{\alpha\beta}(t)\,d\tau. \tag{41}$$

It is then an easy matter to generalize the technique developed by Dupree[19] and Wolff[20] to obtain as the final result

$$\left[\frac{\partial\varphi_\alpha(\mathbf{p},t)}{\partial t}\right]_{\text{p.p.}}=\frac{2\pi}{\hbar\Omega^2}\sum_{s_1,s_2}\sum_{\beta,\mathbf{p}_2}\sum_{\mathbf{k}}\left|\frac{V_{\alpha\beta}(\mathbf{k})}{\mathcal{E}\{\mathbf{k},[E_\alpha(\mathbf{p}_1+\hbar\mathbf{k})-E_\alpha(\mathbf{p}_1)]/\hbar\}}\right|^2[G_{\alpha s_1}{}^+(\mathbf{k},\,\mathbf{p}_1+\tfrac{1}{2}\hbar\mathbf{k})G_{\beta s_2}{}^-(\mathbf{k},\,\mathbf{p}_2-\tfrac{1}{2}\hbar\mathbf{k})$$
$$-G_{\alpha s_1}{}^-(\mathbf{k},\,\mathbf{p}_1+\tfrac{1}{2}\hbar\mathbf{k})G_{\beta s_2}{}^+(\mathbf{k},\,\mathbf{p}_2-\tfrac{1}{2}\hbar\mathbf{k})]\delta[E_\alpha(\mathbf{p}_1)+E_\beta(\mathbf{p}_2)-E_\alpha(\mathbf{p}_1+\hbar\mathbf{k})-E_\beta(\mathbf{p}_2-\hbar\mathbf{k})], \tag{42}$$

where

$$E_\alpha(p)=p^2/2m_\alpha,$$

and

$$\mathcal{E}[\mathbf{k},\omega]=1+\frac{1}{\hbar\Omega}\sum_{\alpha,\mathbf{p}}\frac{V_{\alpha\alpha}(\mathbf{k})\Delta_\alpha(\mathbf{k},\mathbf{p})}{\omega-(\mathbf{k}\cdot\mathbf{p}/m_\alpha)+i\epsilon}. \tag{43}$$

Equation (42) is simply the multicomponent generalization of the quantum Balescu-Lenard equation.[18]

III. PARTICLE-PHOTON SCATTERING

The contributions of particle-photon scattering processes to the kinetic equations are given by

$$\left[\frac{\partial\varphi_\alpha(\mathbf{p})}{\partial t}\right]_{\text{p.s.}}=-2\sum_{kk'}\sum_{\lambda\lambda'}d_{kk'}{}^\alpha\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}\,\mathrm{Im}[h_{\lambda\lambda'}{}^\alpha(\mathbf{k},\,\mathbf{k}',\,\mathbf{p}-\tfrac{1}{2}\hbar(\mathbf{k}'-\mathbf{k}))-h_{\lambda\lambda'}{}^\alpha(\mathbf{k},\,\mathbf{k}',\,\mathbf{p}+\tfrac{1}{2}\hbar(\mathbf{k}'-\mathbf{k}))], \tag{44}$$

and

$$\left[\frac{\partial n_{\mathbf{k}\lambda}}{\partial t}\right]_{\text{p.s.}}=-4\sum_{\alpha,\mathbf{p}}\sum_{\mathbf{k}'\lambda'}{}'\,d_{kk'}{}^\alpha\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}\,\mathrm{Im}h_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p}). \tag{45}$$

Thus, we must obtain an expression for the particle-photon correlation function $h_{\lambda\lambda'}{}^\alpha$. This correlation function obeys the following equation of motion:

$$[\partial/\partial t+i(\mathbf{K}\cdot\mathbf{p}/m_\alpha+\omega_k-\omega_{k'})]h_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p})$$
$$=i\sum_{\beta,\mathbf{p}'}[c_{\beta k}\mathbf{p}'\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\langle\hat{f}_\alpha(\mathbf{K},\mathbf{p})\hat{f}_\beta(\mathbf{k},\mathbf{p}')b_{\mathbf{k}'\lambda'}{}^\dagger\rangle-c_{\beta k'}\mathbf{p}'\cdot\boldsymbol{\varepsilon}_{k'\lambda'}\langle\hat{f}_\alpha(\mathbf{K},\mathbf{p})\hat{f}_\beta(-\mathbf{k}',\,\mathbf{p}')b_{\mathbf{k}\lambda}\rangle]$$
$$-2i\sum_{\beta,\mathbf{p}'}\sum_{\mathbf{l},\nu}[d_{lk}{}^\beta\boldsymbol{\varepsilon}_{\mathbf{l}\nu}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\langle\hat{f}_\alpha(\mathbf{K},\mathbf{p})\hat{f}_\beta(\mathbf{K}-\mathbf{l},\,\mathbf{p}')b_{\mathbf{k}'}{}^\dagger(b_{\mathbf{l}}+b_{-\mathbf{l}}{}^\dagger)\rangle-d_{lk'}{}^\beta\boldsymbol{\varepsilon}_{\mathbf{l}\nu}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}\langle\hat{f}_\alpha(\mathbf{K},\mathbf{p})\hat{f}_\beta(\mathbf{l}-\mathbf{k}',\,\mathbf{p}')(b_{\mathbf{l}\nu}{}^\dagger+b_{-\mathbf{l}\nu})b_{\mathbf{k}\lambda}\rangle]$$
$$-2i\sum_{\mathbf{l}\nu}\hbar c_{\alpha l}\mathbf{K}\cdot\boldsymbol{\varepsilon}_{\mathbf{l}\nu}[\langle\hat{f}_\alpha(\mathbf{K}-\mathbf{l},\,\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l})(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)b_{\mathbf{k}'\lambda'}{}^\dagger b_{\mathbf{k}\lambda}\rangle+\langle\hat{f}_\alpha(\mathbf{K}-\mathbf{l},\,\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{l})(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)b_{\mathbf{k}'\lambda'}{}^\dagger b_{\mathbf{k}\lambda}\rangle]$$
$$+i\sum_{\mathbf{l}\nu}c_{\alpha l}\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{l}\nu}[\langle\hat{f}_\alpha(\mathbf{K}-\mathbf{l},\,\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l})(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)b_{\mathbf{k}'\lambda'}{}^\dagger b_{\mathbf{k}\lambda}\rangle-\langle\hat{f}_\alpha(\mathbf{K}-\mathbf{l},\,\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{l})(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)b_{\mathbf{k}'\lambda'}{}^\dagger b_{\mathbf{k}\lambda}\rangle]$$
$$-i\sum_{\mathbf{l}\nu}\sum_{\mathbf{l}'\nu'}d_{ll'}{}^\alpha\boldsymbol{\varepsilon}_{\mathbf{l}\nu}\cdot\boldsymbol{\varepsilon}_{\mathbf{l}'\nu'}[\langle\hat{f}_\alpha(\mathbf{K}+\mathbf{l}'-\mathbf{l},\,\mathbf{p}+\tfrac{1}{2}\hbar(\mathbf{l}-\mathbf{l}'))(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)\cdot(b_{-\mathbf{l}'\nu'}+b_{\mathbf{l}'\nu'}{}^\dagger)b_{\mathbf{k}'\lambda'}{}^\dagger b_{\mathbf{k}\lambda}\rangle$$
$$-\langle\hat{f}_\alpha(\mathbf{K}+\mathbf{l}'-\mathbf{l},\,\mathbf{p}-\tfrac{1}{2}\hbar(\mathbf{l}-\mathbf{l}'))(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)\cdot(b_{-\mathbf{l}'\nu'}+b_{\mathbf{l}'\nu'}{}^\dagger)b_{\mathbf{k}'\lambda'}{}^\dagger b_{\mathbf{k}\lambda}\rangle]$$
$$=\frac{i}{\hbar\Omega}\sum_{\beta,\mathbf{p}'}\sum_{\mathbf{l}}V_{\alpha\beta}(\mathbf{l})[\langle\hat{f}_{\alpha\beta}(\mathbf{K}-\mathbf{l},\,\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l};\,\mathbf{l},\mathbf{p}')b_{\mathbf{k}'\lambda'}{}^\dagger b_{\mathbf{k}\lambda}\rangle-\langle\hat{f}_{\alpha\beta}(\mathbf{K}-\mathbf{l},\,\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{l};\,\mathbf{l},\,\mathbf{p}')b_{\mathbf{k}'\lambda'}{}^\dagger b_{\mathbf{k}\lambda}\rangle], \tag{46}$$

where $\mathbf{K}=\mathbf{k}'-\mathbf{k}$.

We assume that the correlation functions appearing in the above equation can be approximated by sums of products of φ_α, n_k, $g_\lambda{}^\alpha$, $h_{\lambda\lambda'}{}^\alpha$, $k_{\lambda\lambda'}{}^\alpha$, and $g_{\alpha\beta}$. Thus, for each of the correlation functions appearing above, we take the

[19] T. H. Dupree, Phys. Fluids 4, 696 (1961).

[20] P. A. Wolff, Phys. Fluids 5, 316 (1962); see also in this connection C.-S. Wu, J. Math. Phys. 5, 1701 (1964).

sum of all possible ways of achieving such a breakup. For example, we write

$$\langle \hat{f}_\alpha(\mathbf{K}-\mathbf{l},\, \mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l})(b_{1\nu}+b_{-1\nu}{}^\dagger)b_{\mathbf{k}'\lambda'}{}^\dagger b_{\mathbf{k}'}\rangle = \delta_{1,\mathbf{k}'}\delta_{\nu,\lambda'}(1+n_{\mathbf{k}'\lambda'})g_\lambda{}^\alpha(\mathbf{k},\, \mathbf{p}+\tfrac{1}{2}\hbar\mathbf{k}') + \delta_{1,-\mathbf{k}}n_{\mathbf{k}\lambda}g_\lambda{}^{\alpha*}(\mathbf{k}',\, \mathbf{p}-\tfrac{1}{2}\hbar\mathbf{k}) + \text{higher order correlation}. \quad (47)$$

The first term on the right-hand s ide of the above equation is obtained by our writing

$$\langle \hat{f}_\alpha(-\mathbf{k},\, \mathbf{p}+\tfrac{1}{2}\hbar\mathbf{k}')b_{\mathbf{k}'\lambda'}b_{\mathbf{k}'\lambda'}{}^\dagger b_{\mathbf{k}\lambda}\rangle = \langle b_{\mathbf{k}'\lambda'}b_{\mathbf{k}'\lambda'}{}^\dagger\rangle\langle \hat{f}_\alpha(-\mathbf{k},\, \mathbf{p}+\tfrac{1}{2}\hbar\mathbf{k}')b_{\mathbf{k}\lambda}\rangle + \text{higher order correlation}. \quad (48)$$

The terms denoted "higher order correlation" are clearly at least one order higher in the coupling than the other terms. In a similar manner, we obtain

$$\langle \hat{f}_\alpha(\mathbf{K},\mathbf{p})f_\beta(\mathbf{k},\mathbf{p}')b_{\mathbf{k}'\lambda'}{}^\dagger\rangle = \delta_{\mathbf{p}',\mathbf{p}+\hbar\mathbf{k}'/2}\delta_{\alpha\beta}g_\lambda{}^{\alpha*}(\mathbf{k}',\, \mathbf{p}+\tfrac{1}{2}\hbar\mathbf{k}') + K_{\alpha\beta\lambda}{}^*(\mathbf{k}',\, -\mathbf{k},\, \mathbf{p},\, \mathbf{p}'), \quad (49)$$

$$\langle \hat{f}_\alpha(\mathbf{K},\mathbf{p})f_\beta(\mathbf{k}-\mathbf{l},\, \mathbf{p}')b_{\mathbf{k}'\lambda'}{}^\dagger b_{1\nu}\rangle = \delta_{\mathbf{k}',1}\delta_{\lambda,\nu}n_{\mathbf{k}'\lambda'}\{-\delta_{\alpha\beta}\delta_{\mathbf{p},\mathbf{p}'}\textstyle\sum_s \varphi_{\alpha s}(\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{K})[1-\varphi_{\alpha s}(\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{K})] + g_{\alpha\beta}(\mathbf{K},\mathbf{p},\mathbf{p}')\} + \text{higher order correlation}, \quad (50)$$

$$\langle \hat{f}_\alpha(\mathbf{K}+\mathbf{l}'-\mathbf{l},\, \mathbf{p})b_{1\nu}b_{1'\nu'}{}^\dagger b_{\mathbf{k}'\lambda'}{}^\dagger b_{\mathbf{k}\lambda}\rangle = \varphi_\alpha(\mathbf{p})[\delta_{1,\mathbf{k}'}\delta_{1'\mathbf{k}}n_{\mathbf{k}\lambda}(1+n_{\mathbf{k}'\lambda'}) + \delta_{1,1'}\delta_{\nu,\nu'}\delta_{\mathbf{k},\mathbf{k}'}\delta_{\lambda,\lambda'}(1+n_{\mathbf{k}'\lambda'})] + \text{higher order correlation}, \quad (51)$$

and

$$\langle \hat{f}_{\alpha\beta}(\mathbf{K}-\mathbf{l},\, \mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l};\, \mathbf{l},\, \mathbf{p}')b_{\mathbf{k}'\lambda'}{}^\dagger b_{\mathbf{k}\lambda}\rangle = \delta_{1'\mathbf{K}}\varphi_\alpha(\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{k})h_{\lambda\lambda'}{}^\beta(\mathbf{k},\mathbf{k}',\mathbf{p}') + \text{higher order correlation}, \quad (52)$$

where to obtain Eqs. (49) and (50) we made use of the identity[10]

$$\hat{f}_2(1,2) = \hat{f}_1(1)\hat{f}_1(2) + \hat{g}_2(1,2),$$

where

$$\hat{g}_2(1,2) = -\frac{\hbar^6}{\Omega^2}\sum_{\mathbf{p}}\int d\mathbf{l}\, \exp[(2i/\hbar)(\mathbf{p}_2-\mathbf{p})\cdot(\mathbf{r}_1-\mathbf{r}_2) + i\mathbf{l}\cdot(\mathbf{p}_1-\mathbf{p}_2)]\hat{f}_1(\mathbf{r}_2+\tfrac{1}{2}\hbar\mathbf{l},\, \mathbf{p}). \quad (53)$$

This superposition ansatz is identical (although somewhat different in form) to that used by Ron[11] for the electron-phonon system.

With the use of the above ansatz, a closed set of equations is obtained when the higher order correlation terms are neglected. Note that one of the higher order correlation terms was written explicitly as $K_{\alpha\beta\lambda}$ in Eq. (49). These terms are connected with self-energy contributions. We will return to this topic later.

In order to simplify the equation somewhat more, we will take the system to be isotropic. In this case, the terms containing $g_\lambda{}^\alpha$ can be ignored because the lowest order term in $g_\lambda{}^\alpha(\mathbf{k},\mathbf{p})$ is proportional to $(\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})$ as will be demonstrated in Sec. IV. The contribution of this term will vanish because isotropy dictates that factors such as $\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}$ and $\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}$ must appear in even powers. The terms that contain $k_{\lambda\lambda'}{}^\alpha$ can also be ignored to this order because $k_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p})$ is proportional to $d_{kk'}{}^\alpha$.

The resulting equation is

$$(\partial/\partial t + i(\mathbf{K}\cdot\mathbf{p}/m_\alpha) + i\omega_k - i\omega_{k'})h_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p}) - \frac{i}{\hbar\Omega}V_{\alpha\beta}(\mathbf{K})\Delta_\alpha(\mathbf{K},\mathbf{p})\sum_{\beta,\mathbf{p}'}h_{\lambda\lambda'}{}^\beta(\mathbf{k},\mathbf{k}',\mathbf{p}') = -2id_{kk'}{}^\alpha\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}I_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p}) + 2i\sum_{\beta,\mathbf{p}'}d_{kk'}{}^\beta\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}g_{\alpha\beta}(\mathbf{K},\mathbf{p},\mathbf{p}')(n_{\mathbf{k}\lambda}-n_{\mathbf{k}'\lambda'}), \quad (54)$$

where

$$I_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p}) = \textstyle\sum_s[n_{\mathbf{k}\lambda}(1+n_{\mathbf{k}'\lambda'})G_{\alpha s}{}^+(\mathbf{K},\mathbf{p}) - n_{\mathbf{k}'\lambda'}(1+n_{\mathbf{k}\lambda})G_{\alpha s}{}^-(\mathbf{K},\mathbf{p})]. \quad (55)$$

The second term on the left-hand side of Eq. (54) has been retained because it corresponds to the Vlassov term in the linearized equation of motion for $\hat{f}_\alpha(\mathbf{K},\mathbf{p})$. Thus, it represents the screening of the particles. The last term on the right-hand side has been retained because it involves an integral over $g_{\alpha\beta}$. The long-range nature of the Coulomb potential can cause such a term to make an important contribution. In fact, such terms also give rise to screening effects as will be shown in a later section.

We assume that the Bogoliubov assumption holds for the correlation function $h_{\lambda\lambda'}{}^\alpha$, as well as $g_{\alpha\beta}$. A similar assumption was made by Ron concerning the electron-phonon correlation function. The equation for the asymptotic correlation function $\tilde{h}_{\lambda\lambda'}{}^\alpha$ is then given by

$$(\epsilon + i(\mathbf{K}\cdot\mathbf{p}/m_\alpha) + i\omega_k - i\omega_{k'})\tilde{h}_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p}) - \frac{i}{\hbar\Omega}\sum_{\beta\mathbf{p}'}V_{\alpha\beta}(\mathbf{K})\Delta_\alpha(\mathbf{K},\mathbf{p})\tilde{h}_{\lambda\lambda'}{}^\beta(\mathbf{k},\mathbf{k}',\mathbf{p}') = -2id_{kk'}{}^\alpha\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}I_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p}) + 2i\sum_{\beta,\mathbf{p}'}d_{kk'}{}^\beta\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}\tilde{g}_{\alpha\beta}(\mathbf{K},\mathbf{p},\mathbf{p}')(n_{\mathbf{k}\lambda}-n_{\mathbf{k}'\lambda'}), \quad (56)$$

where again

$$\tilde{h}_{\lambda\lambda'}{}^{\alpha}(\mathbf{k},\mathbf{k}',\mathbf{p})=\lim_{\epsilon\to0+}\epsilon\int_0^\infty e^{-\epsilon t}h_{\lambda\lambda'}{}^{\alpha}(\mathbf{k},\mathbf{k}',\mathbf{p})dt\,. \tag{57}$$

From the above equation we easily obtain

$$\begin{aligned}\tilde{h}_{\lambda\lambda'}{}^{\alpha}(\mathbf{k},\mathbf{k}',\mathbf{p})=&\frac{2d_{kk'}{}^{\alpha}\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}I_{\lambda\lambda'}{}^{\alpha}(\mathbf{k},\mathbf{k}',\mathbf{p})}{\omega_{k'}-\omega_k-(\mathbf{K}\cdot\mathbf{p}/m_\alpha)+i\epsilon}\\&-\sum_{\beta,\mathbf{p}'}\frac{2d_{kk'}{}^{\beta}\epsilon_{\mathbf{k}\lambda}\cdot\epsilon_{\mathbf{k}'\lambda'}\tilde{g}_{\alpha\beta}(\mathbf{K},\mathbf{p},\mathbf{p}')(n_{\mathbf{k}\lambda}-n_{\mathbf{k}'\lambda'})}{\omega_{k'}-\omega_k-(\mathbf{K}\cdot\mathbf{p}/m_\alpha)+i\epsilon}+2z_\alpha V(\mathbf{k})\Delta_\alpha(\mathbf{K},\mathbf{p})\{\hbar\Omega\mathcal{E}[\mathbf{K},\,\omega_{k'}-\omega_k](\omega_{k'}-\omega_k-(\mathbf{K}\cdot\mathbf{p}/m_\alpha)+i\epsilon)\}^{-1}\\&\times\left[-\sum_{\beta,\mathbf{p}'}\frac{z_\beta d_{kk'}{}^{\beta}\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}I_{\lambda\lambda'}{}^{\beta}(\mathbf{k},\mathbf{k}',\mathbf{p}')}{\omega_{k'}-\omega_k-(\mathbf{K}\cdot\mathbf{p}'/m_\beta)+i\epsilon}+\sum_{\beta,\mathbf{p}'}\sum_{\gamma,\mathbf{p}''}\frac{z_\beta d_{kk'}{}^{\gamma}\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}\tilde{g}_{\beta\gamma}(\mathbf{K},\mathbf{p}',\mathbf{p}'')(n_{\mathbf{k}\lambda}-n_{\mathbf{k}'\lambda'})}{\omega_{k'}-\omega_k-(\mathbf{K}\cdot\mathbf{p}'/m_\beta)+i\epsilon}\right].\end{aligned} \tag{58}$$

The contribution of particle-photon scattering to the kinetic equations is then obtained by the substitution of the above expression into Eqs. (44) and (45). The results of these substitutions are

$$\begin{aligned}\left[\frac{\partial n_{\mathbf{k}\lambda}}{\partial t}\right]_{\mathrm{p.s.}}=&-\sum_{\alpha,\mathbf{p}}\sum_{\mathbf{k}'\lambda'}8\pi\hbar[d_{kk'}{}^{\alpha}\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}]^2I_{\lambda\lambda'}{}^{\alpha}(\mathbf{k},\mathbf{k}',\mathbf{p})\delta[E_\alpha(\mathbf{p}+\hbar\mathbf{K})+\hbar\omega_k-E_\alpha(\mathbf{p})-\hbar\omega_{k'}]\\&+\sum_{\alpha,\mathbf{p}}\sum_{\beta,\mathbf{p}'}\sum_{\mathbf{k}',\lambda'}8d_{kk'}{}^{\alpha}d_{kk'}{}^{\beta}(\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'})^2(n_{\mathbf{k}\lambda}-n_{\mathbf{k}'\lambda'})\,\mathrm{Im}\left[\frac{\tilde{g}_{\alpha\beta}(\mathbf{K},\mathbf{p},\mathbf{p}')}{\omega_{k'}-\omega_k-(\mathbf{K}\cdot\mathbf{p}/m_\alpha)+i\epsilon}\right]\\&+\sum_{\alpha,\mathbf{p}}\sum_{\mathbf{k}',\lambda'}8z_\alpha d_{\lambda\lambda'}{}^{\alpha}V(\mathbf{K})\Delta_\alpha(\mathbf{K},\mathbf{p})(\epsilon_{\mathbf{k}\lambda}\cdot\epsilon_{\mathbf{k}'\lambda'})^2\,\mathrm{Im}\Bigg\{[\hbar\Omega\mathcal{E}[\mathbf{K},\,\omega_{k'}-\omega_k](\omega_{k'}-\omega_k-(\mathbf{K}\cdot\mathbf{p}/m_\alpha)+i\epsilon)]^{-1}\\&\times\left[-\sum_{\beta,\mathbf{p}'}\frac{z_\beta d_{kk'}{}^{\beta}I_{\lambda\lambda'}{}^{\beta}(\mathbf{k},\mathbf{k}',\mathbf{p})}{\omega_{k'}-\omega_k-(\mathbf{K}\cdot\mathbf{p}'/m_\beta)+i\epsilon}+\sum_{\beta,\mathbf{p}'}\sum_{\gamma,\mathbf{q}}\frac{z_\beta d_{kk'}{}^{\gamma}(n_{\mathbf{k}\lambda}-n_{\mathbf{k}'\lambda'})\tilde{g}_{\beta\gamma}(\mathbf{K},\mathbf{p}',\mathbf{q})}{\omega_{k'}-\omega_k-(\mathbf{K}\cdot\mathbf{p}'/m_\beta)+i\epsilon}\right]\Bigg\},\end{aligned} \tag{59}$$

and

$$\begin{aligned}\left[\frac{\partial\varphi_\alpha(\mathbf{p})}{\partial t}\right]_{\mathrm{p.s.}}=&-\sum_{k,k'}16\pi\hbar[d_{kk'}{}^{\alpha}\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}]^2I_{\lambda\lambda'}{}^{\alpha}(\mathbf{k},\,\mathbf{k}',\,\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{K})\delta[E_\alpha(\mathbf{p}+\hbar\mathbf{K})+\hbar\omega_k-E_\alpha(\mathbf{p})-\hbar\omega_{k'}]\\&+\sum_{\mathbf{k},\mathbf{k}'}\sum_{\lambda,\lambda'}\sum_{\beta,\mathbf{p}'}8d_{kk'}{}^{\alpha}d_{kk'}{}^{\beta}(\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'})^2(n_{\mathbf{k}\lambda}-n_{\mathbf{k}'\lambda'})\\&\times\mathrm{Im}\left[\frac{\tilde{g}_{\alpha\beta}(\mathbf{K},\,\mathbf{p}+\frac{1}{2}\hbar\mathbf{K},\,\mathbf{p}')}{\omega_{k'}-\omega_k-(\mathbf{K}/m_\alpha)\cdot(\mathbf{p}+\hbar\mathbf{K}/2)+i\epsilon}-\frac{\tilde{g}_{\alpha\beta}(\mathbf{K},\,\mathbf{p}-\frac{1}{2}\hbar\mathbf{K},\,\mathbf{p}')}{\omega_{k'}-\omega_k-(\mathbf{K}/m_\alpha)\cdot(\mathbf{p}-\frac{1}{2}\hbar\mathbf{K})+i\epsilon}\right]\\&+\sum_{\mathbf{k},\mathbf{k}'}\sum_{\lambda,\lambda'}8z_\alpha d_{kk'}{}^{\alpha}(\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'})^2V(\mathbf{K})\,\mathrm{Im}[\hbar\Omega\mathcal{E}(\mathbf{K},\,\omega_k'-\omega_k)]^{-1}\\&\times\left[\frac{\Delta_\alpha(\mathbf{K},\,\mathbf{p}+\frac{1}{2}\hbar\mathbf{K})}{\omega_{k'}-\omega_k-(\mathbf{K}/m_\alpha)\cdot(\mathbf{p}+\frac{1}{2}\hbar\mathbf{K})+i\epsilon}-\frac{\Delta_\alpha(\mathbf{K},\,\mathbf{p}-\frac{1}{2}\hbar\mathbf{K})}{\omega_{k'}-\omega_k-(\mathbf{K}/m_\alpha)\cdot(\mathbf{p}-\frac{1}{2}\hbar\mathbf{K})+i\epsilon}\right]\\&\times\left[-\sum_{\beta,\mathbf{p}'}\frac{z_\beta d_{kk'}{}^{\beta}I_{\lambda\lambda'}{}^{\beta}(\mathbf{k},\mathbf{k}',\mathbf{p})}{\omega_{k'}-\omega_k-(\mathbf{K}\cdot\mathbf{p}'/m_\beta)+i\epsilon}+\sum_{\beta,\mathbf{p}'}\sum_{\gamma,q}\frac{z_\beta d_{kk'}{}^{\gamma}\tilde{g}_{\beta\gamma}(\mathbf{K},\mathbf{p}',\mathbf{q})}{\omega_{k'}-\omega_k-(\mathbf{K}\cdot\mathbf{p}'/m_\beta)+i\epsilon}\right],\end{aligned} \tag{60}$$

where the formula

$$(x+i\epsilon)^{-1}=Px^{-1}i\pi\delta(x)$$

has been used to obtain the delta function. The leading terms in Eqs. (59) and (60) yield the results of Osborn and Klevans[6] when

$$\omega_k=(k^2c^2+\omega_p{}^2)^{1/2}$$

is replaced by

$$\omega_k=kc+\omega_p{}^2/2kc\,,$$

and these terms also yield the nonrelativistic limit of Dreicer's[7] results when ω_k is taken to be kc. The remaining terms represent corrections arising from correlations. These terms also have the interesting property that they yield contributions that are not proportional to

$$\delta[E_\alpha(\mathbf{p}+\hbar\mathbf{K})+\hbar\omega_k-E_\alpha(\mathbf{p})-\hbar\omega_{k'}].$$

These terms are similar to terms obtained by Kohn and Luttinger,[12] Mangeney,[4] and Michel.[13] The other terms can be identified, in the classical limit, with the corrections due to the shielding clouds of the particles.[2,21]

Some of the higher order correlation terms that were neglected in the derivation of Eq. (54) can be shown to yield self-energy corrections. It is not very difficult to pick out the appropriate terms. One way is to look at the equations of motion for these higher order correlation functions and to pick out terms proportional to $h_{\lambda\lambda'}{}^\alpha(k,k',p)$. A simpler method is to consider the equation of motion or higher random-phase approximation[16,22] methods for deriving dispersion relations. Then we simply pick out the corresponding terms in Eq. (54). In particular, we can include the lowest photon self-energy correction by retaining the terms corresponding to Eq. (49). The equation of motion for $h_{\lambda\lambda'}{}^\alpha$ is then given by

$$\begin{aligned}(\partial/\partial t+i(\mathbf{K}\cdot\mathbf{p}/m_\alpha)+i\omega_k-i\omega_{k'})h_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p})-\frac{i}{\hbar\Omega}V_{\alpha\beta}(\mathbf{K})\Delta_\alpha(\mathbf{K},\mathbf{p})\sum_{\beta,\mathbf{p}'}h_{\lambda\lambda'}{}^\beta(\mathbf{k},\mathbf{k}',\mathbf{p}')\\ -i\sum_{\beta,\mathbf{p}'}[c_{\beta k}(\mathbf{p}'\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})K_{\alpha\beta\lambda}{}^*(\mathbf{k}',-\mathbf{k},\mathbf{p},\mathbf{p}')-c_{\beta k'}(\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'})K_{\alpha\beta\lambda}(\mathbf{k},-\mathbf{k}',\mathbf{p},\mathbf{p}')]\\ =-2id_{kk'}{}^\alpha\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}I_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p})+2i\sum_{\beta,\mathbf{p}'}d_{kk'}{}^\beta\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}g_{\alpha\beta}(\mathbf{K},\mathbf{p},\mathbf{p}')(n_{\mathbf{k}\lambda}-n_{\mathbf{k}'\lambda'}),\end{aligned}\tag{61}$$

where

$$K_{\alpha\beta\lambda}(\mathbf{k},\mathbf{k}',\mathbf{p},\mathbf{p}')=\langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{k}',\mathbf{p};\mathbf{k}',\mathbf{p}')b_{\mathbf{k}}\rangle-\delta_{\mathbf{k},-\mathbf{k}'}\delta_{\lambda,\lambda'}\varphi_\alpha(\mathbf{p})g_\lambda{}^\beta(\mathbf{k},\mathbf{p}).\tag{62}$$

The procedure at this point is to consider the equation for $K_{\alpha\beta\lambda}$ and to solve for the asymptotic function $\tilde{K}_{\alpha\beta\lambda}$, retaining only those terms proportional to $h_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p})$. The results of this procedure are the following:

$$\tilde{K}_{\alpha\beta\lambda}{}^*(\mathbf{k}',-\mathbf{k},\mathbf{p},\mathbf{p}')=[\omega_{k'}-(\mathbf{k}\cdot\mathbf{p}'/m_\beta)-(\mathbf{K}\cdot\mathbf{p}/m_\alpha)+i\epsilon]^{-1}c_{\beta k}(\mathbf{p}'\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})\Delta_\beta(\mathbf{k},\mathbf{p}')h_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p}),\tag{63}$$

and

$$\tilde{K}_{\alpha\beta\lambda}(\mathbf{k},-\mathbf{k}',\mathbf{p},\mathbf{p}')=[\omega_k-(\mathbf{k}'\cdot\mathbf{p}'/m_\beta)+(\mathbf{K}\cdot\mathbf{p}/m_\alpha)-i\epsilon]^{-1}c_{\beta k'}(\mathbf{p}'\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'})\Delta_\beta(\mathbf{k}',\mathbf{p}')h_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p}).\tag{64}$$

With the use of the above equations and Eq. (61), we obtain the following equation for $\tilde{h}_{\lambda\lambda'}{}^\alpha(k,k',p)$:

$$\begin{aligned}\{\epsilon+i(\mathbf{K}\cdot\mathbf{p}/m_\alpha)+i[\omega_k+\delta_{\mathbf{k}}(\omega_{k'}-(\mathbf{K}\cdot\mathbf{p}/m_\alpha))]-i[\omega_{k'}+\delta_{\mathbf{k}'}(\omega_k+(\mathbf{K}\cdot\mathbf{p}/m_\alpha))]\}h_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p})\\ -\frac{i}{\hbar\Omega}\Delta_\alpha(\mathbf{K},\mathbf{p})\sum_{\beta,\mathbf{p}'}V_{\alpha\beta}(\mathbf{K})h_{\lambda\lambda'}{}^\beta(\mathbf{k},\mathbf{k}',\mathbf{p}')=-2id_{kk'}{}^\alpha\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}I_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p})\\ +2i\sum_{\beta,\mathbf{p}'}d_{kk'}{}^\beta\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}g_{\alpha\beta}(\mathbf{K},\mathbf{p},\mathbf{p}')(n_{\mathbf{k}\lambda}-n_{\mathbf{k}'\lambda'}),\end{aligned}\tag{65}$$

where

$$\delta_{\mathbf{k}}(\omega)=-\sum_{\alpha,\mathbf{p}}\frac{c_{\alpha k}{}^2(\mathbf{p}\cdot\epsilon_{\mathbf{k}\lambda})^2\Delta_\alpha(\mathbf{k},\mathbf{p})}{\omega-(\mathbf{k}\cdot\mathbf{p}/m_\alpha)+i\epsilon}.\tag{66}$$

The quantity $\delta_{\mathbf{k}}(\omega_{k'}-(\mathbf{K}\cdot\mathbf{p}/m_\alpha))$ gives the energy shift for the photon with momentum $\hbar\mathbf{k}$. These corrections correspond to the phonon self-energy corrections obtained by Michel[13] in his analysis of the electron-phonon system. Higher order corrections to the photon self-energy and self-energy corrections for the particles can be obtained in a similar manner.

IV. SINGLE EMISSION-ABSORPTION

The correlation function which corresponds to single emission-absorption processes is

$$g_\lambda{}^\alpha(\mathbf{k},\mathbf{p})=\langle\hat{f}_\alpha(-\mathbf{k},\mathbf{p})b_{\mathbf{k}\lambda}\rangle.$$

[21] M. N. Rosenbluth and N. Rostoker, Phys. Fluids 5, 776 (1962).
[22] R. K. Nesbet, J. Math. Phys. 6, 621 (1965).

The process of bremsstrahlung will be included in the contribution of $g_\lambda{}^\alpha$. The equation of motion for $g_\lambda{}^\alpha$ is given by

$$
\begin{aligned}
(\partial/\partial t-i(\mathbf{k}/m_\alpha)\cdot\mathbf{p}+i\omega_k)g_\lambda{}^\alpha(\mathbf{k},\mathbf{p}) &= i\sum_{\beta,\mathbf{p}'}c_{\beta k}(\mathbf{p}'\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})\langle\hat{f}_\alpha(-\mathbf{k},\mathbf{p})\hat{f}_\beta(\mathbf{k},\mathbf{p}')\rangle \\
&\quad -2i\sum_{\beta,\mathbf{p}'}\sum_{\mathbf{k}'\lambda'}d_{kk'}{}^\beta\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}\langle\hat{f}_\alpha(-\mathbf{k},\mathbf{p})\hat{f}_\beta(\mathbf{k}-\mathbf{k}',\mathbf{p})(b_{\mathbf{k}'\lambda'}+b_{-\mathbf{k}'\lambda'}{}^\dagger)\rangle \\
&\quad -\left(\frac{i}{2}\right)\sum_{\mathbf{k}'\lambda'}c_{\alpha k}(\hbar\mathbf{k}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'})[\langle\hat{f}_\alpha(-\mathbf{k}-\mathbf{k}',\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{k}')(b_{\mathbf{k}'\lambda'}+b_{-\mathbf{k}'\lambda'}{}^\dagger)b_{\mathbf{k}\lambda}\rangle \\
&\quad -\langle\hat{f}_\alpha(-\mathbf{k}-\mathbf{k}',\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{k}')(b_{\mathbf{k}'\lambda'}+b_{-\mathbf{k}'\lambda'}{}^\dagger)b_{\mathbf{k}\lambda}\rangle] \\
&\quad -i\sum_{\mathbf{k}'\lambda'}c_{\alpha k}(\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})[\langle\hat{f}_\alpha(-\mathbf{k}-\mathbf{k}',\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{k}')(b_{\mathbf{k}'\lambda'}+b_{-\mathbf{k}'\lambda'}{}^\dagger)b_{\mathbf{k}\lambda}\rangle \\
&\quad -\langle\hat{f}_\alpha(-\mathbf{k}-\mathbf{k}',\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{k}')(b_{\mathbf{k}'\lambda'}+b_{-\mathbf{k}'\lambda'}{}^\dagger)b_{\mathbf{k}\lambda}\rangle]+i\sum_{\mathbf{k}',\lambda'}\sum_{\mathbf{l},\nu}d_{kk'}{}^\alpha\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}\cdot\boldsymbol{\varepsilon}_{\mathbf{l}\nu} \\
&\quad \times[\langle\hat{f}_\alpha(-\mathbf{k}+\mathbf{l}-\mathbf{k}',\mathbf{p}+\tfrac{1}{2}\hbar(\mathbf{k}'-\mathbf{l}))(b_{\mathbf{k}'\lambda'}+b_{-\mathbf{k}'\lambda'}{}^\dagger)(b_{-\mathbf{l}\nu}+b_{\mathbf{l}\nu}{}^\dagger)b_{\mathbf{k}\lambda}\rangle \\
&\quad -\langle\hat{f}_\alpha(-\mathbf{k}+\mathbf{l}-\mathbf{k}',\mathbf{p}-\tfrac{1}{2}\hbar(\mathbf{k}'-\mathbf{l}))(b_{\mathbf{k}'\lambda'}+b_{-\mathbf{k}'\lambda'}{}^\dagger)(b_{-\mathbf{l}\nu}+b_{\mathbf{l}\nu}{}^\dagger)b_{\mathbf{k}\lambda}\rangle] \\
&= \frac{i}{\hbar\Omega}\sum_{\beta,\mathbf{p}'}\sum_{\mathbf{l}}V_{\alpha\beta}(\mathbf{l})[\langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{l},\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l};\mathbf{k}',\mathbf{p}')b_{\mathbf{k}\lambda}\rangle-\langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{l},\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{k}';\mathbf{k}',\mathbf{p}')b_{\mathbf{k}\lambda}\rangle]. \qquad (67)
\end{aligned}
$$

We again truncate this equation using the same method as in the previous section. The assumption of isotropy allows terms with factors $\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}$ to be ignored. Also all correlation functions are assumed to be small compared to products of φ_α and $n_{\mathbf{k}\lambda}$. The result of these approximations is

$$
\begin{aligned}
(\partial/\partial t-i(\mathbf{k}/m_\alpha)\cdot\mathbf{p}+i\omega_k)g_\lambda{}^\alpha(\mathbf{k},\mathbf{p})-i[\Delta_\alpha(\mathbf{k},\mathbf{p})/\hbar\Omega]\sum_{\beta,\mathbf{p}'}V_{\alpha\beta}(\mathbf{k})g_\lambda{}^\alpha(\mathbf{k},\mathbf{p}') &= ic_{\alpha k}(\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})J_{\alpha\lambda}(\mathbf{k},\mathbf{p}) \\
+i\sum_{\beta,\mathbf{p}'}c_{\beta k}(\mathbf{p}'\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})g_{\alpha\beta}(-\mathbf{k},\mathbf{p},\mathbf{p}')+\frac{i}{\hbar\Omega}\sum_{\beta,\mathbf{p}'}\sum_{\mathbf{l}}V_{\alpha\beta}(\mathbf{l})[K_{\alpha\beta\lambda}(\mathbf{k},\mathbf{k}',\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{k}',\mathbf{p}')&-K_{\alpha\beta\lambda}(\mathbf{k},\mathbf{k}',\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{k}',\mathbf{p}')], \qquad (68)
\end{aligned}
$$

where

$$J_{\alpha\lambda}(\mathbf{k},\mathbf{p})=\textstyle\sum_s[(1+n_{\mathbf{k}\lambda})G_{\alpha s}{}^+(\mathbf{k},\mathbf{p})-n_{\mathbf{k}\lambda}G_{\alpha s}{}^-(\mathbf{k},\mathbf{p})]. \qquad (69)$$

The above equation has the same form as Eq. (54). The higher order correlation function $K_{\alpha\beta\lambda}$ has been retained at this point because it is responsible for the bremsstrahlung contribution. The $g_{\alpha\beta}$ term gives rise to photon self-energy terms, as well as screening effects.

For the moment we will ignore the $K_{\alpha\beta\lambda}$ terms in order to obtain an expression for $\tilde{g}_\lambda{}^\alpha$. The equation for the asymptotic function $\tilde{g}_\lambda{}^\alpha$ is then given by

$$
\begin{aligned}
(\omega_k-(\mathbf{k}/m_\alpha)\cdot\mathbf{p}-i\epsilon)\tilde{g}_\lambda{}^\alpha(\mathbf{k},\mathbf{p})-[\Delta_\alpha(\mathbf{k},\mathbf{p})/\hbar\Omega]\sum_{\beta,\mathbf{p}'}V_{\alpha\beta}(\mathbf{k})\tilde{g}_\lambda{}^\beta(\mathbf{k},\mathbf{p}') \\
=c_{\alpha k}(\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})J_{\alpha\lambda}(\mathbf{k},\mathbf{p})+\sum_{\beta,\mathbf{p}'}c_{\beta k}(\mathbf{p}'\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})\tilde{g}_{\alpha\beta}(-\mathbf{k},\mathbf{p},\mathbf{p}'). \qquad (70)
\end{aligned}
$$

The above equation is easily solved for $\tilde{g}_\lambda{}^\alpha$. The result is

$$
\begin{aligned}
\tilde{g}_\lambda{}^\alpha(\mathbf{k},\mathbf{p}) &= \frac{c_{\alpha k}(\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})J_{\alpha\lambda}(\mathbf{k},\mathbf{p})}{\omega_k-(\mathbf{k}/m_\alpha)\cdot\mathbf{p}-i\epsilon}+\sum_{\beta,\mathbf{p}'}\frac{c_{\beta k}(\mathbf{p}'\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})\tilde{g}_{\alpha\beta}(-\mathbf{k},\mathbf{p},\mathbf{p}')}{\omega_k-(\mathbf{k}/m_\alpha)\cdot\mathbf{p}-i\epsilon}+z_\alpha\Delta_\alpha(\mathbf{k},\mathbf{p})V(\mathbf{k})\{\hbar\Omega\mathcal{E}[\mathbf{k},\omega_k](\omega_k-(\mathbf{k}/m_\alpha)\cdot\mathbf{p}-i\epsilon)\}^{-1} \\
&\quad \times\sum_{\beta,\mathbf{p}'}\left[\frac{z_\beta c_{\beta k}(\mathbf{p}'\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})J_{\beta\lambda}(\mathbf{k},\mathbf{p}')}{\omega_k-(\mathbf{k}/m_\beta)\cdot\mathbf{p}'-i\epsilon}+\sum_{\gamma,\mathbf{p}''}\frac{c_{\gamma k}z_\beta(\mathbf{p}''\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})\tilde{g}_{\beta\gamma}(-\mathbf{k},\mathbf{p}',\mathbf{p}'')}{\omega_k-(\mathbf{k}/m_\beta)\cdot\mathbf{p}'-i\epsilon}\right]. \qquad (71)
\end{aligned}
$$

The fact that only the imaginary part of $g_\lambda{}^\alpha$ is required results in considerable simplification because the delta function (which becomes the Dirac function in the limit $\Omega\to\infty$), $\delta(\omega_k-(\mathbf{k}/m_\alpha)\cdot\mathbf{p})$, is always zero. Therefore,

$$
\begin{aligned}
\mathrm{Im}g_\lambda{}^\alpha(\mathbf{k},\mathbf{p}) &= \sum_{\beta,\mathbf{p}'}\frac{c_{\beta k}(\mathbf{p}'\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})\,\mathrm{Im}\tilde{g}_{\alpha\beta}(-\mathbf{k},\mathbf{p},\mathbf{p}')}{\omega_k-(\mathbf{k}/m_\alpha)\cdot\mathbf{p}}+z_\alpha\Delta_\alpha(\mathbf{k},\mathbf{p})V(\mathbf{k})\{\hbar\Omega\mathcal{E}[\mathbf{k},\omega_k](\omega_k-(\mathbf{k}/m_\alpha)\cdot\mathbf{p}-i\epsilon)\}^{-1} \\
&\quad \times\sum_{\gamma,\mathbf{p}''}\sum_{\beta,\mathbf{p}'}\frac{c_{\gamma k}z_\beta(\mathbf{p}''\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})\,\mathrm{Im}\tilde{g}_{\beta\gamma}(-\mathbf{k},\mathbf{p}',\mathbf{p}'')}{\omega_k-(\mathbf{k}/m_\beta)\cdot\mathbf{p}'}. \qquad (72)
\end{aligned}
$$

A further simplification results if only the largest contribution to $\tilde{g}_{\alpha\beta}(\mathbf{k},\mathbf{p},\mathbf{p}')$ is considered. This contribution, which arises directly from the Coulomb interaction, is proportional to $\delta(\mathbf{k}\cdot\mathbf{p}/m_\alpha-\mathbf{k}\cdot\mathbf{p}'/m_\beta)$. Since $g_{\alpha\beta}$ has the property that

$$\mathrm{Im}g_{\alpha\beta}(\mathbf{k},\mathbf{p},\mathbf{p}')=-\mathrm{Im}g_{\beta\alpha}(\mathbf{k},\mathbf{p}',\mathbf{p}), \tag{73}$$

such terms will not contribute to the second term on the right-hand side of Eq. (72). (This is seen by carrying out the transformation $\beta\leftrightarrow\gamma$ and $\mathbf{p}'\leftrightarrow\mathbf{p}''$.)

The contribution of $g_\lambda{}^\alpha$ to the kinetic equations is then given by

$$\left[\frac{\partial n_{\mathbf{k}\lambda}}{\partial t}\right]_{\mathrm{s.e.}}=\sum_{\alpha,\mathbf{p}}\sum_{\beta,\mathbf{p}'}\frac{2c_{\alpha k}c_{\beta k}(\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})(\mathbf{p}'\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})}{\omega_k(\omega_k-(\mathbf{k}/m_\alpha)\cdot\mathbf{p})}\times\mathrm{Im}\tilde{g}_{\alpha\beta}(-\mathbf{k},\mathbf{p},\mathbf{p}'), \tag{74}$$

and

$$\left[\frac{\partial\varphi_\alpha(\mathbf{p})}{\partial t}\right]_{\mathrm{s.e.}}=-\sum_{\beta,\mathbf{p}'}\sum_{\mathbf{k}\lambda}\frac{4\omega_k c_{\alpha k}c_{\beta k}(\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})(\mathbf{p}'\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})}{\omega_k{}^2-((\mathbf{k}/m_\alpha)\cdot\mathbf{p})^2}\times\mathrm{Im}\tilde{g}_{\alpha\beta}(\mathbf{k},\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{k},\mathbf{p}'). \tag{75}$$

The term on the right-hand side of Eq. (74) can be seen to be zero by the same considerations which were used to eliminate the second term on the right-hand side of Eq. (72). Therefore, as expected, there is no radiation at this order.

Clearly Eq. (75) represents an additional interparticle force [cf. Eq. (32)]. This is the force arising from the exchange of virtual photons by the particles. There are similar terms in Eq. (60) arising from the scattering of a single photon by two particles.

The lowest order photon self-energy correction can be obtained by our considering in more detail the equation for $\tilde{g}_{\alpha\beta}(-\mathbf{k},\mathbf{p},\mathbf{p}')$ (see Sec. VI). This equation has the following form:

$$[\mathbf{k}\cdot(\mathbf{p}/m_\alpha-\mathbf{p}'/m_\beta)+i\epsilon]\tilde{g}_{\alpha\beta}(-\mathbf{k},\mathbf{p},\mathbf{p}')=c_{\beta k}(\mathbf{p}'\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})\Delta_\beta(\mathbf{k},\mathbf{p}')\tilde{g}_\lambda{}^\alpha(\mathbf{k},\mathbf{p})+\cdots, \tag{76}$$

where $\cdots$ denotes those terms that are not proportional to $g_\lambda{}^\alpha(\mathbf{k},\mathbf{p})$. The result of the substitution of the above expression for $\tilde{g}_{\alpha\beta}$ into Eq. (70) is

$$\left[\omega_k+\delta_{\mathbf{k}}\left(\frac{\mathbf{k}\cdot\mathbf{p}}{m_\alpha}\right)-\frac{\mathbf{k}\cdot\mathbf{p}}{m_\alpha}-i\epsilon\right]\tilde{g}_\lambda{}^\alpha(\mathbf{k},\mathbf{p})-\frac{\Delta_\alpha(\mathbf{k},\mathbf{p})}{\hbar\Omega}\sum_{\beta,p'}V_{\alpha\beta}(\mathbf{k})\tilde{g}_\lambda{}^\beta(\mathbf{k},\mathbf{p}')$$
$$=c_{\alpha k}(\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})J_{\alpha\lambda}(\mathbf{k},\mathbf{p})+\sum_{\beta,\mathbf{p}'}c_{\beta k}(\mathbf{p}'\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})\tilde{g}_{\alpha\beta}'(-\mathbf{k},\mathbf{p},\mathbf{p}'), \tag{77}$$

where $\tilde{g}_{\alpha\beta}'$ differs from $\tilde{g}_{\alpha\beta}$ by containing no terms proportional to $\tilde{g}_\lambda{}^\alpha$. The self-energy correction does not affect the vanishing of certain terms in the expression for $\mathrm{Im}\tilde{g}_\lambda{}^\alpha$.

In order to obtain radiation, the hierarchy must be truncated at a higher level. Since the bremsstrahlung process involves the interaction of two particles and a photon, it is clear that the correlation function $K_{\alpha\beta\lambda}$ contains information about this process.

The equation of motion for $K_{\alpha\beta\lambda}$ is given in the Appendix. We again apply the superposition ansatz to the correlation functions which appear in this equation. These correlation functions are then written as

$$\begin{aligned}\langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{k}',\mathbf{p};\mathbf{k}',\mathbf{p}')\hat{f}_\gamma(\mathbf{k},\mathbf{q})\rangle=&-\delta_{\alpha\gamma}\delta_{\mathbf{q},\mathbf{p}-\hbar\mathbf{k}'/2}f_{\alpha\beta}(-\mathbf{k}',\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{k};\mathbf{k}',\mathbf{p}')\\&-\delta_{\beta\gamma}\delta_{\mathbf{q},\mathbf{p}'+\hbar(\mathbf{k}'+\mathbf{k})/2}f_{\alpha\beta}(\mathbf{k}+\mathbf{k}',\mathbf{p};-\mathbf{k}-\mathbf{k}',\mathbf{p}'-\tfrac{1}{2}\hbar\mathbf{k})\\&+f_{\alpha\beta\gamma}(-\mathbf{k}-\mathbf{k}',\mathbf{p};\mathbf{k}',\mathbf{p}';\mathbf{k},\mathbf{q}),\end{aligned} \tag{78}$$

$$\langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{k}'-\mathbf{l},\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l};\mathbf{k}',\mathbf{p})(b_{\mathbf{l}}+b_{-\mathbf{l}}{}^\dagger)b_{\mathbf{k}}\rangle=\delta_{\mathbf{l},-\mathbf{k}}n_{\mathbf{k}}f_{\alpha\beta}(-\mathbf{k}',\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{k};\mathbf{k}',\mathbf{p}'), \tag{79}$$

$$\langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{k}'-\mathbf{l},\mathbf{p};\mathbf{k}'+\mathbf{l},\mathbf{p}')b_{\mathbf{k}}\rangle=\delta_{\mathbf{l},-\mathbf{k}-\mathbf{k}'}\varphi_\alpha(\mathbf{p})g_\lambda{}^\beta(\mathbf{k},\mathbf{p}')+\delta_{\mathbf{l},-\mathbf{k}'}\varphi_\beta(\mathbf{p}')g_\lambda{}^\alpha(\mathbf{k},\mathbf{p}), \tag{80}$$

and

$$\begin{aligned}\langle\hat{f}_{\alpha\beta\gamma}(-\mathbf{k}-\mathbf{k}'-\mathbf{l},\mathbf{p};\mathbf{k}',\mathbf{p}';\mathbf{l},\mathbf{q})b_{\mathbf{k}}\rangle=&\,\delta_{\mathbf{l},-\mathbf{k}}f_{\alpha\beta}(-\mathbf{k}',\mathbf{p};\mathbf{k}',\mathbf{p}')g_\lambda{}^\gamma(\mathbf{k},\mathbf{q})\\&+\delta_{\mathbf{l},-\mathbf{k}'}f_{\beta\gamma}(\mathbf{k}',\mathbf{p}';-\mathbf{k}',\mathbf{q})g_\lambda{}^\alpha(\mathbf{k},\mathbf{p})+\delta_{\mathbf{l},-\mathbf{k}-\mathbf{k}'}\varphi_\alpha(\mathbf{p})K_{\gamma\beta}(\mathbf{k},\mathbf{k}',\mathbf{q},\mathbf{p}').\end{aligned} \tag{81}$$

The contributions of the exchange terms arising from $f_{\alpha\beta}(-\mathbf{k}',\mathbf{p};\mathbf{k}',\mathbf{p}')$ are neglected for the same reasons as given in Sec. II. Furthermore, terms containing factors of $g_{\alpha\beta}g_\lambda{}^\alpha$ are neglected because each of the correlation functions is assumed to be small. The resulting equation is given by

$$\left[\frac{\partial}{\partial t}-i(\mathbf{k}+\mathbf{l})\cdot\frac{\mathbf{p}}{m_\alpha}+i\mathbf{l}\cdot\frac{\mathbf{p}'}{m_\beta}+i\omega_k\right]K_{\alpha\beta\lambda}(\mathbf{k},\mathbf{l},\mathbf{p},\mathbf{p}')+\frac{i}{\hbar\Omega}\Delta_\alpha(\mathbf{k}+\mathbf{l},\mathbf{p})\sum_{\gamma,\mathbf{q}}V_{\alpha\gamma}(\mathbf{k})K_{\gamma\beta}(\mathbf{k},\mathbf{l},\mathbf{q},\mathbf{p}')$$

$$-\frac{k}{\hbar\Omega}\Delta_\beta(\mathbf{l},\mathbf{p}')\sum_{\gamma,\mathbf{q}}V_{\gamma\beta}(\mathbf{k})K_{\alpha\gamma}(\mathbf{k},\mathbf{l},\mathbf{p},\mathbf{q})=ic_{\alpha k}[(\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{l})\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}(1+n_{\mathbf{k}\lambda})g_{\alpha\beta}(-\mathbf{l},\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{k},\mathbf{p}')$$

$$-(\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l})\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}n_{\mathbf{k}\lambda}g_{\alpha\beta}(-\mathbf{l},\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{k},\mathbf{p}')]+ic_{\beta k}[(\mathbf{p}'+\tfrac{1}{2}\hbar\mathbf{l})\cdot\boldsymbol{\epsilon}_{\mathbf{k}\lambda}(1+n_{\mathbf{k}\lambda})g_{\alpha\beta}(\mathbf{l}+\mathbf{k},\mathbf{p},\mathbf{p}'+\tfrac{1}{2}\hbar\mathbf{k})$$

$$-(\mathbf{p}'-\tfrac{1}{2}\hbar\mathbf{l})\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}n_{\mathbf{k}\lambda}g_{\alpha\beta}(\mathbf{l}+\mathbf{k},\mathbf{p},\mathbf{p}'-\tfrac{1}{2}\hbar\mathbf{k})]+\frac{i}{\hbar\Omega}V_{\alpha\beta}(\mathbf{l})\sum\nolimits_s[G_{\beta s}^{+}(\mathbf{l},\mathbf{p}')g_\lambda^{\alpha}(\mathbf{k},\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{l})-G_{\beta s}^{-}(\mathbf{l},\mathbf{p}')g_\lambda^{\alpha}(\mathbf{k},\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l})]$$

$$+\frac{i}{\hbar\Omega}V_{\alpha\beta}(-\mathbf{k}-\mathbf{l})\sum\nolimits_s[G_{\alpha s}^{-}(\mathbf{l}+\mathbf{k},\mathbf{p})g_\lambda^{\beta}(\mathbf{k},\mathbf{p}'+\tfrac{1}{2}\hbar(\mathbf{l}+\mathbf{k}))-G_{\alpha s}^{+}(\mathbf{l}+\mathbf{k},\mathbf{p})g_\lambda^{\beta}(\mathbf{k},\mathbf{p}'-\tfrac{1}{2}\hbar(\mathbf{l}+\mathbf{k}))].\quad(82)$$

The equation of motion for $K_{\alpha\beta\lambda}$ given above and the equations of motion for g_λ^α and $g_{\alpha\beta}$ given by Eqs. (68) and (35), respectively, constitute a closed set of equations. We have not solved this set of equations as it stands. However, it is very easy to obtain an approximate solution if we ignore the screening terms. In this case, the equation for the asymptotic function $\tilde{K}_{\alpha\beta\lambda}$ becomes

$$\left[\mathbf{l}\cdot\frac{\mathbf{p}'}{m_\beta}-(\mathbf{l}+\mathbf{k})\cdot\frac{\mathbf{p}}{m_\alpha}+\omega_k-i\epsilon\right]\tilde{K}_{\alpha\beta\lambda}(\mathbf{k},\mathbf{l},\mathbf{p},\mathbf{p}')=c_{\alpha k}[(\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{l})\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}(1+n_{\mathbf{k}\lambda})\tilde{g}_{\alpha\beta}(-\mathbf{l},\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{k},\mathbf{p}')$$

$$-(\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l})\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}n_{\mathbf{k}\lambda}\tilde{g}_{\alpha\beta}(-\mathbf{l},\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{k},\mathbf{p}')]+c_{\beta k}[(\mathbf{p}'+\tfrac{1}{2}\hbar\mathbf{l})\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}(1+n_{\mathbf{k}\lambda})\tilde{g}_{\alpha\beta}(\mathbf{l}+\mathbf{k},\mathbf{p},\mathbf{p}'+\tfrac{1}{2}\hbar\mathbf{k})$$

$$-(\mathbf{p}'-\tfrac{1}{2}\hbar\mathbf{l})\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}n_{\mathbf{k}\lambda}\tilde{g}_{\alpha\beta}(\mathbf{l}+\mathbf{k},\mathbf{p},\mathbf{p}'-\tfrac{1}{2}\hbar\mathbf{k})]+\frac{V_{\alpha\beta}(\mathbf{l})}{\hbar\Omega}\sum\nolimits_s[G_{\beta s}^{+}(\mathbf{l},\mathbf{p}')\tilde{g}_\lambda^{\alpha}(\mathbf{k},\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{l})-G_{\beta s}^{-}(\mathbf{l},\mathbf{p}')\tilde{g}_\lambda^{\alpha}(\mathbf{k},\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l})]$$

$$+\frac{V_{\alpha\beta}(-\mathbf{l}-\mathbf{k})}{\hbar\Omega}\sum\nolimits_s[G_{\alpha s}^{-}(\mathbf{l}+\mathbf{k},\mathbf{p})\tilde{g}_\lambda^{\beta}(\mathbf{k},\mathbf{p}'+\tfrac{1}{2}\hbar(\mathbf{l}+\mathbf{k}))-G_{\alpha s}^{+}(\mathbf{l}+\mathbf{k},\mathbf{p})\tilde{g}_\lambda^{\beta}(\mathbf{k},\mathbf{p}'-\tfrac{1}{2}\hbar(\mathbf{l}+\mathbf{k}))].\quad(83)$$

The expression for $\tilde{g}_{\alpha\beta}$ in the absence of the screening terms is given by

$$\tilde{g}_{\alpha\beta}(\mathbf{k},\mathbf{p},\mathbf{p}')=\left[\mathbf{k}\cdot\left(\frac{\mathbf{p}}{m_\alpha}-\frac{\mathbf{p}'}{m_\beta}\right)-i\epsilon\right]^{-1}\frac{V_{\alpha\beta}(\mathbf{k})}{\hbar\Omega}\sum_{s_1,s_2}[G_{\alpha s_1}^{+}(\mathbf{k},\mathbf{p})G_{\beta s_2}^{-}(\mathbf{k},\mathbf{p}')-G_{\alpha s_1}^{-}(\mathbf{k},\mathbf{p})G_{\beta s_2}^{+}(\mathbf{k},\mathbf{p}')].\quad(84)$$

Finally, the expression taken for $\tilde{g}_\lambda^\alpha$ is

$$\tilde{g}_\lambda^\alpha(\mathbf{k},\mathbf{p})=(\omega_k-\mathbf{k}\cdot\mathbf{p}/m_\alpha-i\epsilon)^{-1}c_{\alpha k}(\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})J_{\alpha\lambda}(\mathbf{k},\mathbf{p}).\quad(85)$$

When the above expressions are substituted into Eq. (83) and the resulting expression for $\tilde{K}_{\alpha\beta\lambda}$ is substituted into Eq. (68), we obtain for the bremsstrahlung contributions to the kinetic equations the following expressions:

$$\left[\frac{\partial n_{\mathbf{k}\lambda}}{\partial t}\right]_{\mathrm{b}}=\sum_{\alpha,\beta}\sum_{s_1,s_2}\sum_{\mathbf{p}_1,\mathbf{p}_2}\sum_{\mathbf{q}_1,\mathbf{q}_2}\Gamma_{\alpha\beta\lambda}(\mathbf{p}_1,\mathbf{q}_1;\mathbf{p}_2,\mathbf{q}_2,\mathbf{k})[(1+n_{\mathbf{k}\lambda})F_{\alpha s_1}(\mathbf{p}_1,\mathbf{q}_1)F_{\beta s_2}(\mathbf{p}_2,\mathbf{q}_2)-n_{\mathbf{k}\lambda}F_{\alpha s_1}(\mathbf{q}_1,\mathbf{p}_1)F_{\beta s_2}(\mathbf{q}_2,\mathbf{p}_2)]$$

$$-\frac{4\pi^2}{\Omega^3\omega_k}\sum_{\alpha,\beta}\sum_{s_1,s_2}\sum_{\mathbf{p}_1,\mathbf{p}_2}\sum_{\mathbf{q}_1,\mathbf{q}_2}\left|V_{\alpha\beta}\left(\frac{\mathbf{p}_2-\mathbf{p}_2'}{\hbar}\right)\right|^2\frac{e_\alpha^2}{m_\alpha^2}[1(+n_{\mathbf{k}\lambda})\Lambda_\alpha^2(\mathbf{k},\mathbf{q}_1+\tfrac{1}{2}\hbar\mathbf{k})-n_{\mathbf{k}\lambda}\Lambda_\alpha^2(\mathbf{k},\mathbf{q}_1-\tfrac{1}{2}\hbar\mathbf{k})](\mathbf{p}_1'\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})^2$$

$$\times[F_{\alpha s_1}(\mathbf{p}_1,\mathbf{q}_1)F_{\beta s_2}(\mathbf{p}_2,\mathbf{q}_2)-F_{\alpha s_1}(\mathbf{q}_1,\mathbf{p}_1)F_{\beta s_2}(\mathbf{q}_2,\mathbf{p}_2)]\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{q}_1-\mathbf{q}_2)\delta[E_\alpha(\mathbf{p}_1)+E_\beta(\mathbf{p}_2)-E_\alpha(\mathbf{q}_1)-E_\beta(\mathbf{q}_2)]$$

$$+\sum_{\alpha,\beta}\sum_{s_1,s_2}\sum_{\mathbf{p}_1,\mathbf{p}_2}\sum_{\mathbf{q}_1,\mathbf{q}_2}[\Theta_{\alpha\beta\lambda}^{(1)}(\mathbf{p}_1,\mathbf{q}_1;\mathbf{p}_2,\mathbf{q}_2;\mathbf{k})(1+n_{\mathbf{k}\lambda})-\Theta_{\alpha\beta\lambda}^{(2)}(\mathbf{p}_1,\mathbf{q}_1;\mathbf{p}_2,\mathbf{q}_2;\mathbf{k})n_{\mathbf{k}\lambda}]$$

$$\times[F_{\alpha s_1}(\mathbf{p}_1,\mathbf{q}_1)F_{\beta s_2}(\mathbf{p}_2,\mathbf{q}_2)-F_{\alpha s_1}(\mathbf{q}_1,\mathbf{p}_1)F_{\beta s_2}(\mathbf{q}_2,\mathbf{p}_2)]$$

$$\times\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{q}_1-\mathbf{q}_2)\delta[E_\alpha(\mathbf{p}_1)+E_\beta(\mathbf{p}_2)-E_\alpha(\mathbf{q}_1)-E_\beta(\mathbf{q}_2)],\quad(86)$$

and

$$\left[\frac{\partial\varphi_\alpha}{\partial t}\right]_b=\sum_{\beta,\mathbf{k},\lambda}\sum_{s_1,s_2}\sum_{\mathbf{q}_1,\mathbf{p}_2,\mathbf{q}_2}\{\Gamma_{\alpha\beta\lambda}(\mathbf{p}_1,\mathbf{q}_1;\,\mathbf{p}_2,\mathbf{q}_2,\mathbf{k})[n_{\mathbf{k}\lambda}F_{\alpha s_1}(\mathbf{q}_1.\mathbf{p}_1)F_{\beta s_2}(\mathbf{q}_2,\mathbf{p}_2)-(1+n_{\mathbf{k}\lambda})F_{\alpha s_1}(\mathbf{p}_1,\mathbf{q}_1)F_{\beta s_2}(\mathbf{p}_2,\mathbf{q}_2)]$$
$$+\Gamma_{\beta\alpha\lambda}(\mathbf{q}_1,\mathbf{p}_1;\,\mathbf{q}_2,\mathbf{p}_2,\mathbf{k})[(1+n_{\mathbf{k}\lambda})F_{\alpha s_1}(\mathbf{q}_1,\mathbf{p}_1)F_{\beta s_2}(\mathbf{q}_2,\mathbf{p}_2)-n_{\mathbf{k}\lambda}F_{\alpha s_1}(\mathbf{p}_1,\mathbf{q}_1)F_{\beta s_2}(\mathbf{p}_2,\mathbf{q}_2)]\}$$
$$+\frac{8\pi^2e_\alpha^2}{\Omega^3m_\alpha^2}\sum_{\beta,\mathbf{k}}\sum_{s_1,s_2}\sum_{\mathbf{p}_2,\mathbf{q}_1,\mathbf{q}_2}\left|V_{\alpha\beta}\left(\frac{\mathbf{p}_2-\mathbf{q}_2}{\hbar}\right)\right|^2[(1+n_{\mathbf{k}\lambda})\Lambda_\alpha(\mathbf{k},\,\mathbf{p}_1'+\tfrac{1}{2}\hbar\mathbf{k})(\mathbf{p}_1'\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})^2-n_{\mathbf{k}\lambda}\Lambda_\alpha(\mathbf{k},\,\mathbf{p}_1-\tfrac{1}{2}\hbar\mathbf{k})(\mathbf{p}_1\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})^2]$$
$$\times[F_{\beta s_1}(\mathbf{q}_1,\mathbf{p}_1)F_{\beta s_2}(\mathbf{q}_2,\mathbf{p}_2)-F_{\alpha s_1}(\mathbf{p}_1,\mathbf{q}_1)F_{\beta s_2}(\mathbf{p}_2,\mathbf{q}_2)]\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_1'-\mathbf{p}_2')\delta[E_\alpha(\mathbf{p}_1)+E_\beta(\mathbf{p}_2)-E_\alpha(\mathbf{q}_1)-E_\beta(\mathbf{q}_2)]$$
$$+2\sum_{\beta,\mathbf{k}}\sum_{s_1,s_2}\sum_{\mathbf{p}_2,\mathbf{q}_1,\mathbf{q}_2}[\Theta_{\alpha\beta\lambda}^{(1)}(\mathbf{p}_1,\mathbf{q}_1;\,\mathbf{p}_2,\mathbf{q}_2;\,\mathbf{k})(1+n_{\mathbf{k}\lambda})-\Theta_{\alpha\beta\lambda}^{(2)}(\mathbf{p}_1,\mathbf{q}_1;\,\mathbf{p}_2,\mathbf{q}_2,\mathbf{k})n_{\mathbf{k}\lambda}]$$
$$\times[F_{\alpha s_1}(\mathbf{q}_1,\mathbf{p}_1)F_{\beta s_2}(\mathbf{q}_2,\mathbf{p}_2)-F_{\alpha s_1}(\mathbf{p}_1,\mathbf{q}_1)F_{\beta s_2}(\mathbf{p}_2,\mathbf{q}_2)]\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{q}_1-\mathbf{q}_2)\delta[E_\alpha(\mathbf{p}_1)+E_\beta(\mathbf{p}_2)-E_\alpha(\mathbf{q}_1)-E_\beta(\mathbf{q}_2)],\quad(87)$$

where

$$F_{\alpha s}(\mathbf{p},\mathbf{q})=\varphi_{\alpha s}(\mathbf{p})[1-\varphi_{\alpha s}(\mathbf{q})],\quad(88)$$

$$\Gamma_{\alpha\beta\lambda}(\mathbf{p}_1,\mathbf{q}_1;\,\mathbf{p}_2,\mathbf{q}_2,\mathbf{k})=(2\pi^2/\Omega^3\omega_k)[\sigma_{\alpha\beta\lambda}(\mathbf{p}_1,\mathbf{q}_1;\,\mathbf{p}_2,\mathbf{q}_2,\mathbf{k})+\sigma_{\beta\alpha\lambda}(\mathbf{p}_2\mathbf{q}_2;\,\mathbf{p}_1,\mathbf{q}_1,\mathbf{k})]^2\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{q}_1-\mathbf{q}_2-\hbar\mathbf{k})$$
$$\times\delta[E_\alpha(\mathbf{q}_1)+E_\beta(\mathbf{q}_2)+\hbar\omega_k-E_\alpha(\mathbf{p}_1)-E_\beta(\mathbf{p}_2)],\quad(89)$$

$$\sigma_{\alpha\beta\lambda}(\mathbf{p}_1,\mathbf{q}_1;\,\mathbf{p}_2,\mathbf{q}_2,\mathbf{k})=\left(\frac{e_\alpha}{m_\alpha}\right)V_{\alpha\beta}\left(\frac{\mathbf{p}_2-\mathbf{p}_2'}{\hbar}\right)[(\mathbf{p}_1\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})\Lambda_\alpha(\mathbf{k},\,\mathbf{p}_1-\tfrac{1}{2}\hbar\mathbf{k})-(\mathbf{q}_1\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})\Lambda_\alpha(\mathbf{k},\,\mathbf{q}_1+\tfrac{1}{2}\hbar\mathbf{k})],\quad(90)$$

$$\Lambda_\alpha(\mathbf{k},\mathbf{p})=[E_\alpha(\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{k})-E_\alpha(\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{k})-\hbar\omega_k]^{-1},\quad(91)$$

$$\Theta_{\alpha\beta\lambda}^{(1)}(\mathbf{p}_1,\mathbf{q}_1;\,\mathbf{p}_2,\mathbf{q}_2,\mathbf{k})=\frac{4\pi^2}{\Omega^3\omega_k}V_{\alpha\beta}\left(\frac{\mathbf{p}_2-\mathbf{q}_2+\hbar\mathbf{k}}{\hbar}\right)V_{\alpha\beta}\left(\frac{\mathbf{q}_1-\mathbf{p}_1}{\hbar}\right)\left(\frac{e_\alpha e_\beta}{m_\alpha m_\beta}\right)(\mathbf{q}_2\cdot\boldsymbol{\epsilon}_{\mathbf{k}\lambda})$$
$$\times[(\mathbf{q}_1\cdot\boldsymbol{\epsilon}_{\mathbf{k}\lambda})\Lambda_\alpha(\mathbf{k},\,\mathbf{q}_1+\tfrac{1}{2}\hbar\mathbf{k})-(\mathbf{p}_1\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})\Lambda_\alpha(\mathbf{k},\,\mathbf{p}_1-\tfrac{1}{2}\hbar\mathbf{k})]\Lambda_\beta(\mathbf{k},\,\mathbf{q}_2-\tfrac{1}{2}\hbar\mathbf{k}),\quad(92)$$

and

$$\Theta_{\alpha\beta\lambda}^{(2)}(\mathbf{p}_1,\mathbf{q}_1;\,\mathbf{p}_2,\mathbf{q}_2,\mathbf{k})=\frac{4\pi^2}{\Omega^3\omega_k}V_{\alpha\beta}\left(\frac{\mathbf{p}_2-\mathbf{q}_2-\hbar\mathbf{k}}{\hbar}\right)V_{\alpha\beta}\left(\frac{\mathbf{q}_1-\mathbf{p}_1}{\hbar}\right)\left(\frac{e_\alpha e_\beta}{m_\alpha m_\beta}\right)(\mathbf{p}_2\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})$$
$$\times[(\mathbf{q}_1\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})\Lambda_\alpha(\mathbf{k},\,\mathbf{q}_1+\tfrac{1}{2}\hbar\mathbf{k})-(\mathbf{p}_1\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})\Lambda_\alpha(\mathbf{k},\,\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{k})]\Lambda_\beta(\mathbf{k},\,\mathbf{p}_2+\tfrac{1}{2}\hbar\mathbf{k}).\quad(93)$$

The leading terms in Eqs. (86) and (87) are the normal bremsstrahlung contributions to the kinetic equations. These terms agree with the results of Osborn and Klevans.[6] The remaining terms are rather unusual because of the absence of a delta function conserving the unperturbed energies.[14,15] These terms are similar to terms in Eqs. (59) and (60). They are equivalent to the bremsstrahlung terms obtained by Mangeney.[4] Another interesting aspect of these terms is that they vanish when the particles are in equilibrium (independent of the nature of the photon distribution). It has been suggested that these terms are present because the particles in the system are always interacting and never completely isolated.[14,15] Briefly stated, the source of these terms is the appearance of factors of the form $(x-a-i\epsilon)^{-1}(x-b-i\epsilon)^{-1}$. The imaginary part of this quantity is given by

$$\pi\delta(x-a)P(x-b)^{-1}+\pi\delta(x-b)P(x-a)^{-1}.$$

One of the delta functions will conserve unperturbed energies, and the other will not.

V. DOUBLE EMISSION-ABSORPTION

We will next briefly discuss the contributions that arise from the correlation function

$$k_{\lambda\lambda'}{}^{\alpha}(\mathbf{k},\mathbf{k}',\mathbf{p})=\langle\hat{f}_\alpha(\mathbf{K},\mathbf{p})b_{\mathbf{k}\lambda}b_{-\mathbf{k}'\lambda'}\rangle\quad(\mathbf{K}=\mathbf{k}'-\mathbf{k}).$$

This function contains information about two-photon processes.

The exact equation of motion for this quantity is given in the Appendix. With the use of the truncation procedure and the assumptions of isotropy of the correlation functions, we obtain

$$(\partial/\partial t+i(\mathbf{K}\cdot\mathbf{p}/m_\alpha)+i\omega_k+i\omega_{k'})k_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p})-\frac{i}{\hbar\Omega}\Delta_\alpha(\mathbf{K},\mathbf{p})\sum_{\beta,\mathbf{q}}V_{\alpha\beta}(\mathbf{K})k_{\lambda\lambda'}{}^\beta(\mathbf{k},\mathbf{k}',\mathbf{q})$$
$$=-2id_{kk'}{}^\alpha\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}L_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p})+2i\sum_{\beta,\mathbf{q}}d_{kk'}{}^\beta\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}(1+n_{\mathbf{k}\lambda}+n_{\mathbf{k}'\lambda'})g_{\alpha\beta}(\mathbf{K},\mathbf{p},\mathbf{q}), \quad (94)$$

where

$$L_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p})=\sum_s[n_{\mathbf{k}\lambda}n_{\mathbf{k}'\lambda'}G_{\alpha s}{}^+(\mathbf{K},\mathbf{p})-(1+n_{\mathbf{k}\lambda})(1+n_{-\mathbf{k}'\lambda'})G_{\alpha s}{}^-(\mathbf{K},\mathbf{p})]. \quad (95)$$

The asymptotic solution (making the Bogoliubov assumption) is then given by

$$\tilde{k}_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p})=2\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}(\mathbf{K}\cdot\mathbf{p}/m_\alpha+\omega_k+\omega_{k'}-i\epsilon)^{-1}\Big\{-d_{kk'}{}^\alpha L_{\lambda\lambda'}{}^\alpha(\mathbf{k},\mathbf{k}',\mathbf{p})+\sum_{\beta,\mathbf{q}}d_{kk'}{}^\beta(1+n_{\mathbf{k}\lambda}+n_{\mathbf{k}'\lambda'})\tilde{g}_{\alpha\beta}(\mathbf{K},\mathbf{p},\mathbf{q})$$
$$+\Delta_\alpha(\mathbf{K},\mathbf{p})[\hbar\Omega\mathcal{E}(\mathbf{K},\omega_k+\omega_{k'})]^{-1}\sum_{\beta\mathbf{q}}V_{\alpha\beta}(\mathbf{K})\left[\frac{L_{\lambda\lambda'}{}^\beta(\mathbf{k},\mathbf{k}',\mathbf{q})}{\mathbf{K}\cdot\mathbf{q}/m_\beta+\omega_k+\omega_{k'}-i\epsilon}-\sum_{\gamma,\mathbf{p}'}\frac{d_{kk'}{}^\gamma(1+n_{\mathbf{k}\lambda}+n_{\mathbf{k}'\lambda'})\tilde{g}_{\beta\gamma}(\mathbf{K},\mathbf{q},\mathbf{p}')}{\mathbf{K}\cdot\mathbf{q}/m_\beta+\omega_k+\omega_{k'}-i\epsilon}\right]\Big\}. \quad (96)$$

Considerable simplification results because the delta function $\delta(\omega_k+\omega_{k'}+\mathbf{K}\cdot\mathbf{p}/m_\beta)$ is always zero.

Even more simplification results if $\omega_k+\omega_{k'}+\mathbf{K}\cdot\mathbf{p}/m_\beta$ is approximated by $\omega_k+\omega_{k'}$. The contributions of $h_{\lambda\lambda'}{}^\alpha$ to the kinetic equations are then given by

$$[\partial n_{\mathbf{k}\lambda}/\partial t]_{\mathrm{d.e.}}=0$$

and

$$[\partial\varphi_\alpha(\mathbf{p})/\partial t]_{\mathrm{d.e.}}=\sum_{\beta,\mathbf{q}}\sum_{\mathbf{k},\mathbf{k}'}\sum_{\lambda,\lambda'}\frac{8d_{kk'}{}^\alpha d_{kk'}{}^\beta(\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'})^2}{(\omega_k+\omega_{k'})}(1+n_{\mathbf{k}\lambda}+n_{\mathbf{k}'\lambda'})\,\mathrm{Im}\tilde{g}_{\alpha\beta}(\mathbf{K},\,\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{K},\,\mathbf{q}), \quad (97)$$

where we have made use of the identities

$$\mathrm{Im}\tilde{g}_{\alpha\beta}(\mathbf{K},\mathbf{p},\mathbf{q})=-\mathrm{Im}\tilde{g}_{\alpha\beta}(-\mathbf{K},\,\mathbf{p},\,\mathbf{q})$$

and

$$\mathrm{Im}\tilde{g}_{\alpha\beta}(\mathbf{K},\mathbf{p},\mathbf{q})=-\mathrm{Im}\tilde{g}_{\beta\alpha}(\mathbf{K},\mathbf{q},\mathbf{p}).$$

Similar techniques to those used in the last section can be applied to obtain higher order effects such as the self-energy and the double bremsstrahlung contributions. Clearly, the double bremsstrahlung contribution is obtained by considering the correlation function

$$\langle\hat{f}_{\alpha\beta}(\mathbf{K}-\mathbf{l},\,\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l};\,\mathbf{l},\,\mathbf{q})b_{\mathbf{k}\lambda}b_{-\mathbf{k}'\lambda'}\rangle.$$

In this case, photon self-energy corrections will arise from the first two terms on the right-hand side of Eq. (A2). The correlation functions in these terms are $K_{\alpha\beta\lambda}(-\mathbf{k}',\,\mathbf{k},\,\mathbf{p},\,\mathbf{p}')$ and $K_{\alpha\beta\lambda}(\mathbf{k},\,-\mathbf{k}',\,\mathbf{p},\,\mathbf{p}')$.

VI. ELECTROMAGNETIC CORRECTIONS TO THE TWO-PARTICLE CORRELATION FUNCTION

In the earlier calculation (Sec. II) of the contribution of $g_{\alpha\beta}$, all effects of the transverse field were ignored. In this section we will consider the correction terms arising from the presence of the particle-photon interaction. If the truncation procedure is applied to Eq. (8), the equation of motion for $g_{\alpha\beta}$ becomes

$$\left[\frac{\partial}{\partial t}+i\mathbf{k}\cdot\left(\frac{\mathbf{p}_1}{m_\alpha}-\frac{\mathbf{p}_2}{m_\beta}\right)\right]g_{\alpha\beta}-\frac{i}{\hbar\Omega}\Delta_\alpha(\mathbf{k},\mathbf{p}_1)\sum_{\gamma,\mathbf{p}}V_{\alpha\gamma}(\mathbf{k})g_{\gamma\beta}(\mathbf{k},\mathbf{p},\mathbf{p}_2)+\frac{i}{\hbar\Omega}\Delta_\beta(\mathbf{k},\mathbf{p}_2)\sum_{\gamma,\mathbf{p}}V_{\beta\gamma}(\mathbf{k})g_{\alpha\gamma}(\mathbf{k},\mathbf{p}_1,\mathbf{p})$$
$$+ic_{\alpha k}(\mathbf{p}_1\cdot\boldsymbol{\varepsilon}_{k\lambda})\Delta_\alpha(\mathbf{k},\mathbf{p}_1)[g_\lambda{}^\beta(\mathbf{k},\mathbf{p}_2)+g_\lambda{}^{\beta*}(-\mathbf{k},\,\mathbf{p}_2)]-ic_{\beta k}(\mathbf{p}_2\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})\Delta_\beta(\mathbf{k},\mathbf{p}_2)[g_\lambda{}^\alpha(-\mathbf{k},\,\mathbf{p}_1)+g_\lambda{}^{\alpha*}(\mathbf{k},\mathbf{p}_1)]$$
$$-i\Delta_\alpha(\mathbf{k},\mathbf{p}_1)\sum_{\mathbf{l},\lambda,\lambda'}\boldsymbol{\varepsilon}_{\mathbf{l}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{l}-\mathbf{k}',\lambda}[h_{\lambda\lambda'}{}^\beta(\mathbf{l},\,\mathbf{l}-\mathbf{k},\,\mathbf{p}_2)+h_{\lambda\lambda'}{}^{\beta*}(-\mathbf{l},\,\mathbf{k}-\mathbf{l},\,\mathbf{p}_2)+k_{\lambda\lambda'}{}^\beta(\mathbf{l},\,\mathbf{l}-\mathbf{k},\,\mathbf{p}_2)+k_{\lambda\lambda'}{}^{\beta*}(-\mathbf{l},\,\mathbf{k}-\mathbf{l},\,\mathbf{p}_2)]$$
$$+i\Delta_\beta(\mathbf{k},\mathbf{p}_2)\sum_{\mathbf{l},\lambda,\lambda'}\boldsymbol{\varepsilon}_{\mathbf{l}\lambda}\cdot\epsilon_{\mathbf{l}-\mathbf{k},\lambda'}[h_{\lambda\lambda'}{}^\alpha(-\mathbf{l},\,\mathbf{k}-\mathbf{l},\,\mathbf{p}_1)+h_{\lambda\lambda'}{}^{\alpha*}(\mathbf{l},\,\mathbf{l}-\mathbf{k},\,\mathbf{p}_1)+k_{\lambda\lambda'}{}^\alpha(-\mathbf{l},\,\mathbf{k}-\mathbf{l},\,\mathbf{p}_1)+k_{\lambda\lambda'}{}^{\alpha*}(\mathbf{l},\,\mathbf{l}-\mathbf{k},\,\mathbf{p}_1)]$$
$$=\frac{i}{\hbar\Omega}\sum_{s_1,s_2}V_{\alpha\beta}(\mathbf{k})[G_{\alpha s_1}{}^+(\mathbf{k},\mathbf{p}_1)G_{\beta s_2}{}^-(\mathbf{k},\mathbf{p}_2)-G_{\alpha s_1}{}^-(\mathbf{k},\mathbf{p}_1)G_{\beta s_2}{}^+(\mathbf{k},\mathbf{p}_2)]. \quad (98)$$

Let us begin by considering the contribution of the correlation functions $g_\lambda{}^\alpha$. It follows from Eq. (71) that

$$\tilde{g}_\lambda{}^\beta(\mathbf{k},\mathbf{p}_2)+g_\lambda{}^{\beta*}(-\mathbf{k},\mathbf{p}_2)=\sum_s c_{\beta k}\mathbf{p}_2\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\omega_k{}^{-1}[G_{\beta s}{}^+(\mathbf{k},\mathbf{p}_2)+G_{\beta s}{}^-(\mathbf{k},\mathbf{p}_2)]$$
$$+\sum_{\gamma,q} c_{\gamma k}\mathbf{q}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\omega_k{}^{-1}[\tilde{g}_{\beta\gamma}(-\mathbf{k},\mathbf{p}_2,\mathbf{q})+\tilde{g}_{\beta\gamma}{}^*(\mathbf{k},\mathbf{p}_2,\mathbf{q})],\quad(99)$$

where $\omega_k-\mathbf{k}\cdot\mathbf{p}/m_\alpha$ has been approximated by ω_k. There is a similar expression for $g_\lambda{}^\alpha(-\mathbf{k},\mathbf{p}_1)+g_\lambda{}^{\alpha*}(\mathbf{k},\mathbf{p}_1)$. Clearly, the contributions of these terms to $\tilde{g}_{\alpha\beta}$ will have the form $(\mathbf{p}_1\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})(\mathbf{p}_2\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})F_{\alpha\beta}$, where $F_{\alpha\beta}$ is independent of the angles $\mathbf{p}_1\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}/p_1$ and $\mathbf{p}_2\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}/p_2$. Therefore, $g_{\alpha\beta}$ can be written as

$$g_{\alpha\beta}=g_{\alpha\beta}{}^1+g_{\alpha\beta}{}^2,\quad(100)$$

where $g_{\alpha\beta}{}^2(\mathbf{k},\mathbf{p}_1,\mathbf{p}_2)$ is the part of $g_{\alpha\beta}$ contributed by the $g_\lambda{}^\alpha$ terms. (This separation can be made only for an isotropic system.) The equation of motion for $\tilde{g}_{\alpha\beta}{}^2$ is given by

$$[\mathbf{k}\cdot(\mathbf{p}_1/m_\alpha-\mathbf{p}_2/m_\beta)-i\epsilon]\tilde{g}_{\alpha\beta}{}^2(\mathbf{k},\mathbf{p}_1,\mathbf{p}_2)+(2/\omega_k)c_{\alpha k}\mathbf{p}_1\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\Delta_\alpha(\mathbf{k},\mathbf{p}_1)\sum_{\gamma,\mathbf{q}} c_{\gamma k}\mathbf{q}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\tilde{g}_{\gamma\beta}{}^2(\mathbf{k},\mathbf{q},\mathbf{p}_2)$$
$$-(2/\omega_k)c_{\beta k}\mathbf{p}_2\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\Delta_\beta(\mathbf{k},\mathbf{p}_2)\sum_{\gamma,\mathbf{q}} c_{\gamma k}\mathbf{q}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\tilde{g}_{\alpha\gamma}{}^2(\mathbf{k},\mathbf{p}_1,\mathbf{q})$$
$$=-(2/\omega_k)c_{\alpha k}c_{\beta k}(\mathbf{p}_1\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})(\mathbf{p}_2\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})\sum_{s_1,s_2}[G_{\alpha s_1}{}^+(\mathbf{k},\mathbf{p}_1)G_{\beta s_2}{}^-(\mathbf{k},\mathbf{p}_2)-G_{\alpha s_1}{}^-(\mathbf{k},\mathbf{p}_1)G_{\beta s_2}{}^+(\mathbf{k},\mathbf{p}_2)].\quad(101)$$

The above equation has precisely the same structure as the equation for $\tilde{g}_{\alpha\beta}$ that results from Eq. (35). Its solution can be obtained by similar methods.

If the result is then substituted into Eq. (74) we obtain

$$\left[\frac{\partial\varphi_\alpha(\mathbf{p}_1)}{\partial t}\right]_{\mathrm{s.e.}}=\frac{2\pi}{\hbar\Omega^2}\sum_{\beta,\mathbf{p}_2}\sum_{\mathbf{k},\lambda}\sum_{s_1,s_2}\left|\frac{4\pi e_\alpha e_\beta(\mathbf{p}_1\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})(\mathbf{p}_2\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda})}{m_\alpha m_\beta(\omega_k{}^2+2\omega_k\delta_k(\mathbf{k}\cdot\mathbf{p}_1/m_\alpha+\hbar^2k^2/2m_\alpha))}\right|^2[G_{\alpha s_1}{}^+(\mathbf{k},\mathbf{p}_1+\tfrac{1}{2}\hbar\mathbf{k})G_{\beta s_2}{}^-(\mathbf{k},\mathbf{p}_2-\tfrac{1}{2}\hbar\mathbf{k})$$
$$-G_{\alpha s_1}{}^-(\mathbf{k},\mathbf{p}_1+\tfrac{1}{2}\hbar\mathbf{k})G_{\beta s_2}{}^+(\mathbf{k},\mathbf{p}_2-\tfrac{1}{2}\hbar\mathbf{k})]\delta[E_\alpha(\mathbf{p}_1)+E_\beta(\mathbf{p}_2)-E_\alpha(\mathbf{p}_1+\hbar\mathbf{k})-E_\beta(\mathbf{p}_2-\hbar\mathbf{k})].\quad(102)$$

The presence of the self-energy factor δ_k in the denominators is due to the screening terms in Eq. (101). If these terms are neglected, the answer is changed only by δ_k being replaced by zero. The factor $\omega_k{}^2+2\omega_k\delta_k$ has the appearance of a self-energy correction [cf., Eq. (77)], except that the expression would seem to be missing a term $\delta_k{}^2$. In fact, the factor $\omega_k{}^2+2\omega_k\delta_k$ is the proper expression [i.e., the self-energy correction is actually given by $(\omega_k{}^2+2\omega_k\delta_k)^{1/2}-\omega_k$], and the earlier results obtained in Eqs. (65) and (77) are approximations corresponding to the first two terms of the expansion of $(\omega_k{}^2+2\omega_k\delta_k)^{1/2}$.[23] This should also apply to the results obtained by Michel.[13] The difference of $\delta_k{}^2$ is very small. Therefore, the preceding discussion is relevant only to the theoretical interpretation.

Equation (103) is the quantum-mechanical version of the classical Fokker-Planck equation obtained by Simon[1] and by Aamodt, Eldridge, and Rostoker.[5] This term in the kinetic equations arises from the magnetic forces between moving charges. From a more quantum-mechanical point of view, it is due to the potential arising from the exchange of virtual quasiphotons by the particles. This point of view allows us to obtain this term, including the self-energy correction, by a "Golden Rule" or "guessing" method similar to that used by Wyld and Pines to obtain Eq. (42).[24]

It is much more difficult to treat the corrections to $\tilde{g}_{\alpha\beta}$ that are due to the correlation functions $h_{\lambda\lambda'}{}^\alpha$ and $k_{\lambda\lambda'}{}^\alpha$. In this case we do not have the convenient symmetry that existed for $g_\lambda{}^\alpha$. If $g_{\alpha\beta}{}^1$ is split into the sum of a large part which expresses the correlations arising from direct Coulomb interactions and a small part $g_{\alpha\beta'}{}^1$ which contains the effects of $h_{\lambda\lambda'}{}^\alpha$ and $k_{\lambda\lambda'}{}^\alpha$, and if all screening terms are neglected we obtain

$$\tilde{g}_{\alpha\beta'}{}^1(\mathbf{k},\mathbf{p}_1,\mathbf{p}_2)=[\mathbf{k}\cdot(\mathbf{p}_1/m_\alpha-\mathbf{p}_2/m_\beta)-i\epsilon]^{-1}\sum_{l\lambda\lambda'}4d_{kl}{}^\alpha d_{kl}{}^\beta\boldsymbol{\varepsilon}_{\mathbf{l}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{l}-\mathbf{k}\lambda'}$$
$$\times\left\{\frac{(1+n_1+n_{1-\mathbf{k}})}{(\omega_l+\omega_{l-k})}[G_{\alpha s_1}{}^+(\mathbf{k},\mathbf{p}_1)G_{\beta s_2}{}^-(\mathbf{k},\mathbf{p}_2)-G_{\alpha s_1}{}^-(\mathbf{k},\mathbf{p}_1)G_{\beta s_2}{}^+(\mathbf{k},\mathbf{p}_2)]\right.$$
$$\left.+\frac{\Delta_\beta(\mathbf{k},\mathbf{p}_2)I_{\lambda\lambda'}{}^\alpha(-\mathbf{l},\mathbf{k}-\mathbf{l},\mathbf{p}_1)}{\mathbf{k}\cdot\mathbf{p}_1/m_\alpha+\omega_l-\omega_{l-k}-i\epsilon}-\frac{\Delta_\alpha(\mathbf{k},\mathbf{p}_1)I_{\lambda\lambda'}{}^\beta(-\mathbf{l},\mathbf{k}-\mathbf{l},\mathbf{p}_2)}{\mathbf{k}\cdot\mathbf{p}_2/m_\beta+\omega_l-\omega_{l-k}+i\epsilon}\right\},\quad(103)$$

[23] The approximations made in obtaining Eqs. (65) and (77) involved our neglecting terms proportional to $k_{\lambda\lambda'}{}^\alpha$ and $g_\lambda{}^{\alpha*}(-\mathbf{k},\mathbf{p})$. The neglect of these terms is equivalent to the neglect of the admixture of $b_{-\mathbf{k}\lambda}{}^\dagger$ with $b_{\mathbf{k}\lambda}$ in the higher random-phase approximation method for obtaining dispersion relations (Ref. 16). This approximation results in linear instead of quadratic dispersion relations.

[24] H. W. Wyld and D. Pines, Phys. Rev. **127**, 1851 (1962).

where the following approximate expressions have been used for $\tilde{k}_{\lambda\lambda'}{}^{\alpha}$ and $\tilde{h}_{\lambda\lambda'}{}^{\alpha}$:

$$\tilde{k}_{\lambda\lambda'}{}^{\alpha}(\mathbf{k},\mathbf{k}',\mathbf{p}) = -2d_{kk'}{}^{\alpha}\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}(\omega_k+\omega_{k'})^{-1}L_{\lambda\lambda'}{}^{\alpha}(\mathbf{k},\mathbf{k}',\mathbf{p}), \tag{104}$$

and

$$\tilde{h}_{\lambda\lambda'}{}^{\alpha}(\mathbf{k},\mathbf{k}',\mathbf{p}) = \frac{2d_{kk'}{}^{\alpha}\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}I_{\lambda\lambda'}{}^{\alpha}(\mathbf{k},\mathbf{k}',\mathbf{p})}{\omega_{k'}-\omega_k-\mathbf{K}\cdot\mathbf{p}/m_\alpha+i\epsilon}. \tag{105}$$

The quantities $L_{\lambda\lambda'}{}^{\alpha}$ and $I_{\lambda\lambda'}{}^{\alpha}$ are defined in Eqs. (95) and (55).

The quantity $\tilde{g}_{\alpha\beta}{}^{1'}$ gives the correlations which arise from the scattering of a single photon by two particles and by the exchange of two virtual photons. In this case, the expression depends on the photon distribution $n_{\mathbf{k}\lambda}$.

VII. SUMMARY

A method is presented for deriving kinetic equations for a homogeneous, isotropic, multicomponent system of charged particles coupled to the electromagnetic field. A hierarchy of equations is introduced by use of the equations of motion for the Wigner distribution operators and quasiphoton creation and annihilation operators. The basic quantities in this method are certain correlation functions that are related to the physical processes of particle-particle scattering, particle-photon scattering, and single and double emission-absorption of photons by the particles. The contributions of these various correlation functions to the kinetic equations are obtained by use of the Bogoliubov method for the truncation of the hierarchy and the subsequent introduction of irreversibility.

The kinetic equations that are obtained contain the results of Osborn and Klevans[6] and Dreicer.[7] In addition, terms are obtained that reflect the correlations between the particles. These correlation effects include screening, photon self-energies, and other many-body effects. Corrections to the two-particle correlation function are obtained that correspond to interactions between the particles mediated by the photons.

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APPENDIX

The exact equation of motion for $K_{\alpha\beta\lambda}(\mathbf{k},\mathbf{k}',\mathbf{p},\mathbf{p}')$ is

$$
\begin{aligned}
&[\partial/\partial t - i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{p}/m_\alpha + i(\mathbf{k}'\cdot\mathbf{p}'/m_\beta) + i\omega_k]K_{\alpha\beta\lambda}(\mathbf{k},\mathbf{k}',\mathbf{p},\mathbf{p}')\\
&= i\sum_{\gamma,\mathbf{q}} c_{\gamma k}\mathbf{q}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{k}',\,\mathbf{p},\,\mathbf{k}',\,\mathbf{p}')\hat{f}_\gamma(\mathbf{k},\mathbf{q})\rangle - i\sum_{\gamma,\mathbf{q}}\sum_{\mathbf{l},\nu}{}' 2d_{kl}{}^{\gamma}\boldsymbol{\varepsilon}_{\mathbf{l}\nu}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{k}',\,\mathbf{p},\,\mathbf{k}',\,\mathbf{p}')\hat{f}_\gamma(\mathbf{k}-\mathbf{l},\,\mathbf{q})(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)\rangle\\
&\quad+\frac{i}{\hbar\Omega}\sum\nolimits_{\mathbf{l}} V_{\alpha\beta}(\mathbf{l})[\langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{k}'-\mathbf{l},\,\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l};\,\mathbf{k}'+\mathbf{l},\,\mathbf{p}'-\tfrac{1}{2}\hbar\mathbf{l})b_{\mathbf{k}\lambda}\rangle - \langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{k}'-\mathbf{l},\,\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{l};\,\mathbf{k}'+\mathbf{l},\,\mathbf{p}'+\tfrac{1}{2}\hbar\mathbf{l})b_{\mathbf{k}\lambda}\rangle]\\
&\quad-\frac{i\hbar}{2}\sum_{\mathbf{l},\nu} c_{\alpha l}(\mathbf{l}+\mathbf{k}+\mathbf{k}')\cdot\boldsymbol{\varepsilon}_{\mathbf{l}\nu}[\langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{k}'-\mathbf{l},\,\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l},\,\mathbf{k}',\,\mathbf{p}')(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)b_{\mathbf{k}\lambda}\rangle\\
&\qquad\qquad+\langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{k}'-\mathbf{l},\,\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{l},\,\mathbf{k}',\,\mathbf{p}')(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)b_{\mathbf{k}\lambda}\rangle]\\
&\quad-\frac{i\hbar}{2}\sum_{\mathbf{l},\nu} c_{\beta l}(\mathbf{l}-\mathbf{k}')\cdot\boldsymbol{\varepsilon}_{\mathbf{l}\nu}[\langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{k}',\,\mathbf{p},\,\mathbf{k}'-\mathbf{l};\,\mathbf{p}'+\tfrac{1}{2}\hbar\mathbf{l})(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)b_{\mathbf{k}\lambda}\rangle\\
&\qquad\qquad+\langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{k}',\,\mathbf{p},\,\mathbf{k}'-\mathbf{l};\,\mathbf{p}'-\tfrac{1}{2}\hbar\mathbf{l})(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)b_{\mathbf{k}\lambda}\rangle]\\
&\quad-i\sum_{\mathbf{l},\nu} c_{\alpha l}\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{l}\nu}[\langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{k}'-\mathbf{l},\,\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l};\,\mathbf{k}',\,\mathbf{p}')(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)b_{\mathbf{k}\lambda}\rangle - \langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{k}'-\mathbf{l},\,\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{l};\,\mathbf{k}',\,\mathbf{p}')(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)b_{\mathbf{k}\lambda}\rangle]\\
&\quad-i\sum_{\mathbf{l},\nu} c_{\beta l}\mathbf{p}'\cdot\boldsymbol{\varepsilon}_{\mathbf{l}\nu}[\langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{k}',\,\mathbf{p};\,\mathbf{k}'-\mathbf{l},\,\mathbf{p}'+\tfrac{1}{2}\hbar\mathbf{l})(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)b_{\mathbf{k}\lambda}\rangle - \langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{k}',\,\mathbf{p};\,\mathbf{k}'-\mathbf{l},\,\mathbf{p}'-\tfrac{1}{2}\hbar\mathbf{l})(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)b_{\mathbf{k}\lambda}\rangle]\\
&\quad+i\sum_{\nu,\nu'}\sum_{\mathbf{l},\mathbf{l}'} d_{ll'}{}^{\alpha}\boldsymbol{\varepsilon}_{\mathbf{l}\nu}\cdot\boldsymbol{\varepsilon}_{\mathbf{l}'\nu'}[\langle\hat{f}_{\alpha\beta}(\mathbf{l}'-\mathbf{l}-\mathbf{k}-\mathbf{k}',\,\mathbf{p}+\tfrac{1}{2}\hbar(\mathbf{l}-\mathbf{l}');\,\mathbf{k}',\,\mathbf{p}')(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)(b_{-\mathbf{l}'\nu'}+b_{\mathbf{l}'\nu'}{}^\dagger)b_{\mathbf{k}\lambda}\rangle\\
&\qquad\qquad-\langle\hat{f}_{\alpha\beta}(\mathbf{l}'-\mathbf{l}-\mathbf{k}-\mathbf{k}',\,\mathbf{p}-\tfrac{1}{2}\hbar(\mathbf{l}-\mathbf{l}');\,\mathbf{k}',\,\mathbf{p}')(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)(b_{-\mathbf{l}'\nu'}+b_{\mathbf{l}'\nu'}{}^\dagger)b_{\mathbf{k}\lambda}\rangle]\\
&\quad+i\sum_{\nu,\nu'}\sum_{\mathbf{l},\mathbf{l}'} d_{ll'}{}^{\beta}\boldsymbol{\varepsilon}_{\mathbf{l}\nu}\cdot\boldsymbol{\varepsilon}_{\mathbf{l}'\nu'}[\langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{k}',\,\mathbf{p};\,\mathbf{k}'+\mathbf{l}'-\mathbf{l},\,\mathbf{p}'+\tfrac{1}{2}\hbar(\mathbf{l}-\mathbf{l}'))(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)(b_{-\mathbf{l}'\nu'}+b_{\mathbf{l}'\nu'}{}^\dagger)b_{\mathbf{k}\lambda}\rangle\\
&\qquad\qquad-\langle\hat{f}_{\alpha\beta}(-\mathbf{k}-\mathbf{k}',\,\mathbf{p};\,\mathbf{k}'+\mathbf{l}'-\mathbf{l},\,\mathbf{p}'-\tfrac{1}{2}\hbar(\mathbf{l}-\mathbf{l}'))(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^\dagger)(b_{-\mathbf{l}'\nu'}+b_{\mathbf{l}'\nu'}{}^\dagger)b_{\mathbf{k}\lambda}\rangle]
\end{aligned}
$$

$$+\frac{i}{\hbar\Omega}\sum_{\gamma,\mathbf{q}}\sum_{\mathbf{l}} V_{\alpha\gamma}(\mathbf{l})[\langle\hat{f}_{\alpha\beta\gamma}(-\mathbf{k}-\mathbf{k}'-\mathbf{l},\, \mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l};\, \mathbf{k}',\, \mathbf{p}';\, \mathbf{l},\, \mathbf{q})b_{\mathbf{k}\lambda}\rangle-\langle\hat{f}_{\alpha\beta\gamma}(-\mathbf{k}-\mathbf{k}'-\mathbf{l},\, \mathbf{p}-\tfrac{1}{2}\hbar\mathbf{l};\, \mathbf{k}',\, \mathbf{p}';\, \mathbf{l},\, \mathbf{q})b_{\mathbf{k}\lambda}\rangle]$$

$$+\frac{i}{\hbar\Omega}\sum_{\gamma,\mathbf{q}}\sum_{\mathbf{l}} V_{\beta\gamma}(\mathbf{l})[\langle\hat{f}_{\alpha\beta\gamma}(-\mathbf{k}-\mathbf{k}',\, \mathbf{p};\, \mathbf{k}'-\mathbf{l},\, \mathbf{p}'+\tfrac{1}{2}\hbar\mathbf{l};\, \mathbf{l},\, \mathbf{q})b_{\mathbf{k}\lambda}\rangle-\langle\hat{f}_{\alpha\beta\gamma}(-\mathbf{k}-\mathbf{k}',\, \mathbf{p};\, \mathbf{k}'-\mathbf{l},\, \mathbf{p}'-\tfrac{1}{2}\hbar\mathbf{l};\, \mathbf{l},\, \mathbf{q})b_{\mathbf{k}\lambda}\rangle]. \quad \text{(A1)}$$

The exact equation of motion for $k_{\lambda\lambda'}{}^{\alpha}(\mathbf{k},\mathbf{k}',\mathbf{p})$ is

$$[\partial/\partial t+i(\mathbf{K}\cdot\mathbf{p}/m_\alpha+\omega_k+\omega_{k'})]k_{\lambda\lambda'}{}^{\alpha}(\mathbf{k},\mathbf{k}',\mathbf{p})$$

$$=i\sum_{\beta,\mathbf{p}'}[c_{\beta k}\mathbf{p}'\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\langle\hat{f}_\alpha(\mathbf{K},\mathbf{p})\hat{f}_\beta(\mathbf{k},\mathbf{p}')b_{-\mathbf{k}'\lambda'}\rangle+c_{\beta k'}\mathbf{p}'\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}\langle\hat{f}_\alpha(\mathbf{K},\mathbf{p})\hat{f}_\beta(-\mathbf{k}',\,\mathbf{p})b_{\mathbf{k}\lambda}\rangle]$$

$$-2i\sum_{\beta,\mathbf{p}'}\sum_{\mathbf{l},\nu}[d_{lk}{}^{\beta}\boldsymbol{\varepsilon}_{\mathbf{l}\nu}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}\langle\hat{f}_\alpha(\mathbf{K},\mathbf{p})\hat{f}_\beta(\mathbf{k}-\mathbf{l},\,\mathbf{p}')(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^{\dagger})b_{-\mathbf{k}'\lambda'}\rangle+d_{lk'}{}^{\beta}\boldsymbol{\varepsilon}_{\mathbf{l}\nu}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}'\lambda'}\langle\hat{f}_\alpha(\mathbf{K},\mathbf{p})\hat{f}_\beta(\mathbf{l}-\mathbf{k}',\,\mathbf{p})b_{\mathbf{k}\lambda}(b_{\mathbf{l}\nu}{}^{\dagger}+b_{-\mathbf{l}\nu})\rangle]$$

$$-2i\sum_{\mathbf{l},\nu}\hbar c_{\alpha l}\mathbf{K}\cdot\boldsymbol{\varepsilon}_{\mathbf{l}\nu}[\langle\hat{f}_\alpha(\mathbf{K}-\mathbf{l},\,\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l})(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^{\dagger})b_{\mathbf{k}\lambda}b_{-\mathbf{k}'\lambda'}\rangle+\langle\hat{f}_\alpha(\mathbf{K}-\mathbf{l},\,\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{l})(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^{\dagger})b_{\mathbf{k}\lambda}b_{-\mathbf{k}'\lambda'}\rangle]$$

$$+i\sum_{\mathbf{l},\nu}c_{\alpha l}\mathbf{p}\cdot\boldsymbol{\varepsilon}_{\mathbf{l}\nu}[\langle\hat{f}_\alpha(\mathbf{K}-\mathbf{l},\,\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l})(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^{\dagger})b_{\mathbf{k}\lambda}b_{-\mathbf{k}'\lambda'}\rangle-\langle\hat{f}_\alpha(\mathbf{K}-\mathbf{l},\,\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{l})(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^{\dagger})b_{\mathbf{k}\lambda}b_{-\mathbf{k}'\lambda'}\rangle]$$

$$-i\sum_{\mathbf{l},\mathbf{l}'}\sum_{\nu,\nu'}d_{ll'}{}^{\alpha}\boldsymbol{\varepsilon}_{\mathbf{l}\nu}\cdot\boldsymbol{\varepsilon}_{\mathbf{l}'\nu'}[\langle\hat{f}_\alpha(\mathbf{K}+\mathbf{l}'-\mathbf{l},\,\mathbf{p}+\tfrac{1}{2}\hbar(\mathbf{l}-\mathbf{l}'))(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^{\dagger})(b_{-\mathbf{l}'\nu'}+b_{\mathbf{l}'\nu'}{}^{\dagger})b_{\mathbf{k}\lambda}b_{-\mathbf{k}'\lambda'}\rangle$$
$$-\langle\hat{f}_\alpha(\mathbf{K}+\mathbf{l}'-\mathbf{l},\,\mathbf{p}-\tfrac{1}{2}\hbar(\mathbf{l}-\mathbf{l}'))(b_{\mathbf{l}\nu}+b_{-\mathbf{l}\nu}{}^{\dagger})(b_{-\mathbf{l}'\nu'}+b_{\mathbf{l}'\nu'}{}^{\dagger})b_{\mathbf{k}\lambda}b_{-\mathbf{k}'\lambda'}\rangle]$$

$$=\frac{i}{\hbar\Omega}\sum_{\beta,\mathbf{p}'}\sum_{\mathbf{l}}V_{\alpha\beta}(\mathbf{l})[\langle\hat{f}_{\alpha\beta}(\mathbf{K}-\mathbf{l},\,\mathbf{p}+\tfrac{1}{2}\hbar\mathbf{l};\,\mathbf{l},\,\mathbf{p}')b_{\mathbf{k}\lambda}b_{-\mathbf{k}'\lambda'}\rangle-\langle\hat{f}_{\alpha\beta}(\mathbf{K}-\mathbf{l},\,\mathbf{p}-\tfrac{1}{2}\hbar\mathbf{l};\,\mathbf{l},\,\mathbf{p}')b_{\mathbf{k}\lambda}b_{-\mathbf{k}'\lambda'}\rangle]. \quad \text{(A2)}$$