$Ca^{42}(d,p)Ca^{43}$ Reaction at 7.0- and 7.2-MeV Bombarding Energies*

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The $Ca^{42}(d,p)Ca^{43}$ reaction has been studied at 7.00- and 7.20-MeV bombarding energies using the MIT-ONR electrostatic generator in connection with the multiple-gap spectrograph. Proton spectra were observed at 23 scattering angles from 7.5 to 172.5 deg with an over-all energy resolution of 12 keV. Eighty transitions were observed up to an excitation energy of 5.0 MeV. Angular distributions showing stripping character were analyzed in the distorted-wave approximation to yield l_n values and spectroscopic strengths. A sum-rule analysis indicated that all of the available $1f_{7/2}$, $1d_{3/2}$, and $2p$ strengths were observed. The beginnings of the $1g_{9/2}$, $1f_{5/2}$, and $2d_{5/2}$ transitions were recorded. Anomalies observed in the angular distributions to the four lowest excited states are discussed. The results are compared with other data on Ca^{42} and Ca^{43} and are discussed in terms of the shell model with residual interactions. The data suggest that neither the neutron nor the proton $1d_{3/2}$ shell is filled in the Ca^{42} ground state.

I. INTRODUCTION

THE present experiment is a reinvestigation of the $Ca^{42}(d,p)Ca^{43}$ reaction which was previously studied[1] in this Laboratory. The older experiment used the MIT single-gap spectrograph[2] for momentum-analyzing the protons, whereas the present measurements were made with the MIT multiple-gap spectrograph.[3] The new instrumentation, combined with a higher target enrichment (94% compared with 64% in Ref. 1), has made it possible to obtain exposures with 10 to 30 times the proton yield as compared to the earlier data, although the same energy resolution was maintained. The present data, therefore, emphasize the weaker transitions, anomalies in the angular distributions, and transitions to higher lying Ca^{43} states. Presumably all of the available $1f_{7/2}$, $1d_{3/2}$, and $2p$ strengths were observed. The beginnings of the $1g_{9/2}$, $1f_{5/2}$, and $2d_{5/2}$ transitions were recorded.

The experimental procedures and results are described in Sec. II, and the distorted-wave (DW) analysis is presented in Sec. III. In Sec. IV, a number of angular distributions from weak transitions are discussed. Section V deals with a sum-rule analysis of the data. A comparison of the present observations with the (d,d') results of Belote *et al.*[4] and with the $Ca^{44}(p,d)$ data of Conlon *et al.*[5] is undertaken in Sec. VI. The nuclear-structure information contained in the present data is discussed in Sec. VII.

II. EXPERIMENTAL PROCEDURES AND RESULTS

A. Targets

The targets were fabricated by vacuum evaporation of $CaCo_3$ (Ref. 6) from an oven made of tantalum. The isotopic composition of the $CaCO_3$ is given in Table I. The backings were carbon and Formvar foils of about 20 $\mu g/cm^2$ thicknesses. The Ca targets had thicknesses from about 15 to about 25 $\mu g/cm^2$. The impurities present in each target were detected by means of elastic deuteron scattering. The target used for the 7.0-MeV experiment (see below) contained traces of Zn, Cd, Ta, and Hg. The target used in the 7.2-MeV run was free of Cd and Zn.

B. Elastic Scattering

Elastic scattering of deuterons from Ca^{42} was observed at 7.5, 7.0, and 3.0 MeV. A newly constructed set of entrance slits[7] was employed in these experiments. The slits determine the solid angle of acceptance of the spectrograph gaps forward of 90 deg in a manner approximately proportional to $\sin^4(\theta/2)$, where θ is the laboratory scattering angle. The slits can be removed or put in place remotely without breaking the vacuum. In practice, this slit system allows the simultaneous observation of elastic scattering at laboratory angles from 172.5 to 22.5 deg. The slit scattering at

TABLE I. Isotopic composition of the targets.[a]

Experiment	Percentage abundances Ca^{40}	Ca^{42}	Ca^{44}
$Ca^{42}(d,p)$	4.96	93.7	1.18
$Ca^{42}(d,d)$	5.75	90.9	3.01

[a] These figures are the numbers quoted on the mass-analysis cards supplied by Oak Ridge (Ref. 6).

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[1] C. K. Bockelman, C. M. Braams, C. P. Browne, W. W. Buechner, R. D. Sharp, and A. Sperduto, Phys. Rev. **107**, 176 (1957).

[2] C. P. Browne and W. W. Buechner, Rev. Sci. Instr. **27**, 899 (1956).

[3] H. A. Enge and W. W. Buechner, Rev. Sci. Instr. **34**, 155 (1963).

[4] T. A. Belote, J. H. Bjerregaard, Ole Hansen, and G. R. Satchler, Phys. Rev. **138**, B1067 (1965).

[5] T. W. Conlon, B. F. Bayman, and E. Kashy (to be published).

[6] Obtained from the Stable Isotopes Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee.

[7] H. J. Young, M.Sc. thesis, MIT, 1965 (unpublished).

more forward angles is too strong to allow reliable data to be obtained.

The apertures of the slit system were calibrated, relative to the apertures of the gaps in the backward quadrant, by the observation of elastic scattering from Au (Ref. 8), Sn (Ref. 7), Cd, and Ca under conditions such that the Rutherford expression can be assumed valid. For Ca, 3.0-MeV deuterons from the molecular beam were used, and the shape of the angular distribution was found to be identical to that of Cd(d,d). The Cd and Ca scattering yields were observed simultaneously, since Cd was present as an impurity in the Ca target. The integration of the molecular beam was investigated separately[8] and was found to give consistent results.

For each (d,p) experiment three different exposures were made in immediate sequence on the same set of plates and on the same target; 3-MeV elastic scattering using the molecular beam, and two (d,p) exposures (see below). The 7-MeV elastic-scattering yield was measured relative to the 3-MeV elastic-scattering yield in a separate experiment. The yield scales of the various exposures were calibrated relative to each other by means of the beam charges collected in the Faraday cup and from the calibration of the slit system used in the elastic-scattering exposures. Assuming the 3-MeV Ca(d,d) scattering to be Rutherford scattering, an absolute cross-section scale was established.

The results for the 7.00-MeV (d,d) experiment on Ca^{42} are shown in Fig. 1 in comparison with an optical-model prediction. The error on the absolute cross-section scale is less than $\pm 20\%$. Relative errors are indicated by error flags on the data points.

C. The (d,p) Reactions

The (d,p) experiments at 7.00-, 7.10-, and 7.20-MeV bombarding energies were performed utilizing the MIT-ONR electrostatic generator and the multiple-gap spectrograph.[3] In each case, two of the available three zones on the nuclear emulsions (Eastman Kodak 50-μ NTB) were covered with Al foils, thick enough to prevent charged particles other than protons from registering in the emulsions. These two zones were used for measuring (d,p) spectra; one zone for the main exposure, the second zone for an 8 to 10 times shorter exposure. The third, uncovered, zone was used for the elastic-scattering measurements. In the 7.00-MeV experiment, the full energy range of the spectrograph was employed, covering transitions up to approximately 7-MeV excitation in Ca^{43}. This exposure gives information mainly on the stronger transitions. The over-all energy resolution was 12 keV. The 67.5-deg proton spectrum is shown in Fig. 2, where it is seen that the Zn impurities provide some background.

[8] J. R. Comfort (private communication).

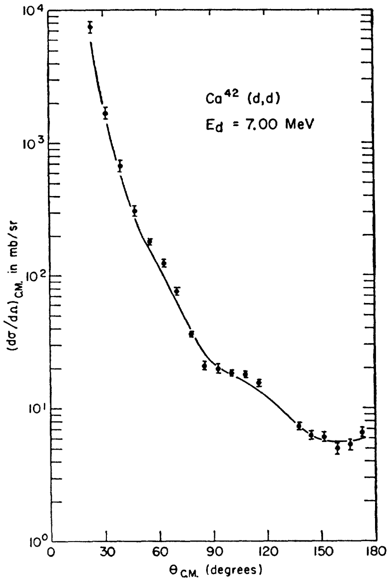

FIG. 1. Angular distribution of deuterons scattered elastically by Ca^{42} at 7.00 MeV. The statistical errors are indicated by error flags on the data points. The curve is the optical-model fit to the data.

The ground-state Q value was found to be 5.707 ± 0.012 MeV, in good agreement with the value reported by Braams.[9] The excitation energies obtained in this experiment are given in Table II. Above level No. 83, we observe some 50 levels, but none are quoted in Table II because the energy resolution, although sufficient to show that most of these proton groups are composite, is not good enough to resolve the groups from each other.

The angular distributions are shown in Figs. 3 through 7 in comparison with DW predictions. Impurity groups from $Ca^{40}(d,p)$ and $Ca^{44}(d,p)$ were sought using the Q values previously measured in this Laboratory.[10–12] If such groups contributed more than 25% of a Ca^{43} group, no yield data for Ca^{43} were extracted.

A cleaner target and a longer exposure were used in the 7.20-MeV experiment. The specific purpose of this run was to study weak, low-lying transitions. Levels up to No. 32 were recorded. The spectrum measured at a laboratory angle of 67.5 deg is shown in Fig. 2. Transitions stronger than 5 μb/sr would have been observed. No new levels were found. A number of the angular distributions for the weak transitions are shown in Sec. IV. The 7.10-MeV data were used in only one case (level No. 2, Sec. IV).

III. ANALYSIS

A. Elastic Deuteron Scattering

The observed 7.00-MeV elastic deuteron scattering was analyzed with the optical-model code ABACUS.[13]

[9] C. M. Braams, Phys. Rev. **105**, 1023 (1957).
[10] T. A. Belote, A. Sperduto, and W. W. Buechner, Phys. Rev. **139**, B80 (1965).
[11] C. M. Braams, Phys. Rev. **103**, 1310 (1956).
[12] W. R. Cobb and D. B. Guthe, Phys. Rev. **107**, 181 (1957).
[13] E. H. Auerbach, Brookhaven National Laboratory Report No. 6562, 1962 (unpublished).

TABLE II. $Ca^{42}(d,p)Ca^{43}$ results. The first column in the table gives the level numbers. Missing numbers refer to levels (see Table VI) that were not observed in the present work but which are known from other experiments. The excitation energies obtained here are given in column 2. The errors in the excitation energies are estimated to be ± 10 keV. Column 3 lists the spectroscopic strength for $l_n = 0, 1, 2, 3$, or 4 derived from the DW analysis of the cross sections. The $\pm 20\%$ error in the absolute cross-section scale is also assigned to the strengths. Spins and parities are given in column 4. A sharp minimum at back angles (a dip) for an $l_n = 1$ transition is typical of a $2p_{1/2}$ transfer, whereas the lack of such a minimum signifies a $2p_{3/2}$ transfer. The dip effect is not very pronounced in the present case, and only the assignments for levels (10) and (56) have been made on this basis. In the case of $l_n = 0$ and $l_n = 2$ transfers, the last column gives the shell-model orbit into which the neutron was assumed to be captured (see also the discussion in Sec. V of the text).

Level number	E_x (MeV) (± 0.010)	$l=0$	$l=1$	$(2j+1)S_{lj}$ $l=2$	$l=3$	$l=4$	J^π	Comments
0	0				5.5		$\frac{7}{2}^-$	
1	0.373						$\frac{5}{2}^-$	
2	0.593		0.21				$\frac{3}{2}^-$	2 dips[a]
3	0.990			0.52			$\frac{3}{2}^+$	$1d_{3/2}$[a]
4	1.393			(0.12)			$\frac{3}{2}\ (\frac{1}{2})^+$	$1d_{3/2}$[a]
5	1.676							a
6	1.899							
7	1.928							
8	1.954	0.13					$\frac{1}{2}^+$	$2s_{1/2}$
10	2.041		3.0				$\frac{3}{2}^-$	no dip[b]
12	2.096		0.06				$(\frac{1}{2})\ \frac{3}{2}^-$	2 dips[a,b]
14	2.219							
15	2.246							
16	2.269							
17	2.404							
18	2.523							
19	2.607		0.29				$\frac{1}{2}, \frac{3}{2}^-$	(dip)
20	2.669				(0.14)			
21	2.693							
22	2.758	0.01					$\frac{1}{2}^+$	$2s_{1/2}$[a,b]
23	2.843							
24	2.874		0.20				$\frac{1}{2}, \frac{3}{2}^-$	(dip)
25	2.939		0.24				$\frac{3}{2}, \frac{1}{2}^-$	(no dip)
26	3.022							
27	3.045							
28	3.071							
29	3.091							
30	3.191							
32	3.287		0.21				$\frac{3}{2}, \frac{1}{2}^-$	(no dip)[a]
33	3.314		0.05				$\frac{3}{2}, \frac{1}{2}^-$	
34	3.352							
35	3.376							
36	3.417							
37	3.566		0.24				$\frac{3}{2}, \frac{1}{2}^-$	(no dip)
38	3.604	0.01					$\frac{1}{2}^+$	$3s_{1/2}$[b]
39	3.655							
40	3.705							
41	3.737							
42	3.772							
43	3.783							
44	3.810				(0.16)			
45	3.864		0.05				$\frac{1}{2}, \frac{3}{2}^-$	(dip)
46	3.898							
47	3.916					1.19	$\frac{9}{2}^+$	b
48	3.958							
49	3.978							
50	4.017							
51	4.048							
52	4.078							
53	4.089							
54	4.124							
55	4.148							
56	4.196		0.88				$\frac{1}{2}^-$	dip[b]
57	4.239		0.12				$\frac{1}{2}, \frac{3}{2}^-$	(dip)
58	4.268			0.04			$\frac{5}{2}^+$	$2d_{5/2}$[b]
59	4.298	0.01					$\frac{1}{2}^+$	$3s_{1/2}$[b]
60	4.324							
61	4.370							
62	4.401							
63	4.429							
64	4.460				0.36		$\frac{5}{2}^-$	b
65	4.498							
66	4.533							

[a] Transition strengths were extracted at both 7.0 and 7.2 MeV.
[b] The J^π assignment is based on the present data.

TABLE II (*continued*)

Level number	E_x (MeV) (± 0.010)	$l=0$	$(2j+1)S_{lj}$ $l=1$	$l=2$	$l=3$	$l=4$	J^π	Comments
67	4.585							
68	4.609							
69	4.638							
70	4.705							
71	4.736							
72	4.758							
73	4.783							
74	4.796							
75	4.826							
76	4.854							
77	4.874							
78	4.897		0.22				$\frac{1}{2}, \frac{3}{2}^-$	(dip)
79	4.922							
80	4.944							
81	4.982		0.05				$\frac{1}{2}, \frac{3}{2}^-$	
82	5.008							
83	5.028		0.21				$\frac{1}{2}, \frac{3}{2}^-$	(dip)

An optical potential of the form

$$V_{\rm opt}=-V(e^x+1)^{-1}+4iW_D(d/dx')(e^{x'}+1)^{-1}+V_c(r,r_c), \quad (1)$$

with

$$x=(r-r_0A^{1/3})/a, \quad x'=(r-r_0'A^{1/3})/a', \quad r_c=r_{0c}A^{1/3},$$

was used, where V_c is the Coulomb potential from a homogeneously charged sphere of radius r_c. The parameter search was defined by a least-squares criterion. The search was started from the parameter set used by Belote *et al.*[14] for $Ca^{46}(d,p)$ at 7.0 MeV. This parameter set, in fact, originated from the "average Z" set of

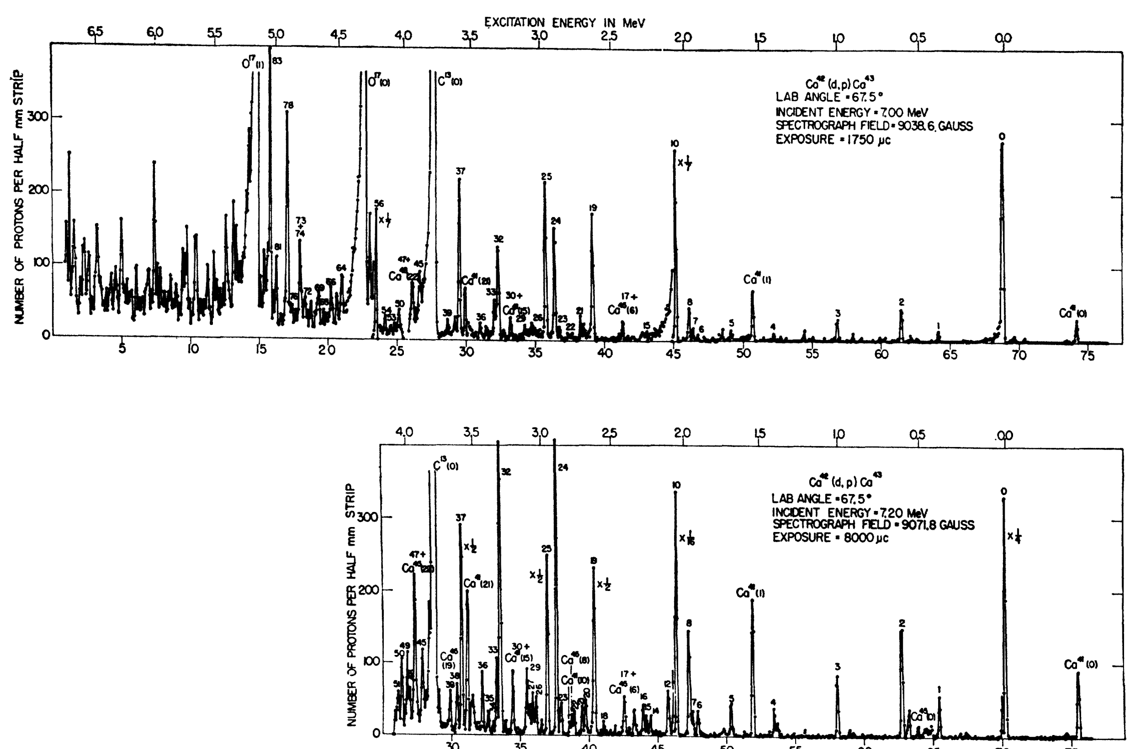

FIG. 2. Measured proton spectra at a laboratory angle of 67.5 deg for deuteron bombarding energies of 7.00 and 7.20 MeV. The number of proton tracks per 0.5-mm strip across the exposed zone is plotted versus plate distance in centimeters. The excitation energy scale is determined by the spectrograph calibration. Proton groups corresponding to levels in Ca^{43} are labeled with the numbers used to identify these states in Table II. Several contaminant groups are also identified.

[14] T. A. Belote, H. Y. Chen, Ole Hansen, and J. Rapaport, Phys. Rev. 142, 624 (1966).

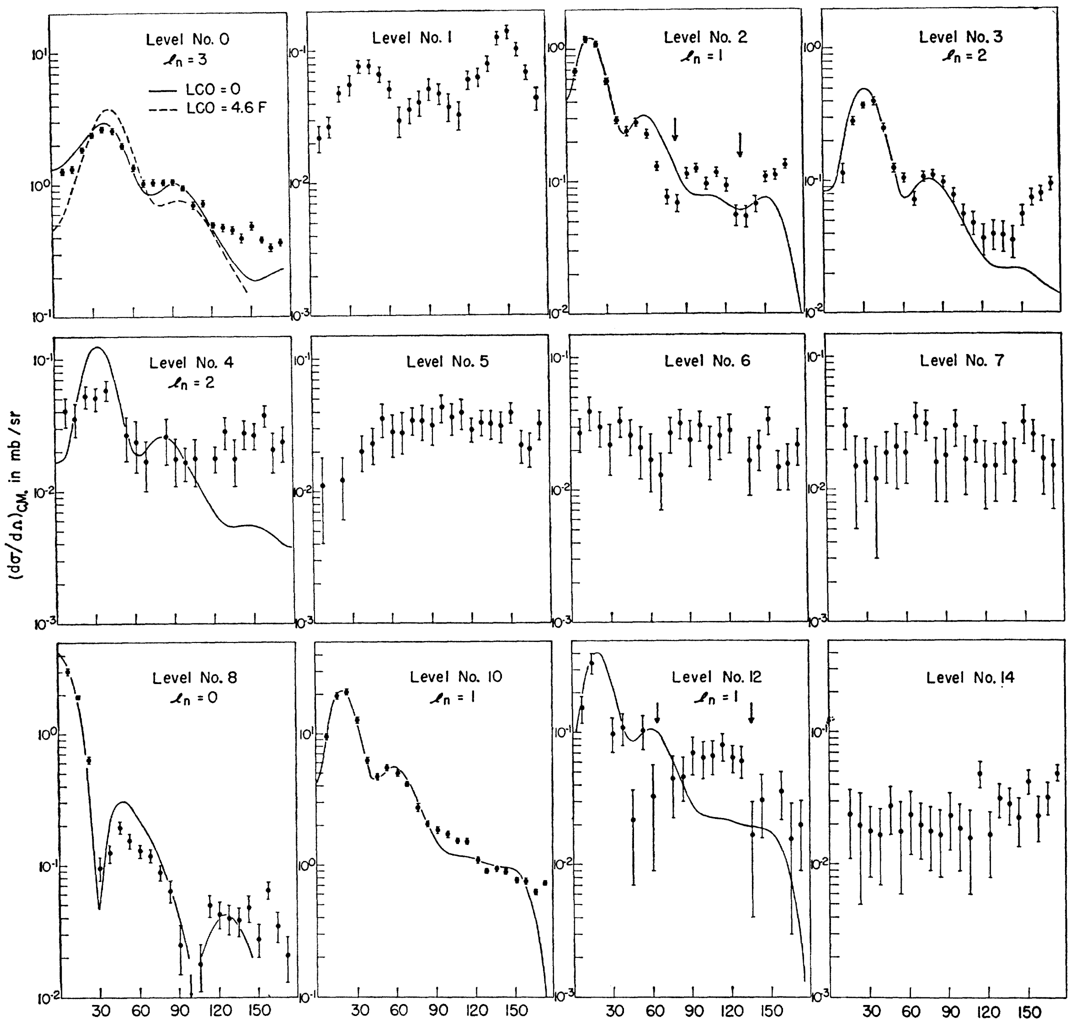

FIG. 3. Angular distributions of protons from $Ca^{42}(d,p)Ca^{43}$. The distributions are labeled with the numbers used to identify the corresponding states in Table II. The statistical errors are indicated by error flags on the data points. The curves are the DW predictions using the parameters of Table III. The areas under the calculated curves were matched to the areas under the experimental curves. The sharp back-angle minimum, characteristic of $2p_{1/2}$ transfers, is not very prominent in the present data; where observed, it is indicated by an arrow.

Bassel *et al.*[15] The values of r_0 and r_{0c} were kept constant; the rest of the parameters were allowed to vary. Two different routes for the search were used, both of which yielded the same best-fit set of parameters given in Table III.

TABLE III. Optical-model parameters.

Particle	V (MeV)	W_D (MeV)	r_0	a	r_0'	a'	r_{0c}
			(all in fermis)				
d	113.2	12.9	1.0	0.753	1.50	0.659	1.3
p	52	10.5	1.25	0.65	1.25	0.47	1.25
n	a		1.25	0.65			

[a] Adjusted to give to the transferred neutron a binding energy of $Q(d,p)+2.23$ MeV.

[15] R. H. Bassel, R. M. Drisko, G. R. Satchler, L. L. Lee, Jr., J. P. Schiffer, and B. Zeidman, Phys. Rev. **136**, B960 (1964).

The elastic deuteron scattering at 7.50 MeV was predicted from this parameter set and subsequently was compared with experimental data. The agreement was very good up to a laboratory angle of 150 deg, after which angle the prediction underestimated the actual cross section by up to 25% at 172.5 deg. We consider this agreement satisfactory.

B. DW Analysis of the (d,p) Reactions

The angular distributions from $Ca^{42}(d,p)$ were calculated by means of the code JULIE[16]; the optical-model parameters given in Table III were used; and the

[16] R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. 3240 (Office of Technical Services, Washington, D. C., 1963).

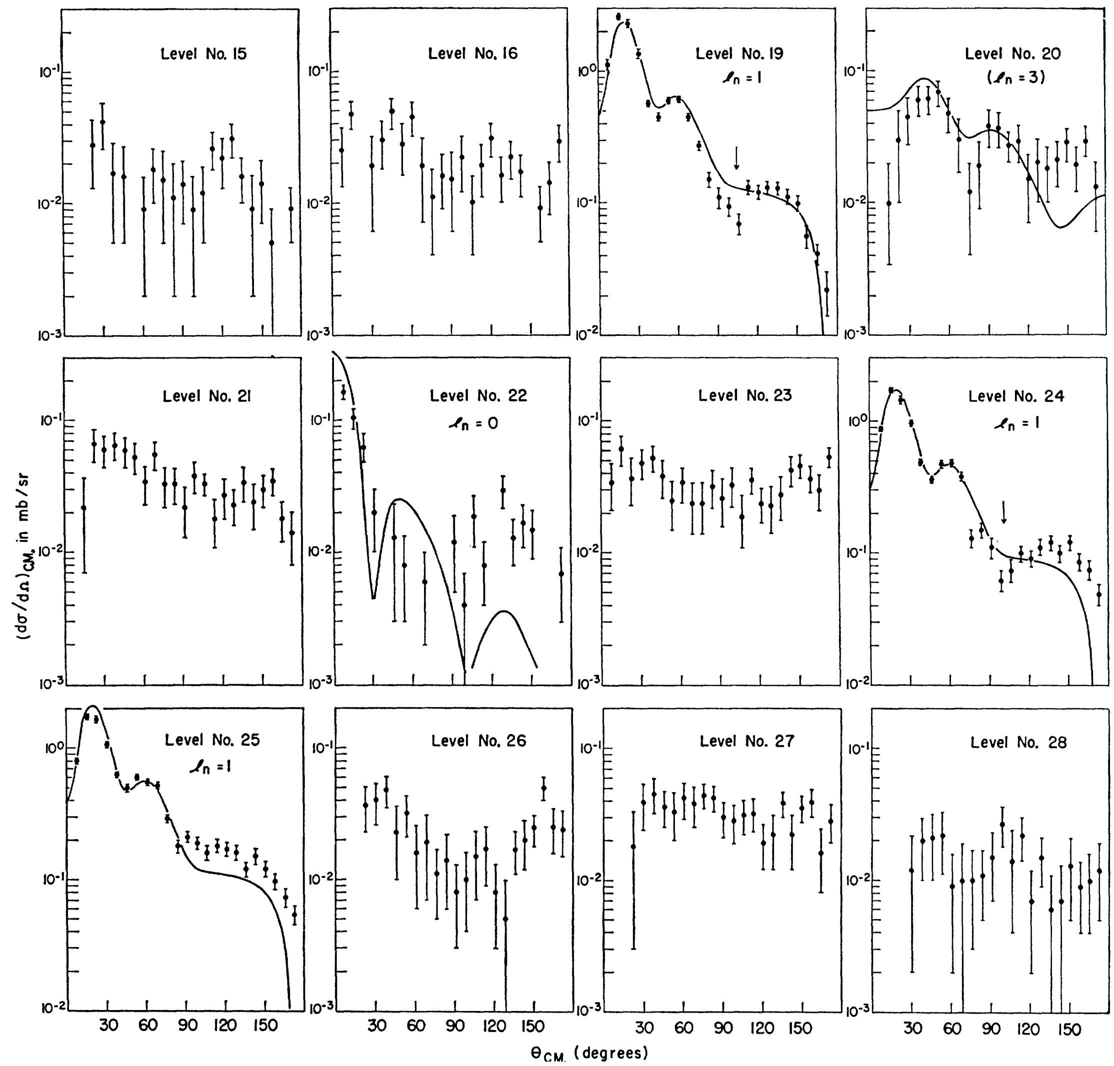

FIG. 4. Angular distributions of protons from $Ca^{42}(d,p)Ca^{43}$. See caption to Fig. 3.

proton parameters were taken from Perey.[17] The calculations were performed in the zero-range approximation,[18] and no lower cutoff was used on the radial integrals. The predicted distributions are shown in comparison with experimental data in Figs. 3 through 7. The l_n values (values of the transferred orbital angular momentum) of Table II were assigned by comparing the predicted and observed angular-distribution shapes. The spectroscopic strengths given in the table were derived by matching the experimental differential cross sections, summed over angles, to the calculated cross-section sums. The calculated cross-section sums are plotted versus Q value in Fig. 8. This graph was used for interpolation in Q.

The agreement between predicted and observed shapes (Figs. 3–7) is good for the strong transitions, but often unsatisfactory for the weak ones. Cases of strong deviations between experiment and DW theory are discussed in Sec. IV. The 7.20-MeV data yielded l_n values and spectroscopic strengths in agreement with values from the 7.00-MeV exposure.

A number of effects, not included in standard DW calculations, have been investigated by Lee *et al.*[19] for the $Ca^{40}(d,p)Ca^{41}$ reaction at $E_d = 11$ MeV. For the $1f_{7/2}$ ground-state transition they found that finite-range effects decreased the predicted cross section by about 35% relative to the zero-range prediction. A spin-orbit coupling term in the bound-state wave function (form factor) of 25 times the Thomas value increased the $1f_{7/2}$ cross section approximately 25%, whereas spin-orbit terms in the elastic-scattering potentials did not affect the predicted cross sections by

[17] F. G. Perey, Phys. Rev. **131**, 745 (1963).
[18] G. R. Satchler, Nucl. Phys. **55**, 1 (1964).
[19] L. L. Lee, Jr., J. P. Schiffer, B. Zeidman, G. R. Satchler, R. M. Drisko, and R. H. Bassel, Phys. Rev. **136**, B971 (1964).

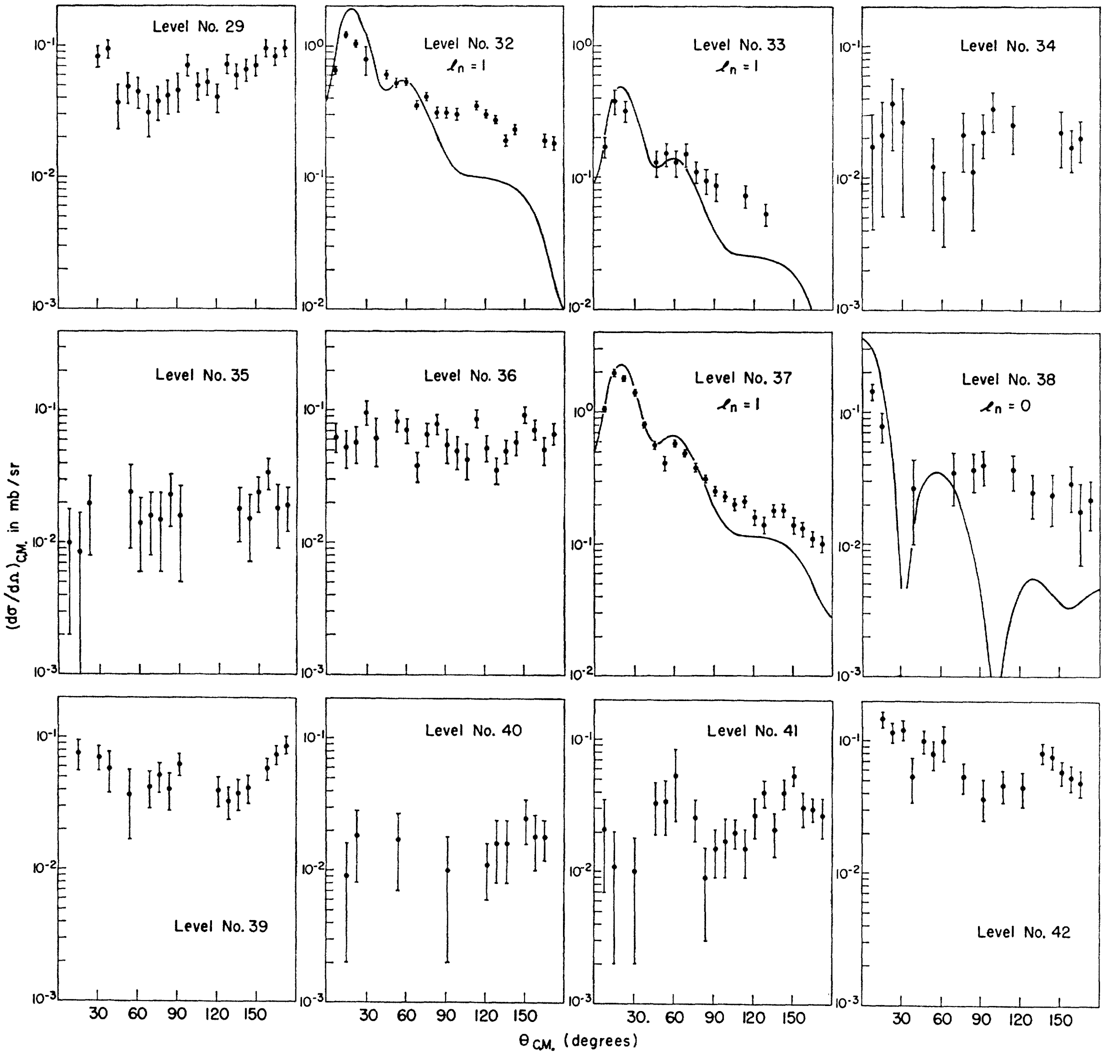

FIG. 5. Angular distributions of protons from $Ca^{42}(d,p)Ca^{43}$. See caption to Fig. 3.

more than approximately 2%. The finite-range effects could be simulated by inserting a lower cutoff on the radial integrals in a zero-range calculation. The cutoff radius fell somewhere between zero and the nuclear radius. The effects on the first strong $2p_{3/2}$ transition were qualitatively similar but were reduced in magnitude to less than 5% change from the zero-range value.

Figures 9 and 10 show finite-range calculations[20] for the $1f_{7/2}$ and first strong $2p_{3/2}$ (d,p) transfers on Ca^{42} at 7.00-MeV bombarding energy. The solid curves were calculated with a Gaussian-type interaction of range 1.5 F, the same as employed in Ref. 19. The dashed curves are the zero-range predictions with no lower cutoff, whereas the dot-dash curves were calculated with a lower cutoff of 4.1 F, and the zero-range interaction. The results obtained at 11-MeV bombarding energy apparently also hold true at 7 MeV; the finite-range calculation predicts a 30% lower cross section in the $1f_{7/2}$ case than does the zero-range calculation. The effect on $2p_{3/2}$ transfer is approximately equal to 2%.

The influence on the total cross section of a lower cutoff in the radial integrals was investigated. The lower cutoff radius was varied from 0 to 6 F. In the case of a $2p$ transfer, the cross section was insensitive to the choice of the lower cutoff radius, whereas in the case of a $1f$ transfer to the Ca^{43} ground state, the cross section decreased by a factor of 2 when the lower cutoff radius was varied from 3.5 to 4.5 F; outside this region, the cross section showed little dependence on the lower cutoff radius.

The spin-orbit coupling effects discussed in Ref. 19 presumably are valid at the lower bombarding energy employed here.

[20] These calculations were performed for us by Dr. G. R. Satchler at Oak Ridge National Laboratory.

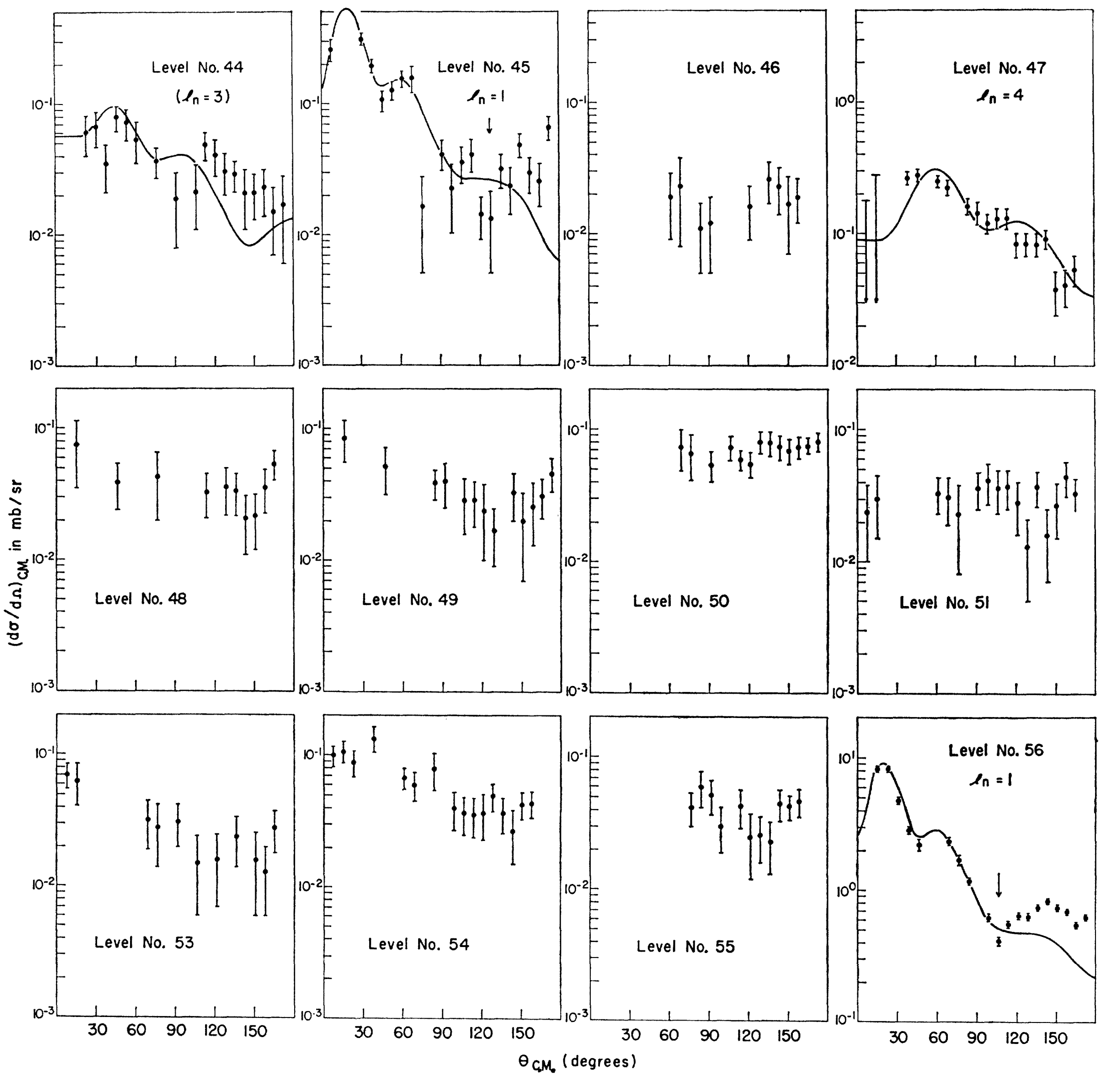

FIG. 6. Angular distributions of protons from $Ca^{42}(d,p)Ca^{43}$. See caption to Fig. 3.

We may therefore summarize this part of the DW analysis as follows: In the cases of $1f_{7/2}$, $2p_{3/2}$, and, presumably, $2p_{1/2}$, $2d_{5/2}$, and $1g_{9/2}$ transfers, the zero-range DW calculations can be used without lower cutoff or spin-orbit effects, because the various effects are either small ($2p$ cases) or they tend to cancel each other; in the cases of $1d_{3/2}$ and $1f_{5/2}$ transfers, the standard DW approach may lead to spectroscopic factors that are too small by up to a factor of two in the worst ($1f_{5/2}$) case.[19]

In the present experiment, a strong fragmentation of the p strength was observed. The single-particle form of the bound-state wave function used in the DW calculations, therefore, seems questionable for these cases.[21,22] In accordance with Sherr *et al.*,[22] the form factor for the transition to the predominantly $(1f_{7/2})^3\ \frac{3}{2}^-$ state at 0.594 MeV in Ca^{43} was calculated with a binding energy equal to that of the unperturbed $2p_{3/2}$ level (see Table V). No appreciable change was observed in the shape of the predicted angular distribution, whereas the spectroscopic factor derived was 20% smaller than the one quoted in Table II. In accordance with Pinkston and Satchler,[21] a form factor was computed, using a slightly increased well radius. The binding energy equaled the separation energy. An increase in r_0 from 1.25 to 1.35 F gave results identical with those obtained by using the unperturbed single-particle binding energy.

IV. ANGULAR-DISTRIBUTION ANOMALIES

A. The 0.373-MeV $\frac{5}{2}^-$ State

The angular distribution for this state has been discussed elsewhere.[23] It was suggested in Ref. 23 that

[21] N. Austern, Phys. Rev. **136**, B1743 (1964); W. T. Pinkston and G. R. Satchler, Nucl. Phys. **72**, 641 (1965).

[22] R. Sherr, E. Rost, and M. E. Rickey, Phys. Rev. Letters **12**, 420 (1964); E. Rost, B. Bayman, and R. Sherr, Bull. Am. Phys. Soc. **9**, 458 (1964).

[23] T. A. Belote, W. E. Dorenbusch, Ole Hansen, and J. Rapaport, Nucl. Phys. **73**, 321 (1965).

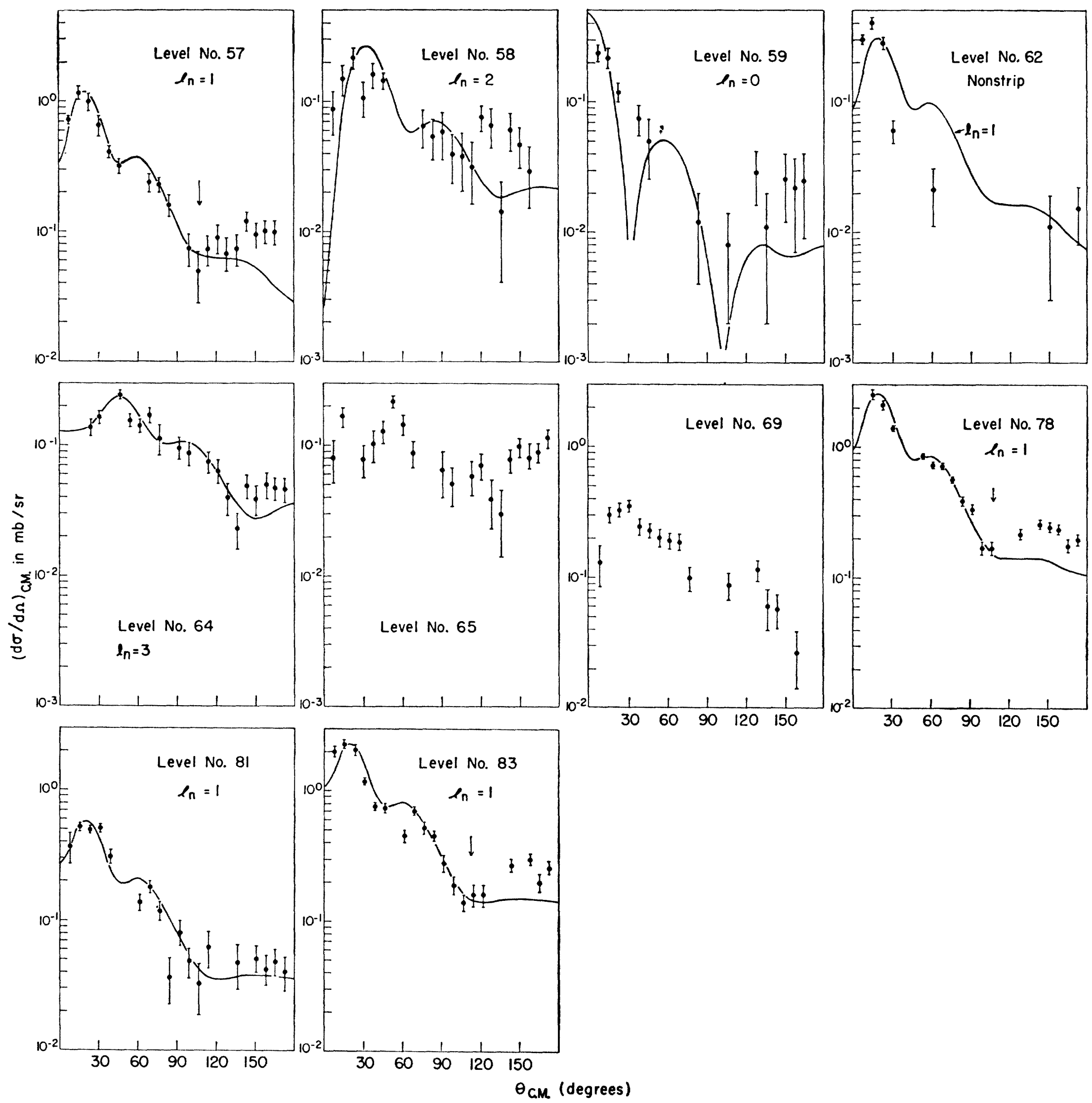

FIG. 7. Angular distributions of protons from $Ca^{42}(d,p)Ca^{43}$. See caption to Fig. 3.

the excitation of the $\frac{5}{2}^-$ state occurs mainly via a second order mechanism: (d,p) to the Ca^{43} ground state followed by inelastic scattering of the outgoing proton, exciting the $\frac{5}{2}^-$ state.

B. The 0.594-MeV $\frac{3}{2}^-$ State and the 2.097-MeV State

The (d,p) transition to the 0.594-MeV $\frac{3}{2}^-$ state in Ca^{43} has been reported as the only existing exception to the rule found by Lee and Schiffer[24] that $2p_{1/2}$ transitions exhibit a sharp minimum at back angles, whereas $2p_{3/2}$ transitions do not. However, it can be seen from the data of Fig. 11 that this angular distribution has not one but two dips at 85 and 130 deg, respectively. Both dips are found at all three bombarding energies. The DW prediction gives a particularly bad fit to this transition, whereas the fit to the 2.050-MeV state (level No. 10), for example, is very good. Since the 0.594-MeV transition is a weak one, going to a state that predominantly belongs to the $(1f_{7/2})^3$ configuration, it is likely that the two dips observed here are of a different origin than are those observed by Lee and Schiffer for the strong p transitions. The transition to the 2.097-MeV state also displays the double-dip feature (see Fig. 11) and also is a weak transition.

[24] L. L. Lee, Jr. and J. P. Schiffer, Phys. Rev. **136**, B405 (1964).

C. The 0.992-MeV and 1.395-MeV States

The beta decay of K^{43} indicates[25,26] that the states in Ca^{43} at 0.992 and at 1.395 MeV have positive parity and probably $J=\frac{3}{2}$. The $Ca^{44}(p,d)$ work of Ref. 5 supports the $\frac{3}{2}^+$ character of the 0.992-MeV state. An alternative spin of $\frac{1}{2}$ for the 1.395-MeV state cannot be ruled out.

The (d,p) data of Ref. 1, as well as the present data, are in agreement with $J^\pi=\frac{3}{2}^+$ for the 0.992-MeV state (see also Fig. 12). The angular distribution for the 1.395-MeV state does not agree very well with either the $l_n=2$ or $l_n=3$ (Fig. 12) calculated curves and thus does not yield any information on the spin and parity of this state. A spin of $\frac{1}{2}$ for this state seems unlikely, since even a small admixture of $(2s_{1/2})$ or $(3s_{1/2})$ should be evident in the forward-angle behavior of the angular distribution.

If both states have $J^\pi=\frac{3}{2}^+$, it is interesting that the corresponding (d,p) angular distributions are quite different.

The spectroscopic strengths given in Table II were derived under the assumption of a $1d$ neutron transfer; if a $2d$ neutron transfer is assumed, the strengths in Table II are reduced by a factor of approximately 3.

V. STRENGTH FUNCTIONS AND SUM-RULE ANALYSIS

The observed strengths are plotted against excitation energy in Fig. 13. As is usually the trend in this region, the f strength splits into two widely different excitation regions, the $1f_{7/2}$ and the $1f_{5/2}$, whereas no such division is observed between the $2p_{3/2}$ and $2p_{1/2}$ transitions. The division of the d strengths into groups more than 3 MeV apart is believed to reflect a difference in the principal quantum number. With regard to the s strengths, it is not evident whether the division between $2s$ and $3s$ used in the analysis (Table II) is correct. For a given Q value, the DW predicts the $3s$ cross section to be larger than the $2s$ cross section by a factor of about 2. The summed strengths are presented in Table IV. The

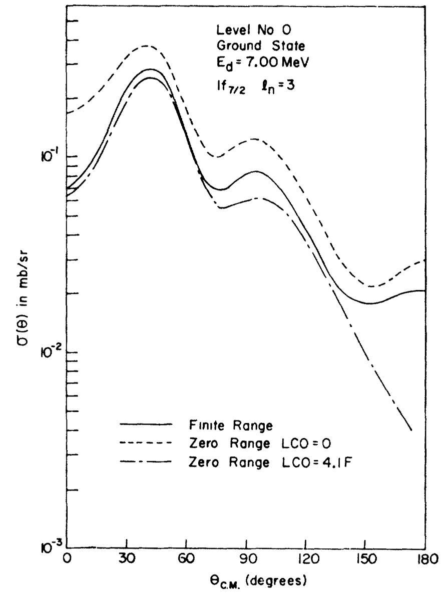

FIG. 9. Effect of finite range on the $l_n=3$ transfer to the Ca^{43} ground state. The curves are the DW predictions for a bombarding energy of 7.00 MeV.

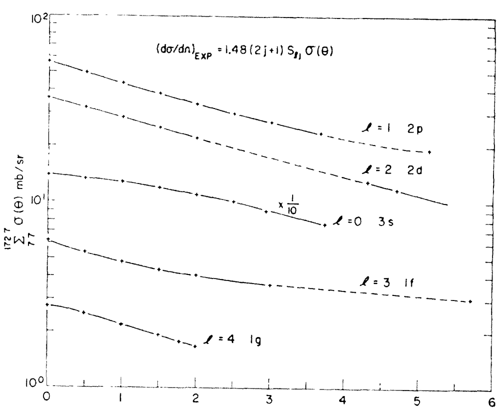

FIG. 8. Dependence of the DW cross-section sum $\Sigma\sigma(\theta)$ on Q and l_n. The summation was carried out over center-of-mass angles corresponding to laboratory angles from 7.5 to 172.5 deg in 7.5-deg intervals.

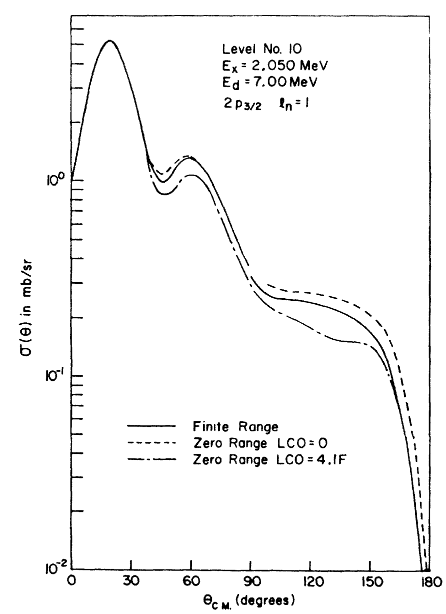

FIG. 10. Effect of finite range on the $l_n=1$ transfer to $Ca^{43}(10)$ at 2.050-MeV excitation. The curves are the DW predictions for a bombarding energy of 7.00 MeV.

[25] N. Benczer-Koller, A. Schwarzschild, and C. S. Wu, Phys. Rev. **115**, 108 (1959).

[26] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington, D. C., 1963).

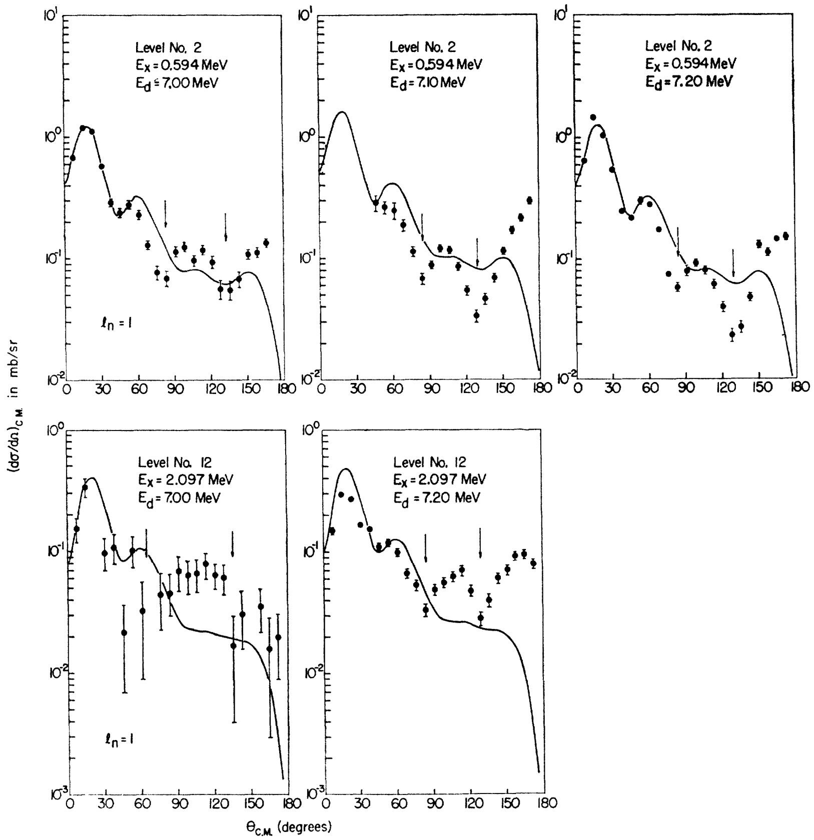

FIG. 11. Proton angular distributions for the levels at 0.594 and 2.097 MeV. The data were taken at bombarding energies of 7.00, 7.10, and 7.20 MeV. The curves are the DW predictions for $2p$ neutron transfer. The minima in the measured distributions at 85 and 130 deg are indicated by arrows.

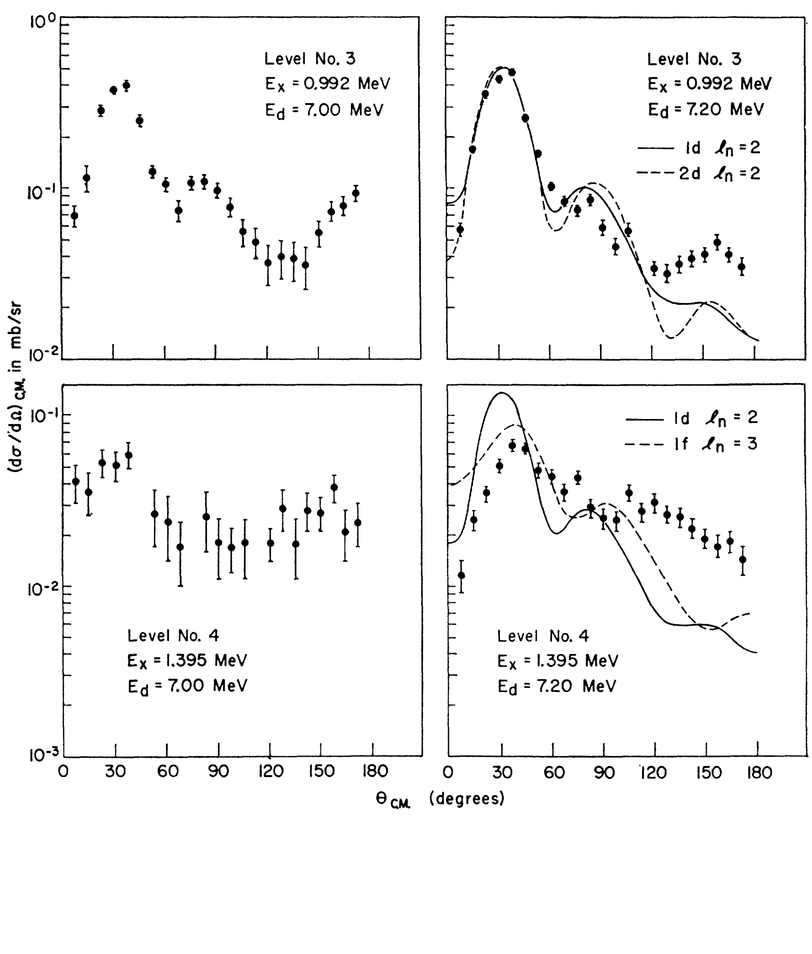

FIG. 12. Proton angular distributions for the states at 0.992 and 1.395 MeV. The data were taken at bombarding energies of 7.00 and 7.20 MeV. The curves are the DW predictions for $l_n = 2$ and 3.

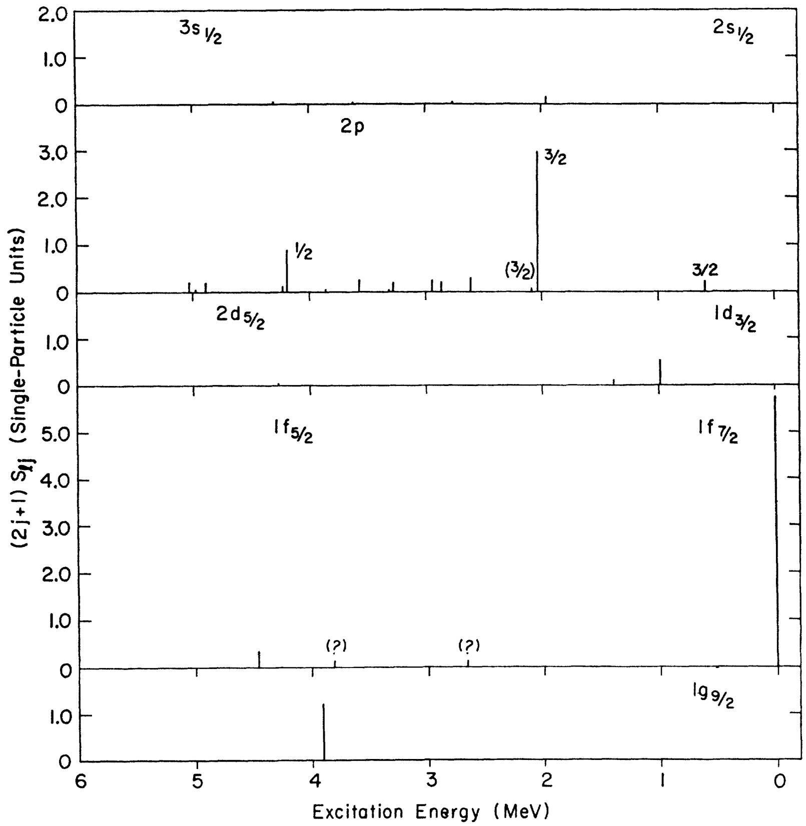

FIG. 13. Strength function for Ca^{43}. The values of $(2j+1)S_{lj}$ from Table II are plotted versus excitation energy. The indicated shell-model assignments are discussed in the text (see also the caption for Table II).

row headed "theory" gives the strengths expected for a doubly closed Ca^{40} core.[27] The agreement of the p strength with the expected value indicates that the DW analysis is reasonable. The $l_n=1$ transitions are less affected by the simplifications made in the DW calculations, and it is known from (p,d) experiments[5] that the $2p$ admixtures in the ground states are small (around 0.1 particle). The fact that $1d$ and $2s$ strengths are observed shows that these shells are not completely filled in $Ca^{42}(0)$.

Application of the energy-weighted sum rule[28]

$$E_{lj}' \sum S_{lj}^a = \sum E_{lj}^a S_{lj}^a \qquad (2)$$

yields the unperturbed single-particle energies E_{lj}' (or center of gravities) from the measured excitation energies E_{lj}^a and spectroscopic factors S_{lj}^a. Using the distinction between $p_{3/2}$ and $p_{1/2}$ states suggested in Table II, we arrive at the numbers given in Table V. The uncertainties quoted on the $E_{1,3/2}'$ and $E_{1,1/2}'$ indicate the limits inside which E' changes for other choices of the $p_{3/2}$ and $p_{1/2}$ assignments. Similar data[10,14,29–31] for other Ca isotopes and for Ar^{41} (an isotone of Ca^{43}) are given in Table V. It is seen that $E_{1,3/2}'$ and $E_{1,1/2}'$ are constant throughout the $1f_{7/2}$ shell calcium isotopes, while both of these excitation

TABLE IV. Summed strengths.

Orbit	$2s_{1/2}$	$1d_{3/2}$	$1f_{7/2}$	$2p_{3/2}$	$2p_{1/2}$	$2p$	$1f_{5/2}$	$1g_{9/2}$	$2d_{5/2}$	$3s_{1/2}$
Expt.	0.14	0.64	5.5	3.96[a]	1.97[b]	6.03[c]	0.66	1.19	0.04	0.02
Theory	0	0	6	4	2	6	6	10	6	2

[a] Levels marked "no dip," "(no dip)," or "two dips" in Table II.
[b] Levels marked "dip" or "(dip)" in Table II.
[c] All $l_n=1$ transitions.

[27] M. H. Macfarlane and J. B. French, Rev. Mod. Phys. **32**, 567 (1960).
[28] S. Yoshida, Nucl. Phys. **38**, 380 (1962).
[29] J. H. Bjerregaard, Ole Hansen, and G. Sidenius, Phys. Rev. **138**, B1097 (1965). Note: The strength quoted in Table I of this reference for level No. 7 at 4012 keV is 0.52. The strength for this transition should be 0.052 (see also Fig. 2 of this reference). The $2p_{3/2}$ energy quoted in Table V of the present paper uses the correct strength (Ole Hansen, unpublished).
[30] E. Kashy, A. Sperduto, H. A. Enge, and W. W. Buechner, Phys. Rev. **135**, B865 (1964).
[31] E. Kashy, A. M. Hoogenboom, and W. W. Buechner, Phys. Rev. **124**, 1917 (1961).

TABLE V. Unperturbed single-particle excitation energies (MeV). The positions of the single-particle states listed in the first column were calculated using Eq. (2). For Ca^{43}, the results given in Table II were used in the calculation. For the other nuclei, the energies were calculated from data presented in the indicated references.

Configuration	$_{18}Ar_{23}^{41}$ [a]	$_{20}Ca_{21}^{41}$ [b]	$_{20}Ca_{23}^{43}$ (Present work)	$_{20}Ca_{25}^{45}$ [c]	$_{20}Ca_{27}^{47}$ [d]	$_{20}Ca_{29}^{49}$ [e]
$1f_{7/2}$	0	0	0		0	...
$2p_{3/2}$	1.35±0.15	2.07	2.19±0.09		2.15±0.10	0
$2p_{1/2}$	3.38±0.15	4.13	4.23±0.20		4.02±0.07	2.03
$1f_{5/2}$	≥5.6	5.50	>4.5		≥5.5	3.95
$1g_{9/2}$	>6.1	>5.0	>4.0		>5.6	≥4.02
$2d_{5/2}$	>4.9	>6.8	>4.3		>6.0	>4.4
$3s_{1/2}$	≥4.8	>6.8	>4.3		>6.0	>6.0
$Q_0(d,p)$ (MeV)	3.87	6.14	5.71	5.19	5.05	2.93

[a] Reference 31. [b] Reference 10. [c] Reference 12. [d] References 14 and 29. [e] Reference 30.

energies are lower by 1 MeV in Ar^{41} and by 2 MeV in Ca^{49}; their difference is 2 MeV inside ±130 keV in all cases. The binding of the last added neutron is also fairly constant throughout the $1f_{7/2}$ calcium isotopes, whereas it drops by 1 MeV in Ar^{41} and by 2 MeV in Ca^{49}.

VI. COMPARISON WITH OTHER EXPERIMENTS

In Table VI we have collected the information we know of pertinent to the Ca^{43} level scheme.[1,4,5,9,25,32–34] Compared with the earlier (d,p) work of Bockelman *et al.*[1] and of Braams,[9] we have identified one new level at 3.314 MeV (level No. 33); otherwise, the agreement is excellent. Level No. 33 at forward angles coincides with level No. 17 of Ca^{41} (Ref. 10), but the intensity of the transition, as well as the observed kinematics, shows that the major component of the observed proton peak in the present experiment belongs to mass 43 and not to mass 41.

Level No. 5 has been assigned $J^\pi=\frac{11}{2}^-$, belonging to the $(1f_{7/2})^3$ configuration.[4] The (d,p) transition to this level is of nonstripping character. The 7.20-MeV data (not shown) support this conclusion. Thus, the present data are consistent with the interpretation of the (d,d') reaction data of Ref. 4. The 2.071-MeV level, probably of $\frac{15}{2}^-$ character,[4] was not observed in the present experiment. The (d,d') results suggest $J^\pi=\frac{9}{2}^-$ for the 2.097-MeV level, in conflict with the $l_n=1$ double-dip character of the (d,p) transition. The characters of the 1.934- and 2.252-MeV states excited in the (d,d') experiment are in question; both states are excited by nonstripping transitions in the present experiment. According to Ref. 4, the 1.934-MeV state might be an alternative candidate for the $\frac{9}{2}^-$ assignment.

VII. DISCUSSION

A total of 84 states up to 5.03-MeV excitation have been identified in Ca^{43}. Of these, 73 may be characterized as having $1f_{7/2}$ or $2p$ stripping character (16 states) or as being nonstripping transitions.

One may attempt to account for these 73 states by considering the degrees of freedom associated with three neutrons moving in $1f_{7/2}$, $2p_{3/2}$, or $2p_{1/2}$ orbits outside an inert Ca^{40} core. A total of 46 states may be constructed in such a model, covering up to about 9-MeV excitation (see also Ref. 35). Therefore, it is clear that extra degrees of freedom are present in the Ca^{43} spectrum.

Next, one may take into account states arising from the coupling of the neutrons to the low-lying excited states in Ca^{40}. Coupling to the 3.34-MeV 0^+ state in Ca^{40} gives rise to another 46 states, the lowest of which probably would be at about 2-MeV excitation. Adding the states arising from coupling to the Ca^{40} 3^- state at 3.73 MeV and to the 2^+ state at 3.90 MeV, one may be able to construct an appropriate number of Ca^{43} levels in the first 5 MeV of excitation. An appreciable mixing among the $\frac{3}{2}^-$ states and among the $\frac{1}{2}^-$ states must occur in order to account for the 15 observed $l_n=1$ (d,p) transitions.

The occurrence of low-lying $l_n=2$ (d,p) transitions has been observed previously from Ca^{40} (Ref. 10), Ca^{42} (Ref. 1), and Ca^{46} (Ref. 14). In all cases the (d,p) transitions lead to a $\frac{3}{2}^+$ state which is also excited in neutron pickup. In the case of Ca^{43}, one may therefore write the wave function[5] for level No. 3 as

$$|Ca^{43}(3)J^\pi=\tfrac{3}{2}^+;\,T=\tfrac{3}{2},\,T_z=\tfrac{3}{2}\rangle = (\tfrac{4}{5})^{1/2}\{1d_{3/2}^{-1};\,T=\tfrac{1}{2},\,T_z=-\tfrac{1}{2}\} \times\{1f_{7/2}^4;\,T=2,\,T_z=2\} - (\tfrac{1}{5})^{1/2}\{1d_{3/2}^{-1};\,T=\tfrac{1}{2},\,T_z=\tfrac{1}{2}\} \times\{1f_{7/2}^4;\,T=2,\,T_z=1\}. \quad (3)$$

In a (d,p) reaction, this state may only be reached if $Ca^{42}(0)$ contains some fraction of four-particle, two-hole configurations:

$$|Ca^{42}(0)0^+;1,1\rangle = a\{1f_{7/2}^2;1,1\} + b[\{1d_{3/2}^{-2};\,T=1\}\{1f_{7/2}^4;\,T=2\}]_{T=1}. \quad (4)$$

From the spectroscopic strength quoted in Table II

[32] T. Lindqvist and A. C. G. Mitchell, Phys. Rev. 93, 1535 (1954).

[33] F. P. Cranston, Jr., D. H. White, and A. J. Smith, Bull. Am. Phys. Soc. 9, 717 (1964).

[34] J. H. Bjerregaard, P. F. Dahl, Ole Hansen, and G. Sidenius, Nucl. Phys. 51, 641 (1964).

[35] B. J. Raz and M. Soga, Phys. Rev. Letters 15, 924 (1965).

TABLE VI. Ca^{43} levels below 3.60-MeV excitation. The table presents a summary of available data on levels in Ca^{43}. Level numbers are assigned in column 1 in order of increasing excitation energy. The evidence for levels above level No. 37 is supplied by the present experiment alone (see Table II). Column 2 lists the values of excitation energy that seem best to the present authors. For levels reported in Ref. 9, the listed values have been recalculated on the basis of a $B\rho$ value of 331.750 kG-cm for Po^{210} alpha particles. For levels observed in the present experiment and not reported in Ref. 9, the values of Table II are listed. Spins and parities are given in column 3. The last columns indicate the excitation mode by which the level in question was observed.

Level number	E_x (MeV)	J^π	Decay	Level reported from (p,p')	(d,d')	(d,p)	(d,α)	(p,d)
0	0	$\frac{7}{2}^-$	a, b, c	d	e	d, f, g	h	i
1	0.373	$\frac{5}{2}^-$	a, b, c	d	e	d, g	h	
2	0.594	$\frac{3}{2}^-$	a, b, c	d	e	d, f, g	h	i
3	0.992	$\frac{3}{2}^+$	a, b, c	d		d, f, g	h	i
4	1.395	$\frac{3}{2}\ (\frac{1}{2})^+$	a, b, c	d		d, g	h	
5	1.680	$(11/2^-)$		d	e	(d), g	h	
6	1.906			d		d, g		
7	1.934	...		d	e	d, g		
8	1.959	$\frac{1}{2}^+$		d		d, f, g		
9	1.987					d		
10	2.050	$\frac{3}{2}^-$	c	d	e	d, f, g		
11	2.071	$(15/2^-)$		d	e			
12	2.097	$\frac{3}{2}\ (\frac{1}{2})^-$		d	e	g		
13	(2.109)			(d)		d		
14	2.227		c	d		d, g		
15	2.252	...		d	e	d, g		
16	2.269			(d)		g		
17	2.411			d		(d), g		
18	2.523					f, g		
19	2.610	$\frac{1}{2}, \frac{3}{2}^-$		(d)		d, f, g		
20	2.676			d		d, g		
21	2.699			d		g		
22	2.756	$\frac{1}{2}^+$		d		g		
23	2.847			d		(d), g		
24	2.883	$\frac{1}{2}, \frac{3}{2}^-$		(d)		d, f, g		
25	2.950	$\frac{3}{2}, \frac{1}{2}^-$		d		d, f, g		
26	3.030			d		g		
27	3.050			d		(d), g		
28	3.077			d		g		
29	3.097			d		d, g		
30	3.197			d		g		
31	3.282			d		g		
32	3.296	$\frac{3}{2}, \frac{1}{2}^-$		d		d, f, g		
33	3.314	$\frac{3}{2}, \frac{1}{2}^-$				g		
34	3.352			(d)		g		
35	3.376			(d)		g		
36	3.422			d		d, g		
37	3.566	$\frac{3}{2}, \frac{1}{2}^-$				f, g		

[a] Reference 32; [b] Reference 25; [c] Reference 33; [d] Reference 9; [e] Reference 4; [f] Reference 1; [g] Present work; [h] Reference 34; [i] Reference 5.

for level No. 3, one obtains

$$b^2=0.27\pm0.07.$$

It is assumed that all configuration components in Eq. (4) have spin zero.

The value of a in Eq. (4) may be obtained from the observed ground-state (d,p) strength. Neglecting possible five-particle, two-hole configurations in $Ca^{43}(0)$, one obtains from Table II

$$a^2=0.91\pm0.23.$$

The value of b^2 seems rather large. In $Ca^{40}(d,p)Ca^{41}$, the $(1f_{7/2})^2(1d_{3/2})^{-2}$ component was found to give a coefficient $b^2=0.60$, whereas pickup data[36] on Ca^{40} yield $b^2=0.15$. The available $Ca^{42}(He^3,\alpha)Ca^{41}$ data[37] are inconclusive with respect to the magnitude of b^2. The source of these inconsistencies in the (d,p) strengths for the low-lying $\frac{3}{2}^+$ states is not clear; it may be connected with the oversimplified form-factor assumptions made in the DW analysis.

It is interesting to speculate about the nature of the second positive-parity state (level No. 4). According to Conlon *et al.*,[5] a second $\frac{3}{2}^+$ state may be constructed in Ca^{43} by coupling the $1d_{3/2}$ hole to a $(1f_{7/2})^4$ $T=1$ configuration. This state would be expected to lie at low excitation energy. If one tentatively associates level No. 4 of Ca^{43} with this configuration, it follows that, since this state is excited in $Ca^{42}(d,p)Ca^{43}$, there will be $(1d_{3/2})^{-2}(1f_{7/2})^4$ components in $Ca^{42}(0)$ with $T=1$ for the $1f_{7/2}$ part. Since a $(1f_{7/2})^4$ $T=1$ configuration cannot have spin zero, this would mean that "unpaired" configurations are present in the Ca^{42} ground state. Further information on the Ca^{43} hole states may be obtained through a study of the $K^{41}(\alpha,d)Ca^{43}$ and

[36] C. Glashausser, M. Kondo, M. E. Rickey, and E. Rost, Phys. Letters 14, 113 (1965).

[37] R. Bock, H. H. Duhm, and R. Stock, Phys. Letters 18, 61 (1965).

$K^{41}(He^3,p)Ca^{43}$ reactions. The first of these should excite $\frac{3}{2}^+$ hole states in Ca^{43} that are coupled to $1(f_{7/2})^4$ $T=1$ configurations more strongly than those coupled to $(1f_{7/2})^4$ $T=2$ configurations.

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High-Resolution Study of the $Fe^{54}(t,p)Fe^{56}$ Reaction*

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The $Fe^{54}(t,p)Fe^{56}$ reaction was studied with 12-MeV incident tritons using the Aldermaston tandem Van de Graaff and multigap spectrograph. The angular distributions of the protons are characterized by sharp structure. For eight cases where the spin and parity of the final state are known, the position of the first maximum agrees very well with predictions of the plane-wave Born approximation. These positions were then used for assigning spins and parities to many other levels. The only known unnatural parity state is excited two orders of magnitude less strongly than the strongly excited states, and has an unusual angular distribution. The collective 3^- state (4.51 MeV) is the most strongly excited state in the spectrum; it is shown that this is very difficult to explain. Other very strongly excited states are the ground and first excited states, 1^- states at 5.19 and 7.06 MeV, and a 6^+ (or 7^-) state at 5.15 MeV. The spin and parity assignments of all states up to 4.5 MeV are discussed in the light of other evidence; the principal correction is to change the assignment of the 3.122-MeV state from 5^+ to 5^-.

I. INTRODUCTION

It has been established for some time that (p,t) and (t,p) reactions, at least above 10 MeV, proceed predominantly by a direct-interaction mechanism in which two neutrons are transferred.[1] Some of the interesting advantages of using these reactions for nuclear-structure studies have been pointed out by Newns, Yoshida, Hintz, and others.[2,1] Unfortunately, application of the (p,t) reaction for such purposes has been hindered by the lack of energy resolution in accelerators with high enough energy to use such a large negative Q reaction, and (t,p) reaction studies have been limited by the difficulties of using highly radioactive tritium as a source material in accelerators. However, a 12-MeV triton beam was recently developed by the Aldermaston group and has been widely used in studies of (t,p), (t,d) and (t,α) reactions. The work reported here is one of these studies. It concerns the reaction $Fe^{54}(t,p)Fe^{56}$ which has a Q value of about 12.03 MeV.

II. PROCEDURES AND RESULTS

A thin target of Fe^{54} was bombarded with 12.01-MeV tritons from the Aldermaston Tandem Generator, and the emitted protons were analyzed and detected in the multi-angle magnetic spectrograph. The method has been described previously.[3] The target thickness was not measured, so that absolute cross sections were not

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