Radiative Muon Capture and the Giant Dipole Resonance in Ca^{40}†

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We have calculated the photon spectrum and total rate for radiative muon capture in Ca^{40} using the giant dipole resonance (GDR) model used by Foldy and Walecka to significantly improve theoretical calculations of ordinary muon capture rates. In this model we relate the dipole parts of the nuclear matrix elements relevant for radiative muon capture to integrals over experimental photo-absorption cross sections in the region dominated by the giant dipole resonance. The remaining other multipole and velocity parts of the nuclear matrix elements are evaluated in the closure approximation using harmonic oscillator wave functions. We present numerical results for the photon spectrum as calculated in the GDR model for various values of the weak-interaction coupling constants and of the average maximum photon energy k_m which is used in the other multipole and velocity parts of the calculation. The most important effect of the GDR calculation is to reduce the dipole contribution to the capture rate, thus giving an absolute radiative rate some 40% lower and a relative rate (ratio of radiative and ordinary rates) some 20% lower than that obtained in the closure–harmonic-oscillator model. As a consequence, the GDR model requires a larger value of the induced pseudoscalar coupling constant g_P to reproduce a given spectrum than does the closure–harmonic-oscillator model. Finally, we compare our results with the data of Conversi *et al.*, who found by interpreting their data in the closure–harmonic-oscillator model that $g_P=(13.3\pm2.7)g_A$, where g_A is the axial vector coupling constant. We find that the GDR calculation requires $g_P=(16.5\pm3.1)g_A$ for a fit to the experiment, where we have assumed $k_m=88$ MeV and have taken currently accepted values for the other coupling constants. Alternatively, by taking the Goldberger-Treiman value $g_P\cong 7g_A$ and varying the induced tensor coupling constant g_T we obtain a fit to the data for $g_T\gtrsim 35g_V$. As these results are quite sensitive to k_m, we give in addition results for other values of k_m.

I. INTRODUCTION

RECENTLY, Conversi, Diebold, and di Lella[1] reported results of a measurement of the photon spectrum for the radiative muon capture process in Ca^{40}. Such a measurement is of particular interest in that the radiative spectrum is quite sensitive to g_P, the induced pseudoscalar coupling constant of the weak interaction. Conversi *et al.*[1] found that $g_P=(13.3\pm2.7)g_A$, where g_A is the axial vector coupling constant, based on an interpretation of their data in terms of the theoretical formulas of Rood and Tolhoek[2] (RT) for radiative muon capture and of Luyten, Rood, and Tolhoek[3] (LRT) for ordinary muon capture. These formulas are based on the closure approximation and the assumption that the initial nucleus can be adequately characterized by a shell-model harmonic-oscillator wave function without spin-orbit coupling. In view of the discrepancy between the measured value of g_P and the theoretical value $g_P\cong 7g_A$ of Goldberger and Treiman,[4] it is of interest to try to improve the nuclear physics in the theoretical calculation of radiative capture.

Foldy and Walecka[5] (FW) have shown that for ordinary muon capture one can significantly improve the results of the closure–harmonic-oscillator model by relating the dipole parts of the relevant nuclear matrix elements to integrals over the photo-absorption cross section, in the giant-dipole-resonance region, of the initial nucleus. The other multipole parts of the matrix elements are evaluated in the closure–harmonic-oscillator model as before. In this article we show that one can extend the calculation of FW to radiative muon capture. In fact in this way, which we call the giant dipole resonance or GDR model, one is able to obtain from experimental photo-absorption cross-section data an even larger proportion of the capture rate in the case of radiative capture than in ordinary capture.

In the first section we outline the derivation of the general formulas for radiative muon capture and their evaluation in the closure–harmonic-oscillator model, following RT. We then show in detail in the following section how the ideas of FW can be extended to radiative muon capture and thus we obtain formulas, analogous to those obtained for ordinary capture by FW, which relate the dipole parts of the nuclear matrix elements appropriate for radiative muon capture to integrals over experimental photo-absorption cross sections. In the next section we present numerical results for both the absolute and the relative radiative capture rates, as calculated in the two models, for several different sets of coupling constants. We find that the most important effect of our calculation is to decrease the dipole contribution to the capture rate from its value in the closure–harmonic-oscillator model. Thus we obtain for the standard set of coupling constants defined below an absolute radiative capture rate some 40% lower than that given by the closure–harmonic-oscillator model and a relative rate some 20% lower, which implies that to fit a given set of experi-

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[1] M. Conversi, R. Diebold, and L. di Lella, Phys. Rev. 136, B1077 (1964).
[2] H. P. C. Rood and H. A. Tolhoek, Nucl. Phys. 70, 658 (1965).
[3] J. R. Luyten, H. P. C. Rood, and H. A. Tolhoek, Nucl. Phys. 41, 236 (1963).
[4] M. L. Goldberger and S. B. Treiman, Phys. Rev. 111, 354 (1958).
[5] L. L. Foldy and J. D. Walecka, Nuovo Cimento 34, 1026 (1964).

mental data one would need a larger value of g_P with the giant-dipole-resonance model than with the closure–harmonic-oscillator model. The GDR model also adds a high-energy tail to the photon spectrum, but this seems to be unimportant numerically.

Finally we examine in the last section the effect of our calculation on the interpretation of the experimental results of Conversi *et al.*[1] As explained in more detail below, because of difficulties associated with the experimental counter resolution it does not seem possible to sufficiently separate theory and experiment so as to allow direct comparison of the results of the GDR calculation and the original data. We therefore compare the GDR results with the theoretical spectrum given by the formulas of RT which gave the best fit to the data. On this basis we find that we need $g_P=(16.5\pm3.1)g_A$ to give agreement of theory with experiment, where the uncertainty is purely of experimental origin. To obtain this result we assumed standard values for the other coupling constants and in particular that the second-class induced tensor coupling constant g_T is zero. Alternatively, we may assume the Goldberger-Treiman value of about $7g_A$ for g_P and vary g_T. In this way we obtain the result $g_T\gtrsim35g_V$ for a fit to the experimental results. The specific numerical values obtained for g_P and g_T are rather sensitive to k_m, the average value of the maximum photon energy which is used in the other multipole part of the calculation. For those given above we assumed the value $k_m=88$ MeV in accordance with the result of Conversi *et al.*[1] Figure 7 shows results for some other choices. These numerical results are also somewhat limited by the nature of the comparison of theory and experiment which was required and so should be interpreted with care.

II. GENERAL FORMULAS FOR THE RADIATIVE MUON CAPTURE RATE

We outline here the derivation of the general formula for radiative muon capture in a nucleus following for the most part the work of RT to which the reader is referred for details and for references to earlier calculations of radiative muon capture. The basic weak interaction Hamiltonian for ordinary muon capture on a proton we take as[6]

$$H_W=\frac{G}{\sqrt{2}}\bar{u}_n\left(g_V\gamma^\alpha+\frac{ig_M}{2m}\sigma^{\alpha\beta}q_\beta+\frac{g_S}{m_\mu}q^\alpha+g_A\gamma^\alpha\gamma_5+\frac{g_P}{m_\mu}q^\alpha\gamma_5+\frac{ig_T}{2m}\sigma^{\alpha\beta}q_\beta\gamma_5\right)u_p\bar{u}_\nu\gamma_\alpha(1-\gamma_5)u_\mu+\text{H.c.},\quad(1)$$

where $q^\alpha=n^\alpha-p^\alpha$ with n^α and p^α the neutron and proton four momenta, and where m is the nucleon mass, m_μ is the muon mass, and u_n, u_p, u_ν, u_μ are the wave functions for the neutron, proton, neutrino, and muon. The coupling constants g_V, g_A, g_M, g_P, g_T, g_S, are measured in units of the Fermi constant G, which we take as $1.01\times10^{-5}/m_p{}^2$, and correspond, respectively, to vector, axial vector, weak magnetism, induced pseudoscalar, and second-class induced tensor and scalar couplings. We will neglect the q^2 dependence of all of the coupling constants except g_P as it is unknown for g_T and g_S and as we expect it to be quite small for g_A, g_V, and g_M in the region of q^2 of interest. For g_V and g_M we know that the effect of including the actual form factors as obtained from the conserved-vector-current theory is only a few percent. We do not have such precise information for g_A. However, theoretical estimates[7,8] and preliminary results of high-energy neutrino experiments[9] seem to indicate that g_A also varies at most a few percent over the range of q^2 appropriate for radiative muon capture. Finally, we obtain the form factor for the induced pseudoscalar term by assuming dominance of the one-pion contribution in accordance with the argument of Goldberger and Treiman.[4] The capture interaction for a nucleus can now be obtained in the usual approximation as a sum over captures by individual nucleons.

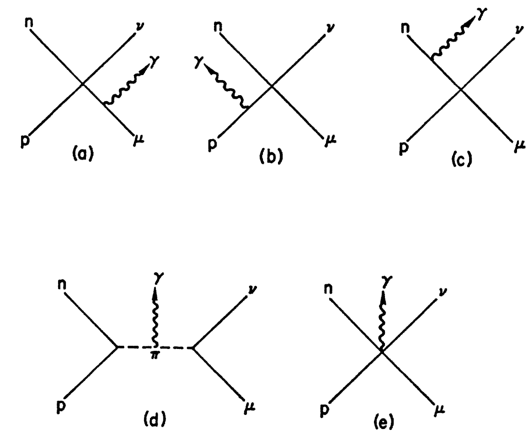

FIG. 1. Diagrams contributing to radiative muon capture on a proton. Diagrams (b) and (c) include radiation from the anomalous magnetic moment of the proton and neutron.

To obtain the capture interaction for radiative muon capture one essentially adds a photon line in all possible ways to the diagram corresponding to Eq. (1). The dominant contribution comes from Fig. 1(a) corresponding to radiation by the muon. RT also include contributions from Fig. 1(b) in which the proton radiates (including radiation from its anomalous magnetic moment), Fig. 1(c) in which the neutron radiates from

[6] The notation is that of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1964). The coupling constants are defined so that they agree in the nonrelativistic limit with those of LRT. We also use the units $\hbar=c=1$.

[7] A. Fujii and H. Primakoff, Nuovo Cimento **12**, 327 (1959).

[8] Ching-Hung Woo, Phys. Rev. Letters **12**, 308 (1964); **11**, 385 (1963).

[9] J. S. Bell, J. Lovseth, and M. Veltman, in *Proceedings of the Sienna International Conference on Elementary Particles*, edited by G. Bernardini and G. P. Puppi (Società Italiana di Fisica, Bologna, 1963), Vol. 1, p. 584.

its anomalous magnetic moment, and Fig. 1(d) corresponding to radiation from the intermediate pion in the diagram leading to the induced pseudoscalar term. Finally there is the contribution of Fig. 1(e) which comes from the requirement of gauge invariance, i.e., from the replacement $p^\alpha \to p^\alpha - eA^\alpha$ in Eq. (1). In each case the induced pseudoscalar term is written as

$$g_P(q^2) \equiv g_P^{N,L} = g_P(m_\pi^2 + \mathrm{m}_\mu^2)/(m_\pi^2 - q^2), \quad (2)$$

where the Goldberger-Treiman agrument gives $g_P \cong 7g_A$, and where q^2 is the four-momentum transfer at the weak vertex. Thus, for diagrams 1(a) and 1(e) $g_P(q^2) \equiv g_P^L$ given by Eq. (2) with $q^\alpha = \mu^\alpha - \nu^\alpha - k^\alpha$, and for diagrams 1(b), 1(c), and 1(d) $g_P(q^2) \equiv g_P^N$ given by Eq. (2) with $q^\alpha = \mu^\alpha - \nu^\alpha$ where μ^α, ν^α, and k^α are the four-momenta of the muon, neutrino, and photon, respectively.

RT now make a nonrelativistic reduction of the contributions of each of these diagrams, including terms to first order in the momentum of the initial nucleon, and thus obtain an effective nonrelativistic Hamiltonian for radiative muon capture completely analogous to that used in ordinary muon capture. One now treats the effective Hamiltonian as a single-particle operator, sums over individual nucleons, and considers the matrix element of the result between nuclear states. The square of this matrix element summed over final and averaged over initial states leads to the following expression for the number of photons in an energy interval dk:

$$N(k)dk = (\alpha m_\mu G^2/4\pi^2) \times |\varphi_\mu|^2 dk \sum_{\lambda=\pm1} \{I_r^2(k,\lambda) + I_{\mathrm{vel}}^2(k,\lambda)\}. \quad (3)$$

Here we have taken the muon wave function out of the nuclear matrix elements and use[5,10] $|\varphi_\mu|^2 = (Zm_\mu\alpha)^3 R/\pi$ with $\alpha = 1/137$ and for Ca^{40}, $Z = 20$ and $R = 0.44$. The sum on $\lambda = \pm 1$ is a sum over right and left circular polarizations of the emitted photon. The nuclear matrix elements are buried in the functions $I_r^2(k,\lambda)$, which contains the major nuclear matrix elements, and $I_{\mathrm{vel}}^2(k,\lambda)$, which contains terms proportional to the momentum of the initial nucleon, the so-called velocity terms. These functions are given by

$$I_r^2(k,\lambda) = \int_{-1}^{1} dy \sum_{\bar{a}b} \frac{k(k_m^{ab} - k)^2}{m_\mu^3} \theta(k_m^{ab} - k)(G_r^2)_{ab} \times |\langle b| \textstyle\sum_i \tau_-(i) \exp(-i\mathbf{s}_{ab}\cdot\mathbf{r}_i)|a\rangle|^2 \quad (4)$$

and, in the closure approximation with certain other conditions given below,

$$I_{\mathrm{vel}}^2(k,\lambda) = \frac{k(k_m - k)^2}{m_\mu^3} \theta(k_m - k) \int_{-1}^{1} dy\, G_{\mathrm{vel}}^2 \times [Z - \sum_{\bar{a}b} |\langle b| \sum_i \tau_-(i) \exp(-i\mathbf{s}_{ab}\cdot\mathbf{r}_i)|a\rangle|^2]. \quad (5)$$

In these expressions $y = \hat{k}\cdot\hat{p}$ and $\mathbf{s}_{ab} = (\mathbf{k} + \boldsymbol{\nu})_{ab}$. The sum on $\bar{a}b$ indicates an average over initial nuclear states $|a\rangle$ and a sum over final nuclear states $\langle b|$. The maximum photon energy k_m^{ab} in the transition $|a\rangle \to |b\rangle$ is given by the equation $k_m^{ab} = \mathrm{m}_\mu - (m_n - m_p) - E_{\mathrm{BE}} - (E_b - E_a)$ in which E_a and E_b are the energies of the nuclear states and E_{BE} is the initial binding energy of the muon. This equation is just the equation expressing conservation of energy for the case in which the neutrino is emitted with negligible energy. The average value of k_m^{ab} is denoted by k_m. The step function $\theta(x)$, defined by $\theta(x \leq 0) = 0$ and $\theta(x > 0) = 1$, simply incorporates in a mathematically convenient way the requirement that the transition probability be zero for photon energies greater than the maximum allowed by energy-momentum conservation. G_r^2 and G_{vel}^2 are rather complicated functions of the coupling constants, of k_m^{ab}, and of y and are given in the Appendix.

To obtain Eq. (4) we had to assume the following relations:

$$\sum_{\bar{a}b} (k_m^{ab} - k)^2 \theta(k_m^{ab} - k) |\langle b| \sum_i \tau_-(i) \exp(-i\mathbf{s}_{ab}\cdot\mathbf{r}_i)\boldsymbol{\sigma}(i)|a\rangle|^2 = 3 \sum_{\bar{a}b} (k_m^{ab} - k)^2 \theta(k_m^{ab} - k) |\langle b| \sum_i \tau_-(i) \exp(-i\mathbf{s}_{ab}\cdot\mathbf{r}_i)|a\rangle|^2, \quad (6)$$

$$\sum_{\bar{a}b} (k_m^{ab} - k)^2 \theta(k_m^{ab} - k) \mathbf{A}\cdot\langle b| \sum_i \tau_-(i) \exp(-i\mathbf{s}_{ab}\cdot\mathbf{r}_i)\boldsymbol{\sigma}(i)|a\rangle\, \mathbf{B}^*\cdot\langle b| \sum_i \tau_-(i) \exp(-i\mathbf{s}_{ab}\cdot\mathbf{r}_i)\boldsymbol{\sigma}(i)|a\rangle^* = \sum_{\bar{a}b} \mathbf{A}\cdot\mathbf{B}^* (k_m^{ab} - k)^2 \theta(k_m^{ab} - k) |\langle b| \sum_i \tau_-(i) \exp(-i\mathbf{s}_{ab}\cdot\mathbf{r}_i)|a\rangle|^2, \quad (7)$$

where $\mathbf{A}$ and $\mathbf{B}$ are arbitrary vectors. These relations have been assumed in almost all previous calculations of ordinary and radiative muon capture. They are true in the closure approximation, which allows us to replace k_m^{ab} by k_m and take it out of the sum on b, as well as in the group summation method of LRT if we assume that the nuclear wave functions can be characterized by shell-model wave functions without spin-orbit coupling and that the initial nucleus has doubly closed shells.[3] These same assumptions, with the additional requirement that the initial nucleus have equal

[10] J. C. Sens, Phys. Rev. 113, 679 (1958).

numbers of neutrons and protons, allow us to obtain Eq. (5) for $I_{\rm vel}^2(k,\lambda)$.[2,3] There is also evidence that these equations are valid under assumptions less restrictive than those of the closure or group-summation approximations. In particular, FW have given an extensive discussion, based on Wigner supermultiplet theory, of the validity of the relations for ordinary muon capture which are analogous to Eqs. (6) and (7). With only minor modifications their arguments are applicable to radiative muon capture. Thus one can show that, for initial nuclei which are isospin singlets and which have a mass number which is a multiple of four, Eqs. (6) and (7) hold exactly, provided that spin-dependent forces can be neglected.

We make the one further assumption, that we can replace k_m^{ab} in the expression for $(G_r^2)_{ab}$ by an appropriate average value k_m, since it appears only in the combination $k_m^{ab}/2m$ which is $\sim 5\%$ of the leading terms. With this approximation we can lump all of the nuclear physics into the quantities

$$I_n=\int_{-1}^{1} y^n dy \sum_{\bar{a}b} \frac{k(k_m^{ab}-k)^2}{m_\mu^3}\theta(k_m^{ab}-k) \times|\langle b|\sum_i \tau_-(i)\exp(-i\mathbf{s}_{ab}\cdot\mathbf{r}_i)|a\rangle|^2 \quad (8)$$

which can now be evaluated using various nuclear models.

RT evaluate I_n in what we shall call the closure-harmonic-oscillator model, which uses the closure approximation and shell-model wave functions with harmonic-oscillator radial parts for the nuclear wave functions. In this model I_n is given by

$$I_n=\frac{k(k_m-k)^2}{m_\mu^3}\theta(k_m-k)\int_{-1}^{1} y^n dy\, M_r^2, \quad (9)$$

where, for Ca^{40}, with $\eta^2\equiv(sb)^2=b^2[k_m^2-2k(k_m-k)\times(1-y)]$, where the oscillator parameter $b=2.03$ F, we have[2,5]

$$M_r^2=20\left\{1-\left[1+\frac{\eta^4}{8}-\frac{\eta^6}{80}+\frac{\eta^8}{640}\right]e^{-\frac{1}{2}\eta^2}\right\}. \quad (10)$$

III. RADIATIVE CAPTURE RATE IN THE GDR MODEL

FW have shown that one can obtain a result for the ordinary capture rate, which is in much better agreement with experiment than that obtained in the closure–harmonic-oscillator model, by using the experimental photo-absorption cross section for Ca^{40} in the region dominated by the giant dipole resonance to evaluate the dipole part of the nuclear matrix elements. We proceed now to extend this model to radiative muon capture, following essentially the development given for ordinary capture in Secs. 4 and 5 of FW.

We define the total isospin raising and lowering operators $T_\pm=\frac{1}{2}\sum_i \tau_x(i)\pm i\tau_y(i)$ in terms of the single particle isospin operator $\boldsymbol{\tau}(i)$ and use the commutation relation $\tau_\pm(i)=\mp\frac{1}{2}[T_\pm,\tau_z(i)]$ to replace the τ_- in the nuclear matrix element in I_n by τ_z, thus obtaining

$$I_n=\frac{1}{2}\int_{-1}^{1} y^n dy \sum_{\bar{a}b'} \frac{k(k_m^{ab}-k)^2}{m_\mu^3}\theta(k_m^{ab}-k)|\langle b'|\sum_i \tau_z(i)\exp(-i\mathbf{s}_{ab}\cdot\mathbf{r}_i)|a\rangle|^2$$
$$=2\pi\int_{-1}^{1} y^n dy \sum_{\bar{a}b'} \frac{k(k_m^{ab}-k)^2}{m_\mu^3}\theta(k_m^{ab}-k)\sum_{l,m}|\langle b'|\sum_i \tau_z(i) j_l(s_{ab}r_i) Y_{lm}(\Omega_r)|a\rangle|^2. \quad (11)$$

We have assumed that isospin is a good quantum number and that $T_-|a\rangle=0$. The index b' represents the $T_z=0$ component of the isospin triplet of which b is the $T_z=-1$ component. Thus, we are summing over $T=1$ states of the initial nucleus rather than the final one. The second formula is obtained by expanding the exponential in multipoles.[11] We now consider the dipole part of I_n, $I_n^{\rm Dip}$. FW have shown that the nuclear matrix element can be written to within a few percent as the product of an unretarded part obtained by the replacement $j_1(s_{ab}r_i)\to s_{ab}r_i/3$ times an elastic form factor $F_{\rm el}$. With this approximation we have

$$I_n^{\rm Dip}=\frac{1}{6}\int_{-1}^{1} y^n dy \sum_{\bar{a}b'} \frac{k(k_m^{ab}-k)^2}{m_\mu^3}\theta(k_m^{ab}-k) \times|F_{\rm el}|^2(s_{ab})^2|\langle b'|\sum_i \tau_z(i)\mathbf{r}(i)|a\rangle|^2, \quad (12)$$

where FW give

$$|F_{\rm el}|^2=\left[1-\tfrac{1}{4}\eta^2+\frac{\eta^4}{80}\right]^2 e^{-\frac{1}{2}\eta^2}. \quad (13)$$

The nuclear matrix element is now exactly that which appears in the expression for the cross section for photo-absorption by a nucleus in the unretarded dipole approximation,

$$\sigma_\gamma(E)=\tfrac{1}{3}\pi^2\alpha\sum_{\bar{a}b'}(E_{b'}-E_a) \times|\langle b'|\sum_i \tau_z(i)\mathbf{r}(i)|a\rangle|^2\delta(E-E_{b'}+E_a). \quad (14)$$

[11] Notation is that of A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1957).

Thus we have

$$I_n^{\mathrm{Dip}}=\int_{-1}^{1} y^n dy\frac{1}{2\pi^2\alpha}\int_0^{E_m} dE\left\{\frac{\sigma_\gamma(E)}{E}\frac{k(E_m-E-k)^2}{m_\mu^3}\times\theta(E_m-E-k)\,|F_{\mathrm{el}}(k_m^{ab}=E_m-E)|^2\times[(E_m-E)^2-2k(E_m-E-k)(1-y)]\right\}, \quad (15)$$

where $E_m=k_m^{ab}+(E_{b'}-E_a)$ is the maximum photon energy for a radiative muon capture between states of an isotopic multiplet, i.e., $|b'\rangle\rightarrow|b\rangle$. FW give $E_m=110$ MeV for Ca^{40}. Since $\sigma_\gamma(E)$ is dominated by the giant dipole resonance at $E=19.6$ MeV[12] we take $|F_{\mathrm{el}}|^2$ out of the integral on E and evaluate it at $E=E_{\mathrm{res}}$. This approximation changes the final results by less than 2%. Therefore, we can write the dipole contribution to I_n as

$$I_n^{\mathrm{Dip}}=\int_{-1}^{1} y^n dy|F_{\mathrm{el}}|^2\{Q_1(k)+Q_2(k)(1-y)\}, \quad (16)$$

where

$$Q_1(k)=\frac{k}{2\pi^2\alpha m_\mu^3}\int_0^{E_m} dE\frac{\sigma_\gamma(E)}{E}\theta(E_m-E-k)\times(E_m-E-k)^2(E_m-E)^2, \quad (17)$$

$$Q_2(k)=\frac{-k^2}{\pi^2\alpha m_\mu^3}\int_0^{E_m} dE\frac{\sigma_\gamma(E)}{E}\theta(E_m-E-k)\times(E_m-E-k)^3. \quad (18)$$

To evaluate Q_1 and Q_2 we used the photoneutron cross section data of Baglin *et al.*[12] which extend up to 30 MeV. Available evidence[13,14] seems to indicate that the photoproton cross section has about the same shape as the photoneutron cross section, so to obtain the total cross section we assumed that the shapes were the same and scaled $\sigma_{\gamma n}(E)$ so as to give the correct integrated cross section for $\sigma_{\gamma p}$, as given by Hayward.[14] The greatest uncertainty in the cross section data is the contribution in the region $E>30$ MeV, but fortunately Q_1 and Q_2 are not particularly sensitive to this uncertainty. In fact for each fixed value of k there is a definite limit, given by the step function, to the values of E which contribute to the integrals for Q_1 and Q_2. Physically, this limit comes from the fact that conservation of energy fixes the maximum amount of energy which can be transferred to the residual nucleus. Thus for example, for $k\geq 80$ MeV only those values of $E\leq 30$ MeV will contribute and we can calculate the integrals for Q_1 and Q_2 exactly with the available data for $\sigma_\gamma(E)$. Even for $k<80$ MeV, the $1/E$ and other factors in the integrands decrease fairly rapidly with increasing E, and thus make the results for Q_1 and Q_2 much less dependent on possible high-energy contributions to $\sigma_\gamma(E)$ than would be, say, the integrated cross section. The final results for $Q_1(k)$ and $Q_2(k)$ are plotted in Fig. 2.

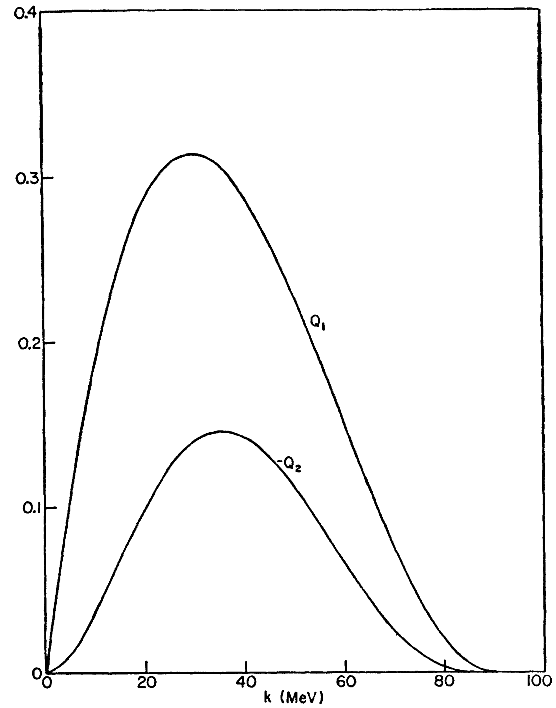

FIG. 2. The integrals Q_1 and Q_2 over the experimental photo-absorption cross section for Ca^{40}, as defined in Eqs. (17) and (18).

Thus we obtain the dipole contribution to the radiative capture rate from Eqs. (16), (3), (4), and (A1) and (A2) of the Appendix with Q_1 and Q_2 from Fig. 2 and with k_m in $|F_{\mathrm{el}}|^2$ and G_r^2 evaluated at $k_m^{\mathrm{res}}=90.4$ MeV. We evaluate the contributions of other multipoles in the closure–harmonic-oscillator model, as they are now hopefully corrections to the leading dipole term. Consequently, to obtain the part of I_n resulting from other multipoles, i.e., I_n^{OM}, we simply replace M_r^2 in Eq. (9) by $M_r^2-(M_r^2)_D$, where $(M_r^2)_D$ is the dipole contribution to M_r^2 calculated in the closure–harmonic-oscillator model, which for Ca^{40} is [5]

$$(M_r^2)_D=10\eta^2\left[1-\tfrac{1}{4}\eta^2+\frac{\eta^4}{80}\right]^2 e^{-\frac{1}{2}\eta^2}. \quad (19)$$

The velocity terms, which are small corrections to the other terms, are evaluated exactly as in the closure–harmonic-oscillator model, i.e., from Eq. (5). In accordance with the closure approximation, we evaluate k_m for the velocity and other multipole terms at some average value. This method of evaluating the radiative capture rate we call the giant dipole resonance or GDR model.

As suggested by RT it might be possible to eliminate

[12] J. E. E. Baglin and B. M. Spicer, Nucl. Phys. 54, 549 (1964).
[13] J. C. Hafele, F. W. Bingham, and J. S. Allen, Phys. Rev. 135, B365 (1964).
[14] Evans Hayward, Rev. Mod. Phys. 35, 324 (1963).

some of the dependence on the nuclear model by considering the ratio of radiative to ordinary muon capture. Thus we give the results for the ordinary capture rate in the two models considered. From LRT the ordinary muon capture rate is

$$\Lambda_{\mu c}=(m_\mu^2/2\pi)|\varphi_\mu|^2\{[G_F^2+3G_{GT}^2]M^2 + (\nu^3_{av}/mm_\mu^2)[G_Vg_V+g_A(G_A-G_P)]Q\}, \quad (20)$$

where G_F, G_{GT}, G_V, G_A, G_P are conventional combinations of the coupling constants[15] which depend on the average neutrino energy ν_{av}, which we took as 85 MeV.[3,5] The induced pseudoscalar term $g_P(q^2)$ which appears in G_{GT} and G_P, was evaluated according to Eq. (2) for $q^2=m_\mu^2-2m_\mu\nu_{av}$. The nuclear matrix elements are given by the dimensionless numbers M^2 (main terms) and Q (velocity terms) which for Ca^{40} in the closure–harmonic-oscillator model are, from LRT, $Q=14.54$ and $M^2=3.54$.

In the model of FW we write $M^2=|F_{el}|^2(M_V^2)_{UD}+(M_V^2)_{OM}$. The contribution of other multipoles evaluated in the closure–harmonic-oscillator model is $(M_V^2)_{OM}=1.67$.[5,16] We evaluate the unretarded dipole contribution in the same way as Q_1 and Q_2 and find $(M_V^2)_{UD}=2.52$ which, with $|F_{el}|^2=0.408$, gives $M^2=2.70$. Our value of $(M_V^2)_{UD}$ is a few percent lower than that obtained by FW because we have used somewhat more recent data for $\sigma_\gamma(E)$ which were taken with high enough resolution to pick out some of the structure in the resonance peak.

Before proceeding to the results for the specific case Ca^{40}, we should point out that the general techniques described above are applicable not only to Ca^{40} but also to other nuclei which have $T_-|a\rangle=0$, e.g., C^{12}, O^{16}, and He^4, and for which Eqs. (6) and (7) and the analogous equations leading to Eq. (5) can be assumed valid. In fact, we might expect the giant-dipole-resonance model to be even better for some of these lighter nuclei than it is for Ca^{40} because the dipole contribution accounts for a larger proportion of the capture rate in the lighter nuclei than in Ca^{40}, thus decreasing the importance of the less-well-known contributions of the other multipoles.

IV. RESULTS

We have calculated the radiative capture rate in Ca^{40}, evaluating the dipole part from the giant dipole resonance according to the formulas above, for various values of the coupling constants and of k_m, the average value of the maximum photon energy. In general our results are qualitatively the same as those obtained by RT in the closure–harmonic-oscillator model, namely we obtain a spectrum of the general shape of curve (III) of Fig. 3. We find the radiative rate to be quite sensitive to the induced pseudoscalar coupling constant and somewhat less so to the second-class induced tensor coupling. An increase in either g_P or g_T increases the magnitude of the radiative rate. The result is also sensitive to the value taken for k_m, because of the factor $(k_m-k)^2$, but less so in the giant dipole resonance calculation than in the closure–harmonic-oscillator calculation, because in the former the factor appears only in the velocity and other multipole terms which together amount to only about half of the total rate.[17]

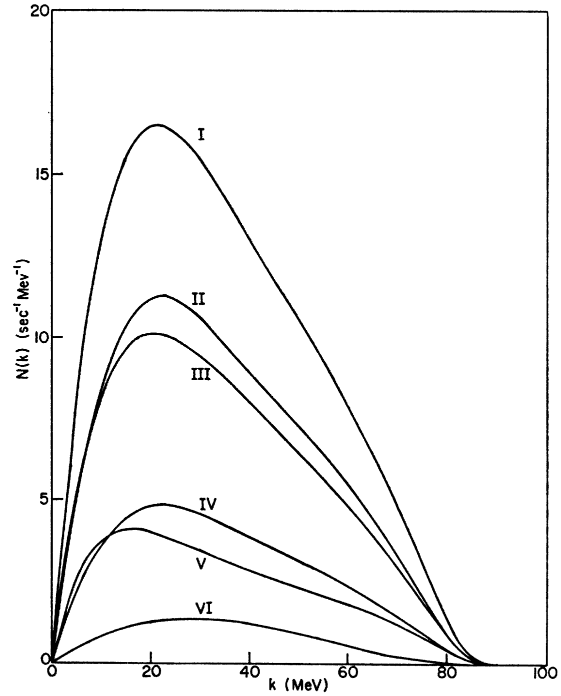

FIG. 3. Radiative capture rate $N(k)$ in Ca^{40} for $k_m=88$ MeV and $g_V=1.0$, $g_M=3.71$, $g_A=-1.18$, $g_P=7g_A$, $g_T=0$, $g_S=0$. (I) Total rate in closure–harmonic-oscillator model. (II) Dipole contribution to I. (III) Total rate in giant-dipole-resonance model. (IV) Dipole contribution to III. (V) Contribution of other multipoles to I and III. (VI) Contribution of velocity terms to I and III.

To illustrate the details of the differences between the two calculations we examine the results obtained with a particular choice of coupling constants, namely

[15] Reference 3, p. 239. As mentioned in an errata statement, $-g_T$ should be replaced by $+g_T$ in G_P.

[16] FW actually used 1.41 which is the average of this and the result calculated by summing over various partial transitions. This gives a somewhat better result for the ordinary capture rate than does 1.67. However, as we hope that some of the model dependence will cancel in the ratio of radiative to ordinary capture, we choose the closure result so as to have exactly the same model for ordinary capture as we use for radiative capture.

[17] Actually k_m has a somewhat different meaning in the GDR calculation than in the closure–harmonic-oscillator calculation. In the latter k_m corresponds to the maximum photon energy averaged over all transitions while in the former, if we neglect velocity terms, the average is only over other multipole transitions. Thus to obtain the quantity in the GDR model directly comparable to k_m of the closure calculation one must take an appropriately weighted average of k_m and the k_m^{res} which corresponds to the average maximum photon energy for the dipole transitions. For the most part we will neglect this distinction and simply give results in the GDR model for several values of k_m.

the "standard" set $g_V=1.0$ and $g_M=3.71$ from the conserved-vector-current theory, $g_A=-1.18$ from neutron beta decay,[18] $g_P=7g_A$ from the Goldberger-Treiman argument[4] and $g_T=g_S=0$ corresponding to the assumption that the weak Hamiltonian has definite transformation properties under G parity. For purpose of example, we also assume $k_m=88$ MeV for Ca^{40} in accordance with the result obtained by Conversi *et al.*,[1] based on the closure-harmonic-oscillator model.

Figure 3 shows the absolute radiative muon capture rate calculated with the above choices of coupling constants. A comparison of curve (I) calculated in the closure-harmonic-oscillator model and curve (III) calculated in the giant-dipole-resonance model shows two important differences. In the first place $N(k)_{\text{GDR}}/N(k)_{\text{Closure}}=0.6$ to within a few percent over the whole spectrum up to $k=80$ MeV. Thus, we have effectively reduced the result obtained in the closure calculation by about 40%. This means that to fit a given set of experimental data one now needs a larger value of g_P or a much larger value of g_T than was necessary in the closure–harmonic-oscillator model. Secondly, we observe that we obtain a high-energy tail in the GDR model which was not present in the closure calculation. We can understand this in the following way: In the closure approximation we assume an average value for k_m; then the factor $(k_m-k)^2\theta(k_m-k)$ cuts off the spectrum at $k=k_m$. Consequently, the very high-energy photons from those transitions which have a maximum photon energy somewhat larger than the average value or equivalently those photons which leave the residual nucleus with somewhat lower than average excitation energy are artificially eliminated from the result by the closure approximation. On the other hand, in the GDR model, for the dipole part of the matrix element, it is this part of the spectrum which we know most accurately because it comes from the lower energy parts of $\sigma_\gamma(E)$ which we know quite well. In fact as mentioned earlier, knowledge of $\sigma_\gamma(E)$ up to 30 MeV allows us to obtain the dipole part of the matrix element for $k\geq 80$ MeV exactly, whether or not there are higher energy contributions to $\sigma_\gamma(E)$. Unfortunately, however, we still must rely on the closure approximation to calculate the other multipole and velocity terms. Thus, we have only partially remedied the deficiencies of the straight closure-harmonic-oscillator calculation at the high-energy end of the spectrum. However, it appears that, at least as long as the average and resonance values of k_m are not too far separated, the high-energy tail is so small as to be of interest more as a matter of principle than of practice.

The results in Fig. 3 also show the relative importance of the various contributions to the rate. The dipole part accounts for about 50% of the total rate in the GDR calculation over the whole range of k up to 80 MeV; the other multipole and velocity terms account for about 35–45% and 5–15%, respectively. It is interesting to note that this indicates that the giant-dipole-resonance calculation is better for radiative capture than for ordinary capture since for ordinary capture in Ca^{40} the dipole term amounts to only 34% (with the same coupling constants as above) as opposed to 50%. This probably just reflects the fact that even in the closure-harmonic-oscillator calculation the dipole term makes a larger contribution to radiative capture than to ordinary capture.

Finally, for the total radiative capture rate, obtained by integrating the spectrum, we find, for this standard set of coupling constants, the values 519 $\sec^{-1}$ in the GDR calculation and 841 $\sec^{-1}$ in the closure–harmonic-oscillator calculation. The ordinary capture rates come out $\Lambda_{\mu c}{}^{\text{GDR}}=30.4\times10^5\ \sec^{-1}$ and $\Lambda_{\mu c}{}^{\text{Closure}}=38.9\times10^5\ \sec^{-1}$ compared with the experimental value[10] $\Lambda_{\mu c}{}^{\text{Exp}}=(25.5\pm0.5)\times10^5\ \sec^{-1}$. One can get even better agreement of $\Lambda_{\mu c}{}^{\text{GDR}}$ with experiment by taking, as did FW, the other multipole contribution to be the average of the results of closure and summation over partial transition calculations. This procedure gives $\Lambda_{\mu c}=27.8\times10^5\ \sec^{-1}$, which is essentially the result of FW.

We now consider the relative spectrum for radiative muon capture, i.e., the quantity $R(k)\equiv N(k)/\Lambda_{\mu c}$. We might expect, since the nuclear matrix elements appearing in $N(k)$ and $\Lambda_{\mu c}$ are quite similar, that much of the dependence on the nuclear model would cancel in the ratio. One can see from the results for the relative spectra, which are plotted in Fig. 4, that to a certain extent this is true; the ratio $R(k)_{\text{GDR}}/R(k)_{\text{Closure}}=0.8$ to within a few percent over the entire range of k up to 80 MeV. Thus, the effect of the GDR calculation is to reduce the relative spectrum obtained in the closure approximation by about 20%, compared to a reduction of about 40% in the absolute spectrum.

We can perhaps better understand the important features of the GDR calculation by examining the following simple calculation. We begin by observing from Fig. 3 that in the closure–harmonic-oscillator model the ratios of the dipole, other multipole, and velocity contributions to the total rate are approximately constant. The averages of these ratios at 10-MeV intervals over the range $k=10$ to $k=80$ MeV are, respectively, 0.679, 0.250, and 0.071 and the variations from these averages are typically less than about 0.03. This indicates that the shape of the spectrum obtained from the various contributions is about the same, and so we write $N(k)_{\text{Closure}}=N(k)_{\text{Closure}}{}^{\text{Dip}}+N(k)_{\text{Closure}}{}^{\text{OM}}+N(k)_{\text{Closure}}{}^{\text{Vel}}=(0.679+0.250+0.071)N(k)_{\text{Closure}}$. We further observe that the ratio of the dipole contribution in the GDR calculation to the dipole contribution in the closure-harmonic-oscillator model is also approximately constant, the average value being 0.447 with variations from the average being less than 0.02 except near $k=80$ MeV. Thus, if we simply scale the

[18] C. P. Bhalla, Phys. Letters **19**, 691 (1966).

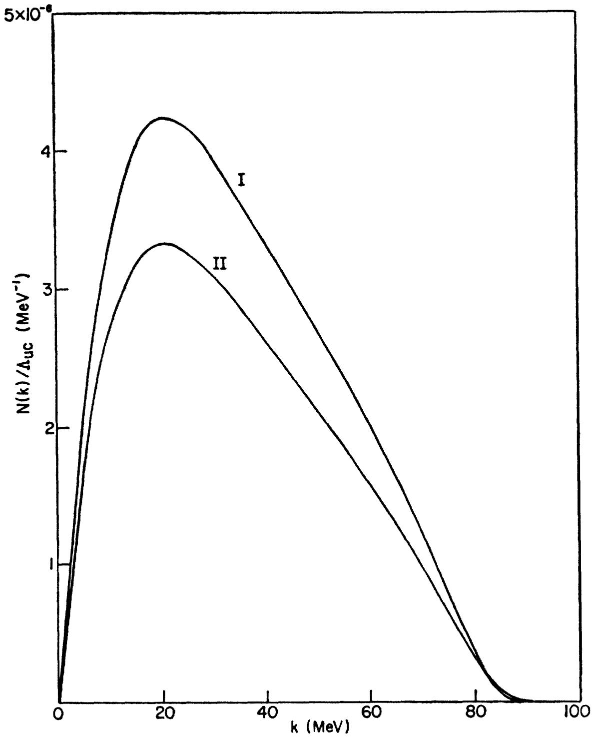

FIG. 4. Relative radiative muon capture rate $N(k)/\Lambda_{\mu c}$ in Ca^{40} for $k_m=88$ MeV and $g_V=1.0$, $g_M=3.71$, $g_A=-1.18$, $g_T=0$, $g_S=0$. (I) Rate in closure-harmonic-oscillator model for $g_P=7g_A$. (II) Rate in giant-dipole-resonance model for $g_P=7g_A$.

dipole contribution to $N(k)_{\text{Closure}}$ by this factor we obtain $N(k)_{\text{GDR}}\cong 0.625N(k)_{\text{Closure}}$. Similarly, for ordinary capture using the values of the matrix elements given in the text after Eq. (20), we have $\Lambda_{\mu c}{}^{\text{Closure}}=(0.487+0.435+0.078)\Lambda_{\mu c}{}^{\text{Closure}}$. The scale factor for the dipole part is 0.551 which leads to $\Lambda_{\mu c}{}^{\text{GDR}}=0.781\times\Lambda_{\mu c}{}^{\text{Closure}}$. Taking the ratio we have $R(k)_{\text{GDR}}/R(k)_{\text{Closure}}=0.800$. The average value of this ratio obtained from the complete calculation is 0.800 with variations from the average of less than 0.02 except near $k=80$ MeV (where the effect of the high-energy tail obtained in the GDR calculation is beginning to be noticeable). Such excellent agreement is of course accidental but it does emphasize that the major feature of the GDR approach is that it provides a way, through appeal to experiment, of correctly scaling the dipole contributions to radiative and ordinary capture rates.

This simple calculation also suggests that even without a detailed theory of the other multipole and velocity contributions one could improve the GDR calculation by finding a way of correctly scaling these contributions. In their calculation of ordinary capture, FW argued that the correct value for the contribution of other multipoles was somewhere between that obtained in the closure approximation and that obtained in the summation over partial transitions. They therefore took an average value for this contribution, which corresponds to scaling the closure approximation result which we used by 0.844. As a rough estimate of the proper scale factor for the multipole term in radiative capture, one could take this same factor, or alternatively, a factor in the same ratio to this as the ratio of the scale factors for the dipole contributions. These estimates lead to $R(k)_{\text{GDR}}/R(k)_{\text{Closure}}=0.82$ and 0.77, respectively, which perhaps gives an indication of the kind of results one might expect from a correct calculation. We note however that our main conclusion remains unchanged, i.e., that the GDR calculation gives a lower relative rate than does the closure–harmonic-oscillator model and thus, other parameters being equal, would require a larger value of g_P to fit a given set of data.

We are now in a position to examine more carefully the extent to which the calculated value of the relative capture rate depends on the specific nuclear model used. We observed above that the ratios used in our simple calculation were very nearly constant over most of the spectrum. This implies that the shapes of the various contributions to the capture rate, as calculated both in the GDR and the closure-harmonic-oscillator model, are all approximately the same and thus apparently the detailed dependence of the nuclear matrix elements on $(sb)^2$ and $(\nu b)^2$, which varies from model to model, is not so important. This conclusion is

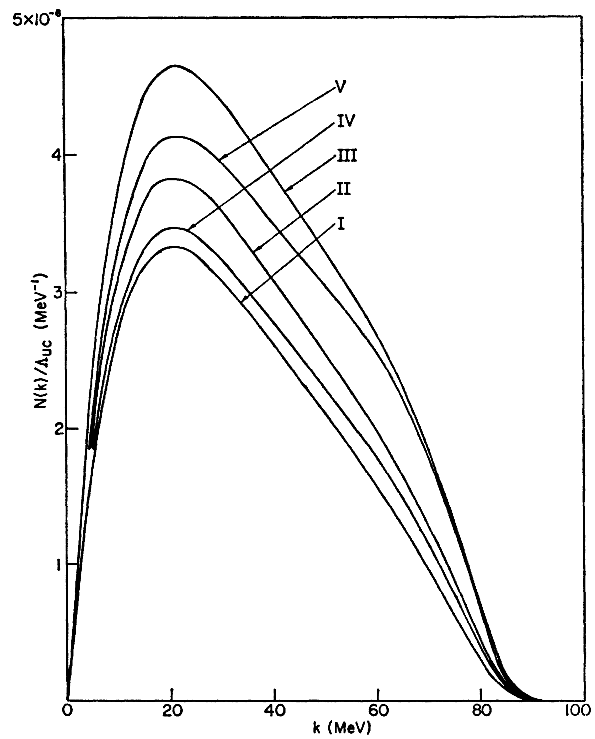

FIG. 5. Relative radiative muon capture rate $N(k)/\Lambda_{\mu c}$ in $Ca^4{}_0$ calculated in the giant-dipole-resonance model for various choices of g_P and g_T and with $k_m=88$ MeV, $g_V=1.0$, $g_M=3.71$, $g_A=-1.18$, and $g_S=0$. (I) $g_P=7g_A$, $g_T=0$. (II) $g_P=10g_A$, $g_T=0$. (III) $g_P=14g_A$, $g_T=0$. (IV) $g_P=7g_A$, $g_T=10g_V$. (V) $g_P=7g_A$, $g_T=35g_V$.

essentially that of RT, as they compared the relative capture rates calculated in several different models, all of which used the closure approximation and which differed mainly in the second and higher order terms in $(sb)^2$ and $(\nu b)^2$ in the expressions for the nuclear matrix elements, and found for Ca^{40} differences of less than 10% in the results. In our calculation, however, there is an additional important feature, i.e., we are able to scale the dipole part of the nuclear matrix elements for radiative capture and, separately, for ordinary capture. These scale factors are not the same because the factors multiplying $\sigma_\gamma(E)$ in Eqs. (17) and (18) are different from those in the analogous equation for ordinary capture. Also the dipole part makes a larger contribution to radiative capture than to ordinary capture.[19] As a result of these two effects, the radiative capture rate is reduced more than the ordinary rate, and so we obtain a relative rate in the GDR model which is some 20% lower than that obtained in the closure-harmonic-oscillator model.

We now illustrate in Figs. 5 and 6 some typical results obtained in the giant-dipole-resonance model with values of the coupling constants and of k_m other than the standard ones of the previous example. Figure 5 shows the effect of varying g_P and g_T. We see that the relative radiative capture rate is quite sensitive to g_P and somewhat less so to g_T[20] and increases as either is increased. Figure 6 shows results obtained with the standard coupling constants for several values of k_m. All of these results are in qualitative agreement with those obtained in the closure–harmonic-oscillator model. Again, as in the example, the most important effect of the GDR calculation is to decrease the closure result for the relative capture rate by a factor which apparently remans fairly constant at about 20–25% for moderate variations of the coupling constants from the standard values. Finally, we observe that, if g_T is actually fairly small, as evidence from ordinary muon capture in H, He^3, and C^{12} and perhaps from the difference in ft values of B^{12} and N^{12} seems to indicate,[21–24] then it will be extremely difficult to obtain information about it from radiative muon capture without a very precise knowledge of both g_P and k_m.

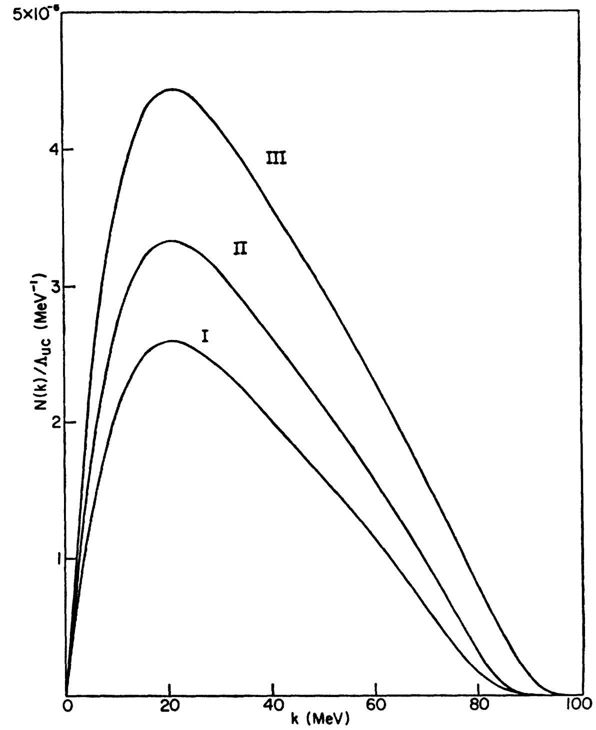

FIG. 6. Relative radiative muon capture rate $N(k)/\Lambda_{\mu c}$ in Ca^{40} calculated in the giant-dipole-resonance model for various values of k_m and with $g_V=1.0$, $g_M=3.71$, $g_A=-1.18$, $g_P=7g_A$, $g_T=0$, and $g_S=0$. (I) $k_m=80$ MeV. (II) $k_m=88$ MeV. (III) $k_m=96$ MeV.

V. COMPARISON WITH EXPERIMENT

Next we examine the experimental results of Conversi *et al.*[1] for radiative muon capture in Ca^{40} in light of our theoretical calculations based on the giant dipole resonance model. Ideally, one would hope to compare directly the measured points on the photon spectrum with the theoretical spectrum. Unfortunately, such a clear separation of the experiment and theory does not seem to be possible, as the quantity one measures is not the actual photon spectrum, but a spectrum which has been somewhat distorted by effects of the finite resolution of the counters used. Consequently, one must numerically fold the experimental counter resolution, with its associated uncertainties, into the theoretical spectrum and then compare the results with the measured number of photons. For the experiment of Ref. 1, the counter resolution function was relatively broad and quite asymmetric and had the effect of decreasing the high-energy part of the spectrum by amounts ranging from 10 to 35%.[25] Rather than carry out such an elaborate program, we use the following rather approximate method which we hope will be sufficient to

[19] As an estimate of the maximum angular momentum which contributes, we have νR for ordinary muon capture and sR for radiative muon capture, where R is a measure of the size of the nucleus in which the capture takes place. For ordinary capture, the value of ν for each particular transition is fixed by energy conservation. As noted before, $\nu_{av}\cong 85$ MeV. However, in radiative capture, since we are dealing with a three-body final state, s is not fixed but is a function of k and y which has a maximum value $k_m\cong\nu_{av}$ at the endpoints of the spectrum and which may go to zero near the center. Therefore, over most of the spectrum $sR<\nu R$ and so we expect that the higher multipole terms should be less important and the dipole terms more important for radiative capture than for ordinary capture. This is the result we obtain from a complete calculation.

[20] This conclusion, of course, depends to a large extent on our assumption that the q^2 dependence of g_T can be neglected while it is quite important for g_P, giving an enhancement of a factor of 3 near the high-energy end of the spectrum.

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[23] Hisao Ohtsubo and Akihiko Fujii, Nuovo Cimento 42, 109 (1966).

[24] A. Bietti, Nuovo Cimento 37, 337 (1965).

[25] R. Diebold (private communication).

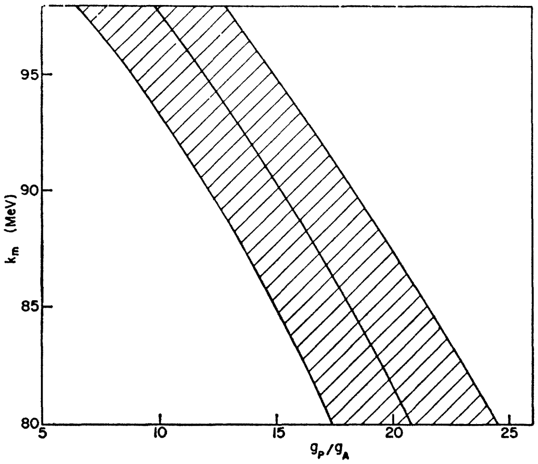

FIG. 7. Region in $k_m - g_P$ plane consistent with experiment of Conversi *et al.*, based on approximate comparison of results of giant-dipole-resonance model and experiment. The other coupling constants were taken as $g_V=1.0$, $g_M=3.71$, $g_A=-1.18$, $g_T=0$, $g_S=0$.

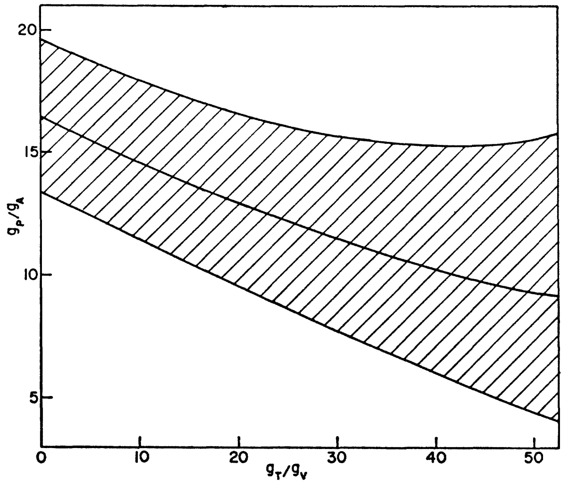

FIG. 8. Region in $g_P - g_T$ plane consistent with experiment of Conversi *et al.*, based on approximate comparison of the results of giant-dipole-resonance model and experiment. The other coupling constants were taken as $g_V=1.0$, $g_M=3.71$, $g_A=-1.18$, and $g_S=0$; k_m was assumed to be 88 MeV.

give at least a general understanding of the effect of the present theoretical calculation on the interpretation of the experimental results. Using the formulas of RT with $g_P=13.3g_A$ and $k_m=88$ MeV, we reconstruct the relative spectrum which, according to Ref. 1, gave the best fit to the experimental points. We then compare the results in the GDR calculation to this spectrum over the range of photon energies 60–80 MeV, which includes most of the experimental range, but avoids points near the very high-energy end of the spectrum, where the high-energy tail obtained in the GDR calculation might affect the results. Since the shape of the spectrum calculated in the two models is about the same, and since the main effect of varying g_P is to increase or decrease the magnitude of the spectrum, we would expect this method of comparison to give at least an approximately correct value of g_P. We, in fact, found that, for a fixed value of k_m, one could find a well-defined value of g_P such that the GDR spectrum and the "experimental spectrum" agreed in most cases to better than 3% over the range 60-80 MeV. Since the fits were, for the most part, equally good over the whole range of k_m from 80 to about 95 MeV, we were unable to find a best value for k_m. This may be in part due to the approximate nature of our comparison procedure and in part due to the fact that the results are not as sensitive to k_m in the GDR calculation as they are in the closure-harmonic oscillator calculation.

In Fig. 7 we show the region in the $k_m - g_P$ plane consistent with experiment, as interpreted in the giant-dipole-resonance model, where we have assumed the standard values for g_V, g_M, and g_A and that $g_T=g_S=0$. The error limits are taken from Ref. 1. For the choice $k_m=88$ MeV, we find $g_P=(16.5\pm3.1)g_A$. Alternatively, we may allow g_T to be nonzero. Figure 8 shows the region in the $g_P - g_T$ plane consistent with experiment under this assumption, where again we have chosen $k_m=88$ MeV and standard values for the other coupling constants. We see that, if we take $g_P \cong 7g_A$, then g_T must be $\gtrsim 35g_V$, which is in rather serious disagreement with the evidence mentioned above, all of which indicates that g_T is small. Finally, it is also possible to obtain a fair fit to experiment by taking $g_P=7g_A$, $g_T=0$, and $g_S=-1.5g_V$, with $k_m=88$ MeV and standard values for g_V, g_M, and g_A. We should re-emphasize that all of the conclusions in this paragraph are based on our approximate method of comparing theory and experiment and therefore should be interpreted with some caution.

ACKNOWLEDGMENTS

The author would like to thank Professor R. J. Oakes and Professor J. D. Walecka for their guidance and encouragement and for reading the manuscript. He is also grateful to Dr. R. Diebold for several discussions of the details of the radiative muon capture experiment of Ref. 1. Computational assistance from the National Science Foundation under grant number NSF-GP948 is also gratefully acknowledged.

APPENDIX

Assuming, as mentioned in the text, that the $k_m{}^{ab}$ dependence of $(G_r{}^2)_{ab}$ can be factored out, we obtain

$$
\begin{aligned}
I_r^2(k,\lambda)=&(g_2^2+2g_3^2+g_{14}^2)I_0+\lambda(g_3^2+g_{14}^2-2g_2g_3)I_1\\
&+\kappa(I_2-I_0)\{(1-x)[g_2(g_{11}+g_9)+g_3(g_7-2g_9-g_{11})+g_{14}(g_6+\lambda g_5)]\\
&+(\gamma+\delta x(1-x))[(1-x)(g_2g_{12}-g_3g_{12}-g_2g_{13})-\lambda x g_3g_{13}]\}\\
&+\kappa(I_1-I_3)(1-x)[\delta x(1-x)(g_2g_{12}-g_3g_{12}-g_2g_{13})-\lambda\delta x^2g_3g_{13}]\\
&-2\kappa[xI_0+(1-x)I_1][\lambda g_7g_2+g_3g_9+g_3g_{11}-(\gamma+\delta x(1-x))g_3g_{12}]\\
&+2\kappa[(1-x)I_0+xI_1][g_3g_7+g_2g_9-g_3g_{11}-(\gamma+\delta x(1-x))g_3g_{12}]\\
&+2\kappa(I_1-I_2)\delta x(1-x)(1-2x)g_3g_{12}+2\kappa(I_1+\lambda I_2)(x-1)g_5g_{14}\\
&+\kappa^2[I_3-I_1-2x(1-x)(I_3-I_4-I_1+I_2)][(\tfrac{1}{2}g_{13}^2-g_{12}g_{13})(2\gamma\delta x(1-x))-\delta x(1-x)g_{11}g_{13}]\\
&+\kappa^2[I_0-I_2-2x(1-x)(I_0-I_1-I_2+I_3)][(\tfrac{1}{2}g_{13}^2-g_{12}g_{13})(\gamma^2+2\gamma\delta x(1-x))\\
&-(\gamma+\delta x(1-x))g_{11}g_{13}]+\kappa^2[I_2-2x(1-x)(I_2-I_3)][2\gamma\delta x(1-x)g_{12}^2]\\
&+\kappa^2[I_0-2x(1-x)(I_0-I_1)][g_{11}^2+g_{12}^2(\gamma^2+2\gamma\delta x(1-x))]+\kappa^2(1-x)^2(I_2+\lambda I_1)g_5^2\\
&+\kappa^2[I_1-2x(1-x)(I_1-I_2)][g_{11}^2-g_{12}^2(\gamma^2+4\gamma\delta x(1-x))]+\kappa^2(1-x)^2(I_1-I_3)g_5g_6\\
&+\kappa^2\tfrac{1}{2}g_7^2[(1-2x+3x^2)I_0+4x(1-x)I_1+(1-x)^2I_2]\\
&+\kappa^2g_7g_9[x(x-1)I_0+2(-2x^2+2x-1)I_1-3x(1-x)I_2]\\
&+\kappa^2\tfrac{1}{2}g_9^2[(3-8x+7x^2)I_0+4x(1-x)I_1+(3x-1)(1-x)I_2],\qquad\text{(A1)}
\end{aligned}
$$

and in the closure approximation

$$
\begin{aligned}
I_{\text{vel}}^2(k,\lambda)=&-2\lambda_+g_{14}g_V\kappa(2\rho-I_0-I_1)-\lambda_+g_2g_A\kappa(1-x)(\tfrac{4}{3}\rho-I_0+I_2)\\
&-2\lambda_+g_3g_A\kappa[2\rho-\tfrac{2}{3}(1-x)\rho-I_0-I_1+\tfrac{1}{2}(1-x)(I_0-I_2)]\\
&+g_3g_P{}^N\varphi\kappa\lambda((8/3)\rho-I_0-I_2)+2\lambda_+\kappa^2(1-x)g_5g_V(\tfrac{2}{3}\rho-I_1-I_2),\qquad\text{(A2)}
\end{aligned}
$$

where

$$
\begin{aligned}
g_2&=-g_A(\lambda_++\varphi)-g_P{}^N\varphi+(g_V+g_S)\lambda\varphi\mu+g_T(\lambda_+\kappa x+\lambda_-\varphi),\\
g_3&=-g_A(\lambda_++\lambda\varphi)+(g_V+g_M)\varphi+g_T\lambda_+\varphi(\kappa x-\varphi),\\
g_5&=g_V\lambda_++g_M\lambda\varphi\mu,\\
g_6&=g_V\lambda_++(g_P{}^N-g_T)\lambda\varphi\mu-g_S\varphi,\\
g_7&=(g_V+g_M)\lambda_++\varphi(\lambda g_P{}^N-g_S),\\
g_9&=(g_V+g_M)\lambda_+,\\
g_{11}&=(g_A+g_T)\lambda_+,\\
g_{12}&=g_P\lambda_-,\\
g_{13}&=g_P{}^N\beta(1-x),\\
g_{14}&=-g_V[\lambda_+(1+\kappa x)+\varphi]+g_S\lambda_-+g_A\lambda\varphi\mu+2g_T\lambda\varphi\mu(\varphi-\kappa(1-x)),
\end{aligned}
$$

$$\kappa=k_m/2m,\quad \varphi=m_\mu/2m,\quad \mu=4.71,\quad \lambda_\pm=\tfrac{1}{2}(1\pm\lambda),$$

$$x=k/k_m,\quad \rho=Zk(k_m-k)^2/m_\mu{}^3,$$

$$g_P{}^N=g_P\frac{m_\pi{}^2+m_\mu{}^2}{m_\pi{}^2-m_\mu{}^2+2m_\mu k_m(1-x)}\equiv g_P\frac{1}{\alpha-\beta x},$$

$$g_P{}^L=g_P\frac{m_\pi{}^2+m_\mu{}^2}{m_\pi{}^2-m_\mu{}^2+2m_\mu k_m-2k_m{}^2x(1-x)(1-y)}\cong g_P[\gamma+\delta x(1-x)(1-y)],$$

$$\alpha=\frac{m_\pi{}^2-m_\mu{}^2+2m_\mu k_m}{m_\pi{}^2+m_\mu{}^2},\quad \beta=\frac{2m_\mu k_m}{m_\pi{}^2+m_\mu{}^2},\quad \gamma=\frac{1}{\alpha},$$

$$\delta=\frac{2k_m{}^2\gamma^2}{m_\pi{}^2+m_\mu{}^2}.$$

The I_n were evaluated from Eq. (8) for the appropriate nuclear model as detailed in the text. The maximum photon energy k_m was taken as an average value for the closure–harmonic-oscillator model and for the other multipole and velocity contributions to the giant-dipole-resonance model and as the resonance value for the dipole contribution to the giant-dipole-resonance calculation.

Equations (A1) and (A2) follow directly from the analogous Eqs. (A2) and (A3) of RT. We have redefined some of the combinations of coupling constants so that they are all independent of y and have absorbed the factor $k(k_m-k)^2\theta(k_m-k)/m_\mu{}^3$ and the integral on y into the formulas, which amounts to replacing y^n in the formulas of RT by I_n. In addition, we kept a few small terms in g_7 and g_9, which were apparently dropped by RT and an additional term in the expansion of $g_P{}^L$. In numerical work, we kept all the I_n as opposed to the approximation of RT: $I_3=I_4=0$, $I_2=\frac{1}{3}I_0$. These additional terms seem to have little or no effect on the final numerical results.

Our numerical calculations also differ somewhat from those of Ref. 1.[25] In particular, we have included the velocity terms, which were not included in the calculation of Ref. 1, and we used $\nu_{\text{av}}=85$ MeV as opposed to $\nu_{\text{av}}=(89/91)k_m$. We also evaluated g_P in the ordinary capture rate at a slightly different value of q^2 than used in Ref. 1. Together, these differences make our results for the relative capture rate in the closure–harmonic-oscillator model typically 2–5% larger than those of Ref. 1.