one sees by going to a neighboring energy where the boundary conditions are not the natural boundary conditions any more. There the Wigner-Eisenbud procedure works since the wave function spans the complete function space. The obtained one-level Breit-Wigner cross section can then be calculated at this energy, and one sees that one can go with the energy through the point of the natural boundary conditions without encountering any problems. This procedure works only as long as the cross section is describable by a one-level formula. In general at the point of the natural boundary conditions indeterminacies will show up. This case has been discussed in Sec. III at length.

Concluding, we believe that the proposed method is particularly suited to the actual computation of cross sections because it leads, without the consideration of intermediate auxiliary quantities, directly to the $S$ matrix. Also, the calculational effort needed for the solution of the nuclear problem seems to us to have been minimized.

## ACKNOWLEDGMENTS

We thank U. Fano for useful discussions and suggestions, and H. A. Weidenmüller for valuable comments.

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# Scattering of 40.5-MeV Alpha Particles by $C^{12}$, $C^{13}$, $N^{14}$, $N^{15}$, $O^{16}$, and $O^{18}$†

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Elastic and inelastic scattering of 40.5-MeV $\alpha$ particles from targets of $C^{12}$, $C^{13}$, $N^{14}$, $N^{15}$, $O^{16}$, and $O^{18}$ was studied. Angular distributions were measured for a large number of excited states. It was found that the shape of the angular distribution depends on the nature of the single-particle transition involved. Six examples of quadrupole transitions involving promotion of a $p_{3/2}$ nucleon to the $p_{1/2}$ shell were found. Although the cross sections varied over a tenfold range, the shapes of the angular distributions remained very similar. Six examples of the dipole transition $p_{1/2} \rightarrow 2s_{1/2}$ and seven examples of the octupole transition $p_{1/2} \rightarrow d_{5/2}$ were also observed. The dipole transitions gave angular distributions of a characteristic and unusual shape. Excitation of the $N^{14}$ levels at 9.41, 9.71, 10.22, and 10.55 MeV suggests that they are all $T=0$. The levels at 6.05, 6.70, 7.40, and 7.60 MeV were not observed; probably they do not exist. A weak level at 10.85 MeV and two strongly excited levels (or groups of levels) at 11.3 and 12.9 MeV were observed in $N^{14}$. The angular distribution of particles scattered from the 4.45-MeV level of $O^{18}$ suggests strongly that this level is 1− rather than 3+. Several unnatural parity states were observed, but no states known to have isotopic spin different from the ground state. The angular distributions for several scattered particle groups were compared with distorted-wave Born approximation calculations and very approximate reduced transition probabilities for excitation of the levels were obtained. For the quadrupole and octupole excitations the results are in reasonably good agreement with values measured by electromagnetic methods.

## I. INTRODUCTION

The inelastic scattering of $\alpha$ particles is a useful method for studying the surface shapes of medium-mass nuclei.[1] The levels most strongly excited are the 2+, 4+, and 3− collective states. It is therefore for the excitation of such levels that most of the angular distributions of scattered particles have been measured. The shapes of the angular distributions are determined by the angular momentum transfer $L$, while the absolute value of the differential cross section depends upon the collective strength of the level excited.

The light nuclei present many opportunities for studying inelastic scattering of $\alpha$ particles from targets for which the structure of the initial and final nuclear states is rather well understood. In many cases the transitions should be almost pure single-particle rather than collective. Large numbers of levels are sufficiently well separated in energy to permit resolution of the corresponding groups of scattered particles.

In the present survey experiment, elastic and inelastic

† This work was supported by the U. S. Atomic Energy Commission and the U. S. Navy, Bureau of Ships.
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scattering of 40.5-MeV $\alpha$ particles from targets of $C^{12}$, $C^{13}$, $N^{14}$, $N^{15}$, $O^{16}$, and $O^{18}$ was studied, using the Berkeley 224-cm (88-in.) spiral-ridge cyclotron. The energy was chosen because it is sufficiently high to avoid the worst complications of compound nuclear effects but sufficiently low to permit good energy resolution to be obtained with silicon detectors.

## II. EXPERIMENTAL METHOD

The elastic and inelastic scattering of 40.5±0.3-MeV helium ions from the Berkeley 224-cm (88-in.) spiral ridge cyclotron was studied with the equipment that has been previously described.[2] The $C^{12}$ target was a 1.61 mg/cm² film made by charring filter paper. Oxygen was removed by heating to about 1400°C *in vacuo*. The $C^{13}$ target was ethylene gas ($C_2H_4$) containing 28.9 at. % of $C^{13}$. Considerable decomposition of the ethylene occurred during the course of the measurements, even though the gas pressure in the target cell remained perfectly constant. The cross sections for $C^{13}$ were therefore obtained by reference to the $C^{12}$ peaks in the energy spectra.

The $N^{14}$, $N^{15}$, $O^{16}$, and $O^{18}$ targets were natural $N_2$ and $O_2$ gas (for $N^{14}$ and $O^{16}$) or separated isotopes of 99.9% isotopic purity ($N^{15}$ and $O^{18}$). The gases were contained in a 7.66-cm-diam cell with a window of Havar[3] foil 0.00025 cm thick. 1024 channels of a Nuclear Data ND-160 analyzer recorded the pulses from a lithium-drifted silicon detector 1.5 mm thick. The energy resolution ranged from 100 to 300 keV, depending on the scattering angle. It was limited by multiple scattering of $\alpha$ particles in the Havar windows and in the target gas. This angular dispersion produces a corresponding energy dispersion which is very serious when the energy of scattered particles is a steep function of scattering angle, as it was in the present work. Spectra taken at angles in the vicinity of 25 deg in the laboratory system are shown in Figs. 1, 2, and 3.

## III. RESULTS

### A. Energy Levels Observed

The energies of inelastic groups were measured[2] only with sufficient accuracy to permit identification of the corresponding excited states.

#### *1. Carbon-12*

Scattering of $\alpha$ particles by $C^{12}$ has been extensively studied.[4–10] The energy levels above about 8 MeV are obscured by a strong continuous spectrum due to four-body break-up, $C^{12}+\alpha\rightarrow 4\alpha$.[11] Except for the very broad 10.1-MeV (0+) level, all known levels up to 14 MeV were observed, including the 0+ level at 7.656 MeV and the unnatural-parity (1+) level at 12.71 MeV.

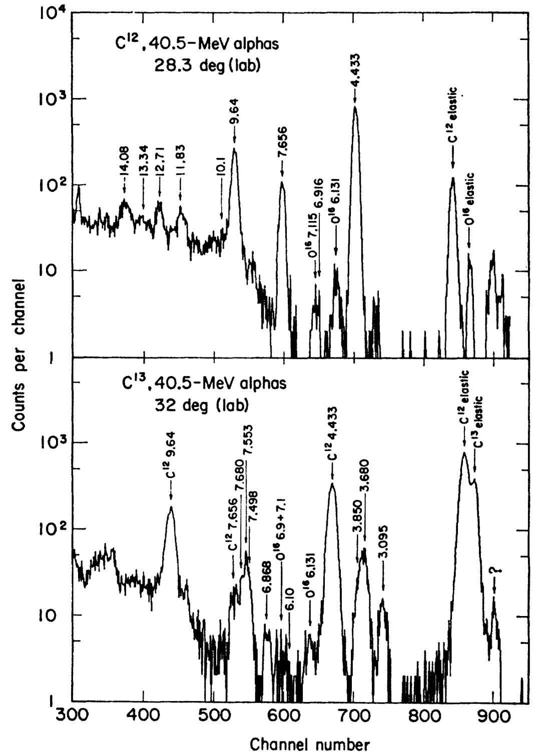


FIG. 1. Energy spectra of 40.5-MeV $\alpha$ particles scattered from $C^{12}$ (upper) and $C^{13}$ (lower).

#### *2. Carbon-13*

The scattering of $\alpha$ particles by $C^{13}$ was observed, but not studied, by Fulbright *et al.*[12] We observed excitation of levels only below 8 MeV since the continuum from the four-body break-up of $C^{12}$ was very large (70% of the target was $C^{12}$). No excitation was observed of the levels at 5.51 and 6.10 MeV reported in the reaction $B^{11}(He^3,p)C^{13}$.[13]

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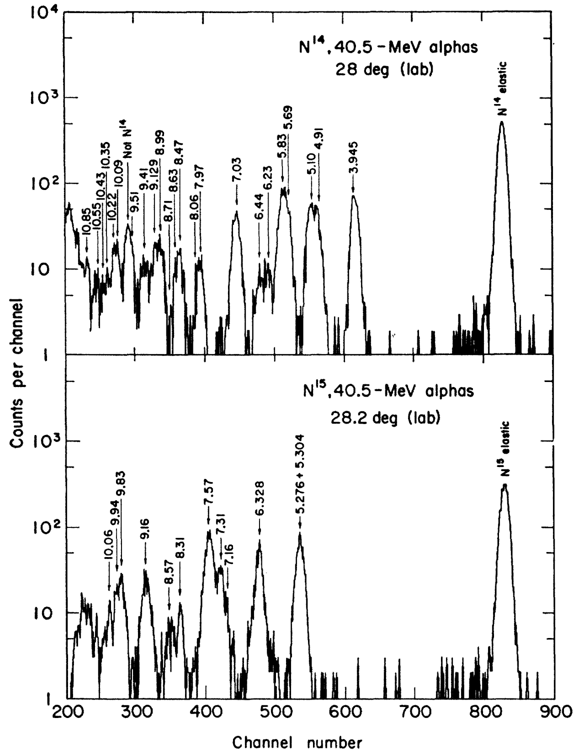


FIG. 2. Energy spectra of 40.5-MeV $\alpha$ particles scattered from $N^{14}$ (upper) and $N^{15}$ (lower).

### 3. *Nitrogen-14*

In $N^{14}$, excitation of the first few levels by inelastic scattering of $\alpha$ particles has been previously reported.[14,15] We observed excitation of many levels up to an excitation of about 13 MeV. No $\alpha$ groups corresponding to the previously reported[16] levels at 6.70, 7.40, and 7.60 MeV were observed, confirming an earlier suggestion[17] that these levels of $N^{14}$ do not in fact exist. Furthermore, the level at 6.05 MeV reported in the reaction $N^{15}(He^3,\alpha)N^{14}$ (Ref. 18) was not observed.

No known $T=1$ levels were definitely observed to be excited. A peak corresponding to an excitation of 9.1 MeV is probably due to excitation of the recently found 2− level at 9.129 MeV[19] rather than to excitation of the 9.17-MeV 2+, $T=1$ level.

Observation of the levels at 9.41, 9.71, 10.22, and 10.55 MeV suggests strongly that they are all $T=0$.

A weakly excited level was observed at 10.85 MeV and two very strong levels (or groups of levels) at 11.3 and 12.9 MeV. Presumably they also are $T=0$.

### 4. *Nitrogen-15*

Alpha-particle scattering by $N^{15}$ was first studied recently at Berkeley.[20] We have observed excitation of many levels up to 15 MeV but without resolving the higher levels. The level reported at 9.77 MeV[21] was not observed.

### 5. *Oxygen-16*

Scattering of $\alpha$ particles by $O^{16}$ has been very widely studied[2,4–6,22–24] for different incident-particle energies. Excitation of levels up to 15 MeV was observed in the present work. The unnatural parity 2− level at 8.870 MeV was quite strongly excited. The 0+ excited levels at 6.052 and 11.26 MeV were not observably populated. The 0− levels at 10.925 and 12.798 MeV (whose excitation is forbidden by conservation of spin and parity) were not observed.

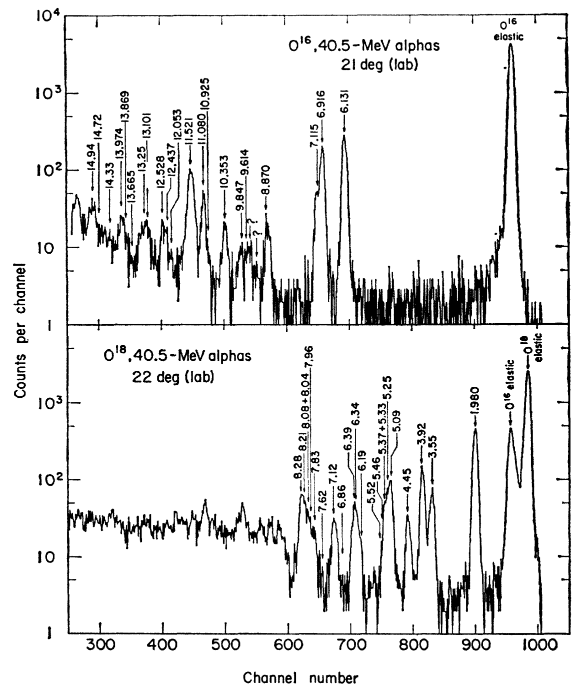


FIG. 3. Energy spectra of 40.5-MeV $\alpha$ particles scattered from $O^{16}$ (upper) and $O^{18}$ (lower).

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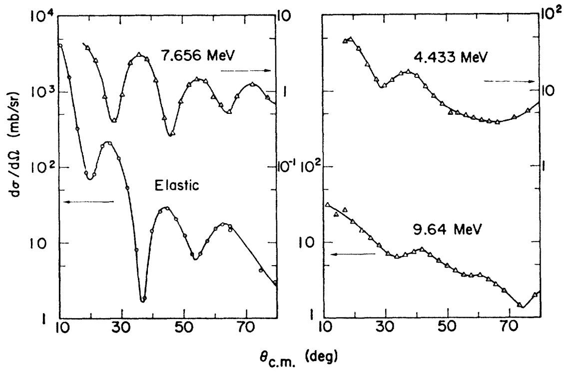


FIG. 4. Angular distributions for elastic and inelastic scattering of 40.5-MeV $\alpha$ particles by $C^{12}$.

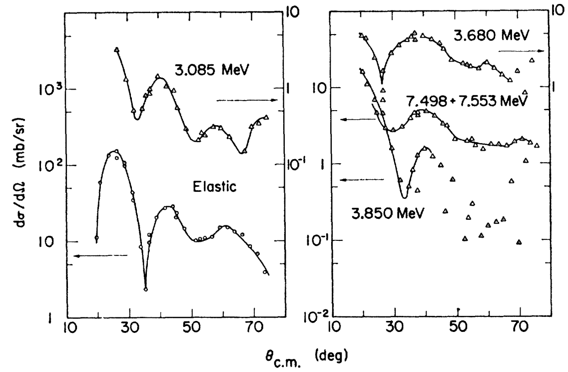


FIG. 5. Angular distributions for elastic and inelastic scattering of 40.5-MeV $\alpha$ particles by $C^{13}$.

### 6. *Oxygen-18*

Many levels below 8 MeV of excitation were observed, but there was no detectable population of the 6.86-MeV level.

## B. Angular Distributions

Elastic and inelastic angular distributions for $C^{12}$, $C^{13}$, $N^{14}$, $N^{15}$, $O^{16}$, and $O^{18}$ are shown in Figs. 4 to 9.[25] Statistical errors were in nearly all cases smaller than random and systematic errors (imperfect measurement of target gas pressure and temperature, errors of background subtraction, difficulties of resolving peaks from one another, and beam-integration errors). All these sources of error produced an uncertainty of about $\pm 6\%$ in the cross sections of levels excited with average intensity. For weakly excited or imperfectly resolved levels the errors were naturally much greater, but are very difficult to estimate. The angular uncertainty in all cases is $\pm 0.5$ deg, due mainly to multiple scattering in the gas cell windows.

A computer program was used to make least-squares fits of Gaussian peaks to resolve the individual contributions of imperfectly resolved doublets and triplets in the experimental energy spectra, for instance the 4.91–5.10-MeV and 5.69–5.83-MeV doublets of $N^{14}$.[26]

### 1. *Carbon-12*

The curves for the 4.433 MeV 2+ and 7.656 MeV 0+ levels (see Fig. 4) oscillate clearly out of phase with that of the elastic group, in agreement with the normal Blair phase rule,[1] but the angular distribution for the 2+ level shows no strong oscillations beyond 50 deg.

The oscillatory behavior of the angular distribution for the 9.64-MeV 3− level is unusually weak, but this is perhaps due to the uncertainty in the subtraction of the large background from the four-body break-up. The absence of the first minimum (which would be at about 20 deg in this case) is characteristic of $L=3$ transitions.

[25] Tables of differential cross sections are available in University of California Radiation Laboratory Report No. UCRL-16573 (unpublished).
[26] We are indebted to Dr. A. Springer for writing this program.

### 2. *Carbon-13*

The angular distribution for the 3.085-MeV $\frac{1}{2}+$ level (see Fig. 5) is in phase with that of the elastic group as required by the phase rule for an odd-$L$ transition with change of parity. Previous tests of the phase rule have involved the excitation of more or less collective transitions. Thus it is interesting to observe that the rule is clearly obeyed for this pure single-particle transition.

For the (7.498–7.553)-MeV unresolved doublet, the negative-parity level (7.553 MeV, $\frac{5}{2}-$) was responsible for very approximately two-thirds of the combined cross section. The 3.850-MeV $\frac{5}{2}+$ level was excited much less strongly than the 3.680-MeV $\frac{3}{2}-$ level, but it appears to obey the phase rule at small angles.

### 3. *Nitrogen-14*

The 3.945-MeV 1+ and 7.03-MeV 2+ levels (see Fig. 6) give angular distributions that oscillate out of

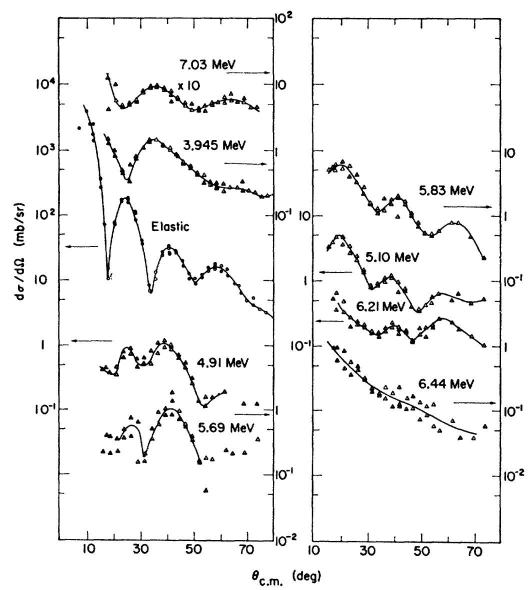


FIG. 6. Angular distributions for elastic and inelastic scattering of 40.5-MeV $\alpha$ particles by $N^{14}$.

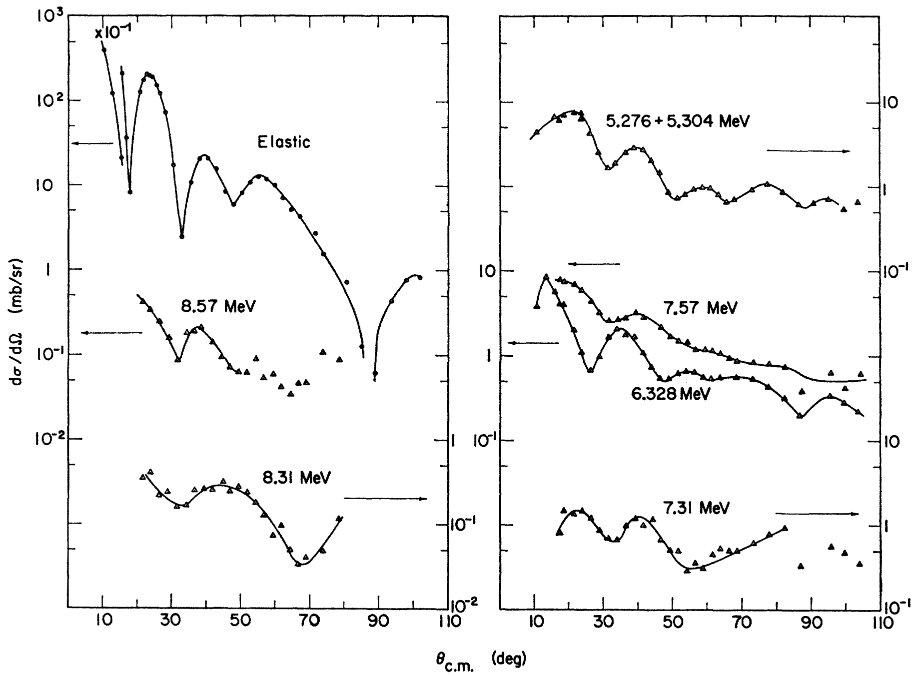


FIG. 7. Angular distributions for elastic and inelastic scattering of 40.5-MeV $\alpha$ particles by $N^{15}$.

phase with the elastic cross section; their shapes are characteristic of $L=2$ transitions. The angular distributions for the 4.91-MeV 0−, 5.10-MeV 2−, 5.69-MeV 1− and 5.83-MeV 3− levels are in phase with the elastic angular distribution; these levels are excited by $L=1$ or $L=3$ transitions (see below). The 6.21-MeV 1+ level gives rise to an angular distribution very similar in shape to those of the 5.10- and 5.83-MeV negative-parity levels. It, and the 6.44-MeV 3+ level, are believed to be excited by a second-order process (the differential cross sections are about one-tenth of those for the 5.10- and 5.83-MeV levels).

### 4. *Nitrogen-15*

Angular distributions for the (5.276-MeV $\frac{5}{2}+$, 5.304-MeV $\frac{1}{2}+$) unresolved doublet, 7.31-MeV $\frac{3}{2}+$ and 7.57-MeV $\frac{7}{2}+$ levels (see Fig. 7) are in phase with the elastic angular distribution, consistent with the positive parity of all these levels. By comparing the slopes at small angles, and the relative heights of the maxima, we assign an $L=1$ transition to the 7.31-MeV level and $L=3$ to the other two. The 8.31-MeV ($\frac{1}{2}+$ or $\frac{3}{2}+$) and 8.57-MeV ($\frac{5}{2}+$) levels are weakly excited, and while their angular distributions are consistent with a positive

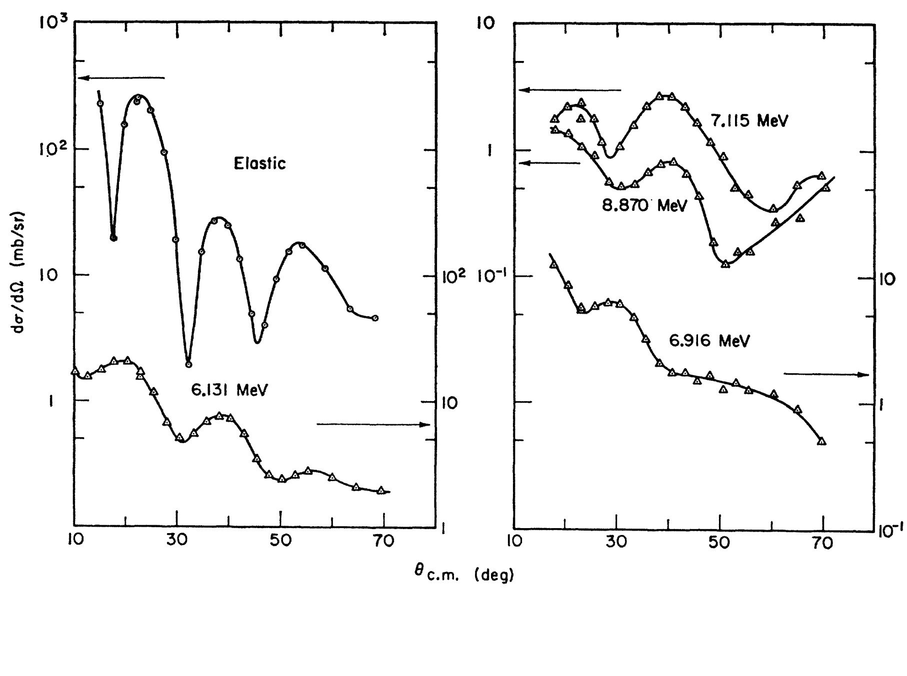


FIG. 8. Angular distributions for elastic and inelastic scattering of 40.5-MeV $\alpha$ particles by $O^{16}$.

parity assignment for these levels, the curves are not accurate enough to give definite information about the $L$ value for the transitions.

The 6.328-MeV ($\frac{3}{2}-$) level gives an angular distribution that at small angles looks clearly like an $L=2$ transition.

### 5. *Oxygen-16*

The cross sections for the 6.131-MeV 3−, 7.115-MeV 1− and 8.870-MeV 2− levels (see Fig. 8) are in phase with the angular distribution of the elastic group, in agreement with their negative parity. Comparison between their curves suggests an $L=3$ transition for the 6.131- and 8.870-MeV levels and an $L=1$ transition for the 7.115-MeV level. The 8.870-MeV 2− unnatural parity level is, as expected,[4,23,27] only weakly excited.

Excitation of the 6.916-MeV 2+ level gives an angular distribution that is not exactly out of phase with the elastic cross section, but it is consistent with the positive parity of the level when the curve is contrasted with the angular distributions of the known negative-parity levels.

An unresolved group was observed at an excitation of 13.1 MeV but the angular distribution is not plotted in Fig. 8. That we observed excitation of levels at all in this energy region confirms the existence of the 2+, $T=0$ level recently found at 13.1 MeV.[9]

### 6. *Oxygen-18*

The 5.09-MeV 3−, the (6.34-MeV–6.39-MeV) unresolved doublet and the (7.96-MeV–8.04-MeV–8.08-MeV) unresolved triplet (see Fig. 9) have angular distributions that are in phase with the elastic curve, corresponding to odd-$L$ transitions with a change of parity. The transition is clearly $L=3$ for excitation of the 5.09-MeV level. The 7.96-MeV and 8.08-MeV members of the triplet seem to be only weakly excited. The angular distributions for the 1.98-MeV and 3.92-MeV 2+ levels are almost exactly out of phase with the elastic angular distribution, confirming the positive parity of the levels. The oscillations of their angular distributions are much stronger than those for any other levels so far observed among the light nuclei: they resemble the angular distributions obtained in the excitation of strongly collective quadrupole levels in somewhat heavier nuclei.

The angular distribution for excitation of the 4.45-MeV level is clearly due to an $L=1$ transition. The spin of the level must therefore be 1− rather than 3+ or 3−.[28–31]

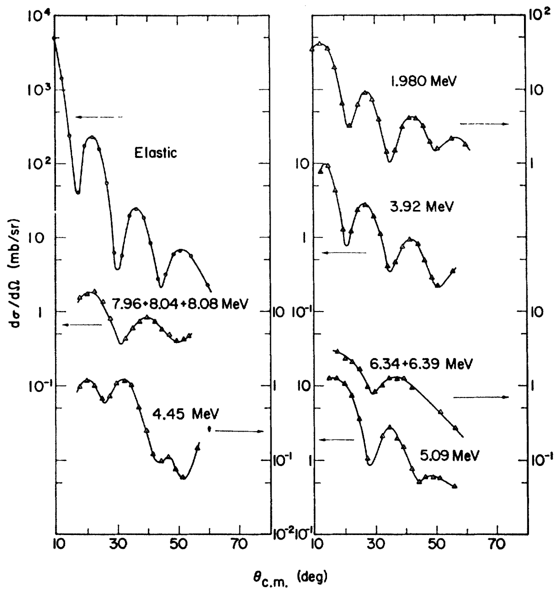


FIG. 9. Angular distributions for elastic and inelastic scattering of 40.5-MeV α particles by $O^{18}$.

## IV. DISCUSSION

The angular distributions seem to fall into groups depending on the particular dominant single-particle transition involved in the excitation. In this respect the angular distributions for excitation of levels in the light nuclei differ from those for excitation of collective levels in heavier nuclei, where the shape of the angular distribution depends almost entirely on the angular momentum transfer, and where the difference between an $L=1$ or 3 or an $L=2$ or 4 angular distribution is rather small except at small angles.

### 1. $p_{3/2}\rightarrow p_{1/2}$, $L=2$ *Transition*

The $p_{3/2}\rightarrow p_{1/2}$ transition might involve a change of 0 or 2 units of angular momentum. If the interaction between an α particle and a nucleon is spin-independent, the transition will involve only the orbital angular momentum of the nucleon, with no change of spin. The change in the total angular momentum must be $\frac{3}{2}\pm\frac{1}{2}=1$ or 2 units. Hence in the absence of spin-flip and of parity change, $L$ can only be 2.

Six transitions have been observed that are believed to involve the promotion of a $p_{3/2}$ nucleon into the $p_{1/2}$ shell. In four cases ($C^{12}$ 2+, 4.433-MeV; $C^{13}$ $\frac{5}{2}-$, 7.553-MeV; $C^{13}$ $\frac{3}{2}-$, 3.680-MeV; and $N^{15}$ $\frac{3}{2}-$, 6.328-MeV levels) conservation of angular momentum and parity requires that the excitations be $L=2$. Since the angular distributions of the scattered α particles are very much

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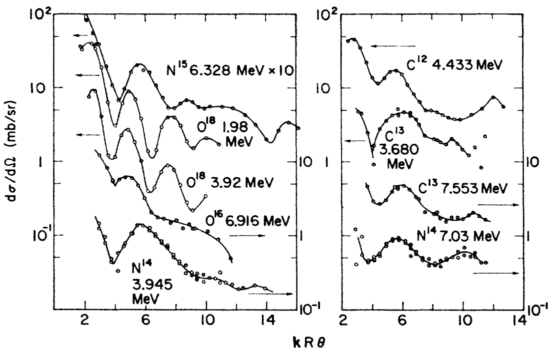


FIG. 10. $L=2$ angular distributions for inelastic scattering of 40.5-MeV $\alpha$ particles.

alike in all six cases, it seems quite probable that all six transitions are quadrupole.

Figure 10 shows the differential cross sections for inelastic scattering of $\alpha$ particles from the 4.433-MeV $2+$ level of $C^{12}$, the 3.680-MeV $\frac{3}{2}-$ and 7.553-MeV $\frac{5}{2}-$ levels of $C^{13}$, the 3.945-MeV $1+$ and 7.03-MeV $2+$ levels of $N^{14}$ and the 6.328-MeV $\frac{3}{2}-$ level of $N^{15}$ plotted as a function of the parameter

$$kR\theta=\tfrac{1}{2}(k_i+k_f)(1.3A^{1/3}+1.2)\theta_{\text{c.m.}},$$

where $k_i$ and $k_f$ are the center-of-mass wave numbers of the incident particle and of the outgoing particle at 0 deg. All these levels give rise to angular distributions of about the same shape. Figure 10 also shows the angular distributions for excitation of the 6.916-MeV $2+$ level of $O^{16}$ and the 1.98-MeV and 3.92-MeV $2+$ levels of $O^{18}$ for which the shape is quite different.

The shell-model components with the largest amplitudes in the wave functions of the 4.433-MeV level of $C^{12}$, the 3.945-MeV and 7.03-MeV levels of $N^{14}$ and the 6.328-MeV level of $N^{15}$ are $s^4p^8(p_{3/2}{}^{-1}p_{1/2})_2$,[32] $s^4p^{12}(p_{3/2}{}^{-1}p_{1/2}{}^{-1})_{1,2}$,[33,34] and $s^4p^{12}p_{3/2}{}^{-1}$,[35] respectively. Thus these four levels can be excited by promotion of a $p_{3/2}$ nucleon into the $p_{1/2}$ shell. Since the angular distributions for excitation of the 3.680-MeV $\frac{3}{2}-$ and 7.553-MeV $\frac{5}{2}-$ levels of $C^{13}$ resemble those of the other four, these levels of $C^{13}$ should be largely $s^4p^8[(p_{3/2}{}^{-1}p_{1/2})_2p_{1/2}]_{3/2,5/2}$.

The angular distributions for the two $2+$ levels of $O^{18}$ and for the 6.916-MeV $2+$ level of $O^{16}$ are quite different from the others shown in Fig. 10. Since the $p_{1/2}$ shell is filled in these two nuclei, the promotion $p_{3/2}\rightarrow p_{1/2}$ is impossible. The angular distributions thus seem to be dependent upon the details of the transition and not only upon the angular momentum change.

The configuration given above for the $\frac{3}{2}-$ and $\frac{5}{2}-$ levels of $C^{13}$ at 3.680- and 7.553-MeV, respectively, corresponds to coupling of the $p_{1/2}$ odd neutron to a $C^{12}$ core in the 4.433-MeV $2+$ excited state. If the coupling is weak, the angular distributions for the two levels should be similar in shape to that for the excitation of the $2+$ state in $C^{12}$. Further, the reduced transition probabilities, $B(E2)\downarrow$ of the two $C^{13}$ levels should be equal to that for the $2+$ $C^{12}$ level.

The angular distributions for the three levels have roughly the same shape. At the maximum near 40 deg, the sum of the cross sections for the two $C^{13}$ levels is about 11 mb/sr whereas the corresponding $C^{12}$ cross section is 18 mb/sr. The cross sections do not obey either the $(2J+1)$ rule or the center-of-gravity theorem.[36] The rather large splitting (3.87 MeV) between the two levels shows, of course, that the coupling is in fact rather strong.

### 2. $p_{1/2}\rightarrow 2s_{1/2}$, $L=1$ Transition

The promotion $p_{1/2}\rightarrow 2s_{1/2}$ necessarily involves a dipole transition ($L=1$). Although there is a general similarity between the inelastic scattering and electric transitions of the same multipole order,[37] there are certain important differences in the special case of a dipole transition.

Protons and neutrons interact in a different way with an electromagnetic field (since their electric charge and magnetic moments are different). The interaction Hamiltonian can therefore be written explicitly in terms of the third component of isospin:

$$H_{\text{int}}=-\sum_{i=1}^{A}\left[\frac{e}{2c}\mathbf{v}_i\cdot\mathbf{a}(\mathbf{r}_i)+\tfrac{1}{2}(\mu_p+\mu_n)\boldsymbol{\sigma}_i\cdot\boldsymbol{\mathcal{H}}\right]$$

$$-\sum_{i=1}^{A}\left\{t_{3_i}\left[\frac{e}{c}\mathbf{v}_i\cdot\mathbf{a}(\mathbf{r}_i)+\tfrac{1}{2}(\mu_p-\mu_n)\boldsymbol{\sigma}_i\cdot\boldsymbol{\mathcal{H}}\right]\right\}$$

$$=H_0+H_1.$$

Here $\mathbf{r}_i$, $\mathbf{v}_i$, $\boldsymbol{\sigma}_i$, and $t_{3_i}$ are the coordinate, velocity, spin vector and the third component of isospin for the $i$th nucleon. $\boldsymbol{\mathcal{H}}$ is the magnetic field vector, $\mathbf{a}(\mathbf{r}_i)$ the vector potential and $\mu_p$ and $\mu_n$ the proton and neutron magnetic moments.

The first sum ($H_0$) is a scalar in isospin space, but $H_1$ is the third component of a vector, producing different isospin selection rules. On making a multipole expansion of $H_{\text{int}}$, it is found that $H_0$ contains almost no $E1$ part provided that $kR\ll 1$.[38] Electric dipole transitions therefore occur predominantly through the isospin vector part $H_1$; $H_0$ leads only to motion of the nuclear center of mass.

Under the assumption that the particles can be represented by plane waves, and that they interact with the nucleons through a zero-range force, the inelastic

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scattering operator has the same form as the spin-independent part of the electric operator of the same multipole order.[37] However, under the further assumption that the particles interact identically with protons and neutrons, there will be nothing to correspond to $H_1$, and hence dipole inelastic scattering excitations will be forbidden provided that $QR \ll 1$ (where $Q$ is the momentum transfer).

Nevertheless, several examples of dipole excitation were observed in the present work. This may be due to breakdown of any of the assumptions mentioned above, but most probably the excitation arises from the failure of the plane-wave assumption that is to be expected for strongly interacting particles such as α particles.

Excitation through a multiple-scattering process is also possible. It has previously been postulated for excitation of the 5.80-MeV 1− level in $Ne^{20}$,[20,39] by an octupole transition to the 7.17-MeV 3− level followed by a quadrupole transition to the 1− level. The angular distribution for formation of the 1− level was out of phase with the elastic angular distribution, as might be expected for a double transition to a negative parity state. However, of the dipole levels studied in the present work only the 4.45-MeV level of $O^{18}$ gave an anomalous angular distribution. There is as yet no evidence that multiple transitions are involved in excitation of the 7.115-MeV 1− level of $O^{16}$, even though the strong $E3$ transition to the 6.131-MeV 3− level[40] and the strong $E2$ transition from the 3− to the 1− level[41] should make this a particularly favorable case.

Angular distributions of α particles corresponding to excitation of the 3.085-MeV $\frac{1}{2}+$ level of $C^{13}$ the 4.91-MeV 0− and 5.69-MeV 1− levels of $N^{14}$, the 7.31-MeV $\frac{3}{2}+$ level of $N^{15}$, the 7.115-MeV 1− level of $O^{16}$ and the 4.45-MeV 1− level of $O^{18}$ are shown in Fig. 11. The differential cross sections are plotted as a function of the parameter $kR\theta$. Except for the 3.085-MeV level of $C^{13}$, for which the data are poor at small angles, the angular distributions have a very characteristic and unusual shape—there is little or no decrease in cross section between the first and second maxima. The shell-model components with the largest amplitudes for these levels are: $s^4p^8s_{1/2}$(3.085 MeV, $C^{13}$),[42] $s^4p^8(p_{1/2}s_{1/2})_{0,1}$, (4.91 MeV and 5.69 MeV, $N^{14}$),[33,34] $s^4p^{12}[(p_{1/2}^{-1}s_{1/2})_1p_{1/2}^{-1}]_{3/2}$, (7.31 MeV, $N^{15}$),[35] and $s^4p^{12}(p_{1/2}^{-1}s_{1/2})_1$, (7.115 MeV, $O^{16}$).[32,43] Thus these excited states can be formed by the single-nucleon transition $p_{1/2} \rightarrow 2s_{1/2}$.

Sebe[42] has calculated wave functions for the positive parity levels of $C^{13}$, assuming that the odd neutron can be in the $2s_{1/2}$, $1d_{5/2}$, or $1d_{3/2}$ shell model states coupled to a $C^{12}$ core that can be either in the 0+ ground state or in the 2+ 4.433-MeV state. According to this model, the 3.085-MeV $\frac{1}{2}+$ state is almost pure $2s_{1/2}$ with the $C^{12}$ core in the ground state. The 6.860-MeV $\frac{5}{2}+$ level of $C^{13}$ is only weakly excited at most angles, consistent with Sebe's wave function, according to which excitation both of the $C^{12}$ core and of the odd neutron would be required. The 3.850-MeV $\frac{5}{2}+$ level should be excited quite readily by a $p_{1/2} \rightarrow d_{5/2}$ transition of the odd neutron, but for some reason the peak is apparently only a minor component of the 3.680–3.850-MeV unresolved doublet. The positive-parity levels of $C^{13}$ can be formed only by excitation of $p$-shell nucleons into the $sd$ shell, whereas the negative-parity levels can be formed by processes related to the very strong excitation of the $C^{12}$ 4.433-MeV quadrupole level.

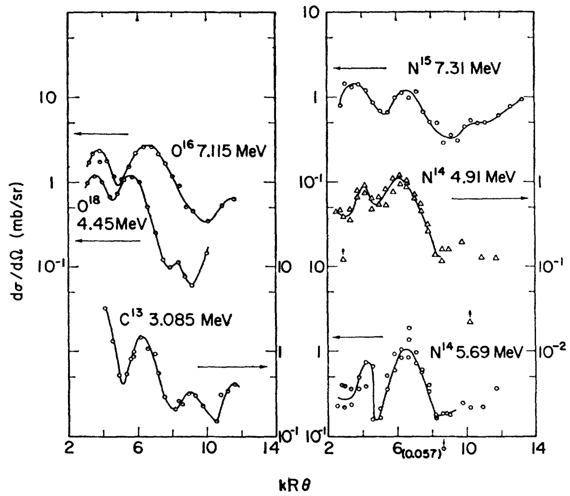


FIG. 11. $L=1$ angular distributions for inelastic scattering of 40.5-MeV α particles.

In earlier experiments with 65-MeV alpha particles,[2] the characteristic $L=1$ shape observed in the present work for the angular distribution corresponding to excitation of the 7.115-MeV 1− level of $O^{16}$ was not observed. Its angular distribution differed from the $L=3$ shape only in the absence of a sharp peak at 20°.

### 3. $p_{1/2} \rightarrow d_{5/2}$, $L=3$ Transition

The promotion of a $p_{1/2}$ nucleon to the $d_{5/2}$ shell could involve an orbital angular momentum transfer of 1 or 3 units. Seven examples of this transition have been identified: some of them are required by angular momentum and parity conservation to be $L=3$ (e.g., the excitation of the 6.131-MeV 3− level of $O^{16}$). The similarity of the seven angular distributions suggests that these transitions are all octupole.

Figure 12 shows the angular distributions of α particles scattered from the 5.10-MeV 2− and 5.83-MeV 3− levels of $N^{14}$, the 5.276-MeV $\frac{5}{2}+$ and 7.57-MeV

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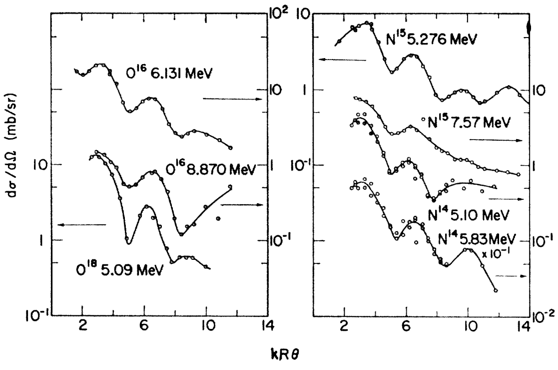


FIG. 12. $L=3$ angular distributions for inelastic scattering of 40.5-MeV $\alpha$ particles.

$\frac{7}{2}+$ levels of $N^{15}$, the 6.131-MeV 3− and 8.870-MeV 2− levels of $O^{16}$ and the 5.09-MeV 3− level of $O^{18}$. The shell model components with the largest amplitudes in the wave functions of these levels are: $s^4p^8(p_{1/2}d_{5/2})_{2,3}$ ($N^{14}$, 5.10 and 5.85 MeV),[33,34] $s^4p^8[(p_{1/2}d_{5/2})_3p_{1/2}]_{5/2,7/2}$ ($N^{15}$, 5.276 and 7.57 MeV),[35] and $s^4p^{12}(p_{1/2}^{-1}d_{5/2})_{2,3}$ ($O^{16}$, 8.870 and 6.131 MeV).[32,43] Thus the most important single-particle transition in the excitation of these levels should be $p_{1/2}\rightarrow d_{5/2}$.

In the case of the 8.870-MeV 2− level of $O^{16}$, either octupole or dipole excitation would have to be accompanied by a spin-flip, so it is perhaps surprising that at small angles the angular distribution resembles rather closely those of the non-spin-flip transitions, although at larger angles substantial differences are observed. The differential cross section for formation of the 2− level begins to rise at large angles: this behavior is characteristic of the more complex excitation processes. In scattering experiments at 65 MeV,[2] excitation of the 2− level gave an angular distribution that seemed closer to the $L=1$ shape than to $L=3$.

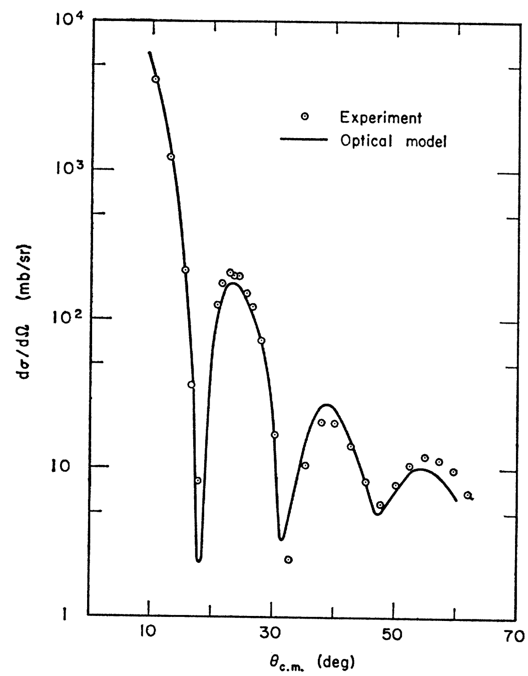


FIG. 13. Optical model fit to elastic scattering of 40.5-MeV $\alpha$ particles by $N^{15}$.

In an earlier publication,[20] the excitation of the 5.276- and 7.57-MeV levels of $N^{15}$ was considered as an example of weak coupling of an odd nucleon (in this case a $p_{1/2}$ proton hole) to an octupole core vibration, the core in this instance being $O^{16}$. The octupole reduced transition probabilities, $B(E3)\downarrow/e^2$, were found to be nearly equal for these two levels in $N^{15}$ to that for the 6.131-MeV octupole state of $O^{16}$. The weak-coupling model appears to give a better account of these octupole states than it did for the quadrupole levels of $C^{13}$ discussed above. The coupling is in fact somewhat weaker in $N^{15}$: the splitting is only 2.29 MeV compared with 3.87 MeV in $C^{13}$.

In addition to the octupole levels, one might expect to find in $N^{15}$ doublets of levels based on the strongly excited 1− and 2+ levels of $O^{16}$. The $\frac{1}{2}+$ and $\frac{3}{2}+$ levels of $N^{15}$ at 5.304 and 7.31 MeV are possible candidates for the dipole doublet. The 5.304-MeV level was not resolved in the present work; the 7.31-MeV level, however, does give an angular distribution similar to that of the 7.115-MeV 1− level of $O^{16}$.

TABLE I. Parameters for optical potential.

| Nucleus | $-V$ (MeV) | $-W$ (MeV) | $a$ (F) | $b$ (F) |
|---|---|---|---|---|
| $C^{12}$ | 34.44 | 9.62 | 0.486 | 0.700 |
| $C^{13}$ | 37.89 | 10.43 | 0.519 | 0.500 |
| $N^{14}$ | 35.05 | 9.48 | 0.550 | 0.700 |
| $N^{15}$ | 45.09 | 15.07 | 0.586 | 0.500 |
| $O^{16}$ | 45.71 | 15.23 | 0.606 | 0.500 |
| $O^{18}$ | 46.60 | 15.70 | 0.551 | 0.500 |

The only quadrupole level found in $N^{15}$ (the 6.328-MeV $\frac{3}{2}-$ level) appears to be related to the $p_{3/2}\rightarrow p_{1/2}$ single-particle transition rather than to the 6.916-MeV 2+ level of $O^{16}$.

The $L=3$ excitation of the 3− level of $C^{12}$ gives an angular distribution markedly different from the octupole excitations considered in the present section. In $C^{12}$, the single-particle transition with the largest amplitude is $p_{3/2}\rightarrow d_{5/2}$ rather than $p_{1/2}\rightarrow d_{5/2}$.[32] As in the case of the quadrupole levels discussed above, the detailed shape of an octupole angular distribution seems to depend on the details of the transition as well as on the angular momentum transfer.

### 4. *Two-Nucleon Transitions*

Several transitions, corresponding to the promotion of two nucleons from one shell into another were observed.

In $C^{12}$, the 7.656-MeV 0+ level is excited with surprising strength, considering that it can be populated only by the promotion of two or four nucleons from the $p_{3/2}$ ground state into the higher shells. The strong excitation of the $C^{12}$ 4.433-MeV 2+ level shows that the individual $p_{3/2} \rightarrow p_{1/2}$ quadrupole transitions are collectively enhanced.

The configurations $s^4p^8(s_{1/2}^2)_1$ and $s^4p^8(s_{1/2}d_{5/2})_3$ of the 6.21- and 6.44-MeV levels of $N^{14}$ [34] require that these states be formed from the ground state through the two-nucleon promotions $p_{1/2}^2 \rightarrow 2s_{1/2}^2$ and $p_{1/2}^2 \rightarrow 2s_{1/2}d_{5/2}$. The cross sections are indeed rather small, as are those for formation of the 8.99-MeV $1+s^4p^{12}(p_{3/2}^{-2})_1$ and 8.963-MeV $5+s^4p^8(d_{5/2}^2)_5$ levels, which could be populated through the transitions $p_{3/2}^2 \rightarrow p_{1/2}^2$ and $p_{1/2}^2 \rightarrow d_{5/2}^2$.

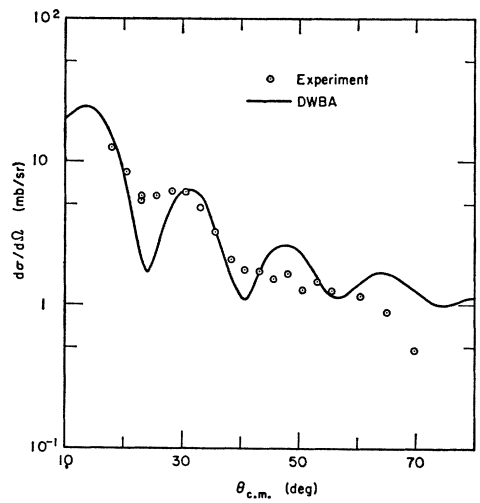


FIG. 14. Distorted-wave Born approximation (DWBA) angular distribution for excitation of the 6.916-MeV 2+ level of $O^{16}$ by inelastic scattering of 40.5-MeV α particles. (The fit is representative of the worst results.)

## V. REDUCED TRANSITION PROBABILITIES

Comparison of experimental cross sections with distorted-wave Born approximation (DWBA) calculations permits the extraction of reduced transition probabilities. The potential used in the present work was of the form

$$V = V_c - V_R[1+\exp((r-R)/a)]^{-1} + iW[1+\exp((r-R)/b)]^{-1},$$

where $V_c$, $V_R$, and $W$ are the Coulomb, real and imaginary parts of the potential, respectively. The parameters $a$ and $b$ are related to the diffuseness of the real and imaginary potentials.

The appropriate values of the parameters $V_R$, $W$, $a$, and $b$ were obtained from an optical-model analysis of the elastic angular distribution, using a computer program described by Pehl.[44] Reasonably good fits to the experimental elastic angular distributions were obtained with the parameters shown in Table I. As a typical example, Fig. 13 shows the calculated and experimental elastic angular distributions for $N^{15}$.

With the parameters shown in Table I, and using a derivative Woods-Saxon nuclear form factor, inelastic angular distributions were calculated in DWBA. The calculated angular distributions were never a particularly good fit to the experimental results: probably more detailed calculations with microscopic wave functions are needed, although results of some preliminary attempts were not particularly encouraging. Nevertheless, comparison of the calculated and experimental angular distributions was made in order to extract extremely approximate values of the reduced quadrupole and octupole transition probabilities connecting the target ground state and each excited state. The values thus obtained are given in Tables II and III. Two fairly typical comparisons of calculated and experimental angular distributions are shown in Figs. 14 and 15.

As Table II shows, the present experiment yielded values for the reduced electric quadrupole transition

TABLE II. Reduced electric quadrupole transition probabilities.

| Level | $J\pi$ | $B(E2)\downarrow/e^2$ ($F^4$) | Other values, comments |
|---|---|---|---|
| $C^{12}$, 4.433 MeV | 2+ | 13 | 9.6 $F^4$ [a] |
| $C^{13}$, 3.680 MeV | $\frac{3}{2}-$ | ~6 | DWBA fit not good |
| 7.553 MeV | $\frac{5}{2}-$ | ~6 | DWBA fit not good |
| $N^{14}$, 3.945 MeV | 1+ | 6.5 | 6.21±0.37 $F^4$ [b] |
| 7.03 MeV | 2+ | 3.3 | 3.3 $F^4$ [b] |
| $N^{15}$, 6.328 MeV | $\frac{3}{2}-$ | 4.9 | |
| $O^{16}$, 6.916 MeV | 2+ | 7.7 | 4.3±1.1 $F^4$ [c] |
| $O^{18}$, 1.980 MeV | 2+ | 8.4 | {7.2±0.14 $F^4$ [d]<br>8.0±3.6 $F^4$ [e] |
| 3.92 MeV | 2+ | 3.0 | $0.16_{-0.009}^{+0.014}$ $F^4$ [f] |

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[b] G. R. Bishop, M. Bernheim, and P. Kossanyi-Demay, Nucl. Phys. **54**, 353 (1964).
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TABLE III. Reduced electric octupole transition probabilities.

| Level | $J\pi$ | $B(E3)\downarrow/e^2$ ($F^6$) | Other values, comments |
|---|---|---|---|
| $C^{12}$, 9.64 MeV | 3− | 50–70 | DWBA fit poor |
| $N^{14}$, 5.10 MeV | 2− | 40 | 104±21 $F^6$ [a] |
| 5.83 MeV | 3− | 60 | 108±6 $F^6$ [a] |
| $N^{15}$, 5.276 MeV | $\frac{5}{2}+$ | 60 | |
| 7.57 MeV | $\frac{7}{2}+$ | 60 | |
| $O^{16}$, 6.131 MeV | 3− | 90 | $210_{-16}^{+25}$ $F^6$ [b] |
| $O^{18}$, 5.09 MeV | 3− | 40 | |

[a] G. R. Bishop, M. Bernheim, and P. Kossanyi-Demay, Nucl. Phys. **54**, 353 (1964).
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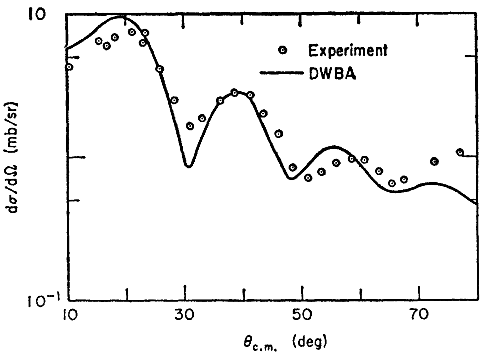


FIG. 15. Distorted-wave Born approximation (DWBA) angular distribution for excitation of the 5.276-MeV $\frac{5}{2}+$ level of $N^{15}$ by inelastic scattering of 40.5-MeV $\alpha$ particles. (The fit is representative of the best results.)

probabilities $B(E2)\downarrow/e^2$ in surprisingly good agreement with values obtained by other workers by electromagnetic measurements (except for the 3.92-MeV level of $O^{18}$). The $B(E3)\downarrow/e^2$ values of Table III are lower by a factor of about 2 than the three electromagnetic results.

In view of the profound difference between the inelastic scattering dipole operator and the electric dipole operator, there is little point in comparing the inelastic scattering dipole transition probabilities with the electromagnetic values, especially since the use of a collective nuclear model for the distorted wave calculations makes little sense for the dipole states. Nevertheless, it is interesting that the inelastic scattering cross sections for excitation of the 7.115-MeV and 4.45-MeV 1− levels of $O^{16}$ and $O^{18}$ are respectively $10^4$ and $10^6$ times greater than would be expected from the electromagnetic dipole transition probabilities!

For the self-conjugate nucleus $O^{16}$, the isospin vector part $H_1$ of the electromagnetic Hamiltonian vanishes, so apart from the differences due to the greater absorption of $\alpha$ particles (and therefore greater departure from the plane-wave approximation) the inelastic scattering and electromagnetic transitions should be formally similar. That they are not is presumably due to the very great distortion of the $\alpha$ particle waves.

The DWBA calculation gave an angular distribution for excitation of the 4.45-MeV 1− level of $O^{18}$ that was in very poor agreement with the experimental results. Figure 11 shows that although the angular distribution for this level has the equal maxima that seem to be characteristic of the dipole transitions, the values of $kR\theta$ at which they fell are substantially lower than for any other of the dipole excitations. As already mentioned, double excitation through the 5.09-MeV 3− level might be important.

Further studies of the very interesting dipole excitations are in progress.

## ACKNOWLEDGMENTS

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