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Scattering of Slow Electrons from Helium Atoms*

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The effect of Pauli exchange on the elastic scattering of a fermion from a many-fermion system has been studied in the context of the "no-polarization" approximation. Calculations of the low-energy elastic-scattering cross sections of electrons on helium atoms have been performed in order to investigate various effects in this approximation. It is shown that the "post" and "prior" forms give essentially identical results when calculated exactly with correct spatially correlated target wave functions. The results also indicate the importance of including all exchange terms and using accurate representations of the target wave function.

I. INTRODUCTION

WE have been interested in the inclusion of exchange effects arising from the Pauli exclusion principle in the elastic scattering of a fermion from a many-fermion system. In this paper we discuss the approximation in which the effect of polarization of the target by the incoming particle is neglected. This approximation is obviously best at low energies for tightly bound target systems.

In the elastic-scattering "no-polarization" approximation, the Hamiltonian for the many-body system reduces to an effective one-body Hamiltonian in which the interaction between the incident fermion and the target particle is given by a nonlocal potential. The nonlocality in the effective one-body potential arises from exchange terms due to the exclusion principle. In the derivation of this potential two forms are developed (Secs. II, III, and IV), which differ in two of the three exchange terms. It is also easy to see (Sec. V) that when a target wave function which can be written as an antisymmetrized product of a single-particle state with an $(N-1)$-particle state is used, the differing exchange terms must vanish. These developments give rise to questions about the importance of using correlated target wave functions, and the necessity for the inclusion of all exchange terms. Further we must investigate the difference between the alternative effective potentials we have developed and decide which is to be preferred.

In order to gain some insight into these questions we have applied the formalism to the scattering of slow electrons by helium atoms (Secs. VII and VIII). The electron-helium problem is particularly favorable for application since the basic two-body interaction is known exactly and the target wave function can be obtained as accurately as desired.[1] With this advantage we can perform an exact calculation within the framework of the no-polarization approximation. Individual effects or approximations can then be isolated and their relative importance studied.

II. DEVELOPMENT OF THE FORMALISM

Let us consider the scattering of a fermion from a target with N identical fermions. The Hamiltonian for the system is given by

$$H(0,1,\cdots,N)=\mathcal{H}(1,\cdots,N)+h_0, \tag{1}$$

where

$$\mathcal{H}(1,\cdots,N)=\sum_{i=1}^{N} T_i+\sum_{\substack{j=1\\ i>j}}^{N} V_{i,j}+\sum_i W_i \tag{2}$$

and

$$h_0=T_0+\sum_{j=1}^{N} V_{0,j}+W_0. \tag{3}$$

In the above equations T is the kinetic-energy operator, V_{ij} are the two-body interactions between the identical fermions, and W is a potential in which all the particles move. The coordinates are taken to represent position, spin, and isotopic spin. The Hamiltonian is, of course, symmetric under particle exchange.

The wave function for the entire system may be written as

$$\Psi(0,1,\cdots,N)=\sum_\nu \mathcal{A}\phi_\nu(1,\cdots,N)\psi_\nu(0), \tag{4}$$

where $\mathcal{A}$ is the antisymmetrization operator and $\phi_\nu(1,\cdots,N)$ are the normalized and antisymmetrized eigenfunctions of the target Hamiltonian,

$$\mathcal{H}(1,\cdots,N)|\phi_\nu(1,\cdots,N)\rangle=\mathcal{E}_\nu|\phi_\nu(1,\cdots,N)\rangle. \tag{5}$$

In the scattering problem we wish to solve the equation

$$H|\Psi(0,1,\cdots,N)\rangle=E|\Psi(0,1,\cdots,N)\rangle \tag{6}$$

under the scattering boundary conditions. Since we assume the N-body problem to be solved, we attempt to solve for the one-body functions $\psi_\nu(0)$. We may express this formally as

$$\langle\phi_\mu(1,\cdots,N)|H|\Psi(0,1,\cdots,N)\rangle =E\langle\phi_\mu(1,\cdots,N)|\Psi(0,1,\cdots,N)\rangle. \tag{7}$$

The equations symbolically written in Eq. (7) are a system of coupled one-body equations for $\psi_\nu(0)$, whose solution is more difficult than we care to attempt. For low-energy elastic scattering we neglect polarization of

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[1] E. A. Hylleraas, Z. Physik 54, 347 (1929).

the target by writing the wave function of the entire system as

$$\Psi(0,1,\cdots,N)=\mathcal{A}\phi(1,\cdots,N)\psi(0)\,, \tag{8}$$

where $\phi(1,\cdots,N)$ is the antisymmetric, normalized ground-state wave function of the N-body target system. For convenience we have dropped the state subscript $\nu=0$, since we are taking into account only the lowest eigenstate.

Instead of Eq. (7), we now look at the much simpler equation,

$$\begin{aligned}\langle\phi(1,\cdots,N)|H|\mathcal{A}\phi(1,\cdots,N)\psi(0)\rangle\\ =E\langle\phi(1,\cdots,N)|\mathcal{A}\phi(1,\cdots,N)\psi(0)\rangle.\end{aligned} \tag{9}$$

III. OPERATING TO THE LEFT

We may simplify Eq. (9) through the use of the Hermitian adjoint of Eq. (5) to replace $\langle\phi(1,\cdots,N)|\mathcal{H}$ by $\mathcal{E}_0\langle\phi(1,\cdots,N)|$. With this substitution, Eq. (9) becomes

$$\begin{aligned}\langle\phi(1,\cdots,N)|h_0|\mathcal{A}\phi(1,\cdots,N)\psi(0)\rangle\\ =\epsilon\langle\phi(1,\cdots,N)|\mathcal{A}\phi(1,\cdots,N)\psi(0)\rangle,\end{aligned} \tag{10}$$

where ϵ is the single-particle energy,

$$\epsilon\equiv(E-\mathcal{E}_0)\,. \tag{11}$$

Since $\phi(1,\cdots,N)$ is an antisymmetric function we may write

$$\Psi(0,1,\cdots,N)=\mathcal{A}\phi(1,\cdots,N)\psi(0)=\psi(0)\phi(1,\cdots,N)-\psi(1)\phi(0,2,\cdots,N)-\cdots-\psi(N)\phi(1,\cdots,N-1,0)\,, \tag{12}$$

so that Eq. (10) is then, more explicitly,

$$\begin{aligned}(T_0+\{\sum_{j=1}^{N}\langle\phi(1,\cdots,N)|V_{0j}|\phi(1,\cdots,N)\rangle\}+W_0-\epsilon)|\psi(0)\rangle=\langle\phi(1,\cdots,N)|(T_0+W_0+\sum_{j=1}^{N}V_{0j}-\epsilon)|\psi(1)\phi(0,2,\cdots,N)\rangle\\ +\cdots+\langle\phi(1,\cdots,N)|(T_0+W_0+\sum_{j=1}^{N}V_{0j}-\epsilon)|\psi(N)\phi(1,\cdots,N-1,0)\rangle.\end{aligned} \tag{13}$$

The exchange integrals which appear on the right-hand side of Eq. (13) give rise to three different kinds of terms. Thus Eq. (13) can be further simplified to read

$$\begin{aligned}(T_0+W_0+\mathcal{V}-\epsilon)|\psi(0)\rangle=N\langle\phi(1,\cdots,N)|V_{01}|\psi(1)\phi(0,2,\cdots,N)\rangle\\ +N(T_0+W_0-\epsilon)\langle\phi(1,\cdots,N)|\psi(1)\phi(0,2,\cdots,N)\rangle+N(N-1)\langle\phi(1,\cdots,N)|V_{02}|\psi(1)\phi(0,2,\cdots,N)\rangle,\end{aligned} \tag{14}$$

where

$$\mathcal{V}=\mathcal{V}(0)\equiv\sum_{j=1}^{N}\langle\phi(1,\cdots,N)|V_{0j}|\phi(1,\cdots,N)\rangle=N\langle\phi(1,\cdots,N)|V_{01}|\phi(1,\cdots,N)\rangle. \tag{15}$$

We see that Eq. (14) is a one-body Schrödinger equation with a nonlocal potential, where the nonlocality arises from the antisymmetrization.

IV. OPERATING TO THE RIGHT

We may obtain another equation, differing in form from Eq. (14), taking account of the symmetry of the Hamiltonian. Thus we write[2]

$$H=\mathcal{H}(1,\cdots,N)+h_0=\mathcal{H}(1,\cdots,i-1,0,i+1,\cdots,N)+h_i\equiv\mathcal{H}_i+h_i, \tag{16}$$

where $\mathcal{H}$ without a subscript is identical to $\mathcal{H}_0$. Thus, using Eq. (5) we see that H operating to the right on the wave function, Eq. (8), yields

$$H\mathcal{A}\phi(1,\cdots,N)\psi(0)=\mathcal{E}_0\mathcal{A}\phi(1,\cdots,N)\psi(0)+h_0\phi(1,\cdots,N)\psi(0)-\cdots-h_i\phi(1,\cdots,i-1,0,i+1,\cdots,N)\psi(i)-\cdots. \tag{17}$$

The use of Eq. (17) in Eq. (9) then leads to the "right-hand" result analogous to the "left-hand" result of Eq. (14), viz.,

$$\begin{aligned}(T_0+W_0+\mathcal{V}-\epsilon)|\psi(0)\rangle=N\langle\phi(1,\cdots,N)|V_{01}|\psi(1)\phi(0,2,\cdots,N)\rangle\\ +N\langle\phi(1,\cdots,N)|(T_1+W_1-\epsilon)|\psi(1)\phi(0,2,\cdots,N)\rangle+N(N-1)\langle\phi(1,\cdots,N)|V_{12}|\psi(1)\phi(0,2,\cdots,N)\rangle,\end{aligned} \tag{18}$$

where again $\mathcal{V}$ is given by Eq. (15). In the absence of exchange effects, Eqs. (18) and (14) are identical. However, two of the three exchange terms in these two equations are formally different. The first so-called "direct exchange" integrals, corresponding to the incident particle interacting and exchanging with the same target particle, are identical. The second and third, which we shall call "$(T-\epsilon)$" and "many-body" exchange, differ. In the "left" equation (14) the "many-body" exchange term can be thought of as representing the incident

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[2] M. J. Seaton, Trans. Roy. Soc. (London) A245, 469 (1953).

particle interacting with a given target particle with the ejection of a different target particle, whereas in the "right" equation (18) the exchange or ejected particle interacts with a target particle. If the single-particle wave function ψ is approximated by a plane wave then Eqs. (14) and (18) lead to the "post" and "prior" forms of the scattering amplitude in the Born-Oppenheimer approximation.[3]

Since both equations appear to have equal *a priori* basis, it is of interest to investigate the difference between them.

V. SLATER-DETERMINANT WAVE FUNCTIONS

An interesting special case occurs when the ground-state wave function of the target is written in the form

$$\phi(1,\cdots,N)=\mathcal{A}\phi(i)\chi(1,\,\cdots,\,i-1,\,i+1,\,\cdots,\,N)\,. \qquad (19)$$

In that case the "left-hand" equation (14) has the property that there exists a solution such that

$$\psi(0)=\phi(0)\,, \qquad (20)$$

independent of the energy ϵ. This corresponds to the trivial solution,

$$\begin{aligned}&\mathcal{A}\phi(1,\cdots,N)\psi(0)\\&\quad=\mathcal{A}\phi(0)\phi(i)\chi(1,\,\cdots,\,i-1,\,i+\tfrac{1}{2},\,\cdots,\,N)=0\,,\end{aligned} \qquad (21)$$

in which the entire wave function vanishes. This is the reason that the condition that ψ be orthogonal to ϕ is imposed.[4] If we take a target wave function of the form of Eq. (19), and impose the orthogonality condition

$$\langle\psi(i)\,|\,\phi(i)\rangle=0 \qquad (22)$$

then the "left-hand" equation (14) reduces to

$$\begin{aligned}&(T_0+W_0+\mathcal{V}-\epsilon)\,|\,\psi(0)\rangle\\&\quad=N\langle\phi(1,\cdots,N)\,|\,V_{01}\,|\,\psi(1)\phi(0,2,\cdots,N)\rangle\,,\end{aligned} \qquad (23)$$

that is to say the last two exchange terms on the right in Eq. (14) vanish by virtue of orthogonality.

This already presents a dilemma. If we use target wave functions of the form of Eq. (19), as for example if we use a Slater-determinant wave function, then we must necessarily drop the last two exchange integrals in Eq. (14). However, since the exact eigenfunction of the target Hamiltonian cannot be of this form, we have no estimate of the importance of such exchange terms.

For the "right-hand" equation (18) the assumption of a target wave function of the form of Eq. (19) does not lead to the energy-independent solution, Eq. (20). This occurs because when operating to the right we rearrange and operate on the separate parts of the total antisymmetric wave function. Thus, unless we were dealing with the exact eigenfunction we preclude the possibility of the trivial, or energy-independent, solution.

If the Hamiltonian were of the form

$$H=\textstyle\sum_i(T_i+W_i)=\sum_i h_i\,, \qquad (24)$$

then the exact eigenfunctions could be Slater determinants, in which the single-particle states $\phi_\nu(j)$ were eigenfunctions of the single-particle Hamiltonian $h(i)$. In that case, it is obvious that the "right-" and "left-" hand equations both reduce to the same equation, in which all the exchange integrals vanish and all the single-particle states are necessarily orthogonal.

VI. CORRELATED WAVE FUNCTIONS

We have seen that the difficulties under discussion arise from the fact that we are not dealing with exact eigenfunctions of the target Hamiltonian. If instead of a wave function of the form given by Eq. (19) we take a "correlated" function then the condition given by Eq. (21) cannot occur. In that case Eq. (14) no longer possesses an energy-independent solution and the last two exchange integrals need not be dropped. In the following two sections we shall investigate the use of correlated target wave functions for the particular problem of the elastic scattering of low-energy electrons from helium atoms.

Numerical results will be given which relate to the difference between the "right" and "left" solution, the importance of the "many-body" and "$T-\epsilon$" exchange terms, and the use of different types of correlations.

VII. ELECTRON-HELIUM SCATTERING

In the case of the scattering of electrons by neutral atomic helium the operators in Eqs. (14) and (18) become

$$T_i=-\tfrac{1}{2}\nabla_i^2\,, \qquad (25)$$

$$W_i=-2/r_i\,, \qquad (26)$$

$$V_{ij}=|\mathbf{r}_i-\mathbf{r}_j|^{-1}\,, \qquad (27)$$

in atomic units.

For the target wave function we pick

$$\phi(i,j)=f(\mathbf{r}_i,\mathbf{r}_j)2^{-1/2}[\alpha(i)\beta(j)-\beta(i)\alpha(j)]\,, \qquad (28)$$

where α and β are the usual spin eigenfunctions. The single-particle state ψ can be written as

$$\psi(i)=\chi(\mathbf{r}_i)\alpha(i)\,. \qquad (29)$$

The use of Eqs. (25)–(29), in Eqs. (14) and (18) yields the explicit "left-hand" and "right-hand" equations:

$$\begin{aligned}&\{\nabla_0^2+4/r_0-2\mathcal{V}(\mathbf{r}_0)+2\epsilon\}\chi_L(\mathbf{r}_0)\\&\quad=\int f^*(\mathbf{r}_1,\mathbf{r}_2)\{-2/|\mathbf{r}_0-\mathbf{r}_1|+\nabla_0^2+4/r_0\\&\qquad+2\epsilon-2/|\mathbf{r}_0-\mathbf{r}_2|\}f(\mathbf{r}_0,\mathbf{r}_1)\chi_L(\mathbf{r}_1)d\mathbf{r}_1d\mathbf{r}_2\,,\end{aligned} \qquad (30)$$

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[3] D. R. Bates, A. Fundaminsky, and H. S. W. Massey, Phil. Trans. Roy. Soc. London A243, 13 (1950).

[4] E. Feenberg, Phys. Rev. 42, 17 (1932).

$$\{\nabla_0^2+4/r_0-2\mathcal{V}(\mathbf{r}_0)+2\epsilon\}\chi_R(\mathbf{r}_0)$$
$$=\int f^*(\mathbf{r}_1,\mathbf{r}_2)\{-2/|\mathbf{r}_0-\mathbf{r}_1|+\nabla_1^2+4/r_1$$
$$+2\epsilon-2/|\mathbf{r}_1-\mathbf{r}_2|\}f(\mathbf{r}_0,\mathbf{r}_1)\chi_R(\mathbf{r}_1)d\mathbf{r}_1d\mathbf{r}_2, \quad (31)$$

where

$$\mathcal{V}(\mathbf{r}_0)=2\int |f(\mathbf{r}_1,\mathbf{r}_2)|^2|\mathbf{r}_0-\mathbf{r}_1|^{-1}d\mathbf{r}_1d\mathbf{r}_2. \quad (32)$$

VIII. NUMERICAL RESULTS

Substitution of the partial-wave expansion,

$$\chi(\mathbf{r}_0)=\sum_l r_0^{-1}g_l(r_0)P_l(\cos\theta_0), \quad (33)$$

into Eqs. (30) and (31) leads to integro-differential equations for the functions $g_l(r_0)$ of the form

$$g_l''(r_0)+[K_0^2+4/r_0-2\mathcal{V}(r_0)-l(l+1)/r_0^2]g_l(r_0)$$
$$=\int_0^\infty V_{NL}(r_0,r_1)g_l(r_1)dr_1. \quad (34)$$

For certain types of target wave functions the nonlocal kernel $V_{NL}(r_0,r_0')$ is a sum of separable terms. Equation (34) can then be solved numerically in a noniterative fashion using an extension of the method of Percival.[5] We have done this for s-wave scattering using the following form for the normalized wave functions of the helium atom:

$$\phi(1,2)=(C_0+C_1r_1r_2+C_2|\mathbf{r}_1-\mathbf{r}_2|^2)$$
$$\times e^{-\alpha(r_1+r_2)}2^{-1/2}[\alpha(1)\beta(2)-\beta(1)\alpha(2)]. \quad (35)$$

The above function is spatially correlated as long as C_1 and C_2 are not both zero. The coefficients C_i which minimize the expectation value of the Hamiltonian of the helium atom have been given by Hylleraas.[1] We have considered those cases in which either C_1, C_2 or both are zero.

Using the values of the C_i's given by Hylleraas the spatial function $f(\mathbf{r}_1,\mathbf{r}_2)$ of Eq. (28) is determined. Integration over the target position coordinate $\mathbf{r}_2$ in Eqs. (30)–(32) then gives explicit expressions for the single-particle potential in Eq. (34). With the choice of C_2 equaling zero for example, in the "left-hand" case, the nonlocal part $V_{NL}(r_0,r_0')$ is given by

$$V_{NL}(r_0,r_1)=4\pi\{[|r_0-r_1|-(r_0+r_1)$$
$$+r_0r_1(\nabla_0^2+4/r_0+K_0^2)]F(r_0,r_1)-2r_0r_1W(r_0,r_1)\}, \quad (36)$$

and in the "right-hand" case

$$V_{NL}(r_0,r_1)=4\pi\{[|r_0-r_1|-(r_0+r_1)$$
$$+r_0r_1(\nabla_1^2+4/r_1+K_0^2)]F(r_0,r_1)-2r_0r_1W(r_1,r_0)\}, \quad (37)$$

where

$$F(r_0,r_1)=(C_0^2\pi/\alpha^3)e^{-\alpha(r_0+r_1)}$$
$$\times\{1+(3C_1/2\alpha C_0)(r_0+r_1+2r_0r_1C_1/\alpha C_0)\}, \quad (38)$$

$$W(r_0,r_1)=(C_0^2\pi/\alpha^3)e^{-\alpha(r_0+r_1)}\{V_0(r_0,\alpha)+(C_1/C_0)$$
$$\times[(r_0r_1)V_1(r_0,\alpha)+(C_1/C_0)r_0r_1V_2(r_0,\alpha)]\}, \quad (39)$$

$$V_n(r_0,\alpha)=\int\frac{r_2^ne^{-2\alpha r_2}}{|\mathbf{r}_2-\mathbf{r}_0|}d\mathbf{r}_2. \quad (40)$$

Table I gives the parameters of the target wave functions for the three cases we have calculated. The parameter C_0 is determined from over-all normalization. The uncorrelated wave function of case E_0 is a Slater determinant with its single-particle states given by hydrogenic wave functions characterized by an effective charge α. This wave function has been used previously in calculations of electron-helium scattering.[6] From the discussion given in Sec. V it follows that the calculations using the target function of case E_0 will only involve "direct exchange" terms, the "many-body" and "$(T-\epsilon)$" exchange terms being deleted due to orthogonality. The wave functions of cases E_1 and E_2 represent two types of spatial correlations with the latter having its ground-state energy value closer to the true value of $-5.807a_0^{-2}$.[6]

TABLE I. Ground-state energies and parameters for the various target wave functions considered.

Case	$2\xi_0$	C_1/C_0	C_2/C_0	α
E_0	$-5.695\ a_0^{-2}$	0.0	0.0	1.6875
E_1	-5.712	-0.100	0.0	1.58
E_2	-5.755	0.0	0.111	1.85

In Table II the s-wave phase shifts are given for the cases considered calculated with both the "right" and "left" solutions. Case E_0 does not have formally different "right" and "left" results since, as previously stated, it requires only "direct" exchange terms. Examination of this table shows that for both case E_1 and case E_2 the "right" and "left" phase shifts are essentially identical. This is an interesting result since "post" and "prior" solutions give quite different values when calculated in Born approximation.[3] It should also be noted that the agreement is better for case E_2 supporting the idea that the differences in "right" and "left" come from the fact that we are not dealing with exact eigenfunctions of the target Hamiltonian.

TABLE II. S-wave phase shifts for electron-helium scattering using the "left" and "right" equations.

				$\delta_0(\epsilon)$		
Energy (eV)	Case:	E_0	E_{1L}	E_{1R}	E_{2L}	E_{2R}
0.544		2.84801	2.88489	2.88453	2.85244	2.85240
2.18		2.56731	2.63546	2.63476	2.56434	2.56432
4.90		2.30962	2.39983	2.39880	2.30410	2.30416
16.45		1.79480	1.89469	1.89309	1.79561	1.79559

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[5] R. Marriott, Proc. Phys. Soc. (London) **72**, 121 (1958).

[6] B. L. Moiseiwitsch, Proc. Roy. Soc. (London) **A219**, 102 (1953).

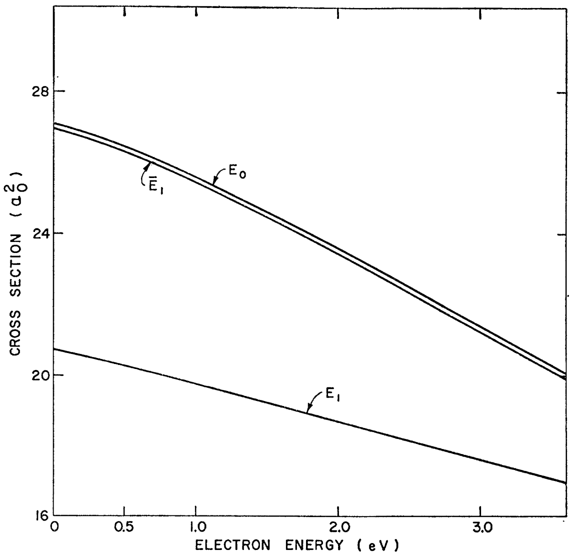

FIG. 1. S-wave scattering cross sections as a function of energy comparing solutions with (case E_1) and without (case $\bar{E}_1$) the "many-body" and "$(T-\epsilon)$" exchange integrals.

The effect of the "many-body" and "$(T-\epsilon)$" exchange for case E_1 is exhibited in Fig. 1. The s-wave cross section for case $\bar{E}_1$ differs significantly from that of E_1, where $\bar{E}_1$ is obtained from E_1 by removing the "many-body" and "$(T-\epsilon)$" exchange integrals. Cases $\bar{E}_1$ and E_0 only differ in the form of the target wave functions used, both having only the "direct" exchange terms. The close agreement between the curves for these two cases indicates that the need for more exact target wave functions (correlated wave functions) is minor when only "direct" exchange terms are involved. However, when all exchange terms are included and different target wave functions are used the results *can* differ significantly (see Fig. 2—cases E_1 and E_2). Thus for the cases considered, the importance of using correct target wave functions is coupled to the inclusion of all contributing exchange integrals.

The s-wave cross section computed from the phase shifts given in Table II are plotted in Fig. 2 for cases E_1 and E_2. These curves show that the different forms of correlation chosen for the target wave function give cross sections which differ by as much as 25%. The recent experiment data of Golden and Bandel[7] have also been included in Fig. 2. To fit the experimental data it would probably be appropriate to add a term due to polarization of the target to the formalism.

In both the left and right equation we have made no approximation except that of no polarization. Therefore, if we use exact eigenfunctions of the target Hamiltonian our result is exact within the no polarization approximation. By using a target wave function which is not an exact eigenfunction we can in some sense allow for polarization effects. This follows from the physical argument that the target is distorted due to the presence of the incident particle. Also, comparing the results of cases E_1 and E_2 with the experimental data of Golden and Bandel (Fig. 2) indicates that some type of correlated function exists which reflects the distortion of the target due to the incident particle and also gives a good fit to the experimental data. However, use of a distorted target wave function in Eq. (8) probably will lead to "post-prior" problems and also it is not clear that all polarization effects [replacement of Eq. (4) by Eq. (8)] can be allowed for in this manner.

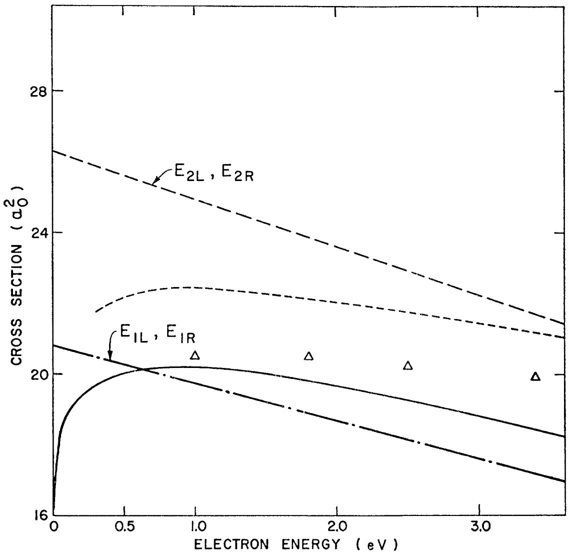

FIG. 2. S-wave and total scattering cross sections as a function of energy. The short dashed curve represents the recent experimental total-cross-section data of Golden and Bandel. The triangles represent the total-cross-section calculations by La Bahn and Callaway. The solid curve represents the s-wave cross section calculated by La Bahn and Callaway with dipole polarization effects considered. The long dashed curve and the dash-dot curve are results of the present calculation.

Calculations[8–10] (La Bahn and Callaway's results are given in Fig. 2) have been made which include a local polarization potential having an asymptotic form of $-\alpha/r^4$. The inclusion of this attractive potential has the effect of increasing the value of the phase shift and thus decreasing the low-energy cross section (see Fig. 2). Consequently, it seems that the best description of the electron-helium problem requires accurate representation of the target wave function, i.e., a correlated wave function,[1] all exchange terms, and then inclusion of target-polarization effects.

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[7] D. Golden and H. Bandel, Phys. Rev. **138**, A14 (1965).

[8] R. La Bahn and J. Callaway, Phys. Rev. **135**, A1539 (1964).

[9] E. Bauer and H. Browne, *Atomic Collision Processes* (North-Holland Publishing Company, Amsterdam, 1964), pp. 16–27.

[10] N. Kestner, J. Jortner, M. Cohen, and S. Rice, Phys. Rev. **140**, A56 (1965).