

Inelastic-Scattering Model for (p, xn) and $(p, p(x-1)n)$ Reactions at Intermediate Energies*

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(Received 17 January 1966)

A semiclassical model has been developed to calculate isobaric ratios of (p, xn) to $(p, p(x-1)n)$ reaction cross sections at energies of 40–100 MeV. It is assumed that these reactions are due to inelastic or charge-exchange scattering processes in which sufficient excitation energy is transferred to the residual nuclei to lead by subsequent particle evaporation to the products of interest. Account is taken of some of the effects of the exclusion principle and of the motion of the struck target nucleons. However, the effects of refraction, reflection, and absorption of the incident and emitted nucleons, as well as that of the relative availability of different target nucleons, are assumed to cancel in the calculation of the isobaric ratio. The results are in moderately good agreement with the available experimental data in the above energy range.

I. INTRODUCTION

NUCLEAR reactions induced by 20–100-MeV protons are known to undergo a striking change in mechanism in this energy interval. At the lower energies the principal mechanism is the formation and subsequent decay of a compound nucleus. At the high-energy end of this interval direct interactions predominate. Monte Carlo cascade calculations^{1,2} have indicated that at the outset the direct process involves the emission of a single prompt nucleon. The residual nucleus may retain enough excitation energy to lead to the subsequent evaporation of additional particles. As the bombarding energy increases the cascade proliferates and the number of promptly emitted nucleons rises.

The calculation of the cross sections of reactions involving the emission of more than two prompt nucleons is so complicated that the Monte Carlo approach is generally used. Reactions involving the emission of only one or two cascade particles are more amenable to analytical calculations. A discussion of the factors entering into such calculations and a summary of previous work above 100 MeV are given in a recent review by Grover and Caretto.³ Additional calculations have been performed at lower energies, including those by Hayakawa *et al.*,⁴ and by Elton and Gomes.⁵

The present work is concerned with a related problem: the calculation of isobaric yield ratios of (p, xn) and $(p, p(x-1)n)$ reactions below 100 MeV for a value of the integer $x \geq 2$. The evaluation of ratios rather than absolute cross sections permits a number of important simplifications in the calculation. We adopt the following model: The incident proton undergoes inelastic or charge-exchange scattering in the diffuse surface of the

nucleus and loses enough energy to permit the subsequent evaporation of the required number of nucleons to give the desired reaction product. A rather similar approach has been used by Remsberg and Miller⁶ in their calculation of the contribution of inelastic scattering to the (p, pn) reaction cross section at 370 MeV. The model is developed in the following section and the results are then compared with the available data in Sec. III.

II. MODEL FOR ISOBARIC RATIOS

We have adopted the following simple model for the evaluation of the isobaric cross-section ratios. It is assumed that the incident proton strikes the target nucleus at a large impact parameter and collides with a nucleon in the diffuse surface of the nucleus. The net result of this interaction is the prompt emission of either the incident or struck proton (inelastic scattering), or of the struck neutron (charge-exchange scattering). It is further assumed that the other nucleon participating in the scattering is captured and transfers its energy to the entire nucleus. At a later time this energy is then dissipated by particle evaporation and photon emission. The suggestion that inelastic scattering occurs through two-body collisions in the diffuse nuclear surface was first made by Eisberg and Igo⁷ in order to account for their (p, p') data at 31 MeV. It was further confirmed by the calculation of Elton and Gomes.⁵

The above assumption permits a major simplification in the computation. As the distance that the incident and emerging nucleons traverse through the nuclear periphery is smaller than the mean free path, the probability for more than a single collision will be small. It is thus reasonable to assume that the factors expressing the attenuation as well as the reflection and refraction of the incident and emerging waves will be approximately equal for both (P, N) and (P, P) cascades and can be neglected in the estimation of the ratio of cross sections. We can then simply express the ratio of

* Supported by the U. S. Atomic Energy Commission.

¹ N. Metropolis, R. Bivins, M. Storm, A. Turkevich, J. M. Miller, and G. Friedlander, *Phys. Rev.* **110**, 185 (1958).

² C. Chen, Z. Fraenkel, G. Friedlander, J. R. Grover, J. M. Miller, and Y. Shimamoto (to be published).

³ J. R. Grover and A. A. Caretto, Jr., *Ann. Rev. Nucl. Sci.* **14**, 51 (1964).

⁴ Hayakawa, Kawai, and Kikuchi, *Progr. Theoret. Phys. (Kyoto)* **13**, 415 (1955).

⁵ L. R. B. Elton and L. C. Gomes, *Phys. Rev.* **105**, 1027 (1957).

⁶ L. P. Remsberg and J. M. Miller, *Phys. Rev.* **130**, 2069 (1963).

⁷ R. M. Eisberg and G. Igo, *Phys. Rev.* **93**, 1039 (1954).

(P, N) and (P, P) cascade cross sections as

$$\frac{\sigma(P, N)}{\sigma(P, P)} = \frac{\langle \sigma \rangle_{pn} \mathfrak{N}}{\langle \sigma \rangle_{pp} \mathfrak{P} + \langle \sigma \rangle_{pn} \mathfrak{N}}. \quad (1)$$

The quantities $\langle \sigma \rangle_{pn}$ and $\langle \sigma \rangle_{pp}$ are the effective nucleon-nucleon scattering cross sections inside the nucleus. They differ from the cross sections for free nucleons because of the momentum distribution of the bound nucleons and the operation of the exclusion principle. In the above expression, \mathfrak{N} and \mathfrak{P} stand for the number of neutrons and protons with which the incident proton can collide in the nuclear surface. As we are interested in the ratio of cross sections rather than in their absolute values we can replace \mathfrak{N} and \mathfrak{P} by N and Z , the number of neutrons and protons in the target nucleus, respectively. This procedure assumes that nuclear structure effects, which would lead to a different availability of nucleons in differing shells,⁸ can be neglected in the estimation of the ratio.

As a result of the (P, P) or (P, N) cascade enough excitation energy must be transferred to the residual nucleus to lead by particle evaporation to the product of interest. It is obvious that the (p, xn) reactions can occur through the evaporation of ($x-1$) neutrons following a (P, N) cascade. On the other hand, the ($p, p(x-1)n$) reactions are due to either (P, P) cascades followed by the evaporation of ($x-1$) neutrons or (P, N) cascades followed by the evaporation of 1 proton and ($x-2$) neutrons [or a deuteron and ($x-3$) neutrons].

It was assumed that the evaporation of a given number of nucleons occurred when the excitation energy of the residual nucleus fell in a specified interval. These intervals were determined by Monte Carlo evaporation calculations using an adaptation of the code by Dostrovsky *et al.*⁹ These calculations showed that the branching ratio F for the evaporation of a given number and type of nucleons varies in the following way with increasing excitation energy: F increases sharply once the reaction becomes energetically possible, goes through a maximum, and falls rapidly towards zero. The full width at half-maximum of this peak was used to define the desired excitation energy interval and an average branching ratio for a given evaporation sequence \bar{F} was computed.

The kinetic energy of the captured nucleon E_c can be related to the center-of-mass scattering angle θ and to the energy of the incident proton prior to collision E_p . It is assumed that for this purpose the target nucleons are at rest. In a nonrelativistic approximation we then have

$$\cos \theta = \pm (1 - 2E_c/E_p). \quad (2)$$

The positive and negative signs refer respectively to the case where either the struck nucleon or the incident

proton is captured by the nucleus. This distinction is of no consequence for p - p scattering because the two cases are indistinguishable. However in the case of p - n scattering, if a given energy transfer corresponds to a scattering angle θ for proton emission, then the same energy transfer corresponds to $\pi - \theta$ for neutron emission. The importance of this distinction depends on the asymmetry of the differential cross section for p - n scattering.

In order to facilitate the use of the available values of the effective cross sections it is convenient to take E_p as the energy of the incident proton inside the nucleus, i.e., $E_p = E_i + S_p + E_F$. Here E_i is the proton energy in the laboratory, S_p is the proton binding energy, and E_F the Fermi energy. Conservation of energy then requires that the energy of the captured nucleon be related to the excitation energy transferred to the nucleus E^* by $E_c = E^* + E_F'$, where E_F' is the Fermi energy of the captured nucleon.

It is seen that a given excitation-energy interval corresponds to a particular range of scattering angles. The probability that a given collision will lead to the eventual evaporation of a particular number of nucleons thus depends on the fraction of the nucleon-nucleon cross section leading to the desired scattering angles. We denote this fraction as A_{pp} for p - p scattering, A_{pn} for p - n scattering with proton emission, and A_{np} for p - n scattering with neutron emission. These quantities can be expressed as

$$A_{pp} = 2\pi \int_{\theta_{\min}}^{\theta_{\max}} \left(\frac{d\sigma}{d\Omega} \right)_{pp} \sin \theta d\theta / \sigma_{pp}, \quad (3)$$

$$A_{pn} \text{ or } A_{np} = 2\pi \int_{\theta_{\min}}^{\theta_{\max}} \left(\frac{d\sigma}{d\Omega} \right)_{pn} \sin \theta d\theta / \sigma_{pn}. \quad (4)$$

Here σ is the free nucleon-nucleon scattering cross section and θ_{\min} and θ_{\max} are the angles corresponding to the minimum and maximum excitation energies assumed to lead to the reaction in question. The use of σ instead of $\langle \sigma \rangle$ in these expressions is consistent with the approximation of stationary target nucleons in the calculation of the scattering kinematics.

The various steps of the calculation can now be combined, resulting in the following expression for a particular isobaric cross-section ratio:

$$\frac{\sigma(p, xn)}{\sigma(p, p(x-1)n)} = \frac{\langle \sigma \rangle_{pn} A_{np} N \bar{F}_{(x-1)n}}{\langle \sigma \rangle_{pp} A_{pp} Z + \langle \sigma \rangle_{pn} A_{pn} N \bar{F}_{(x-1)n}' + \langle \sigma \rangle_{pn} A_{np} N \bar{F}_{p(x-2)n}}. \quad (5)$$

In this equation the average evaporation branching ratios are indexed by the number and type of evaporated nucleons. Thus $\bar{F}_{(x-1)n}$ and $\bar{F}_{p(x-2)n}$ refer, respectively, to the evaporation of ($x-1$) neutrons and of one proton

⁸ P. A. Benioff, Phys. Rev. 119, 324 (1960).

⁹ I. Dostrovsky, Z. Fraenkel, and G. Friedlander, Phys. Rev. 116, 683 (1959).

TABLE I. Parameters entering into the calculation of isobaric ratios.

Target	x	Bombarding energy (MeV)	$\langle\sigma\rangle_{pp}$ (mb)	$\langle\sigma\rangle_{pn}$ (mb)	A_{pp}	A_{pn}	A_{np}	$\bar{F}_{(x-1)n}$	$\bar{F}_{(x-1)n'}$	$\bar{F}_{p(x-2)n}$
Y ⁸⁹	2	85	18.3	28.0	0.232	0.120	0.119	0.795	0.800	0.067
	3	85	18.3	28.0	0.348	0.178	0.178	0.550	0.626	0.180
	4	85	18.3	28.0	0.348	0.182	0.188	0.306	0.480	0.302
Co ⁵⁹	3	99	18.2	26.4	0.363	0.172	0.170	0.041	0.407	0.425
	4	99	18.2	26.4	0.504	0.239	0.242	0.007	0.150	0.244
Ga ⁶⁹	2	56	15.9	36.8	0.457	0.222	0.223	0.471	0.556	0.220

plus $(x-2)$ neutrons (or 1 deuteron plus $x-3$ neutrons) following a (P,N) cascade, while $\bar{F}_{(x-1)n'}$ refers to the evaporation of $(x-1)$ neutrons following a (P,P) cascade. As indicated before, the quantities A_{pp} , A_{pn} , and A_{np} depend on the number of evaporated nucleons.

In order to evaluate Eq. (5) we have used the following information about nucleon-nucleon scattering. The values of $\langle\sigma\rangle_{pp}$ and $\langle\sigma\rangle_{pn}$ have been taken from the calculation by Winsberg and Clements.¹⁰ The differential cross section for p - n scattering was approximated by the empirical relations given by Bertini¹¹:

$$\left(\frac{d\sigma}{d\Omega}\right)_{pn} = A_1 + B_1 \cos^3\theta, \quad 0 \leq \cos\theta \leq 1, \quad (6)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{pn} = A_1 + B_2 \cos^4\theta, \quad -1 \leq \cos\theta \leq 0. \quad (7)$$

The values of the constants A_1 , B_1 , and B_2 were obtained from Bertini.¹¹ The value of σ_{pn} was obtained by integrating these expressions over all angles. The differential cross section for $(p-p)$ scattering is isotropic and σ_{pp} was obtained from the differential cross-section values summarized by Hamada and Johnston.¹² In order to calculate the energy of the incident proton inside the nucleus as well as the Fermi energy of the struck nucleon the depth of the nuclear potential well was evaluated assuming the Fermi gas model with $r_0 = 1.25$ F. The proton binding energy was evaluated for the various target nuclei on the basis of Wapstra's¹³ mass values.

III. COMPARISON WITH EXPERIMENT

The calculation has been performed for comparison with the following data: the $(p,2n)/(p,pn)$, $(p,3n)/(p,p2n)$, and $(p,4n)/(p,p3n)$ cross-section ratios for Y⁸⁹ bombarded with 25–85-MeV protons¹⁴; the $(p,2n)/(p,pn)$ ratio for Ga⁶⁹ bombarded with 25–56-MeV protons,¹⁵ and the $(p,3n)/(p,p2n)$ and $(p,4n)/(p,p3n)$

ratios for Co⁵⁹ bombarded with 60–100-MeV protons.¹⁶ We are aware of the existence of earlier data on additional targets but do not consider these results accurate enough for the present comparison.

The values of the parameters entering into the calculation are listed for the various cross-section ratios at the specified bombarding energies in Table I. The comparison with experiment is shown in Figs. 1 and 2. In the case of Y⁸⁹, Fig. 1, data are given for bombarding energies ranging from those corresponding to the peaks in the excitation functions up to 85 MeV. The mechanism discussed in the present work is expected to apply only at energies well above these peaks as compound-nucleus formation presumably is the important mecha-

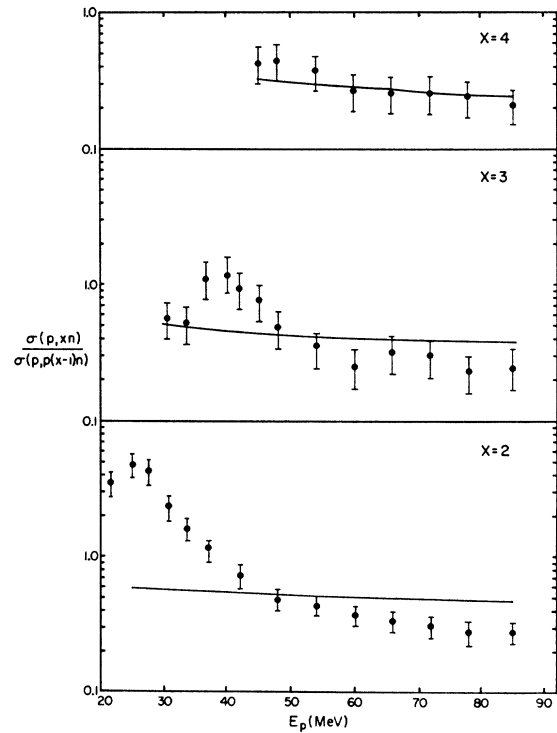


FIG. 1. Isobaric cross-section ratios for proton-induced reactions in Y⁸⁹. The solid line is the result of calculation and the experimental points are from Ref. 14. Results are shown for $x=2$, 3, and 4.

¹⁰ L. Winsberg and T. P. Clements, Phys. Rev. **122**, 1623 (1961).

¹¹ H. W. Bertini, Oak Ridge National Laboratory Report No. ORNL-3383, 1963 (unpublished); Phys. Rev. **131**, 1801 (1963).

¹² T. Hamada and I. D. Johnston, Nucl. Phys. **34**, 382 (1962).

¹³ F. Everling, L. A. Konig, J. H. E. Mattauch, and A. H. Wapstra, Nucl. Phys. **18**, 529 (1960).

¹⁴ G. B. Saha, N. T. Porile, and L. Yaffe, Phys. Rev. **144**, 962 (1966).

¹⁵ N. T. Porile, S. Tanaka, H. Amano, M. Furukawa, S. Iwata, and M. Yagi, Nucl. Phys. **43**, 500 (1963).

¹⁶ R. A. Sharp, R. M. Diamond, and G. Wilkinson, Phys. Rev. **101**, 1493 (1956).

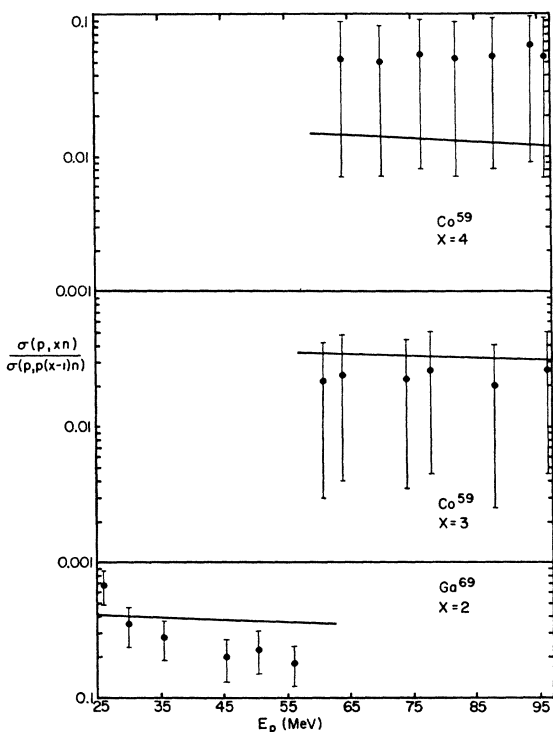


FIG. 2. Isobaric cross-section ratios for proton-induced reactions in Ga^{69} and Co^{59} . See Fig. 1 for details. The experimental values are from Refs. 15 and 16.

nism at peak energies. A look at the excitation functions¹⁴ indicates that the inelastic-scattering mechanism is probably of importance above approximately 40, 55, and 70 MeV for reactions involving respectively the emission of 2, 3, and 4 nucleons. It can be seen in Fig. 1 that experiment and calculation are in moderately good agreement in this energy range. About half the points differ from the calculated curves by less than one standard deviation and only about a quarter differ by more than 2σ . These discrepancies appear to be reasonable in view of the many approximations in the model.

It should also be pointed out that the contribution from the (p,d) pickup reaction has been ignored. This reaction will contribute to the (p,pxn) reactions and could therefore lead to lower isobaric ratios than those calculated. The most detailed information about the (p,d) reaction in the energy range of interest is available for carbon.^{17,18} It is apparent from the fact that the emitted deuterons preferentially populate the low-lying levels of the residual nucleus that the main contribution from the (p,d) reaction is to the (p,pn) cross section. Selove's results¹⁷ indicate that the cross section of the $\text{C}^{12}(p,d)\text{C}^{11}$ reaction at 95 MeV is about 6 mb. The dependence of the (p,d) cross section on target A is not too well known in this energy region but the available results¹⁷ suggest fluctuations of perhaps a factor of 2

for various targets ranging from beryllium to lead. It can be concluded that the (p,d) reaction probably only accounts for about 5–10% of the observed $\text{Y}^{89}(p,pn)$ cross section.

Returning to the comparison between experiment and calculation, it is seen in Fig. 1 that the ratios for $x=2$ and 3 at low energies are strikingly larger than the calculated values. This is the energy region where compound-nucleus formation predominates and one would therefore in general expect neutron emission to be at least as probable as proton emission. The experimental results amply confirm this expectation. In the case of the $(p,4n)$ reaction, however, proton evaporation has four chances to complete with neutron evaporation and so the isobaric ratio for $x=4$ does not attain large values even at low energies.

The behavior of the $(p,2n)/(p,pn)$ ratio for Ga^{69} , shown in Fig. 2, is roughly similar to that for Y^{89} and the same type of agreement with calculation is obtained.

The experimental errors of the isobaric ratios for cobalt are large¹⁹ and a very meaningful comparison with calculation is not possible. The data are only included because the isobaric ratios are nearly an order of magnitude smaller than the corresponding values for Y^{89} . It is seen that the calculation predicts equally low ratios. As shown in Table I this is a consequence of the unusually low values of the branching ratios for the emission of two or three neutrons from Ni^{59} . These low values primarily reflect the occurrence of shell closure in the nickel isotopes.

It is of interest to consider the relevance of the present model at higher energies. It is a well-known fact that the direct knockout of a proton and a neutron makes a major contribution to the (p,pn) reaction cross section at high energies. The knock-out process is also expected to be of importance for the more complex (p,pxn) reactions but of much smaller significance for the (p,xn) reactions. The isobaric ratios are thus expected to decrease with increasing energy as this additional mechanism becomes of importance. For instance, the ratios of (p,xn) to $(p,p(x-1)n)$ reaction cross sections for Ga^{69} at 1.5 GeV are 0.012, 0.015, and 0.003 for $x=2, 3$, and 4, respectively.²⁰ The same ratios¹⁵ at 56 MeV are 0.18, 0.12, and 0.03. Although the latter two values still reflect a contribution from the compound-nuclear process it is obvious that there is an order of magnitude decrease in the isobaric ratio between 50–100 MeV and GeV energies.

¹⁹ The experimental errors of the cobalt data were not directly available. As the authors state that the errors at energies well removed from the peaks in the excitation functions were substantially larger than those at the peaks, the values given for the latter were arbitrarily doubled to arrive at the error bars shown in Fig. 2. The relative yields of the cobalt nuclides have recently been remeasured [A. Ewart and M. Blann, *J. Inorg. Nucl. Chem.* **27**, 967 (1965)]. These results suggest that the cross sections for Co^{57} are about a factor of 3 larger than reported in Ref. 16. This result has been incorporated in Fig. 2.

²⁰ N. T. Porile, *Phys. Rev.* **125**, 1379 (1962).

¹⁷ W. Selove, *Phys. Rev.* **101**, 231 (1956).

¹⁸ P. Cooper and R. Wilson, *Nucl. Phys.* **15**, 373 (1960).

We have not performed a relativistic calculation in order to compare isobaric ratios obtained at high energies with the inelastic-scattering model. However, these ratios are not expected to differ drastically from those calculated for 100-MeV protons. Rensberg and Miller,⁶ in a relativistic treatment of a model substantially similar to ours, thus calculate values of 0.16 and 0.18 for the ratio of the ($p, 2n$) to (p, pn) reaction cross sections for Cr⁵² and Fe⁵⁶ at 370 MeV, respectively. By contrast, the corresponding experimental values are 0.014 and 0.012.

It is apparent from the above discussion that the inelastic-scattering model is not applicable to the calculation of isobaric ratios at proton energies corresponding to the peaks in the excitation functions because of the predominance of the compound-nuclear process. It is also seen that the contribution of the knock-out process invalidates the model at high energies. However there appears to be an intermediate energy region where the inelastic-scattering model seems to be applicable. The precise region of validity undoubtedly depends on the value of x and remains to be delineated.

PHYSICAL REVIEW

VOLUME 146, NUMBER 3

17 JUNE 1966

Elementary-Particle Treatment of Muon Capture in O¹⁶†

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(Received 20 January 1966)

The rate of the muon-capture process in O¹⁶ which leads to the 2⁻ ground state of N¹⁶ is calculated treating the initial and final nuclei as "elementary" particles. Nuclear form factors are evaluated on the basis of the conserved-vector-current and partially-conserved-axial-vector-current hypotheses using experimental data on the corresponding beta-decay and inelastic-electron-scattering processes. The resultant calculated rate $[\Gamma(\mu^- + O^{16}(0^+) \rightarrow N^{16}(2^-) + \nu_\mu)]_{\text{theor}} = (5.8 \pm 2.3) \times 10^3 \text{ sec}^{-1}$ is consistent with the measured value $[\Gamma(\mu^- + O^{16}(0^+) \rightarrow N^{16}(2^-) + \nu_\mu)]_{\text{exper}} = (6.3 \pm 0.7) \times 10^3 \text{ sec}^{-1}$ and is to be contrasted with a calculated rate of $(12 \pm 2) \times 10^3 \text{ sec}^{-1}$ obtained on the basis of a nuclear-model impulse-approximation approach.

I. INTRODUCTION

THE theory of the capture of muons by nuclei based on the fundamental hypotheses of muon-electron universality and conserved polar-vector and partially conserved axial-vector hadron weak currents (CVC and PCAC) has given a satisfactory explanation of the experimental data for muon capture by H, He³, and C¹². There exist, however, discrepancies between calculation and measurement in a few other cases. The asymmetry of the neutrons from muon capture in Ca⁴⁰ and the rate of the ground state \rightarrow ground state muon-capture process $\mu^- + O^{16}(0^+) \rightarrow N^{16}(2^-) + \nu_\mu$ are well-known examples. In this paper we shall discuss the latter.

Previous nuclear-model impulse-approximation calculations of the rate of $\mu^- + O^{16}(0^+) \rightarrow N^{16}(2^-) + \nu_\mu$ are^{1,2}

$$[\Gamma(\mu^- + O^{16}(0^+) \rightarrow N^{16}(2^-) + \nu_\mu)]_{\text{theor}} = (12 \pm 2) \times 10^3 \text{ sec}^{-1}, \quad (1)$$

whereas the measured value is³

$$[\Gamma(\mu^- + O^{16}(0^+) \rightarrow N^{16}(2^-) + \nu_\mu)]_{\text{exper}} = (6.3 \pm 0.7) \times 10^3 \text{ sec}^{-1}. \quad (2)$$

The discrepancy by a factor 2 between theory and experiment is presumably largely due to the inadequacies of the O¹⁶(0⁺) and N¹⁶(2⁻) wave functions provided by the nuclear models. A similar discrepancy is also found between calculated and measured values of the inelastic electron-scattering form factors of O¹⁶.⁴ It is our purpose here to calculate the $\mu^- + O^{16}(0^+) \rightarrow N^{16}(2^-) + \nu_\mu$ rate on the basis of the nuclear-model-independent method developed and used in previous papers for the treatment of nuclear beta decay⁵ and nuclear muon capture.⁶ This method as applied to the present problem consists of treating the O¹⁶(0⁺) and N¹⁶(2⁻) nuclei as "elementary" particles with the nuclear form factors involved evaluated on the basis of the CVC and PCAC hypotheses using experimental data on the corresponding beta decay, $N^{16}(2^-) \rightarrow O^{16}(0^+) + e^- + \bar{\nu}_e$ and inelastic electron-

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¹ I. Duck, Nucl. Phys. **35**, 27 (1962); T. Ericson, J. C. Sens, and H. P. C. Rood, Nuovo Cimento **34**, 51 (1964); V. Gillet and D. A. Jenkins, Phys. Rev. **140**, B32 (1965).

² The value quoted in Eq. (1) is taken from the survey work by H. P. C. Rood (CERN unpublished report, 1965).

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⁵ C. W. Kim and H. Primakoff, Phys. Rev. **139**, B1447 (1965).

⁶ C. W. Kim and H. Primakoff, Phys. Rev. **140**, B566 (1965).