Plasmon Excitation by Charged Particles Outside a Metal Film

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A detailed study is given of the interaction between a metal film and a charged particle outside the film. It is shown in particular that a point charge Ze moving with a given initial velocity u in a direction parallel to the film excites surface plasmons in the film as a result of which a reaction force acts on the moving charge. Provided u is much larger than the Fermi velocity of the metal, and assuming specular reflection for scattering of the metal electrons at film boundaries, the force is given approximately by

$$[(Ze)^2/(2z_0)^2](\tfrac{1}{8}\pi)^{1/2}\alpha^{3/2}e^{-\alpha};\ \alpha=\sqrt{2}\omega_p z_0/u,$$

for $\omega_p d/u\gg 1$ and $\alpha\gg 1$, and

$$[(Ze^2)/(2z_0)^2](\tfrac{1}{2}\pi)^{1/2}\beta^{3/2}e^{-\beta};\ \beta=d\omega_p{}^2 z_0/u,$$

for $\omega_p d/u\ll 1$ and $\beta\gg 1$, where z_0 is the distance between the moving charge and the film, ω_p the volume plasmon frequency, and d the thickness of the film. A brief discussion is also given of the case of diffuse reflection.

1. INTRODUCTION

THE analysis of the energy losses of charged particles in a metal provides a very useful method for the study of the dielectric properties of the metal. A considerable amount of theoretical and experimental work has been devoted to this subject.

For films of infinite thickness, theory predicts a characteristic energy loss of $\hbar\omega_p$ associated with the excitation of volume plasmons.[1] Recent work, the first of which is that of Ritchie, shows that, for finite thickness, boundary effects play an important role in the energy losses, and predicts an additional energy loss of $\hbar\omega_p/\sqrt{2}$ associated with the excitation of surface plasmons.[2] This prediction seems to have gained experimental support.[3]

The difficulty of the analysis is that there exist other mechanisms of energy loss, such as interband transitions, electron-electron scatterings, Bragg reflections (followed by plasmon excitations) and so on, which mix with those arising from the direct plasmon excitations. The existence of a macroscopic variable is required, which behaves differently in different mechanisms of excitation, and hence facilitates the analysis. It is expected that the thickness of a film plays the role of such variable. In fact the thickness dependence of the energy losses has also been observed previously.[4]

However, it seems to us that in previous theories the dielectric properties of a metal have been oversimplified so much that boundary effects are not properly taken into account. We believe for example that the result of a theory should depend on whether we use the assumption of specular reflection or that of diffuse reflection for scattering of the metal electrons at boundaries. Indeed, this occured in the theory of the anomalous skin effect.[5] These two assumptions are likely to lead to a remarkable difference in the thickness dependence of the energy losses.

The purpose of the present paper is to study the interaction between a metal film and a charged particle with particular care being taken of boundary effects. We shall simplify the problem by taking a special configuration in which the charged particle is outside the film.

In Sec. 2 the charged particle will be considered to be at rest. We shall discuss two limiting cases according to whether the film thickness is much larger or smaller than the screening radius of the metal.

Section 3 will be devoted to the dynamical problem in which the charged particle is moving with a given initial velocity in a direction parallel to the film. We shall see that surface plasmons are excited. The possibility of plasmon excitation by a charged particle outside a metal film has already been pointed out by Heidenreich,[6] and experimental work has also been done, but no satisfactory quantitative discussion has ever been given.[7]

2. STATIC POINT CHARGE

Consider a static point charge Ze outside a metal film of thickness d and with plane boundaries of infinite extension. In classical electrostatics,[8] it is usually considered that the electrostatic potential inside the metal

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[1] D. Pines, *Elementary Excitations in Solids* (W. A. Benjamin, Inc., New York, 1963).

[2] R. H. Ritchie, Phys. Rev. **106**, 874 (1957).

[3] C. J. Powell and J. B. Swan, Phys. Rev. **118**, 640 (1960); P. Schmueser, Z. Physik **180**, 105 (1964); A. Otto, *ibid.* **167**, 232 (1962).

[4] C. Kunz, Z. Physik **167**, 53 (1962); Schmueser, Ref. 3.

[5] G. E. Reuter and Sondheimer, Proc. Roy. Soc. (London) **A195**, 336 (1948).

[6] R. D. Heidenreich, J. Appl. Phys. **34**, 964 (1963).

[7] E. C. Shaffer, J. Silcox, and B. M. Siegel, J. Appl. Phys. **35**, 3079 (1964); A. J. F. Metherell, S. L. Cundy, and M. J. Whelan, International Conference on Electron Diffraction and Crystal Defects, Melbourne, 1965 (unpublished).

[8] See, for instance, W. Panofsky and M. Phillips (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1960), 1st ed., p. 44.

film must be a constant, which can be taken to be zero. Thus when the film is thin enough (we have no idea as to how thin the film must be), we can introduce an image point charge $-Ze$ to make the potential zero in the film. The image charge along with the real charge then determines the potential in that part of the region outside the film in which the real charge is located.

However, two objections can be raised against this conclusion. (1) Although the film is very thin, its thickness is finite; it is impossible to make the potential zero everywhere inside the film simply by introducing an image point charge. (2) The conclusion that the potential must be zero inside the film is based on the following argument. The system of conduction electrons in the metal is in a state of statistical equilibrium in the presence of the point charge. No electrostatic potential gradient should exist. Otherwise it induces an electric current, which should not exist in an equilibrium state. But this argument is incorrect.[9] In fact a state of statistical equilibrium exists in a closed system even in the presence of an electric field. A typical example for this may be provided by a metal with impurity atoms. In such a metal the electrostatic potential gradient due to impurity atoms completely balances the chemical potential gradient associated with a nonuniform distribution of metal electrons.

In this section this classical problem will be studied in detail from a microscopic point of view. We shall use a Cartesian coordinate system with the z axis normal to the film and passing through the point charge at $(0, 0, -z_0)$, where z_0 is the distance between the point charge and the film. We divide the whole space into three regions I, II, and III. Region II is occupied by the film, and regions I and III are those outside the film with $z<0$ and $z>d$, respectively.

Let the function $\phi(\mathbf{x})\equiv\phi(x,y,z)$ denote the electrostatic potential. It is determined by a set of Poisson equations and appropriate boundary conditions. These equations are:

$$\begin{aligned}
&\text{(I)} && \Delta\phi^{(1)}(\mathbf{x})=-4\pi Ze\delta(\mathbf{x}-\mathbf{x}_0); \quad \mathbf{x}_0=(0,0,-z_0),\\
&\text{(II)} && \Delta\phi^{(2)}(\mathbf{x})=\lambda^2\phi^{(2)}(\mathbf{x}), \qquad\qquad (2.1)\\
&\text{(III)} && \Delta\phi^{(3)}(\mathbf{x})=0.
\end{aligned}$$

The term $\lambda^2\phi^{(2)}(\mathbf{x})$ represents the screening charge induced in the film. The screening effect has been taken care of by the Thomas-Fermi model, which gives a constant screening radius λ^{-1}.[10]

It is convenient to introduce the Fourier transform of $\phi(\mathbf{x})$ with respect to the variables x and y;

$$\phi(\mathbf{x})=\sum_{\mathbf{q}}\Phi(\mathbf{q},z)e^{i\mathbf{q}\cdot\mathbf{r}},$$

where $\mathbf{r}$ is a vector in the x-y plane; $\mathbf{r}=(x,y,0)$. The Poisson equation for $\phi^{(1)}(\mathbf{x})$ then becomes

$$(\partial^2/\partial z^2)\Phi^{(1)}(\mathbf{q},z)-q^2\Phi^{(1)}(\mathbf{q},z)=-4\pi Ze\delta(z+z_0),$$

whose solution is

$$\Phi^{(1)}(\mathbf{q},z)=(2\pi Ze/q)e^{-q|z+z_0|}+A_{\mathbf{q}}e^{+qz}; \quad z<0,$$

and the Fourier inverse transformation leads to

$$\phi^{(1)}(\mathbf{q})=\sum_{\mathbf{q}}\{(2\pi Ze/q)e^{-q|z+z_0|}+A_{\mathbf{q}}e^{+qz}\}\times e^{i\mathbf{q}\cdot\mathbf{r}}; \quad z<0. \qquad (2.2\text{a})$$

In the same way we obtain

$$\phi^{(2)}(\mathbf{q})=\sum_{\mathbf{q}}\{B_{\mathbf{q}}\exp[-(q^2+\lambda^2)^{1/2}z]+C_{\mathbf{q}}\exp[(q^2+\lambda^2)^{1/2}z]\}e^{i\mathbf{q}\cdot\mathbf{r}}; \quad 0<z<d, \qquad (2.2\text{b})$$

and

$$\phi^{(3)}(\mathbf{q})=\sum_{\mathbf{q}}D_{\mathbf{q}}e^{-qz}e^{i\mathbf{q}\cdot\mathbf{r}}; \quad d<z. \qquad (2.2\text{c})$$

We use the boundary conditions

$$\begin{aligned}
\phi^{(1)}(x,y,0)&=\phi^{(2)}(x,y,0),\\
\phi^{(2)}(x,y,d)&=\phi^{(3)}(x,y,d), \qquad (2.3)\\
(\partial\phi^{(1)}(x,y,z)/\partial z)_{z=0}&=(\partial\phi^{(2)}(x,y,z)/\partial z)_{z=0},\\
(\partial\phi^{(2)}(x,y,z)/\partial z)_{z=d}&=(\partial\phi^{(3)}(x,y,z)/\partial z)_{z=d}.
\end{aligned}$$

The second pair of these relations involves the assumption that no surface charge is present in the microscopic sense.[11]

Substituting (2.2a) to (2.2c) into (2.3), we obtain

$$\begin{aligned}
A_{\mathbf{q}}&=(-2\pi Ze/q)e^{-qz_0}+4\pi Zee^{-qz_0}\frac{[(q^2+\lambda^2)^{1/2}+q]+[(q^2+\lambda^2)^{1/2}-q]\exp\{-2(q^2+\lambda^2)^{1/2}d\}}{[(q^2+\lambda^2)^{1/2}+q]^2-[(q^2+\lambda^2)^{1/2}-q]^2\exp\{-2(q^2+\lambda^2)^{1/2}d\}},\\
B_{\mathbf{q}}&=\frac{4\pi Ze[(q^2+\lambda^2)^{1/2}+q]e^{-qz_0}}{[(q^2+\lambda^2)^{1/2}+q]^2-[(q^2+\lambda^2)^{1/2}-q]^2\exp\{-2(q^2+\lambda^2)^{1/2}d\}},\\
C_{\mathbf{q}}&=\frac{4\pi Ze[(q^2+\lambda^2)^{1/2}-q]e^{-qz_0}\exp\{-2(q^2+\lambda^2)^{1/2}d\}}{[(q^2+\lambda^2)^{1/2}+q]^2-[(q^2+\lambda^2)^{1/2}-q]^2\exp\{-2(q^2+\lambda^2)^{1/2}d\}},\\
D_{\mathbf{q}}&=\frac{4\pi Zee^{-qz_0}2(q^2+\lambda^2)^{1/2}e^{qd}\exp\{-(q^2+\lambda^2)^{1/2}d\}}{[(q^2+\lambda^2)^{1/2}-q]^2-[(q^2+\lambda^2)^{1/2}-q]^2\exp\{-2(q^2+\lambda^2)^{1/2}d\}}.
\end{aligned} \qquad (2.4)$$

[9] Indeed, this argument, when applied directly to a thick film, leads to the trivial conclusion that the potential is zero everywhere outside or inside the film.

[10] C. Kittel, *Quantum Theory of Solids* (John Wiley & Sons, Inc., New York, 1963), p. 105.

[11] The induced charge $\lambda^2\phi^{(2)}(\mathbf{x})$ may be called the surface charge in macroscopic sense.

If we assume that

$$\lambda d \gg 1, \tag{2.5a}$$

and

$$q \ll \lambda, \tag{2.5b}$$

these solutions simplify to

$$A_q = (-2\pi Ze/q)e^{-qz_0} + (4\pi Ze/\lambda)e^{-qz_0},$$
$$B_q = (4\pi Ze/\lambda)e^{-qz_0},$$
$$C_q = (4\pi Ze/\lambda)e^{-qz_0}e^{-2\lambda d},$$
$$D_q = (8\pi Ze/\lambda)e^{-qz_0}e^{-\lambda d + qd}.$$

We then have

$$\phi^{(1)}(\mathbf{x}) = \sum_q (4\pi Ze/\lambda - 2\pi Ze/q)e^{-q(z_0-z)}e^{i\mathbf{q}\cdot\mathbf{r}} + \sum_q (2\pi Ze/q)e^{-q|z_0+z|}e^{i\mathbf{q}\cdot\mathbf{r}}.$$

The assumption $q \ll \lambda$ is justified when $z_0\lambda \gg 1$, i.e., when the distance between the point charge and the film is much larger than the screening radius. The inverse Fourier transformation is easily carried out [we replace the sum by the integral $(2\pi)^{-3}\int d^3q$] to give

$$\phi^{(1)}(\mathbf{x}) = \frac{Ze}{\{(z+z_0)^2+r^2\}^{1/2}} - \frac{Ze}{\{(z-z_0)^2+r^2\}^{1/2}} - \frac{2Ze(z-z_0)}{\lambda\{(z-z_0)^2+r^2\}^{3/2}}. \tag{2.6a}$$

The first two terms on the right-hand side clearly represent the classical result. The third term is equivalent to the potential due to a dipole moment $2Ze/\lambda$ located at $z=z_0$. This term is dominant only in the region limited by

$$r < z, \quad |z| < \lambda^{-1}.$$

We see that the classical result is correct for the most part of region I, provided the two distances z and d are much larger than λ^{-1}.

Similarly we find

$$\phi^{(2)}(\mathbf{x}) = \frac{4Ze}{\lambda}\frac{z_0}{\{z^2+r^2\}^{3/2}}e^{-\lambda d}ch\{\lambda(d-z)\}. \tag{2.6b}$$

In particular

$$\phi^{(2)}(x,y,0) \simeq \frac{2Ze}{\lambda}\frac{z_0}{(z_0^2+r^2)^{3/2}},$$

and

$$\phi^{(2)}(x,y,d) \simeq \frac{4Ze}{\lambda}e^{-\lambda d}\frac{z_0}{(z_0^2+r^2)^{3/2}}.$$

These are equivalent to the potentials due to dipole moments $2Ze/\lambda$ and $(4Ze/\lambda)e^{-\lambda d}$ at $z=z_0$ and z_0+d, respectively. From (2.6b) we see that the screening charge is located only in the surface layer of thickness λ^{-1}, a quite understandable result.

Finally we have

$$\phi^{(3)}(\mathbf{x}) = \frac{4Ze}{\lambda}e^{-\lambda d}\frac{z+z_0-d}{\{(z+z_0-d)^2+r^2\}^{3/2}}. \tag{2.6c}$$

This shows that the screening of the point charge is almost complete.

For degenerate electrons in metals, λ is of the order of $10^8(\text{cm}^{-1})$ so that Eqs. (2.5a) and (2.5b) are fulfilled in practice. For nondegenerate electrons, as in semiconductors or in ionized gases, the parameter λ^{-1} is given by the Debye length

$$\lambda^{-1} = (kT/4\pi ne^2)^{1/2}, \tag{2.7}$$

with n the electron density.[12] The magnitude of n is variable. For instance, the value $n \sim 10^{17}$ can easily be realized in semiconductors. At room temperatures then we have $\lambda^{-1} \sim 10^{-6}$ cm. This estimation suggests that the opposite extreme case $\lambda d \ll 1$ is worth consideration.

We take now

$$\lambda d \ll 1,$$
$$q \ll \lambda,$$

and the solutions (2.4) reduce to

$$A_q = O[(q/\lambda)^2, \lambda d],$$
$$B_q \cong C_q \cong (\pi Ze/q)e^{-qz_0},$$
$$D_q \cong (2\pi Ze/q)e^{-qz_0}.$$

These yield then

$$\phi^{(1)}(\mathbf{x}) = \frac{Ze}{\{(z+z_0)^2+r^2\}^{1/2}}; \quad z<0, \tag{2.8a}$$

$$\phi^{(2)}(\mathbf{x}) = \frac{Ze}{\{z_0^2+r^2\}^{1/2}}; \quad 0<z<d, \tag{2.8b}$$

$$\phi^{(3)}(\mathbf{x}) = \frac{Ze}{\{(z+z_0)^2+r^2\}^{1/2}}; \quad d<z. \tag{2.8c}$$

Because of the assumption $q \ll \lambda$, these formulas are valid for $z_0 \gg \lambda^{-1}$ and/or $r \gg \lambda^{-1}$. We conclude that there is no screening at all, and instead the potential is short-circuited in the z direction by the film, a result which is hardly expected in simple electrostatics.

3. MOVING POINT CHARGE

In the same configuration considered above, we now let the point charge move with a given initial velocity u in a direction parallel to the film. We take the x axis to be parallel to the velocity. The moving point charge induces in the film a time-dependent charge-current distribution, which in turn reacts on the point charge to change its energy and momentum.

[12] T. H. Hill, *An Introduction to Statistical Thermodynamics* (Addison-Wesley Publishing Company, Reading, Massachusetts, 1960), p. 32.

The Poisson equations (2.1a) to (2.1c) are replaced by the d'Alembert equations

$$\left(\Delta-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi^{(1)}(\mathbf{x},t)=-4\pi Ze\delta(\mathbf{x}-\mathbf{x}_0)\,, \quad (3.1a)$$

$$\left(\Delta-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi^{(2)}(\mathbf{x},t)=4\pi\int K(\mathbf{x}-\mathbf{x}',\,t-t')\phi^{(2)}(\mathbf{x}',t')\,d^3x'dt'\,, \quad (3.1b)$$

$$\left(\Delta-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi^{(3)}(\mathbf{x},t)=0\,, \quad (3.1c)$$

where $\mathbf{x}_0$ denotes the position vector of the moving charge: $\mathbf{x}_0=(ut,0,-z_0)$. We have used the classical trajectory of the moving charge (the Weizsäcker-Williams approximation), which is justified for small energy-momentum changes. To suppress relativistic effects we assume that u is much smaller than the light velocity c.

The term on the right-hand side of Eq. (3.1b) represents -4π times the charge density induced in the film.[13] The limits of the space integral in that expression depend on the particular boundary conditions for scattering of the metal electrons. In their study of the anomalous skin effect, Reuter and Sondheimer have discussed two extreme boundary conditions.[14] They assume either (i) specular reflection, i.e., that the trajectory of the metal electrons immediately after scattering from a boundary is the mirror reflection in the boundary of their trajectory immediately before scattering, or (ii) diffuse reflection, i.e., that a coherent translational motion of metal electrons completely disappears at the boundary.

If we use the assumption of specular reflection, the limits of the space integral range over the whole space, provided the function $\phi^{(2)}(\mathbf{x},t)$ is also extended over the whole space with the properties,

$$\begin{aligned}\phi^{(2)}(x,y,-z,t)&=\phi^{(2)}(x,y,z,t)\,,\\ \phi^{(2)}(x,y,z+2d,t)&=\phi^{(2)}(x,y,z,t)\,.\end{aligned} \quad (3.2)$$

It should be noted that discontinuity occurs in the quantity $\partial\phi^{(2)}/\partial z$ at $z=0,\pm d,\pm 2d,\cdots$.[15]

On the other hand, if we use the assumption of diffuse reflection, the integral is taken over region II. We shall first treat the case of specular reflection.

[13] By writing the relation in this form, we have neglected the crystal structure of the metal.

[14] Reference 5.

[15] The thickness d has been taken to be infinite in the study of Reuter and Sondheimer, and accordingly the second relation has been irrelevant in their problem.

We define the Fourier expansions

$$\phi^{(1)}(\mathbf{x},t)=\frac{1}{(2\pi)^3}\int dq_x\int dq_y\int d\omega\;\Phi_\omega^{(1)}(\mathbf{q},z)e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}\,,$$

$$\phi^{(2)}(\mathbf{x},t)=\frac{1}{d}\sum_n\frac{1}{(2\pi)^3}\int dq_x\int dq_y\int d\omega\;\Phi_\omega^{(2)}(\mathbf{q},n)\times e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}e^{if_nz}\,,$$

$$\phi^{(3)}(\mathbf{x},t)=\frac{1}{(2\pi)^3}\int dq_x\int dq_y\int d\omega\;\Phi_\omega^{(3)}(\mathbf{q},z)e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}\,,$$

with

$$f_n=n\pi/d;\quad n=0,\pm1,\pm2,\cdots.$$

Equation (3.1a) then gives

$$\frac{\partial^2}{\partial z^2}\Phi_\omega^{(1)}(\mathbf{q},z)-\left(q^2-\frac{\omega^2}{c^2}\right)\Phi_\omega^{(1)}(\mathbf{q},z)=-4\pi Ze\times2\pi\delta(\omega-uq_x)\delta(z+z_0)\,,$$

whose solution is

$$\Phi_\omega^{(1)}(\mathbf{q},z)=(2\pi Ze/q)e^{-q|z+z_0|}2\pi\delta(\omega-uq_x)+A_{\mathbf{q}}\exp\{(q^2-\omega^2/c^2)^{1/2}z\}\,, \quad (3.3a)$$

provided $q^2-\omega^2/c^2>0$. If this quantity is negative, then the factor $(q^2-\omega^2/c^2)^{1/2}$ must be replaced by $-i[(\omega^2/c^2)-q^2]^{1/2}$ to keep only outgoing waves. However, it turns out that this quantity is always positive. This has the important consequence that no electromagnetic waves are emitted by the film in contrast with the case of normal incidence, where, according to Ferrell,[16] the film does emit electromagnetic waves.

Similarly, Eq. (3.1b) leads to

$$\{-f_n^2-q^2+\omega^2/c^2\}\Phi_\omega^{(2)}(\mathbf{q},n)+2\Phi_\omega^{(2)\prime}(\mathbf{q},d)e^{-if_nd}-2\Phi_\omega^{(2)\prime}(\mathbf{q},0)=4\pi K_n(\mathbf{q},\omega)\Phi_\omega^{(2)}(\mathbf{q},n)\,,$$

with

$$K_n(\mathbf{q},\omega)=\int_{-\infty}^{\infty}K(\mathbf{x},t)e^{-if_nz}e^{-i(\mathbf{q}\cdot\mathbf{r}-\omega t)}\,d^3x dt\,,$$

$$\Phi_\omega^{(2)\prime}(\mathbf{q},d)=\int_{-\infty}^{\infty}(\partial\phi^{(2)}(\mathbf{x},t)/\partial z)_{z=d}e^{-i(\mathbf{q}\cdot\mathbf{r}-\omega t)}\,dxdydt\,,$$

$$\Phi_\omega^{(2)\prime}(\mathbf{q},0)=\int_{-\infty}^{\infty}(\partial\phi^{(2)}(\mathbf{x},t)/\partial z)_{z=0}e^{-i(\mathbf{q}\cdot\mathbf{r}-\omega t)}\,dxdydt\,.$$

The solution for $\Phi_\omega^{(2)}(\mathbf{q},n)$ is given by

$$\Phi_\omega^{(2)}(\mathbf{q},n)=\frac{2\Phi_\omega^{(2)\prime}(\mathbf{q},d)e^{-if_nd}-2\Phi_\omega^{(2)\prime}(\mathbf{q},0)}{f_n^2+q^2-(\omega/c)^2+4\pi K_n(\mathbf{q}\cdot\omega)}\,,$$

whence

$$\Phi_\omega^{(2)}(\mathbf{q},z)=\frac{1}{d}\sum_n\frac{\{\Phi_\omega^{(2)\prime}(\mathbf{q},d)e^{-if_nd}-\Phi_\omega^{(2)\prime}(\mathbf{q},0)\}e^{if_nz}}{f_n^2+q^2-(\omega/c)^2+4\pi K_n(\mathbf{q},\omega)}\,. \quad (3.3b)$$

[16] R. A. Ferrell, Phys. Rev. **111**, 1218 (1958).

Finally, Eq. (3.1c) takes the form

$$(\partial^2/\partial z^2)\Phi_\omega^{(3)}(\mathbf{q},z)-(q^2-(\omega/c)^2)\Phi_\omega^{(3)}(\mathbf{q},z)=0,$$

whence

$$\Phi_\omega^{(3)}(\mathbf{q},z)=D_\mathbf{q}\exp\{-[q^2-(\omega/c)^2]^{1/2}z\}. \quad (3.3c)$$

The boundary conditions (2.3) yield

$$(2\pi Ze/q)e^{-qz_0}2\pi\delta(\omega-uq_x)+A_\mathbf{q}=\Phi_\omega^{(2)}(\mathbf{q},0),$$
$$D_\mathbf{q}\exp\{-[q^2-(\omega/c)^2]^{1/2}d\}=\Phi_\omega^{(2)}(\mathbf{q},d),$$
$$-2\pi Zee^{-qz}\times 2\pi\delta(\omega-uq_x)+qA_\mathbf{q}=\Phi_\omega^{(2)\prime}(\mathbf{q},0),$$
$$-[q^2-(\omega/c)^2]^{1/2}D_\mathbf{q}\times\exp\{-[q^2-(\omega/c)^2]^{1/2}d\}=\Phi_\omega^{(2)\prime}(\mathbf{q},d).$$

Substituting (3.3a) to (3.3c) into these equations and eliminating $\Phi_\omega^{(2)\prime}(\mathbf{q},d)$ and $\Phi_\omega^{(2)\prime}(\mathbf{q},0)$, we obtain

$$A_\mathbf{q}+(2\pi Ze/q)e^{-qz_0}2\pi\delta(\omega-uq_x)=-\mu qD_\mathbf{q}e^{-qd}-\nu qA_\mathbf{q}+2\pi\nu Zee^{-qz_0}\times 2\pi\delta(\omega-uq_x),$$
$$D_\mathbf{q}e^{-qd}=-\nu qe^{-qd}D_\mathbf{q}-\mu qA_\mathbf{q}+2\pi\mu Zee^{-qz_0}\times 2\pi\delta(\omega-uq_x),$$

with

$$\nu=\frac{1}{d}\sum_n\frac{e^{-if_n0+}}{f_n^2+q^2+4\pi K_n(\mathbf{q},\omega)}=\frac{1}{d}\sum_n\frac{e^{if_n0+}}{f_n^2+q^2+4\pi K_n(\mathbf{q},\omega)},$$

and

$$\mu=\frac{1}{d}\sum_n\frac{e^{-f_nd}}{f_n^2+q^2+4\pi K_n(\mathbf{q},\omega)}=\frac{1}{d}\sum_n\frac{e^{if_nd}}{f_n^2+q^2+4\pi K_n(\mathbf{q},\omega)},$$

where we have used the fact that $K_n(\mathbf{q},\omega)$ is an even function of n [see Eq. (3.4)], and quantities of the order of $(u/c)^2$ have been neglected.

Solving these equations for $A_\mathbf{q}$, we obtain

$$A_\mathbf{q}=\frac{2\pi Ze}{q}e^{-qz_0}\times 2\pi\delta(\omega-uq_x)\frac{-1+(\nu^2-\mu^2)q^2}{(1+\nu q)^2-(\mu q)^2}.$$

The potential at the position of the moving point charge is obtained by subtracting the self-energy term [the first term on the right-hand side of Eq. (3.3a)] from $\phi^{(1)}(\mathbf{x},t)$, namely,

$$\phi^{(1)}(\mathbf{x}_0,t)=\frac{1}{(2\pi)^3}\int dq_x\int dq_y\int d\omega\, A_\mathbf{q}e^{-qz_0}.$$

The electric force acting on the moving charge is

$$\mathbf{F}=Ze[-\mathrm{grad}\phi^{(1)}(\mathbf{x},t)]_{\mathbf{x}=\mathbf{x}_0}.$$

Thus

$$F_x=Ze[-\partial\phi^{(1)}(\mathbf{x},t)/\partial x]_{\mathbf{x}=\mathbf{x}_0}$$
$$=\frac{(Ze)^2}{2\pi}\int dq_x\int dq_yd\omega\frac{q_x}{q}e^{-2qz_0}\times\delta(\omega-uq_x)\,\mathrm{Im}\frac{(\nu^2-\mu^2)q^2-1}{(1+\nu q)^2-(\mu q)^2},$$

where we have used the reality condition

$$K_{-n}(-\mathbf{q},-\omega)=\text{c.c. of }K_n(\mathbf{q},\omega).$$

In order to derive ν and μ we must evaluate the sum

$$S=\frac{1}{d}\sum_n\left(\frac{e^{-if_nz}}{f_n^2+q^2+4\pi K_n(\mathbf{q},\omega)}\right)_{\omega=uq_x}.$$

The quantity $K_n(\mathbf{q},\omega)$, often referred to as the generalized dielectric function, has been derived by many previous authors.[17] Assuming that the metal electrons are free and degenerate, we have

$$K_n(\mathbf{q},\omega)=(k^2-(\omega/c)^2)\Gamma_n(\mathbf{q},\omega)/k^2, \quad (3.4)$$

with

$$k^2=f_n^2+q^2,$$
$$4\pi\Gamma_n(\mathbf{q},\omega)=(k_0/2k)\{g(x_+)+g(x_-)\},$$

where

$$g(x)=x+\tfrac{1}{2}(1-x^2)\ln((x+1)/(x-1)),$$
$$x_\pm=(k/2k_0)\pm(\omega/kv_0).$$

The parameters λ^{-1}, v_0, and k_0 denote the screening radius, the Fermi velocity, and the Fermi wave number, respectively. It is understood that the logarithmic functions contain proper imaginary parts when their arguments are negative. But we shall be interested in the limit

$$|x_\pm|=|\omega/kv_0|\gg 1,\qquad k/2k_0\ll 1, \quad (3.5)$$

in which case no imaginary part appears. The function $\Gamma_n(\mathbf{q},\omega)$ then simplifies to

$$4\pi\Gamma_n(\mathbf{q},\omega)=k^2\{-(\omega_p/\omega)^2+i\epsilon\}, \quad (3.6)$$

where ϵ is a small real quantity introduced to allow for possible weak damping effect:

$$\epsilon>0 \text{ for } \omega>0,\qquad \epsilon<0 \text{ for } \omega<0. \quad (3.7)$$

The condition (3.5) requires first of all that the velocity u be much larger than v_0.

In this limit the sum S takes the form

$$S=\frac{1}{d}\frac{1}{1-(\omega_p/\omega)^2+i\epsilon}\sum_n\frac{e^{-if_nz}}{f_n^2+q^2}.$$

The sum is evaluated in Appendix A. We have

$$\sum_n\frac{e^{-if_nz}}{f_n^2+q^2}=\frac{1}{q}\frac{ch\{q(z-d)\}}{shdq},\quad\text{for}\quad 0<z<2d$$

whence

$$q\nu=(1-(\omega_p/\omega)^2+i\epsilon)^{-1}\,cthdq,$$
$$q\mu=(1-(\omega_p/\omega)^2+i\epsilon)^{-1}(shdq)^{-1}.$$

[17] The following formulas are quoted from the author's previous work: N. Takimoto, Progr. Theoret. Phys. (Kyoto) **25**, 327 (1961).

Now the reaction force **F** will be calculated in two limiting cases.

(I) Thick film. When the film is thick enough to satisfy

$$e^{-2dq} \ll 1\,, \tag{3.8}$$

i.e., when the film thickness is large compared to the plasmon wavelength, we can ignore μ compared to ν, which is given by

$$q\nu = (1-(\omega_p/\omega)^2+i\epsilon)^{-1}+O(e^{-2\mu q})\,.$$

Then it follows that

$$F_x = -(Ze)^2 \int dq_x \int dq_y \int d\omega \frac{q_x}{q} e^{-2qz_0} \times \delta(uq_x-\omega)\,\mathrm{sgn}(\omega)\,\delta(2-(\omega_p/\omega)^2)\,,$$

where we have used the relation

$$(x+i\epsilon)^{-1} = (P/x) - \pi i\,\mathrm{sgn}(\epsilon)\,\delta(x)\,,$$

and the fact that $\mathrm{sgn}(\epsilon)$ is the same as $\mathrm{sgn}(\omega)$ [see Eq. (3.7)].

The physical meaning of the two delta functions is obvious. The moving charge generates a distribution of electric fields. A resonance occurs between the fields and the collective motions (plasmons) in the metal at the frequencies $\omega = \pm\omega_p/\sqrt{2}$. Associated with the plasmon excitations, the momentum of the moving charge changes by $q_x = \pm\hbar\omega_p/\sqrt{2}u$.

The evaluation of the integrals is elementary; we have

$$F_x = \frac{-(Ze)^2\omega_p}{4u^2} \int_{\omega_p/\sqrt{2}u}^{\infty} \frac{dq}{q} \frac{e^{-2qz_0}}{[1-(\omega_p^2/2u^2q^2)]^{1/2}}$$

$$= \frac{-(Ze)^2}{(2z_0)^2}\frac{\alpha^3}{2} \int_1^{\infty} d\zeta\, \ln(\zeta+[\zeta^2-1]^{1/2})e^{-\alpha\zeta}\,,$$

with

$$\alpha = \sqrt{2}\omega_p z_0/u\,.$$

The assumption (3.8) is justified when

$$\sqrt{2}\omega_p d/u \gg 1\,. \tag{3.9}$$

For $\omega_p \simeq 10^{16}$/sec, which is the plasmon frequency in normal metals, and $u \simeq 10^9$ cm/sec, which complies with the condition (3.5), we have $d \gg 10^{-7}$ cm and $\alpha \simeq 10^7 z_0$. This indicates that the relation

$$\alpha \gg 1 \tag{3.10}$$

holds in practice. With this in mind we shall evaluate the remaining integral. We expand the logarithmic function in the integrand into the Taylor series at $\zeta = 1$. Then carrying out the integration, we arrive at the asymptotic expansion

$$F_x = (-(Ze)^2/(2z_0)^2)(\pi/8)^{1/2} \times \alpha^{3/2} e^{-\alpha}(1-9/8\alpha+\cdots)\,. \tag{3.11}$$

In the same way we can calculate F_z:

$$F_z = \frac{-(Ze)^2}{2\pi} \mathrm{Re} \int dq_x \int dq_y \int d\omega\, e^{-2qz_0} \times \delta(\omega-uq_x) \frac{(\omega_p/\omega)^2-i\epsilon}{2-(\omega_p/\omega)^2+i\epsilon}$$

$$= ((Ze)^2/(2z_0)^2)(1+3/\alpha^2+\cdots)\,. \tag{3.12}$$

We see that the leading term of F_z is the same as in the static case.

(II) Thin film. When the film is sufficiently thin to satisfy

$$e^{-2dq} \simeq 1\,, \tag{3.13}$$

we have

$$q\nu \simeq q\mu \simeq (1-(\omega_p/\omega)^2+i\epsilon)^{-1}(dq)^{-1}\,.$$

It follows then that

$$F_x = \frac{(Ze)^2}{2\pi} \mathrm{Im} \int dq_x \int dq_y \int d\omega \frac{q_x}{q} e^{-2dz_0} \times \delta(\omega-uq_x)(1+2\nu q)^{-1}$$

$$= \frac{-(Ze)^2}{2} \int dq_x \int dq_y \int d\omega \frac{q_x}{q} e^{-2qz_0} \times 2\delta(\omega-uq_x)\,\mathrm{sgn}(\omega)\,\delta(\omega^2-dq\omega_p^2/2)$$

$$= \frac{-(Ze)^2}{(2z_0)^2}\frac{\beta^2}{2} \int_1^{\infty} d\zeta \frac{\zeta^{1/2}}{(\zeta-1)^{1/2}} e^{-\beta\zeta}\,,$$

with

$$\beta = d\omega_p^2 z_0/u^2\,.$$

Evaluating the last integral in the same way as before, we find

$$F_x = \frac{-(Ze)^2}{(2z_0)^2}\frac{\sqrt{\pi}}{2}\beta^{3/2}e^{-\beta}\left\{1+\frac{1}{4\beta}+\cdots\right\}\,, \tag{3.14}$$

provided

$$\beta \gg 1\,.$$

The condition (3.13) goes over into

$$(\omega_p d/u)^2 \ll 1\,. \tag{3.15}$$

For the numerical values considered above, we have $dz_0 \gg 10^{-14}$, and $d \ll 10^{-7}$. The second condition is never realized in practice. It may, however, be realized in metals with extraordinary low plasmon frequencies.

Also we have

$$F_z = \frac{(Ze)^2}{(2z_0)^2}\left\{1+\left(1-\frac{3d}{4z_0}\right)\frac{1}{\beta}+\cdots\right\}\,. \tag{3.16}$$

In the case of diffuse reflection, it is very hard to obtain the general solution of the integral equation (3.1b) for arbitrary values of d. For $d=\infty$, however, one can use the method of Wiener and Hopf.[18] Detailed analysis is shown in Appendix B. The results are

$$F_x \simeq \frac{-(Ze)^2}{(2z_0)^2}\frac{v_0}{u}\frac{2}{\{\ln|r\alpha|/\pi\}^2+1}, \tag{3.17}$$

with r a numerical factor, and

$$F_z = ((Ze)^2/(2z_0)^2)\{1+O(v_0/u)\}, \tag{3.18}$$

for $e^{-\alpha} \ll 1$ and $z_0 \gg u/v_0 k_0$.

4. SUMMARY AND DISCUSSION

The screening of a static point charge outside a metal film was studied in detail. It was shown that, except for a minor correction, the classical result is correct, provided the film thickness d as well as the distance z_0 between the point charge and the film is much larger than the screening radius λ^{-1} of the metal; and that when d is much smaller than λ^{-1}, as may presumably occur in semiconductors, the electrostatic potential due to the point charge is Coulomb-like everywhere in the space, but is short-circuited by the film in the direction normal to it.

The method was then extended to the calculation of the reaction force $\mathbf{F}$ acting on a point charge outside the film, but moving with a given initial velocity u in a direction parallel to the film. Using the assumption of specular reflection, we found among others that the thickness of the film plays a significant role only when the film is sufficiently thin to satisfy

$$d \ll u/\omega_p.$$

This, however, is not realizable in normal metals. We also found that, when the inequalities $d \gg u/\omega_p$ and $\omega_p z_0/u \gg 1$ hold, the z_0 dependence of the reaction force F_x is very sharp. This entails some restriction on the velocity u. Let Δz_0 be the experimental error for the distance z_0. This error is insignificant if $\Delta z_0/z_0 \ll 1$. On the other hand the uncertainty principle leads to

$$\Delta z_0 \Delta v_z \gtrsim \hbar/m,$$

where Δv_z is the uncertainty in the velocity in the z direction. This relation is rewritten as

$$\frac{\Delta v_z}{u} \gtrsim \frac{\hbar\omega_p}{mu^2}\frac{u}{z_0\omega_p}\frac{z_0}{\Delta z_0}.$$

Roughly speaking, the two factors $(u/z_0\omega_p)$ and $(z_0/\Delta z_0)$ will cancel each other, so that

$$\Delta v_z/u \gtrsim \hbar\omega_p/E; \quad E = mu^2/2.$$

Given this relation we may conclude that quantum effects are negligible when

$$\hbar\omega_p \ll E,$$

i.e., when the energy transfer involved is much smaller than the energy of the incident particle.

The calculation based on the assumption of diffuse reflection was done only for the case $d=\infty$. A remarkable difference was obtained in the z_0 dependence of F_x. It is also expected that the d dependence of F_x is quite different from that in the case of specular reflection.

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APPENDIX A

We shall evaluate the sum (we assume $q \neq 0$)

$$I = \frac{1}{d}\sum_n \frac{e^{-if_n z}}{f_n{}^2+d^2}; \quad f_n = n\pi/d.$$

To begin with we notice that it has the integral form

$$I = \frac{1}{\pi}\int_c \frac{e^{-iz\zeta}}{\zeta^2+q^2}\frac{d\zeta}{e^{2id\zeta}-1},$$

where the path of integration c consists of two straight lines in the complex ζ plane, which enclose all poles of the function $[e^{2id\zeta}-1]^{-1}$ in counterclockwise, but not those of the function $[\zeta^2+q^2]^{-1}$.

We then deform the path in such a way that these two lines, one above and the other below the real axis, close themselves by making large semicircles in the upper and lower half-planes, respectively. Each of the poles at $\zeta = \pm iq$ is enclosed in either of these semicircles. If z satisfies the inequality

$$0 < z < 2d,$$

then contribution to the integral vanishes on the perimeters of the semicircles, as their radii go to infinity. Hence the integral with the deformed path is equal to the original one. Evaluation of the integral is now straightforward, and we obtain the result shown in Sec. 3.

APPENDIX B

When we use the assumption of diffuse reflection, we must solve the integral equation

$$\left\{\frac{\partial^2}{\partial z^2} - (q^2-(\omega/c)^2)\right\}\Phi_\omega{}^{(2)}(\mathbf{q},z) = 4\pi\int_0^d K_\omega(\mathbf{q}, z-z')\Phi_\omega{}^{(2)}(\mathbf{q},z')\,dz'.$$

We shall be interested only in the case $d=\infty$. For simplicity we put

$$\Phi_\omega{}^{(2)}(\mathbf{q},z) \equiv g(z),$$

$$K_\omega(\mathbf{q},z) \equiv k(z).$$

[18] Reference 5.

Then we have

$$\left(\frac{d^2}{dz^2}-q^2\right)g(z)=4\pi\int_0^\infty k(z-z')g(z')\,dz', \qquad \text{(B1)}$$

where the term of the order of $(u/c)^2$ has been ignored.

The boundary conditions are

$$(2\pi Ze/q)e^{-qz_0}2\pi\delta(\omega-uq_x)+A_{\mathbf{q}}=g(0),$$
$$-2\pi Ze e^{-qz_0}2\pi\delta(\omega-uq_x)+qA_{\mathbf{q}}=g'(0),$$

whence

$$A_{\mathbf{q}}=-\frac{g'(0)+qg(0)}{g'(0)-qg(0)}\frac{2\pi Ze}{q}e^{-qz_0}2\pi\delta(\omega-uq_x).$$

Thus the knowledge of the ratio $g'(0)/g(0)$ is required for the derivation of the reaction force $\mathbf{F}$.

Let us define the Fourier transform of $k(z)$ through

$$k(z)=(1/2\pi)\int_{-\infty}^\infty K(f)e^{ifz}\,df.$$

The general expression for $K(f)$ is given by Eq. (3.4) with f_n replaced by f, but in the following we shall use the simplified form (3.6) valid under the condition

$$f\ll qu/v_0\ll k_0.$$

It turns out that only those values of q satisfying $q\ll(2z_0)^{-1}$ are relevant. Therefore, the second inequality becomes

$$u/v_0k_0\ll z_0. \qquad \text{(B2)}$$

To comply with the condition $f\ll qu/v_0$ we introduce

$$4\pi K(f)\equiv 4\pi\Gamma_f(\mathbf{q},\omega)=k^2\{-(\omega_p/\omega)^2+i\epsilon\}\times C(f),$$

where $C(f)$ is a convergence factor with the property

$$C(f)\simeq 1 \quad \text{for} \quad f\ll qu/v_0,$$
$$C(f)\simeq 0 \quad \text{for} \quad f\gtrsim qu/v_0.$$

Equation (B1) can be solved with the use of the method of Wiener and Hopf (modified by Reuter and Sondheimer), provided $C(f)$ satisfies some proper requirements. The analytic properties of $C(f)$ are still arbitrary and can be chosen to make the problem as simple as possible. Here we assume that $C(f)$ is regular in the strip $|\mathrm{Im}(f)|<q$ and the equation

$$f^2+q^2+4\pi\Gamma_f(q,\omega)=0$$

has no solution in the same strip. For example we can take

$$C(f)=\left[\frac{(u/v_0)^2q^2}{f^2+(u/v_0)^2q^2}\right]^2.$$

Then the result of Reuter and Sondneimer can be directly used, and we obtain

$$g'(0)/g(0)=-(\nu_{\mathbf{q}}+q),$$

with

$$\nu_{\mathbf{q}}=\frac{1}{\pi}\int_0^\infty df\ln\left[\frac{f^2+q^2+4\pi\Gamma_f(\mathbf{q},\omega)C(f)}{f^2+q^2}\right]$$
$$\simeq\left(\frac{1}{\pi}\right)\int_0^{uq/v_0}\ln\left\{1-\left(\frac{\omega_p}{\omega}\right)^2+i\epsilon\right\}df$$
$$=(1/\pi)(qu/v_0)\ln\{1-(\omega_p/\omega)^2+i\epsilon\}. \qquad \text{(B3)}$$

The reaction force F_x is calculated from

$$F_x=-(1/2\pi)^3\int dq_x\int dq_y\int d\omega\,(-iq_x)e^{-2z_0q}$$
$$\times 2\pi(Ze)^2/q\times 2\pi\delta(\omega-uq_x)\frac{-(\nu_{\mathbf{q}}+q)+q}{-(\nu_{\mathbf{q}}+q)-q}.$$

Since we have

$$\nu_{-\mathbf{q}}=\text{c.c. of }\nu_{\mathbf{q}},$$

the expression for F_x reduces to

$$F_x=\frac{(Ze)^2}{\pi}\int_0^\infty dq\,q^2e^{-2qz_0}$$
$$\times\int_{-\pi}^{\pi}d\theta\,\cos\theta(\mathrm{Im}[\nu_{\mathbf{q}}+2q]^{-1})_{\omega=uq_x}.$$

Now, according to Eq. (B3), we have

$$\mathrm{Im}[q/(\nu_{\mathbf{q}}+2q)]$$
$$=-(v_0/u)\,\mathrm{sgn}(\omega)\delta(\ln\{1-(\omega_p/\omega)^2\}), \quad \text{for} \quad |\omega|>\omega_p$$
$$=\frac{-(u/v_0)\,\mathrm{sgn}(\omega)}{[(u/\pi v_0)\ln\{(\omega_p/\omega)^2-1\}+2]^2+(u/v_0)^2}, \quad \text{for} \quad |\omega|\leqq\omega_p$$

where ω stands for $uq\cos\theta$. In the region $|\omega|>\omega_p$, the expression is nonvanishing only at $|\omega|=\infty$, or $q=\infty$, but because of the factor $\exp(-2z_0q)$ it has no contribution to the integral. Hence

$$F_x=\frac{-2(Ze)^2u}{\pi v_0}\left(\int_0^{\omega_p/u}dq\int_{-\pi/2}^{\pi/2}d\theta+\int_{\omega_p/u}^\infty dq\int_{-\theta_0}^{\theta_0}d\theta\right)qe^{-2z_0q}$$
$$\times([(u/\pi v_0)\ln\{(\omega_p/\omega)^2-1\}+2]^2+(u/v_0)^2)^{-1}$$

where θ_0 is defined by

$$\cos\theta_0=\omega_p/uq; \quad 0<\theta_0<\pi/2.$$

The integrand has a peak at

$$\omega=\omega_p/(1+e^{-2\pi v_0/u})^{1/2}$$
$$\simeq\omega_p/\sqrt{2},$$

i.e., at the surface plasmon frequency. The peak is not sharp in contrast with the case of specular reflection. Namely, diffuse reflection of metal electrons at the boundary broadens the plasmon resonance peak.

We change the integration variables to

$$2z_0q=x\,,\quad \cos\theta=y\,.$$

These yield

$$F_x=\frac{-(Ze)^2}{(2z_0)^2}\frac{4u}{\pi v_0}\bigg(\int_0^{\sqrt{2}\alpha}dx\int_0^1 dy+\int_{\sqrt{2}\alpha}^{\infty}dx\int_0^{\sqrt{2}\alpha/x}dy\bigg)\frac{e^{-x}}{(1-y^2)^{1/2}}\times([(u/v_0\pi)\ln\{(2\alpha^2/x^2y^2)-1\}+2]^2+(u/v_0)^2)^{-1}.$$

For $\exp(-\sqrt{2}\alpha)\ll 1$, the second integrals can be omitted. The first integrals will be evaluated approximately. We notice that the factor with the logarithmic function is a slowly varying function of x and y, and may be replaced by a constant. The evaluation of the remaining integrals is then elementary and we arrive at

$$F_x\simeq\frac{-(Ze)^2v_0}{(2z_0)^2u}\frac{2}{\{\ln(r\alpha^2)/\pi\}^2+1}\,,$$

where r is a numerical factor ($r>0$). The α dependence of r is not clear, but it does not affect qualitative properties of F_x. (Rough estimation shows that r is of the order of 10 and is independent of α.) Thus we see that the α dependence of F_x is much less sharp than in the case of specular reflection.

Similarly, one can derive

$$F_z=(-(Ze)^2/(2z_0)^2)\{1+O(v_0/u)\}\,. \qquad \text{(B4)}$$

We have retained only collective aspect of the dielectric function, when we introduced the factor $C(f)$, but this has been enough to get a rough idea of the difference between specular and diffuse reflection. It is worthwhile to point out that, if the quantity $\Gamma_f(q,\omega)$ were independent of f, as in the Thomas-Fermi model for the static problem, there is no difference at all between two reflections. This is obvious, for the charge-potential relation then reduces to a local one. However, in the problem which concerns us, this quantity depends strongly on f. Indeed, the f dependence is such that a space charge in the specimen has a long-range effect. Therefore, dynamical properties of plasmons are sensitive to surface scattering. Thus it is not surprising that a remarkable difference in F_x exists between specular and diffuse reflection.

Errata

Positron Annihilation in Solid Argon, K. L. ROSE AND S. DEBENEDETTI [Phys. Rev. **138**, A927 (1965)]. An error in the potential used in the numerical computation of the positron wave function was found. This error does not change any of the formulas, but modifies somewhat the calculated curves; the agreement between theory and experiment is not as good as shown in Fig. 6. Corrected curves have appeared in a paper "Theory of Positron Lifetime in Solid Argon: The Effect of Correlation" by E. J. Woll and K. L. Rose, Phys. Rev. **145**, 258 (1966).