to relate φ to its constituents we cannot even attempt to explain this observed change of sign with volume.

Finally, it is possible that if measurements could be made on the fcc phase at pressures below 23 kbar it would be observed that T_c first decreases with applied pressure and then increases. We think that very pure fcc samples would be required in order to make useful observations of this kind. However, as the "pure" fcc phase of lanthanum readily converts to a mixture of the dhcp and fcc phases at room temperature upon even slight straining[13] there seems to be little hope of conducting such measurements. Instead we would prefer to see a repeat of the thermal-expansion measurements of Andres on fcc lanthanum in order to confirm the present interesting situation.

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New Relationship for Superconductors

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A relationship between the energy gap of superconductors at zero temperature and the slope of the critical magnetic field at the critical temperature is presented. Thermodynamical argument gives the dependence of this relationship on the electronic specific heat near the critical temperature and an approximate formula for the latter is proposed. The conclusions are compared with experimental data.

I. INTRODUCTION

Recently, Toxen[1] proposed a relationship between the energy gap of superconductors at zero temperature [$\Delta(0)$] and the slope of the critical magnetic field at the critical temperature (T_c). Toxen showed, for example, that BCS[2] theory (in the weak-coupling limit) satisfies his relation. It was pointed out[3] that the Toxen relationship is, probably, a numerical coincidence. In this paper we amplify the argumentation which is given in Ref. 3. Thus the thermodynamics of Lewis's[4] model is given (without recourse to the two-fluid model) in Sec. II.

We discuss the experimental aspects of the problem in Sec. III and mention explicitly the experimental results which are useful in discriminating between the Toxen relation and the curve of Ref. 3.

II. THERMODYNAMIC DERIVATION OF LEWIS' FORMULA

Lewis[4] derived a relationship between a parameter (α) appearing in an approximate formula for the specific heat of superconductors (C_s), and the slope of the critical field at the critical temperature. Lewis' derivation (as are those of Refs. 5 and 6) is associated with the two-fluid model for superconductors. We shall outline a thermodynamic derivation of the results without recourse to the two-fluid model. A more general formula—for arbitrary specific-heat curve—is also given. It is valid for superconductors which show the Meissner effect.

In what follows we make the usual assumptions with regard to the separability of the electronic and lattice contributions, and also assume that the latter is unaffected by the transition to the superconducting state. These assumptions are listed and discussed, for example, in Ref. 7.

The electronic specific heat for the superconductor is approximated[4] in the vicinity of T_c:

$$C_s/\gamma T_c = A e^{-\alpha(T_c/T)}, \quad T<T_c. \tag{1}$$

Here T is the temperature and γ is given by the electronic specific heat of the normal metal (C_n) at low temperatures, i.e.

$$C_n=\gamma T.$$

Our problem here is to eliminate A and γ in terms of other quantities. At T_c the entropies of the two phases, superconducting and normal, are equal,[4] viz.

$$\int_0^{T_c} C_s \frac{dT}{T} = \gamma T_c,$$

or

$$AE(\alpha)=1, \tag{2}$$

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where

$$E(\alpha)=\int_0^1 \frac{e^{-\alpha/x}}{x}\,dx.$$

This determines A. (Remark: The degree of agreement with the experiment involved here will be discussed in the next section, e.g. Fig. 2.)

In order to eliminate γ we utilize the following general thermodynamic relation[6] (which is valid for superconductors exhibiting the Meissner effect)

$$C_n(T)-C_s(T)=-(T/8\pi)d(H_c^2)/dT. \qquad (3)$$

Here $H_c(T)$ is the critical field. Since $H_c(T_c)=0$, one obtains the Rutgers' relation,

$$C_n(T_c)-C_s(T_c)=-(T_c/4\pi)(dH_c/dT)^2. \qquad (4)$$

On the other hand, by integrating Eq. (3) we get

$$\int_0^{T_c} C_n(T)dT-\int_0^{T_c} C_s(T)dT=-\frac{H_0^2}{8\pi}. \qquad (5)$$

(Remark: The extension of this equation to hard superconductors is complicated by their unknown magnetization curve.)

Substituting Eq. (1) into Eq. (5) and integrating gives,

$$(\gamma T_c/E(\alpha))[e^{-\alpha}-\alpha E(\alpha)]-\tfrac{1}{2}\gamma T_c=\mathrm{H}_0^2/8\pi. \qquad (6)$$

Dividing Eq. (4) by Eq. (6) gives the Lewis result

$$-\frac{T_c}{H_0}\left(\frac{dH_c}{dT}\right)_{T_c}=\left[\frac{e^{-\alpha}-E(\alpha)}{2e^{-\alpha}-(1+2\alpha)E(\alpha)}\right]^{1/2}\equiv F(\alpha), \qquad (7)$$

where $F(\alpha)$ is defined by the above equation. Equation (7) is the basic equation of Ref. 3.

If one does not assume the temperature dependence of $C_s(T)$ as given by Eq. (1), instead of Eq. (7), one obtains the more general formula

$$-\frac{T_c}{H_0}\left(\frac{dH_c}{dT}\right)_{T_c}=\left\{[1-C_s(T_c)/\gamma T_c]\Big/\left[1-\int_0^{T_c} C_s(T)dT/\tfrac{1}{2}\gamma T_c^2\right]\right\}^{1/2}. \qquad (8)$$

This general formula shows directly that the left-hand side is determined by C_s at T_c and in the immediate neighborhood thereof. This is due to the fact that C_s decreases very rapidly with decreasing temperature.

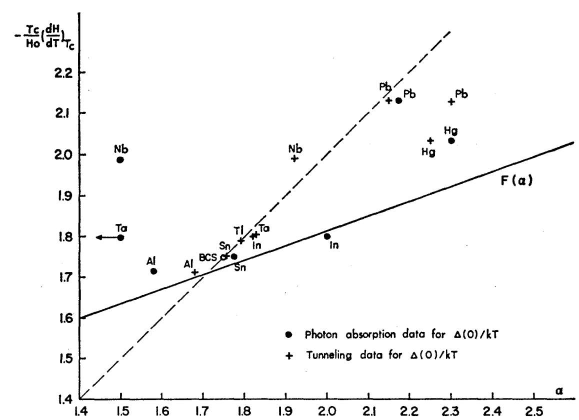

FIG. 1. $F(\alpha)$ as function of α [cf. Eq. (7)]. Experimental points are taken from: abscissas from Ref. 8, ordinates from Ref. 1. The dotted line represents Toxen's relation.

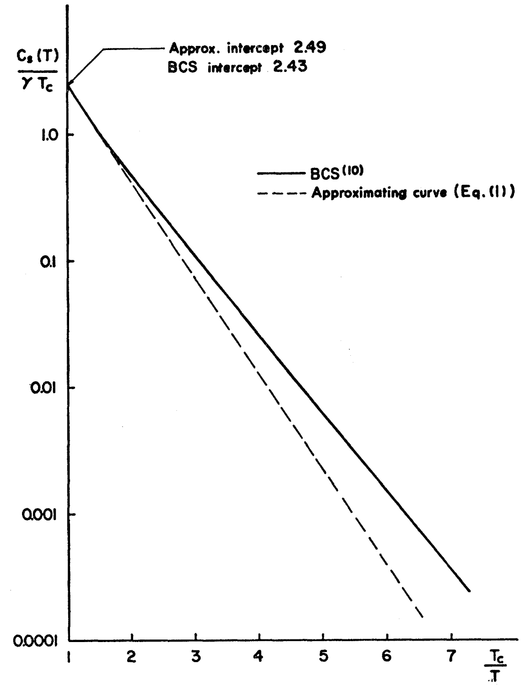

FIG. 2. The specific heat from BCS (Ref. 10) theory as a function of T_c/T. The straight line shows the approximation used for the region close to T_c. The area under the curves,

$$\int_0^{T_c} \{C_s(T)/C_n(T_c)\}dT/T_c,$$

equals 0.74 for BCS (Ref. 10) and 0.75 for approximating curve [Eq. (1)].

III. EXPERIMENTAL ASPECTS OF THE PROBLEM

There are various difficulties in comparing the thermodynamic results given above with the experimental results. The major difficulty lies in the fact that the published experimental values differ considerably among themselves. For the purposes of this paper we choose the energy-gap values as obtained by the photon-absorption and tunneling methods whenever these results are available. The experimental values are taken from Tables 5.1 and 5.5, respectively, in the review article by Douglass and Falicov[8] where the relevant references are given.

A comparison of the theory with experimental results is given in Fig. 1. [Note that the theoretical curve, $F(\alpha)$, is approximated to better than 1% by

$$F(\alpha)\simeq 1.1+(5/14)\alpha, \quad 1.4<\alpha<2.8$$

for the range of α, which is of interest here.] Here the experimental values of α are identified with $\Delta(0)/kT_c$.[3] The agreement with the experimental points is seen to be fair.

It follows from the approximation scheme adopted here [Eqs. (1) and (2)] that for a given $\Delta(0)/kT_c$, $C_s(T_c)$ is determined with *no* adjustable parameters. The agreement with the actual values of $C_s(T_c)$ is surprisingly good for T near T_c (which is the range of interest). Figure 2 shows this for the case of BCS.[9]

In closing this section we would like to remark once again that the scale of our Fig. 1 is large compared with the accuracy of the experimental results available. For example, the values of $2\alpha=2\Delta(0)/kT_c$ for aluminium obtained by photon absorption is 3.16,[8] while the value of 2α obtained from tunneling experiment is 3.37 ± 0.01.[8] This is illustrated in Fig. 1. The difficulties in determining $C_s(T_c)/\gamma T_c$ precisely are well known; here one has to determine the lattice contribution, which in some cases (e.g. indium) are appreciable, and to determine γ.[10]

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IV. REMARKS AND CONCLUSIONS

No attempt was made to fit the experimental points to theory which would refine the results of Ref. 3. The theoretical curve is in good agreement with the experimental result. In particular, in view of the fact that the empirical rule proposed in this paper, namely that C_s near T_c is given by Eq. (2) with α identified with $\Delta(0)/kT_c$, does *not* contain any adjustable parameters. Should further investigations prove our formula to be correct, a more general theory than the BCS weak-coupling-limit theory[1] will be required to account for it. Furthermore, analysis of measurements on hard superconductors along the lines outlined in this paper could shed light on the magnetization of such materials.

To discriminate between the results offered here and Toxen's, better measurements are needed for elements wherein $\alpha>1.85$ and $\alpha<1.55$. A good candidate for this is zinc ($\alpha=1.5$), where no recent value for H_0 is available.

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Energy Loss of Fission Fragments in Light Materials*

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A calibrated silicon detector provided range-energy data for Cf^{252} fission fragments in H_2, D_2, N_2 and Mylar. Bohr's classical stopping-power theory and Lindhard's more recent Thomas-Fermi model both underestimate the measured energy losses. For both theories, the discrepancy with experiment is a monotonic function of the atomic number of the stopping material, at least for the limited range of materials studied.

INTRODUCTION

At the beginning of its range, a fission fragment loses energy chiefly by ionization and excitation of the stopping material. Bohr's classical treatment of electronic energy loss by a charged particle[1] gives

$$-\frac{dE}{dx}=\frac{4\pi z^2e^4}{m_ev^2}NZ_2\ln\left\{\frac{1.123m_ev^3\hbar}{ze^2I}\right\}, \qquad (1)$$

where z and v are the ionic charge and velocity of the particle; m_e and e are the electronic rest mass and charge, N and Z_2 are the atomic density and number of the stopping material; and I is a mean excitation and ionization potential for the stopping material. The derivation of (1) requires that $2ze^2/\hbar v\gg1$ and that $Z_2e^2/\hbar v\ll1$.

Other early treatments resulted in different arguments for the logarithmic term in Eq. (1). Bethe[2] employed the Born approximation which requires that $2ze^2/\hbar v\ll1$, a condition that cannot apply to fission fragments because of their high ionic charges and relatively low velocities. Bloch[3] included the perturbation of the electronic wave function by the incident particle

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