Study of the Reactions $p\bar{p}\rightarrow Y\bar{Y}$ by Regge-Pole Exchange Model

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The reactions $p\bar{p}\rightarrow\Lambda\bar{\Lambda}$, $p\bar{p}\rightarrow(\Lambda\bar{\Sigma}^0+\bar{\Lambda}\Sigma^0)$ and $p\bar{p}\rightarrow\Sigma^+\bar{\Sigma}^-$ are studied using a Regge-pole exchange model. Only the contribution of the leading trajectory, which in this case is K^*, is retained. In analogy with ρ, we assume a linear trajectory for K^* with a slope close to that of ρ, and neglect the t dependence of the residues, after the threshold factor has been extracted out. The energy and angular dependence of the cross sections so obtained are in reasonable agreement with experiment.

INTRODUCTION

It is known that Regge poles have been proposed as a convenient means for the phenomenological description of high-energy scattering. The description of elastic scattering in terms of Regge poles has, however, been plagued with uncertainties arising from the fact that several Regge poles, with unknown trajectory parameters, have to be exchanged.[1] On the other hand, if one considers inelastic and charge-exchange reactions, one has many processes which may be supposed to be dominated by the exchange of a single Regge trajectory, and these may therefore provide more convenient tests for Regge-pole phenomenology. Charge-exchange π^-p and K^-p scattering, and some isobar production processes, which may be supposed to be dominated by the exchange of the ρ trajectory, have thus been described adequately in this fashion.[2,3] This has been all the more encouraging since the absorption model is known to fail rather badly[3,4] for these processes.

The absorption model has also been applied to another set of processes[5,6] dominated by a vector meson exchange—the K^* exchange—namely

(A) $p\bar{p}\rightarrow\Lambda\bar{\Lambda}$,
(B) $p\bar{p}\rightarrow\Lambda\bar{\Sigma}^0+\Sigma^0\bar{\Lambda}$,
(C) $p\bar{p}\rightarrow\Sigma^+\bar{\Sigma}^-$.

Here again the absorption model gives forward peaks that are too broad for reasonable magnitudes of tensor couplings, and predicts cross sections increasing with energy whereas the observed cross sections fall down rapidly with increasing energy. Encouraged by the earlier success of Regge pole exchange model[2,3] in eliminating these difficulties, we proceed to study these processes by Reggeizing the exchanged K^* meson.

The other exchanges possible are those of K and the newly discovered $K^{**}(1400)$ resonance. Because of its lower spin, the K meson has a trajectory much below that of K^*. Then, for K and K^* coupling constants of the same order of magnitude (which happens to be the case when one takes the photoproduction estimate for g_{KNY} and estimates g_{K^*NY} from $g_{\rho NN}$ using symmetry considerations), the former has a much smaller contribution in the GeV range. Thus, similar to the case of absorption model calculations,[5,6] the K exchange can be neglected here as well. There may of course be some complication due to the $K^{**}(1400)$ resonance. The spin-parity assignment for this is still uncertain,[7] but the assignment 2^+ is favored partly in order to associate it to f and A_2 in the same octet. Of course, even with a spin-2 assignment, the K^{**} trajectory is expected to lie below the K^* trajectory in the region $t\approx0$, on the basis of A_2-trajectory estimates.[8] But in this case, K^{**} contribution would not be negligible in a few GeV range, if K^{**} and K^* couplings to baryon-antibaryon systems are of the same order of magnitude. In absence of any knowledge of the K^{**}-coupling strengths, however, we assume this to be small, so that K^*-exchange alone may be supposed to provide the dominant force.

The K^*-exchange contributions to the helicity amplitudes and hence to the differential cross-sections for these three processes are written down in the following section. In accordance with the ρ-trajectory estimates,[2] and the fact that ρ and K^* belong to the same $SU(3)$ multiplet, we assume a linear trajectory $\alpha(t)$ for the K^*, so that the latter can be characterized by a single, parameter, which is the slope α'. The slope parameter is, of course, not completely arbitrary, since its value cannot be far from that of ρ trajectory. Again in analogy with ρ,[2,3] the residues of the K^* trajectory (with the threshold dependence extracted out) are assumed to have weak t dependence. Neglecting this t dependence, the residues can be expressed in terms of the field theoretic coupling constants using the fact that Regge pole exchange becomes identical to the corresponding elementary particle exchange at the pole point. These coupling constants, however, are not known *a priori*[9];

[1] B. M. Udgaonkar, Phys. Rev. Letters **8**, 142 (1962); W. Rarita and R. J. N. Phillips, Phys. Rev. **139**, B1336 (1965). For a review, see, for example, B. M. Udagaonkar, in *Strong Interactions and High Energy Physics* (Oliver and Boyd, London, 1964).
[2] R. K. Logan, Phys. Rev. Letters **14**, 414 (1965).
[3] D. P. Roy, Nuovo Cimento **40A**, 513 (1965); **40A**, 1212 (1965).
[4] V. Barger and M. Ebel, Phys. Rev. **138**, B1148 (1965).
[5] N. J. Sopkovich, Nuovo Cimento **26**, 186 (1962); and Ph.D. thesis, Carnegie Institute of Technology (unpublished).
[6] H. Högaasen and J. Högaasen, Nuovo Cimento **40A**, 560 (1965).
[7] N. Haque *et al.*, Phys. Letters **14**, 338 (1965); S. Focardi *et al.*, *ibid.* **16**, 351 (1965); L. M. Hardy *et al.*, Phys. Rev. Letters **14**, 401 (1965).
[8] R. J. N. Phillips and W. Rarita, Phys. Letters **19**, 598 (1965).
[9] g_{K^*NY} can be estimated from $g_{\rho NN}$ using $SU(3)$, if we assume pure F-type vector coupling for the VBB vertex. But such an assumption is ambiguous. [See, for instance, V. Barger and M. H. Rubin, Phys. Rev. **140**, B1365 (1965).]

and we shall determine them by normalizing the total cross sections for reactions (A) and (B) to the experimental values at a given energy.

THE K^*-EXCHANGE HELICITY AMPLITUDES AND CROSS SECTIONS FOR $p\bar{p}\rightarrow Y\bar{Y}$

To begin with we treat the exchanged K^* meson as an elementary particle. The one-particle exchange contribution to the processes $p\bar{p}\rightarrow Y\bar{Y}$, is illustrated in Fig. 1. Here a, b, c, d, and e refer to the 4-momenta of the particles as indicated. The Feynman amplitude for these processes is given by

$$M=\bar{u}(-a)g_1\left\{\gamma_\mu-\frac{R_1}{m_N+m_Y}\sigma_{\mu\rho}e_\rho\right\}u(-c)\frac{\delta_{\mu\nu}+e_\mu e_\nu/m^2}{m^2-t}$$

$$\times F(t)\bar{u}(d)g_2\{\gamma_\nu+[R_2/(m_N+m_Y)]\sigma_{\nu\tau}e_\tau\}u(b) \quad (1)$$

with

$$\sigma_{\mu\rho}=(1/2i)(\gamma_\mu\gamma_\rho-\gamma_\rho\gamma_\mu).$$

Here g_1, g_2 refer to the rationalized-vector coupling constants at the two vertices and R_1g_1, R_2g_2 are the coupling constants for tensor coupling. m_N, m_Y and m are the nucleon, hyperon, and K^* mass, respectively, and $F(t)$ is the off-mass-shell form factor for the K^*-propagator and the two vertices, which becomes unity at the pole point. If the K^* is now treated as a Regge pole, the factor $F(t)/(m^2-t)$ occurring in (1) is expected to be replaced by[10,11]

$$X=(2\alpha+1)\left[\frac{P_\alpha(-x_t)-P_\alpha(x_t)}{2\sin\alpha\pi}\right]\left(\frac{p_t^2}{m_Nm_Y}\right)^\alpha\beta, \quad (2)$$

where p_t is the c.m. momentum for the t channel. The first two factors arise from the Sommerfeld-Watson transformation of the amplitude in the t channel. The third term brings out the threshold dependence of the residue of the K^*-trajectory, and β is a slowly varying function of t. We shall neglect the t dependence of β.

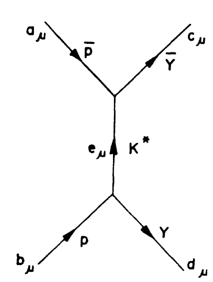

FIG. 1. K^*-exchange diagram for the processes $p\bar{p}\rightarrow Y\bar{Y}$.

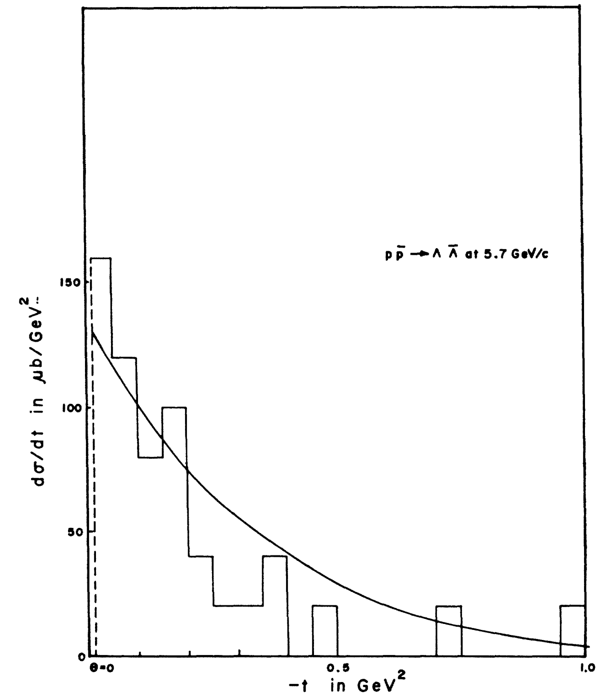

FIG. 2. The differential cross section for the process $p\bar{p}\rightarrow\Lambda\bar{\Lambda}$ at 5.7 GeV/c. The experimental histogram is from Ref. 14 (also quoted in Ref. 6).

Now for large s ($s\gg t$), we have

$$x_t=[s-(m_N^2+m_Y^2)]/2p_t^2. \quad (3)$$

Then the asymptotic approximation[12] for the Legendre function gives

$$P_\alpha(x_t)\simeq\frac{\Gamma(\alpha+\frac{1}{2})2^\alpha}{(\pi)^{1/2}\Gamma(\alpha+1)}\{[s-(m_N^2+m_Y^2)]/2p_t^2\}^\alpha. \quad (4)$$

The factor $\Gamma(\alpha+\frac{1}{2})2^\alpha/(\pi)^{1/2}\Gamma(\alpha+1)$ varies slowly with t [lies in the range 0.9 to 1 for $0<\alpha(t)<1$] and can therefore be absorbed into β. Thus, Eq. (2) now reads

$$X=(2\alpha+1)\frac{(e^{-i\pi\alpha}-1)}{2\sin\pi\alpha}$$

$$\times\{[s-(m_N^2+m_Y^2)]/2m_Nm_Y\}^\alpha\beta. \quad (5)$$

Now, β is determined from the fact that X should coincide with its field-theoretic counterpart at the pole point, which gives

$$\beta=-\tfrac{1}{3}\pi\alpha'\{[s-(m_N^2+m_Y^2)]/2m_Nm_Y\}^{-1},$$

$$\alpha'=\left(\frac{\partial\alpha}{\partial t}\right)_{t=m^2}. \quad (6)$$

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[10] S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. **126**, 2204 (1962); I. J. Muzinich, *ibid.* **130**, 1571 (1963).

[11] M. M. Islam, Nuovo Cimento **30**, 579 (1963); M. M. Islam and R. Pinon, *ibid.* **30**, 837 (1963).

[12] This approximation is fairly good for $x_t>3$. For small t, this corresponds to an incident momentum $P_L>3$ GeV/c, which fixes therefore the lower limit to the range of validity of our result.

Thus from (5) and (6) we get

$$X=\frac{\pi\alpha'}{6}(2\alpha+1)\frac{(1-e^{-i\pi\alpha})}{\sin\pi\alpha}\times\{[s-(m_N^2+m_Y^2)]/2m_Nm_Y\}^{\alpha-1}. \quad (7)$$

Following Muzinich,[10] we normalize the helicity amplitudes ϕ as

$$\phi=(m_Nm_Y/4\pi E)M, \quad (8)$$

where E is the c.m. energy of one of the particles in the direct channel. Then from (1) we get the following expressions for the K^*-exchange helicity amplitudes:

$$\begin{aligned}
\phi_{++,++}&=\frac{X}{16\pi E}\{G_{VV}x_1[1+\sin^2(\theta/2)](+G_{VV}x_3-G_{SS}x_2-G_{4S}{}^1x_4-G_{4S}{}^2x_4)\cos^2(\theta/2)\},\\
\phi_{+-,+-}&=\frac{X\cos^2(\theta/2)}{16\pi E}[G_{VV}x_1G_{VV}x_3-G_{SS}x_2G_{4S}{}^1x_4-G_{4S}{}^2x_4],\\
\phi_{++,-+}=\phi_{--,-+}&=\frac{X\sin(\theta/2)\cos(\theta/2)}{16\pi E}[G_{VV}x_5+G_{VV}x_4-G_{SS}x_4-G_{4S}{}^1x_2-G_{4S}{}^2x_3],\\
\phi_{-+,++}=\phi_{-+,--}&=\frac{X\sin(\theta/2)\cos(\theta/2)}{16\pi E}[G_{VV}x_5-G_{VV}x_4+G_{SS}x_4+G_{4S}{}^1x_2+G_{4S}{}^2x_3],\\
\phi_{+-,-+}&=\frac{X\sin^2(\theta/2)}{16\pi E}[-G_{VV}x_6-G_{VV}x_2+G_{SS}x\ +G_{4S}{}^1x_4+G_{4S}{}^2x_4],\\
\phi_{++,--}&=\frac{X}{16\pi E}\{G_{VV}x_6[1+\cos^2(\theta/2)]+(G_{VV}x_2-G_{SS}x_3-G_{4S}{}^1x_4-G_{4S}{}^2x_4)\sin^2(\theta/2)\},
\end{aligned} \quad (9)$$

where

$$\begin{aligned}
G_{VV}&=g_1g_2[1+R_1+R_2+R_1R_2],\\
G_{SS}&=-g_1g_2\left[\frac{(m_Y-m_N)^2}{m^2}+\frac{[6E^2-m_N^2-m_Y^2+2(E^2-m_N^2)^{1/2}(E^2-m_Y^2)^{1/2}\cos\theta]R_1R_2}{(m_N+m_Y)^2}+R_1(1+R_2)+R_2(1+R_1)\right],\\
G_{4S}{}^1&=g_1g_2\frac{4ER_1(1+R_2)}{m_N+m_Y},\qquad G_{4S}{}^2=g_1g_2\frac{4ER_2(1+R_1)}{m_N+m_Y},
\end{aligned} \quad (10)$$

and

$$\begin{aligned}
x_1&=[(E-m_N)^{1/2}(E+m_Y)^{1/2}+(E+m_N)^{1/2}(E-m_Y)^{1/2}]^2,\\
x_2&=[(E+m_N)^{1/2}(E+m_Y)^{1/2}-(E-m_N)^{1/2}(E-m_Y)^{1/2}]^2,\\
x_3&=[(E+m_N)^{1/2}(E+m_Y)^{1/2}+(E-m_N)^{1/2}(E-m_Y)^{1/2}]^2,\\
x_4&=[(E+m_N)(E+m_Y)-(E-m_N)(E-m_Y)],\\
x_5&=[(E-m_N)(E+m_Y)-(E+m_N)(E-m_Y)],\\
x_6&=[(E-m_N)^{1/2}(E+m_Y)^{1/2}-(E+m_N)^{1/2}(E-m_Y)^{1/2}]^2.
\end{aligned} \quad (11)$$

The other helicity amplitudes are obtained from the above ones simply by parity inversion. In terms of the helicity amplitude, the differential cross section reads

$$\frac{d\sigma}{dt}=\left(\frac{\pi}{pp'}\right)\frac{d\sigma}{d\Omega}=\left(\frac{\pi}{pp'}\right)\frac{1}{(2s_a+1)(2s_b+1)}\sum|\phi|^2=\left(\frac{\pi}{4pp'}\right)\sum|\phi|^2, \quad (12)$$

where the summation is over all initial and final helicity states, and s_a, s_b are spins of the incident particles.

RESULTS AND DISCUSSION

To get a good fit to the angular dependence of the above reaction cross sections, we need a slope for the K^* trajectory of about 0.7 GeV^{-2}, which is quite close to the value 0.64 GeV^{-2} estimated[2] for the ρ trajectory. With this slope, then, a linear trajectory passing through the K^* mass point should have $\alpha(0)\simeq0.4$. Thus the K^* trajectory may be expressed as

$$\alpha(t)=0.4+0.7t. \quad (13)$$

The various mass values are $m=0.89$ GeV, $m_N=0.94$

TABLE I. Energy dependence of the total cross section for the reactions $p\bar{p}\to Y\bar{Y}$.

Incident momentum (GeV/c)	Predicted cross section for $p\bar{p}\to\Lambda\bar{\Lambda}$ (μb)	Experimental cross section for $p\bar{p}\to\Lambda\bar{\Lambda}$ (μb)	Predicted cross section for $p\bar{p}\to\Lambda\bar{\Sigma}^0+\Sigma^0\bar{\Lambda}$ (μb)	Experimental cross section for $p\bar{p}\to\Lambda\bar{\Sigma}^0+\Sigma^0\bar{\Lambda}$ (μb)	Predicted cross section for $p\bar{p}\to\Sigma^+\bar{\Sigma}^-$ (μb)	Experimental cross section for $p\bar{p}\to\Sigma^+\bar{\Sigma}^-$ (μb)	Reference to experimental data
3	117	117±18	83	102±17	75	36±6	15
3.25	102	87±13	75	56±11	68	36±13	16
3.6	87	77±20	67	67±19	61	30±8	15
3.7	82	82±8	65	69±10	58	44±9	16
4	70	39±12	58	46±13	52	24±6	15
5.7	40	40±10	35	30±8	30	37±10	14
7	30	31±13	25	30±5	22	?	16

GeV, $m_\Lambda=1.11$ GeV, and $m_\Sigma=1.19$ GeV. For the joint $\Lambda\Sigma$ production process [reaction (B)], we take for simplicity both the hyperons to have the same average mass $m_Y=1.15$ GeV. For the tensor-to-vector coupling ratio R, we take the following values, which are obtained from $R_{pNN}=4$ on the basis of a pure F-type vector coupling and a D/F ratio $\frac{3}{2}$ for tensor coupling of vector meson octet with the baryon-antibaryon octets [as suggested by $SU(6)$]:

$$R_{K^*p\Lambda}=2.4,\quad R_{K^*p\Sigma^+}=R_{K^*p\Sigma^0}=-0.8.$$

Substituting the above values for the R's,[13] the various mass parameters, and $\alpha(t)$ into Eqs. (7) to (12), we get the differential cross sections for each of the three processes with one unknown over-all multiplicative constant.

Now, normalizing the total cross section for reaction (A) to the experimental value[14] of 40 μb at an incident momentum 5.7 GeV/c, we get $g_{K^*p\Lambda}{}^2\simeq 7$ for the value of the multiplicative constant. Similarly, normalizing

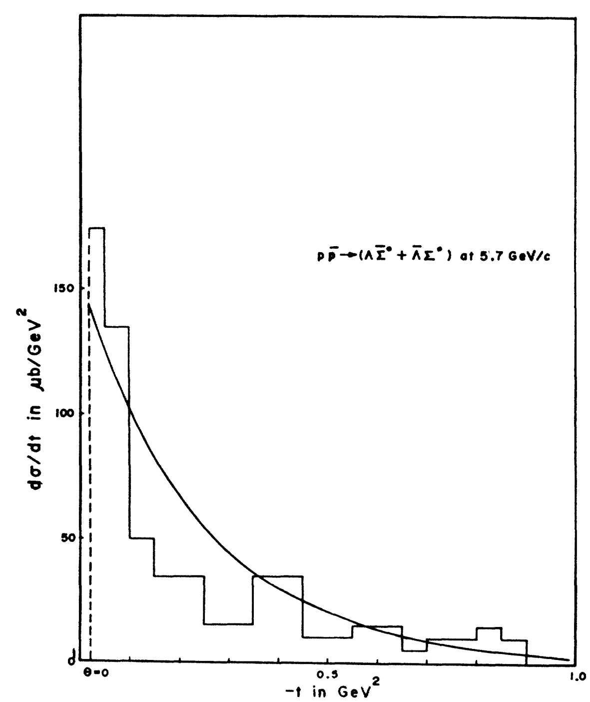

FIG. 3. The differential cross section for the process $p\bar{p}\to\Lambda\bar{\Sigma}^0+\Sigma^0\bar{\Lambda}$ at 5.7 GeV/c. The experimental histogram is from Ref. 14 (also quoted in Ref. 6).

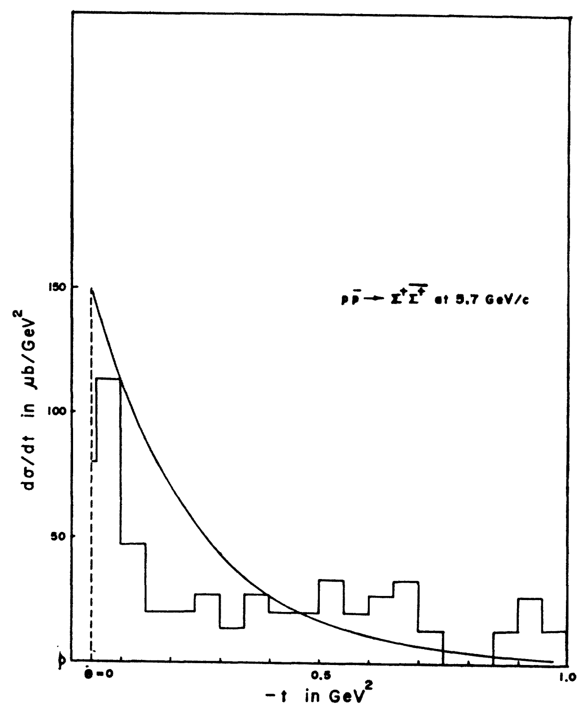

FIG. 4. The differential cross section for the process $p\bar{p}\to\Sigma^+\bar{\Sigma}^-$ at 5.7 GeV/c. The experimental histogram is from Ref. 14 (also quoted in Ref. 6).

[13] We have repeated the calculations with a second set of R's ($R_{K^*p\Lambda}=2$, $R_{K^*p\Sigma^+}=R_{K^*p\Sigma^0}=-2$) corresponding to a D/F ratio 3/1 for tensor coupling. Except for a slight broadening of the forward peaks for processes (B) and (C), the results remain essentially unaltered. Thus the results are not sensitive to the values of the R's. On the other hand, as pointed out in Ref. 6, the absorption model results are quite sensitive to the choice of R and give too broad a forward peak for R outside the range 0 to 1.

[14] R. Bock, A. Cooper, B. R. French, R. Levisetti, D. Ravel, B. Tallini, and S. Zylberajch, in *Proceedings of the 12th International Conference on High-Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965).

the total cross section for reaction (B) to the experimental value[14] of 35 μb at the incident momentum 5.7 GeV/c, we get $g_{K^*p\Lambda}^2 g_{K^*p\Sigma^0}^2 \simeq 24$. Then the over-all multiplicative constant $g_{K^*p\Sigma^+}^2$ for reaction (C) is uniquely known in terms of these. We get

$$g_{K^*p\Sigma^+}^2 = 2g_{K^*p\Sigma^0}^2 = 2g_{K^*p\Lambda}^2 g_{K^*p\Sigma^0}^2 / g_{K^*p\Lambda}^2 \simeq 7 .$$

The resulting differential cross sections for the reactions (A), (B), and (C) at the incident momentum 5.7 GeV/c are plotted in Figs. (2), (3), and (4), respectively, along with the experimental data.[14] The over-all agreement is quite good, except for some weak nonperipheral events observed for the process $p\bar{p} \to \Sigma^+\bar{\Sigma}^-$ which cannot be accounted for in this model. Presumably these nonperipheral events will be further suppressed at a higher energy, where Regge-pole exchanges alone would provide the dominant mechanism.

Finally, we come to the energy dependence of the total reaction cross sections. The total reaction cross sections for the above processes, evaluated at different energies, are given in Table I along with the experimental data of CERN[14,15] and Yale[16] groups. The observed energy dependence of the cross sections for reactions (A) and (B) are reproduced remarkably well by our model. The experimental situation for the reaction (C), however, is somewhat confused. Here the CERN data of Musgrave *et al.*[15] in the 3–4 GeV/c range give a cross section, falling monotonically with energy, which is consistent with the predicted energy dependence. But the 5.7-GeV data of Bock *et al.*[14] gives a somewhat larger cross section. Again, the Yale data[16] indicates a slight increase in the cross section from 3.25 to 3.7 GeV/c. On the face of it, all these could indicate a flat cross section for the reaction (C) in this energy range, which cannot be understood on the basis of a single trajectory exchange.[17] However, the uncertainties in the data are too large to permit any definite conclusion about the trend of the energy dependence. One awaits more data on this process ($p\bar{p} \to \Sigma^+\bar{\Sigma}^-$), especially over a wider energy interval, to decide if the single-trajectory exchange picture needs substantial modification for this process.

To conclude, while more data are needed on the process $p\bar{p} \to \Sigma^+\bar{\Sigma}^-$, we see that the other hyperon-anti-hyperon production processes, in the region of a few GeV, are described quite well in terms of the K^* trajectory exchange.

ACKNOWLEDGMENTS

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[15] B. Musgrave *et al.*, Nuovo Cimento **35**, 735 (1965).

[16] C. Baltay *et al.*, Phys. Rev. **140**, B1027 (1965); C. Baltay *et al.*, in *Proceedings of the 12th International Conference on High-Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965).

[17] Such a behavior, if it persists, may be attributed to an interference arising when one includes the K^{**} exchange. For a consistent description of all three processes, one would then require a $K^{**}p\Sigma$ coupling several times larger than $K^{**}p\Lambda$ coupling. Since these two terms occur with like signs for F-type and with opposite signs for D-type couplings, one can indeed have such a situation for a suitable D/F ratio.

Erratum

Decay Theory of Closely Coupled Unstable States, LYMAN MOWER [Phys. Rev. **142**, 799 (1966)]. On p. 805, line 15 from the bottom, read $\lim_{E\to 0}[\ \ ] = -2i\, E_a I_b(E) - 2i\, E_b I_a(E)$. In Eq. (75) replace h by $\hbar$, and following Eq. (83) read $\bar{E}_{a,n} \equiv E_{a,n} + D_a$. In the line following Eq. (97) read ". . . use of Eq. (86). We may relate ω_σ . . ." On p. 814 the a_{n-2}, a_{n-2} matrix element of g^{-1} should read $E - \bar{E}_{a,n-2} + i\, \Gamma_a/2$.