because of the conservation of isospin. Transforming the $T$ product by $\mathcal{U}^{-1}$ and using $\mathcal{U}^{-1}|0\rangle = |0\rangle$, we get

$$\langle T[-\tfrac{1}{2}\varphi_{\pi^0}(1)+\tfrac{1}{2}\sqrt{3}\varphi_{\eta^0}(1),j_\lambda(2),j_\mu(3)]\rangle_0=0. \quad \text{(A11)}$$

This immediately leads to the well-known relation for the decay amplitudes[33].

$$\mathfrak{M}(\pi^0\to 2\gamma)=\sqrt{3}\mathfrak{M}(\eta^0\to 2\gamma). \quad \text{(A12)}$$

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# $M1$ Photo-Excitation $N\to N^*$ and $SU(6)$ Symmetry

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A direct evaluation of the $M1$ amplitude for the photo-excitation $p\to N^{*+}$ is made from the $\pi^0$ photoproduction cross section at resonance, with result $(1.28\pm0.02)$ times the $SU(6)$ prediction $2\sqrt{2}\mu_p/3$. It is shown that the $N^*$ electro-excitation data also lead to the same value, within larger errors. There is pointed out a strong kinematic form-factor correction to this $SU(6)$ prediction, which occurs in consequence of the $N$-$N^*$ mass difference arising from $SU(6)$ symmetry-breaking interactions. The most direct phenomenological evaluation (based on the three-quark model for the baryonic states) for this correction would reduce the predicted $M1$ amplitude to $0.79\times(2\sqrt{2}\mu_p/3)$.

## 1. INTRODUCTION

WITH the assumption of $SU(6)$ symmetry,[1,2] the baryon octet $B$ and the $(\frac{3}{2}+)$ baryonic decuplet $B^*$ are assigned to the same supermultiplet, the 56-dimensional representation. As a result, their electromagnetic properties are directly related. The amplitude for $M1$ photo-excitation $\gamma N\to N^*$ is then expressible in terms of the proton magnetic moment, as first given by Bég *et al.*,[3]

$$\mathfrak{M}=(p,m=+\tfrac{1}{2}|M_z|N_{3/2}^*,m=+\tfrac{1}{2})=\tfrac{2}{3}\sqrt{2}\mu_p, \quad (1.1)$$

where $\mu_p=2.79\ eh/2Mc$ denotes the total proton magnetic moment. The same result also holds in the relativistic $\tilde{U}(12)$ symmetry scheme.[4] In the limit of exact $\tilde{U}(12)$ symmetry, when the states $B$ and $B^*$ have the same mass, the relation (1.1) holds also for arbitrary momentum transfer $k^2$, so that the same form factor is appropriate for both the transitions $\gamma B\to B$ and $\gamma B\to B^*$. The transitions $\gamma B\to B^*$ are also permitted through $E2$ and $L2$ (longitudinal) electromagnetic interactions. Using $SU(6)_W$ symmetry, a relativistic generalization of $SU(6)$ symmetry for collinear processes, which is also a subgroup symmetry of $\tilde{U}(12)$ symmetry, Harari and Lipkin[5] have shown that the $E2$ and $L2$ transitions are forbidden.

The same results, the relation (1.1) and the vanishing of the $E2$ and $L2$ amplitudes, have also been obtained from the three-quark model for these baryonic states,[6,7] assuming that the $(\frac{1}{2}+)$ baryon octet and the $(\frac{3}{2}+)$ baryonic decuplet both correspond to an $L=0$ configuration. Schematically, the three-quark wave functions for these states take the form

$$\psi=\phi(1,2,3)\chi_S(1,2,3)g(1,2,3), \quad (1.2)$$

where $\phi$, $\chi_S$, $g$ denote the space wave function, the spin wave function for spin $S$, and the unitary spin wave function, respectively. For the $B^*$ states, $S=\frac{3}{2}$, and $\chi_{3/2}$ and $g$ are both symmetric functions; for the $B$ states, $S=\frac{1}{2}$, and $\chi_{1/2}$ and $g$ are both functions of mixed symmetry, their product being taken to give a symmetric function. In the approximation that the quark-quark forces are invariant with respect to the simultaneous spin and unitary-spin transformations of $SU(6)$ symmetry, the space wave functions $\phi(1,2,3)$ are the same for both $B$ and $B^*$ states (antisymmetric if the Pauli principle holds for quarks) and correspond to $L=0$ for the total orbital angular momentum.

The comparison of the prediction (1.1) with experiment is one of the significant tests of $SU(6)$ symmetry, or of its relativistic extensions. The status of this comparison has recently appeared rather obscure. Bég *et al.*[3] have indicated a rather substantial discrepancy between (1.1) and the data. From the detailed analysis of pion photoproduction data which has been carried out by Gourdin and Salin,[8] they deduced the result

$$\mathfrak{M}=(1.6)\times\tfrac{2}{3}\sqrt{2}\mu_p. \quad (1.3)$$

This amplitude corresponds to an experimental cross section more than 2.5 times that predicted with the

[1] F. Gürsey and L. A. Radicati, Phys. Rev. Letters **13**, 173 (1964).
[2] B. Sakita, Phys. Rev. **136**, B1756 (1964).
[3] M. A. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters **13**, 514 (1964).
[4] A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) **A284**, 146 (1965).
[5] H. Harari and H. J. Lipkin, Phys. Rev. **140**, B1617 (1965).
[6] C. Becchi and G. Morpurgo, Phys. Letters **17**, 352 (1965).
[7] R. H. Dalitz, in *High Energy Physics* (Gordon and Breach, Science Publishers, Inc., New York, 1965), p. 251.
[8] M. Gourdin and Ph. Salin, Nuovo Cimento **27**, 193 (1963).

amplitude (1.1). On the other hand, Geshkenbein[9] has compared the pion electroproduction cross section calculated with the $SU(6)$ amplitude (1.1) with the $N^*$ excitation data of Hand[10] and has found excellent agreement (to about 10% accuracy in cross section) for momentum transfers from $k^2=4m_\pi^2$ to 32 $m_\pi^2$, using the same form factor for the $N \to N^*$ amplitude as known for proton and neutron magnetic moments, as required by the relativistic version of (1.1).

Our purpose in this note is to re-examine the question of an empirical estimate for the $M1$ amplitude $\mathfrak{M}$, to clarify the relationship between the comparisons mentioned above, and to illustrate some of the uncertainties which arise from the approximate nature of the $SU(6)$ symmetry.

## 2. THE $M1$ AMPLITUDE FOR $N \to N^*$ AT RESONANCE

The cleanest situation for an estimate of the $M1$ amplitude (1.1) is that provided by data on the $\pi^0$ photoproduction reaction

$$\gamma + p \to p + \pi^0 \tag{2.1}$$

in the neighborhood of the resonance $N^*(1236)$, since this reaction is known to be dominated by $M1$ resonance excitation.

The very low values observed for the total cross section near threshold are already indicative that the $s$-wave photoproduction interaction (2.1) is rather weak. From analysis of the $\pi^0$ and $\pi^+$ photoproduction angular distributions, this $s$-wave amplitude can be deduced as function of the photon energy and it is rather small over the $N^*(1236)$ resonance. For example, from a semiphenomenological analysis of all the available data (based on the dispersion-theoretic calculations for photoproduction), Donnachie and Shaw[11] have obtained a value about $(1.5+2.8i)\times 10^{-3}\, \hbar/m_\pi c$ for the $s$-wave amplitude at the resonance energy; this amplitude would only contribute 2.5 $\mu$b to the total cross section observed (267 $\mu$b at the resonance energy).

With the $M1$ and $E2$ amplitudes for excitation of the resonance denoted by $\mathfrak{M}$ and $\mathcal{E}$, respectively, the $\pi^0$ angular distribution for the resonance process

$$\gamma + p \to N_{3/2}^*(1236) \to p + \pi^0 \tag{2.2}$$

is proportional to

$$A + C\cos^2\theta + \alpha \sin^2\theta \cos 2\phi \tag{2.3}$$

for incident photons with plane polarization (electric vector lying in the plane $\phi=0$), where

$$A = 5|\mathfrak{M}|^2 + 3|\mathcal{E}|^2 + 2\sqrt{3}\mathfrak{M}^*\mathcal{E}, \tag{2.4a}$$

$$C = -3|\mathfrak{M}|^2 + 3|\mathcal{E}|^2 - 6\sqrt{3}\mathfrak{M}^*\mathcal{E}, \tag{2.4b}$$

$$\alpha = -3|\mathfrak{M}|^2 + 3|\mathcal{E}|^2 + 2\sqrt{3}\mathfrak{M}^*\mathcal{E}. \tag{2.4c}$$

From the $\pi^0$ angular distribution for unpolarized photons, McDonald *et al.*[12] obtained the ratio $C/A = -0.6 \pm 0.06$ in the neighborhood of the resonance energy. This ratio corresponds to the value

$$\mathcal{E}/\mathfrak{M} = 0.0 \pm 0.06. \tag{2.5}$$

For the polarization term, Drickey and Mozley[13] and Barbiellini *et al.*[14] obtained the values $0.9\pm0.1$ and $0.9\pm0.06$, respectively, for the ratio $\alpha/C$ in the neighbourhood of the resonance. These values correspond to the estimate

$$\mathcal{E}/M = 0.02 \pm 0.01. \tag{2.6}$$

These estimates for $\mathcal{E}/\mathfrak{M}$ neglect the small contributions possible from the excitation of nonresonant $\pi N$ states; however, they are sufficient to indicate that the contribution of $E2$ transitions to the excitation of the resonant state is exceedingly small, probably not more than 0.1% at the resonant energy.

At resonance, the total cross section for $\pi^0$ photoproduction is given by the expression

$$\sigma(\gamma p \to p\pi^0) = \tfrac{2}{3}(4\pi/k^{*2})(\Gamma_\gamma/\Gamma), \tag{2.7}$$

where $k^*$ denotes the c.m. photon momentum at resonance, $\Gamma_\gamma$ denotes the partial width for the decay $N_{3/2}^*(1236)^+ \to p\gamma$, and $\Gamma$ denotes the total $N_{3/2}^*(1236)$ width, defined by the phase-shift expression

$$\cot\delta_{33} = 2(E^* - E)/\Gamma(E) \tag{2.8}$$

and evaluated at the resonance energy $E^*$. Berkelman and Waggoner[15] have tabulated the $\pi^0$ total cross sections available across the resonance energy, including the values $244\pm3$ $\mu$b at $k_L=295$ MeV, $287\pm3$ $\mu$b at 320 MeV, $226\pm2$ $\mu$b at 360 MeV, and $138\pm2$ $\mu$b at 400 MeV. For the resonance energy $k_L=345$ MeV, interpolation between these values leads to the estimate $267\pm5$ $\mu$b for $\sigma_{\text{tot}}(\gamma,\pi^0)$, where the error includes the uncertainty in the interpolation. Including the $p_{11}$, $p_{13}$ and $p_{31}$ excitations, the total cross section for nonresonant $\pi N$ excitations may be estimated (for example from the analysis of Donnachie and Shaw[11,16]) as between 5 and 10 $\mu$b; for definiteness, we adopt the estimate $7\pm3$ $\mu$b for the nonresonant background, and the estimate

$$\sigma_{\text{res}}(\gamma,\pi^0) = 260 \pm 6\ \mu\text{b}. \tag{2.9}$$

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[9] B. V. Geshkenbein, Phys. Letters **11**, 323 (1965).

[10] L. Hand, Phys. Rev. **129**, 1834 (1963).

[11] A. Donnachie and G. Shaw (private communication, 1965).

[12] W. S. McDonald, V. Z. Peterson, and D. R. Corson, Phys. Rev. **107**, 597 (1959).

[13] D. J. Drickey and R. F. Mozley, Phys. Rev. Letters **8**, 291 (1962).

[14] G. Barbiellini, G. Bologna, J. deWire, G. Diambrini, G. Murtas, and G. Sette, in *Proceedings of the Sienna International Conference on Elementary Particles*, edited by G. Bernardini and G. P. Puppi (Italian Physical Society, Bologna, 1963), Vol. I, p. 516.

[15] K. Berkelman and J. A. Waggoner, Phys. Rev. **117**, 1364 (1960).

[16] A. Donnachie and G. Shaw, Ann. Phys. (N. Y.) (to be published).

A convenient form for the resonant phase shift (2.8) is that used by Gell-Mann and Watson,[17] for which

$$\Gamma(E)=\gamma(qa)^3/[1+(qa)^2]\,, \qquad (2.10)$$

with the parameter values[18–19] $\gamma=127.5$ MeV and $a=0.85/m_\pi$. The width $\Gamma$ at resonance is then 119 MeV. Inserting these values in Eq. (2.7) leads to the result

$$\Gamma_\gamma=0.65(\pm0.02)\text{ MeV}. \qquad (2.11)$$

The radiative width may be expressed directly in terms of the magnetic moment (1.1). The matrix element is

$$V_\gamma=(1/2k)^{1/2}(p,m'|\mathbf{M}|N_{3/2}{}^{*+},m)\cdot\mathbf{k}\times\boldsymbol{\varepsilon}. \qquad (2.12)$$

Since $\Gamma_\gamma$ is the same for every value of the $N^*$ magnetic quantum number $m$, we consider $m=\frac{3}{2}$, for which there is only the one transition, to $m'=\frac{1}{2}$; for this the matrix-element of $\mathbf{M}$ has the value $(\sqrt{\tfrac{3}{2}})\mathfrak{M}$. The width is then given by

$$\Gamma_\gamma=2\pi\int(\tfrac{3}{2}|\mathfrak{M}|^2)(\tfrac{2}{3}k^2)\frac{4\pi k^2dk\,M}{2k(2\pi)^3E_k}\delta(k+E_k-E^*)\,, \qquad (2.13)$$

where $E_k$ denotes the total proton energy, and the factor $\frac{2}{3}$ results from the sum over photon polarizations and the average over photon directions. This expression reduces to

$$\Gamma_\gamma=(\alpha k^{*3}/2ME^*)\mu^{*2}\,, \qquad (2.14)$$

where we have written $\mathfrak{M}=\mu^*(e\hbar/2Mc)$. Equating (2.14) with the result (2.11) leads to the estimate[20]

$$\mu^*=(1.28\pm0.02)\tfrac{2}{3}\sqrt{2}\mu_p. \qquad (2.15)$$

For the case of electroproduction, cross-section formulas have been given by Dalitz and Yennie.[21] For $M1$ excitation of the $N_{3/2}{}^*$ resonance, the formulas (4.11), (4.9), (4.10) of their paper, together with the expression for $X$ given after their Eq. (3.6), and $\sin\sigma$ given by their Eq. (3.7), lead to the cross-section expression (laboratory system)

$$\frac{d^2\sigma}{dp'd\Omega'}=\frac{\alpha|\mathfrak{M}(k^2)|^2\Gamma/2}{\pi[(E^*-E)^2+\Gamma^2/4]}\frac{p'}{p}\left(\frac{(p+p')^2+\mathbf{k}^2}{\mathbf{k}^2-k_0{}^2}\right)\left(\frac{M}{E}\right)^2, \qquad (2.16)$$

where $\mathfrak{M}(k^2)$ is the magnetic-moment amplitude appropriate to the four-momentum transfer $k^2$, and $k_0$, $\mathbf{k}$ denote the energy and momentum transfers from the electron in the laboratory system. With relativistic $SU(6)$ symmetry, $\mathfrak{M}(k^2)$ should have the same form factor as the nucleon magnetic moments, and so it appears natural to write

$$\mathfrak{M}(k^2)=\mathfrak{M}G_{pN^*}(k^2)\,, \qquad (2.17)$$

as assumed by Geshkenbein,[9] with $G_{pN^*}$ equal to the nucleon magnetic form factors observed empirically, given by[22]

$$\frac{G_{Mp}(k^2)}{\mu_p}=\frac{G_{Mn}(k^2)}{\mu_n}=\left(1+\frac{k^2}{36m_\pi{}^2}\right)^{-2}. \qquad (2.18)$$

The expression (2.16) differs from that used by Geshkenbein in two respects. His expression (2) has an additional factor $(M+\epsilon-\epsilon')/E$. This is equal to the quantity $M/E_K$, where $E_K$ is the total energy of the proton before the interaction, in the c.m. frame of the final strongly interacting particles; $\mathbf{K}$ denotes the momentum transferred by the virtual photon, measured in this c.m. frame. This is the relativistic normalization factor for the initial proton wavefunction; it was not given explicitly in the formula (4.11) of Ref. 21, but was tacitly assumed to be contained in the matrix element $\mathbf{J}$. For consistency, and to be in accord with our definition of $\mathfrak{M}$ in the $N^*$ rest frame, Eq. (2.16) should be multiplied by this factor, to give a corrected expression,

$$\frac{d^2\sigma}{dp'd\Omega'}=\frac{\alpha}{\pi}\frac{|\mathfrak{M}(k^2)|^2\Gamma/2}{[(E^*-E)^2+\Gamma^2/4]}\frac{p'}{p}\times\left(\frac{(p+p')^2+\mathbf{k}^2}{\mathbf{k}^2-k_0{}^2}\right)\left(\frac{M}{E}\right)^2\left(\frac{M+k_0}{E}\right). \qquad (2.16')$$

This equation still differs from Geshkenbein's expression by the kinematic factors $(M/E)^2$, where $M$ is the

[17] M. Gell-Mann and K. M. Watson, Ann. Rev. Nucl. Sci. 4, 219 (1954).

[18] These parameter values were obtained with the choice $E^*=1236$ MeV by fitting the phase shifts appropriate to the highest ($k_L=450$ MeV) and lowest ($k_L=270$ MeV) energies considered in Fig. 3. These phase shifts were obtained from the interpolation formulas provided by Donnachie and Shaw (cf. Appendix 1, Ref. 16). Olsson (Ref. 19) has recently given an analysis of all the pion-nucleon total-cross-section data available in terms of a resonance formula of this form, which led to the value $\Gamma=120\pm2$ MeV for the width at resonance. This uncertainty in $\Gamma$ has been included in the error quoted in Eq. (2.11) for $\Gamma_\gamma$.

[19] M. G. Olsson, Phys. Rev. Letters 14, 118 (1965).

[20] This estimate is significantly less than that given by the expressions of Gourdin and Salin (Ref. 8), as evaluated by Bég *et al.* (Ref. 3), despite the fact that their isobar model for the phenomenological analysis of pion photoproduction is parallel in spirit with the treatment adopted here, although apparently more complicated owing to their adoption of a covariant notation. Due to the lack of detail given in their paper, it is difficult to be sure about the reasons for this discrepancy. The $M1$ amplitude for $\pi^0$ photoproduction is $(\sqrt{\tfrac{2}{3}})M_1{}^+$; in consequence, we believe that the first factor in their expression (13) for the total cross section at resonance should be $\frac{2}{3}$ rather than 4/9. Also this equation implies that their fit gives $\sigma_{\text{res}}(\gamma,\pi^0)=0.23$ mb, whereas the best experimental estimate is 0.26 mb. Their estimate for $\lambda_1$ depends on the total width $\Gamma$ adopted for $N^*(1236)$; with the value $\Gamma=119$ MeV, we find the estimate $\lambda_1=2.16$, rather than their value $\lambda_1=2.07$. An increase in $\lambda_1$ would require a corresponding decrease in their estimates for the parameters $C_1$ and $C_2$ occurring in the $M1$ transition amplitude. There is some uncertainty about the value to be used for their parameter $\Gamma_\gamma$. With the cross section $\sigma_{\text{res}}(\gamma,\pi^0)=0.23$ mb, a coefficient 1.60 is obtained in Eq. (2.15); the increase in $\sigma_{\text{res}}(\gamma,\pi^0)$, the factor $\sqrt{\tfrac{2}{3}}$, and the decreases in $C_1$, $C_2$ due to changes in $\lambda_1$ lead to correction of this coefficient by the factors (1.07)×(0.81) ×(0.95) which will lead to a final value of 1.31 for this coefficient, essentially in agreement with our estimate (2.15), within the uncertainties.

[21] R. H. Dalitz and D. R. Yennie, Phys. Rev. 105, 1598 (1957).

[22] See F. M. Pipkin, in *Proceedings of the Oxford International Conference on Elementary Particles* (Rutherford High-Energy Laboratory, Harwell, England, 1966), p. 61.

proton mass and $E$ is the total energy for the strongly interacting particles in their c.m. system; for the nuclear case, whence Geshkenbein obtains his expression, this factor is essentially unity, since the nuclear excitation energies of interest are small relative to the total nuclear mass.

Geshkenbein compared his cross-section expression (2) with the differential cross-sections observed for $N_{3/2}^*$ excitation in inelastic electron-proton scattering by Hand.[10] For five momentum transfer values in the range $k^2 = 4m_\pi^2$ to $32m_\pi^2$, he finds that there is excellent agreement (within the experimental error of about $\pm 10\%$ for each determination) with his expression, assuming the form factor (2.18) and the $SU(6)$ value (1.1) for $\mathfrak{M}$. This comparison shows that the form factor $G_{pN^*}(k^2)$ is in agreement with the nucleon form factor (2.18), as required by relativistic generalization of $SU(6)$ symmetry. Assuming this form factor, and extrapolating to $k^2 = 0$, it also appears to indicate that the $M1$ photo-excitation amplitude is in accord with the $SU(6)$ expectation (1.1).

With our corrected expression (2.16′), the additional factor $M/E$ has the value $1/(1.31)$ for $N_{3/2}^*$ resonance excitation at the resonance energy $E^* = 1236$ MeV. Assuming the form factor $G_{pN^*}$ to be given by the $SU(6)$ expectation (2.18), as verified by Geshkenbein's comparison, agreement with the data of Hand now requires the amplitude $\mathfrak{M}$ to be taken $1.31(\pm 0.1)$ times the $SU(6)$ prediction (1.1). This result from the electroproduction data is in good accord with the estimate (2.15) for $\mathfrak{M}$ obtained directly from the photoproduction data. Of course, this agreement merely indicates that the electroproduction cross sections are in accord with the photoproduction cross sections in the limit $k^2 \rightarrow 0$, as must necessarily be the case. Since these latter cross sections are known with greater accuracy and in far greater detail, the comparison with the electroproduction data does not really add significantly to the accuracy of our knowledge of $\mathfrak{M}$, but it is important to be clear that these data do require a value for $\mathfrak{M}$ significantly larger than the $SU(6)$ prediction.

## 3. FORM FACTORS AND SYMMETRY-BREAKING EFFECTS

With the three-quark wave functions (1.2), the form factors for the $M1$ interactions $N \rightarrow N$ and $N \rightarrow N^*$ are given by the expressions

$$G(\mathbf{K}^2) = \int |\phi(1,2,3)|^2 e^{i\mathbf{K}\cdot\mathbf{r}_1} d^3r_1 d^3r_2 d^3r_3 . \quad (3.1)$$

This expression is essentially nonrelativistic, since the space wave function $\phi$ is given only in the $N$ or $N^*$ rest frame and we do not know how to transform it to another Lorentz frame. However, we may confine our attention to small momentum transfers. Since the proton form factor $G$ corresponds to r.m.s. radius $0.58/m_\pi$, the extent of the spatial distribution associated with $\phi^2$ is presumably of this order of magnitude.

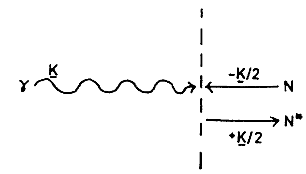


FIG. 1. Definition of the brick-wall frame for the transition $\gamma N \rightarrow N^*$.

The form factor for a magnetic transition $\gamma + A \rightarrow B$ is a function of the four-momentum magnitudes $k^2$, $p_A^2$, and $p_B^2$. It is most convenient to consider the form factor in the brick-wall frame for this interaction, as shown in Fig. 1. When $A$ and $B$ are real physical particles, we have $p_A^2 = m_A^2$ and $p_B^2 = m_B^2$. The energy and momentum transfer are then connected by the relation

$$K_0 = (m_B^2 + \tfrac{1}{4}\mathbf{K}^2)^{1/2} - (m_A^2 + \tfrac{1}{4}\mathbf{K}^2)^{1/2} . \quad (3.2)$$

In this frame, the relation between $\mathbf{K}$ and the covariant momentum transfer $K^2$ is given explicitly by

$$\mathbf{K}^2 = \frac{[K^2 + (m_A + m_B)^2][K^2 + (m_A - m_B)^2]}{2(m_A^2 + m_B^2) + K^2} . \quad (3.3)$$

For a real photon, $K^2 = 0$ and we have

$$\mathbf{K}^2 = (m_B^2 - m_A^2)^2 / 2(m_B^2 + m_A^2) . \quad (3.4)$$

For the transition $N \rightarrow N$, the energy transfer is always $K_0 = 0$ in this frame, and the three-momentum transfer has the magnitude $\sqrt{K^2}$. The form factor $G(K^2)$ is then given by the functional form (3.1); empirically its value is given by Eq. (2.18).

For the transition $N \rightarrow N^*$, the mass difference generated by the $SU(6)$-breaking interaction leads to significant kinematic differences from the $N \rightarrow N$ situation. It is quite possible that these $SU(6)$-breaking interactions may produce relatively weak distortions in the wave functions $\phi$ for the states $N$ and $N^*$; indeed, with the quark model, this appears rather likely to be the case, since the symmetry-breaking interactions appear rather weak (generating mass splittings of order 300 MeV) compared with the strong symmetric forces which give rise to the binding energy (whose magnitude is of order $3M_Q \gtrsim 15$ GeV). Neglecting these distortions, we would then expect the $M1$ form factor to be given by

$$G_{pN^*}(K^2) = G(\mathbf{K}^2) . \quad (3.5)$$

This expectation still neglects the possibility of further modifications to the form factor arising from the relativistic transformation of the wave functions $\phi$ for the $N$ and $N^*$ states in this situation. For the case of a real photon, expression (3.4) leads to $|\mathbf{K}| = 294$ MeV/$c$ for the $N \rightarrow N^*$ transition. Hence, for this case, the $N$ and $N^*$ velocities are only of order $v/c \approx 0.15$, and these corrections would be expected to be only of order $(v/c)^2 \approx 0.02$; we shall not consider them further here. However, this momentum 294 MeV/$c$ is substantial in

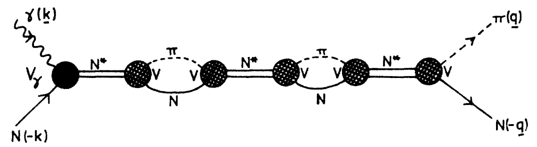


FIG. 2. Typical graph corresponding to the process $\gamma N \to N^*$, showing the final state $\pi N$ scattering. The vertex $V_\gamma$ corresponds to the $M1$ interaction $\gamma N \to N^*$, and the cross-hatched vertices $V$ correspond to the interaction $\pi N \to N^*$. The lines marked $N^*$ denote the $N^*$ propagator $(E-E_0)^{-1}$.

relation to the r.m.s. radius associated with $G(K^2)$. For this momentum transfer, the form factor $G_{pN^*}$ falls from the value $G_{pN^*}(0)=1$ appropriate to the case of exact symmetry $(M^*=M)$ to the value $G_{pN^*}(0)=0.79$, as given by (3.5) with the form factor (2.15).

This reduction from the $SU(6)$ prediction for $\mathfrak{M}$ is a kinematic effect arising ultimately from the existence of $SU(6)$ symmetry-breaking interactions. Our estimate for this reduction is to be regarded only as a first approximation, since it neglects unknown relativistic effects (certainly small for $K^2=0$, but of increasing importance with increasing $K^2$). This effect introduces some uncertainty in our comparison of the phenomenological value for $\mathfrak{M}$ with the $SU(6)$ prediction, in Sec. 2 above. With this symmetry-breaking correction to the $SU(6)$ prediction, the empirical estimate (2.15) for $\mathfrak{M}$ is better expressed as follows:

$$\mathfrak{M}_{\exp} = (1.28/G(\mathbf{K}^2))(\tfrac{2}{3}\sqrt{2}\mu_p G(\mathbf{K}^2)) = (1.62)(0.79\times\tfrac{2}{3}\sqrt{2}\mu_p),$$

this ratio between $\mathfrak{M}_{\text{expt}}$ and the corrected $SU(6)$ prediction being coincidentally close to the value estimated by Bég *et al.*[3] for the ratio between $\mathfrak{M}_{\text{expt}}$ and the exact $SU(6)$ prediction. We should emphasize that this correction represents only one particular contribution from the $SU(6)$ symmetry-breaking interactions. However, it is rather plausible that it may represent the dominant effect, since it is large and has a clear kinematic origin associated with the observed mass-splitting between the $N$ and $N^*$ members of the **56** representation of the $SU(6)$ group and its estimation is based on phenomenological considerations; this effect necessarily exists even if the $SU$-(6)-breaking interactions produce rather weak distortion of the internal wave functions for the members of this supermultiplet.

For $K^2\neq 0$, the relation between $\mathbf{K}^2$ and $K^2$ is given by Eq. (3.3). Over the range of $K^2$ values of physical interest at present, from $K^2=0$ up to $32\, m_\pi^2$, the ratio $\mathbf{K}^2/[K^2+(M^*-M)^2]$ falls only from 0.9815 to 0.983. With the identification of the $N\to N^*$ form factor with $G(\mathbf{K}^2)$, where $G(K^2)$ denotes the nuclear magnetic moment form factor, this relation between $\mathbf{K}^2$ and $K^2$ would lead to some differences from the form factors $G_{pp}(=G_{Ep})$ assumed by Geshkenbein for this transition. However, these differences are small relative to the empirical uncertainties at present; for example, normalizing $G_{pN^*}$ to be unity at $K^2=0$, this identification then leads to the expectation $G_{pN^*}=0.65$ for $K^2=10m_\pi^2$, compared with $G_{pp}=0.61$, and $G_{pN^*}=0.32$ for $K^2=32m_\pi^2$, compared with $G_{pp}=0.28$. This identification already neglects relativistic effects of order $(v/c)^2$, anyway; for $K^2=32m_\pi^2$, we have $(v/c)^2=0.16$, so that the relativistic corrections could well exceed these uncertainties in the expectations appropriate to the form factor $G_{pN^*}$.

The amplitude for $\pi^0$ photoproduction associated with the resonance in the $I=\frac{3}{2}$, $p_{3/2}\,\pi N$ state is given by the sum of the series of graphs of the type shown in Fig. 2, with the result

$$V_{0\gamma}(k)(E-E_0-\Sigma(E))^{-1}V_0(q)\,, \qquad (3.6)$$

where $V_{0\gamma}$ denotes the vertex for the interaction $\gamma N\to N^*$, $V_0$ denotes the vertex $\pi N\to N^*$, and $\Sigma(E)$ denotes the value of the self-energy graph included in this series. The denominator may be rewritten in the form

$$\{E-E_0-\text{Re}\Sigma(E^*)-(E-E^*)\,\text{Re}\Sigma'(E^*) - \text{Re}\Sigma_R(E)-i\,\text{Im}\Sigma(E)\}^{-1}, \qquad (3.7)$$

where the remainder $\Sigma_R(E)$ vanishes quadratically at $E=E^*$, and the resonance energy $E^*$ is defined by the equation

$$E^*=E_0+\text{Re}\Sigma(E^*)\,. \qquad (3.8)$$

The coefficient $Z=[1-\text{Re}\Sigma'(E^*)]$ of $(E-E^*)$ in this denominator is then absorbed by the renormalizations $V_\gamma(k)=V_{0\gamma}/Z^{1/2}$ and $V(q)=V_0/Z^{1/2}$ of the vertex functions, in the usual way. Then, by definition, $\text{Im}\Sigma(E)/Z=\Gamma(E)/2$ where $\Gamma(E)$ is given explicitly by the expression

$$\Gamma(E)=(q/2\pi)(M/E)\,|V(q)|^2 \qquad (3.9)$$

in terms of the renormalized vertex $V(q)$ for the interaction $\pi N\to N^*$. The remainder term $\Delta(E) =\text{Re}\Sigma_R(E)/Z$ represents an $E$-dependent level shift vanishing quadratically at $E=E^*$, which we shall neglect here. This leads finally to the simple Breit-Wigner form for the amplitude,

$$V_\gamma(k)(E-E^*-i\Gamma(E)/2)^{-1}V(q)\,, \qquad (3.10)$$

and thence to the Breit-Wigner expression for $\pi^0$ production cross-section in the $(\frac{3}{2},\frac{3}{2})$ state,

$$\sigma_{33}(\gamma p\to\pi p^0)=\frac{2}{3}\frac{\pi}{k^2}\frac{\Gamma_\gamma(k)\Gamma(E)}{(E-E^*)^2+[\Gamma(E)/2]^2}. \qquad (3.11)$$

A careful discussion of the spin factors appropriate in going from a photo-amplitude such as (3.10) to a cross-section expression such as (3.11) has been given by Goldberger and Watson.[23] We have discussed this

[23] M. L. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons, Inc., New York, 1964), p. 477.

graphical derivation of the Breit-Wigner cross-section formula here in order to emphasize that its use does not depend on the assumptions normally made in its derivation from the nonrelativistic nuclear reaction formalism of Wigner and others.

Comparison of expression (3.9) with the phenomenological expression (2.10) suggests that $V(q)$ should be proportional to $q/[1+(qa)^2]^{1/2}$, i.e. that $V(q)$ should include a factor of the form appropriate for $p$-wave centrifugal barrier penetration to radius $a$; this is the interpretation given to this form by Gell-Mann and Watson.[17] This is not the only interpretation possible; the effect of distant resonances or of nonresonant background can also contribute to the energy dependence of $\Gamma/q^3$. In fact, Donnachie *et al.*[24] have obtained a dispersion-theoretic solution for the phase shift $\delta_{33}$ whose energy dependence is quite closely in agreement with Eqs. (2.8) and (2.10). The radius $a$ in Eq. (2.10) is not necessarily to be interpreted literally as the interaction radius for the $N^*$ system.

For the width $\Gamma_\gamma$, on the other hand, it does appear natural to expect a form factor dependent on the c.m. photon momentum $k$, associated directly with the finite size of the system. Using the expression (2.8) to eliminate $\Gamma(E)$ we obtained from (3.11) the empirical estimate

$$\Gamma_\gamma(k)/k = 3k(E^* - E)\sigma_{33}(\gamma p \rightarrow p\pi^0)/2\pi \sin 2\delta_{33}, \quad (3.12)$$

which has been plotted as function of $k^2$ in Fig. 3. We note that $\Gamma_\gamma/k$ is not proportional to $k^2$, as would be expected with expression (2.4) for a point magnetic-moment interaction but appears to be a slowly varying function of $k^2$ over the resonance region.

From the remarks made above, the form factor expected for $\mathfrak{M}$ would lead to the form

$$\Gamma_\gamma(k)/k = k^2/(1+E^2k^2/18m_\pi^2(E^2+M^2))^4 \quad (3.13)$$

since the momentum $\mathbf{K}$ in the brick-wall frame is related with $k$ by the relation $\mathbf{K}^2 = 2k^2E^2/(M^2+E^2)$. The expression (3.13) has been plotted on Fig. 3; although the denominator does lead to an appreciable damping of the increase in $\Gamma_\gamma/k$ with increasing $k^2$, this prediction does not provide a sufficiently strong effect to give agreement with the experimental data over the resonance region. Of course, the wave function (1.2) describes only the internal wave function for the $N^*$state. The coupling of this state with the $\pi N$ decay channel has been neglected; since the $N^*$ decay rate varies with $E$, the distortion of the internal wavefunction by this coupling will vary with energy, and these additional energy-dependent effects will also have to be included in a more realistic calculation of the $k$ dependence of the $M1$ amplitude $\mathfrak{M}$ for this transition.

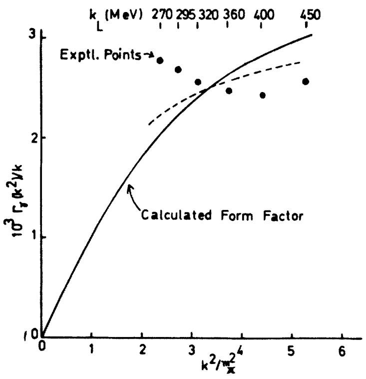


FIG. 3. The quantities $\Gamma_\gamma(k)/k$ deduced from the experimental total cross sections $\sigma(\gamma p \rightarrow p\pi^0)$ and the empirical phase shifts $\delta_{33}$ are plotted as function of $k^2$, where $k$ is the c.m. photon energy. The solid line shows the $k^2$-dependence expected for this quantity, according to the form factor (3.13) deduced from the nucleon magnetic moment form factors. The dashed line shows the $k^2$-dependence for a $p$-wave barrier penetration factor $k^2/(1+(ka)^2)$, with $a=0.85/m_\pi$ as for the pion-nucleon phase shift $\delta_{33}$.

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[24] A. Donnachie, J. Hamilton, and A. T. Lea, Phys. Rev. **135**, B515 (1964).