Parity-Violating Nonleptonic Decay and the Cabibbo Current

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In a current×current theory of weak interactions, the use of a Cabibbo-type current endows parity-violating nonleptonic decay with certain specific properties. These properties do not include octet dominance, but they are sufficient to forbid all $K\to 2\pi$ decays in the limit of $SU(3)$-conserving strong interactions. When electromagnetic corrections are taken into account, the decay modes $K_1^0\to\pi^+\pi^-$ and $K^+\to\pi^+\pi^0$ are allowed, but $K_1^0\to 2\pi^0$ is still forbidden. Consequently the smallness of the rate for $K^+\to\pi^+\pi^0$ relative to both modes of K_1^0 decay can be explained only by an appeal to medium-strong, $SU(3)$-breaking interactions. In the case of nonleptonic hyperon decay, the Cabibbo current×current interaction predicts two sum rules for parity-violating amplitudes. The recent results obtained by Sugawara and Suzuki from current commutation relations can be understood in terms of these sum rules, together with one simple constraint upon the effective Hamiltonian.

1. INTRODUCTION

EVER since the discovery of the $V-A$ interaction[1] for the beta decay of nonstrange particles, it has been attractive to postulate that all weak interactions are obtained by the coupling of a charged current with itself[2]; to allow for parity nonconservation, the weak current must contain both vector and axial-vector parts. The construction of a truly universal theory from this hypothesis has, until recently, met with several obstacles. One, the suppression of strangeness-violating beta decays relative to strangeness-conserving ones, has been removed by Cabibbo[3]; and another, the calculation of the ratio of axial-vector to vector coupling constants, has been successfully completed by Adler and Weisberger.[4] In view of these achievements, it is natural to apply the current×current hypothesis to nonleptonic decay.

Neglecting a small violation of CP, we consider an interaction of the form

$$\mathcal{L}_0=\{(J^++L^+),\ (J+L)\}_{(+)} \tag{1}$$

which generates nonleptonic decay by means of a term

$$\mathcal{L}=\{J^+,J\}_{(+)}. \tag{2}$$

Within the framework of SU(3), the hadronic current is taken to be a charged member of an octet with tensor properties

$$J=(\cos\theta)J_1{}^2+(\sin\theta)J_1{}^3, \tag{3}$$

where θ is the Cabibbo angle.[3] This choice of the interaction and current endows the strangeness-violating part of $\mathcal{L}$,

$$\mathcal{L}_{sv}=(\sin\theta\cos\theta)[\{J_1{}^2,J_3{}^1\}_{(+)}+\{J_2{}^1,J_1{}^3\}_{(+)}] \tag{4}$$

with the following properties:

(a) $\mathcal{L}_{sv}$ is an admixture of the $SU(3)$ representations (8) and (27)

(b) $\mathcal{L}_{sv}$ has U spin equal to unity,

(c) $\mathcal{L}_{sv}$ is symmetric under the Weyl reflection that interchanges the indices 2 and 3.[5] [T-$L(1)$ invariance.[6]]

Although these properties are valid for both the parity-conserving (pc) and parity-violating (pv) parts of $\mathcal{L}_{sv}$, their most interesting consequences apply only to parity-violating decays.

When combined with the additional hypothesis that $\mathcal{L}_{sv}$ transforms like an octet, the current×current interaction forbids all $K\to 2\pi$ decays[7,8] and predicts the Lee-Sugawara triangle[9]

$$\sqrt{3}\langle\Sigma^+|p\pi^0\rangle-\langle\Lambda|p\pi^-\rangle=2\langle\Xi^-|\Lambda\pi^-\rangle \tag{5}$$

for the pv amplitudes of nonleptonic hyperon decay.[8] The first result provides an interesting qualitative explanation for the observed rate of $K^+\to 2\pi$ relative to $K_1{}^0\to 2\pi$, and the second is in very good agreement with

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[1] E. C. G. Sudarshan and R. E. Marshak, in *Proceedings of the Padua-Venice Conference on Mesons and Newly Discovered Particles* (N. Zanichelli, Bologna, Italy, 1957); R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[2] S. Okubo, R. E. Marshak, E. C. G. Sudarshan, W. B. Teusch, and S. Weinberg, Phys. Rev. **112**, 665 (1958).

[3] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[4] W. I. Weisberger, Phys. Rev. Letters **14**, 1047 (1965); S. L. Adler, *ibid.* **14**, 1051 (1965).

[5] We have assumed that $\theta_V=+\theta_A$. Only under this assumption is the $2\leftrightarrow 3$ symmetry valid for the parity-violating part of $\mathcal{L}_{sv}$.

[6] S. P. Rosen, Phys. Rev. **137**, B431 (1965).

[7] N. Cabibbo, Phys. Rev. Letters **12**, 62 (1964).

[8] M. Gell-Mann, Phys. Rev. Letters **12**, 155 (1964).

[9] B. W. Lee, Phys. Rev. Letters **12**, 83 (1964); H. Sugawara, Progr. Theoret. Phys. (Kyoto) **31**, 213 (1964). See also B. Sakita, Phys. Rev. Letters **12**, 362 (1964); S. P. Rosen, *ibid.* **12**, 408 (1964); K. Fujii and D. Ito, Progr. Theoret. Phys. (Kyoto) **30**, 718 (1963); M. Gell-Mann, Ref. 8; and S. Okubo, Phys. Letters **8**, 362 (1964). For a dynamical treatment of pv decays using octet dominance, see Riazuddin, A. H. Zimmerman, and Fayyazuddin, Nuovo Cimento **32**, 1819 (1964).

experiment.[10] However, the assignment of $\mathcal{L}_{sv}$ to an octet also requires the introduction of neutral hadron currents—a feature for which there is neither experimental evidence nor justification elsewhere.[11] It is therefore necessary to determine to what extent the desirable consequences of the octet hypothesis remain valid when octet transformation properties are given up.

Here we wish to show that, in the limit of exact $SU(3)$, the current×current theory forbids $K\to 2\pi$ even when the 27-plet part of $\mathcal{L}_{sv}$ is retained. This result enables us to account for the decay $K^+\to\pi^++\pi^0$ without appealing to electromagnetic interactions. If we assume that all $K\to 2\pi$ decays are induced by a charge-independent strong interaction which breaks both $SU(3)$ and the $3\leftrightarrow 2$ symmetry, then the smallness of the ratio

$$R_K=\Gamma(K^+\to\pi^+\pi^0)/\Gamma(K_1^0\to\pi^+\pi^-) \tag{6}$$

can be attributed to an initial suppression of the 27-component of $\mathcal{L}_{sv}$ relative to the octet. The actual magnitude of R_K provides a measure of the amount by which the 27-plet is suppressed and bears no relation to the electromagnetic interactions.

We shall also show that, without the octet hypothesis for $\mathcal{L}_{sv}$, the pv amplitudes of nonleptonic hyperon decay satisfy two sum rules. One of them has already been derived by Suzuki,[12] and correlates the $\Delta T=\frac{3}{2}$ amplitudes in Ξ and Λ decay. The other expresses the deviation from the Lee-Sugawara triangle in terms of the deviation from $\Delta T=\frac{1}{2}$ in Λ and Σ decay. The results recently obtained by Sugawara[13] and Suzuki[14] from current-commutation relations are special cases of these two sum rules and can be explained in a very simple way.

Meson decays are examined in the next section and hyperon decays in the third. The theory of Sugawara and Suzuki is analyzed in the fourth section. Throughout our discussion, we start with the most general CP-invariant Hamiltonian and introduce the restrictions introduced by the current×current hypothesis one by one.

2. THE $K\to 2\pi$ DECAY INTERACTION

Because K and π mesons are members of the same octet, the effective Hamiltonian for $K\to 2\pi$ must be symmetric under the exchange of any two of the three octets L, M, and N, which represent the mesons.[8] With nonderivative coupling, the most general form of the Hamiltonian will be an admixture of the representations 8, 10, $\overline{10}$, 27 and 64, each occurring with multiplicity one[15]:

$$\mathcal{L}(K\to 2\pi)=\sum_{(n)} g_n\mathcal{L}_{(n)}\,,\quad n=8,10,\overline{10},27,64. \tag{7}$$

In the context of the current×current hypothesis and $SU(3)$ conserving strong interactions, only the octet and 27-plet can appear in $(K\to 2\pi)$. The other representations must be regarded as corrections due to the breaking of $SU(3)$ by medium-strong and electromagnetic interactions.

The specific forms of $\mathcal{L}_{(8)}$ and $\mathcal{L}_{(27)}$ are:

$$\begin{aligned}\mathcal{L}_{(8)}&=(5/4)L_\mu{}^\lambda(MN)+(6/5)[(L\cdot D)_\mu{}^\lambda+(D\cdot L)_\mu{}^\lambda]+\tfrac{1}{2}[27]_\mu{}^\lambda L\,,\\ \mathcal{L}_{(27)}&=(1+p^{\lambda\rho})(1+p_{\mu\tau})\{\tfrac{1}{2}[27]_{\mu\alpha}{}^{\lambda\rho}L_\tau{}^\alpha\\&\quad+\tfrac{1}{2}[27]_{\mu\tau}{}^{\lambda\alpha}L_\alpha{}^\rho+(14/5)L_\mu{}^\lambda D_\tau{}^\rho-\text{trace terms}\}\,,\end{aligned} \tag{8}$$

where

$$\begin{aligned}(A\cdot B)&=A_\beta{}^\alpha B_\alpha{}^\beta;\quad (A\cdot B)_\mu{}^\lambda=A_\beta{}^\lambda B_\mu{}^\beta-\tfrac{1}{3}\delta_\mu{}^\lambda(A\cdot B)\,,\\ [27]_\mu{}^\lambda L&=[27]_{\mu\beta}{}^{\lambda\alpha}L_\alpha{}^\beta\,,\end{aligned} \tag{9}$$

and D and $[27]$ denote symmetric octet and 27-plet formed from the octets M and N. The operators $p^{\lambda\rho}$ and $p_{\mu\tau}$ permute the indices attached to them. For convenience we denote the right-hand sides of Eq. (8) by

$$\mathcal{L}_{(8)}\equiv T_\mu{}^\lambda;\quad \mathcal{L}_{(27)}\equiv T_{\mu\tau}{}^{\lambda\rho}. \tag{10}$$

As pointed out in the introduction, the Cabibbo current forces $\mathcal{L}(K\to 2\pi)$ to have U spin equal to one, and to be symmetric under the Weyl reflection $2\leftrightarrow 3$ [T-$L(1)$ invariance]. The U-spin restriction determines the indices in Eq. (10) and enables us to write down the Hermitian form of $\mathcal{L}(K\to 2\pi)$;

$$\mathcal{L}(K\to 2\pi)=g_{(8)}T_2{}^3+g_{(8)}{}^*T_3{}^2+g_{(27)}T_{21}{}^{31}+g_{(27)}{}^*T_{31}{}^{21}. \tag{11}$$

Next we note that for a normal, pseudoscalar meson octet, say $L_\nu{}^\mu$, the CP transformation gives

$$(CP)L_\nu{}^\mu(CP)^{-1}=-L_\mu{}^\nu. \tag{12}$$

Consequently, CP invariance requires

$$g_{(n)}=-g_{(n)}{}^*\,,\quad n=8,27\,, \tag{13}$$

and leads to an effective Hamiltonian

$$\mathcal{L}(K\to 2\pi)=i|g_{(8)}|(T_2{}^3-T_3{}^2)+i|g_{(27)}|(T_{21}{}^{31}-T_{31}{}^{21})\,, \tag{14}$$

that is antisymmetric under the Weyl reflection $2\leftrightarrow 3$ [i.e., T-$L(2)$ invariant[6]]. It follows that, in a current ×current theory and the limit of exact $SU(3)$, the

[10] N. P. Samios, in Proceedings of the Argonne International Conference on Weak Interactions, 1965 (to be published).

[11] W. Willis, in Proceedings of the Argonne International Conference on Weak Interactions, 1965 (to be published).

[12] M. Suzuki, Phys. Rev. **137**, B1602 (1965). See also D. Bailin, Nuovo Cimento **38**, 1342 (1965).

[13] H. Sugawara, Phys. Rev. Letters **15**, 870 (1965); **15**, 997 (E) (1965).

[14] M. Suzuki, Phys. Rev. Letters **15**, 986 (1965).

[15] K. Itabashi, Phys. Rev. **136**, B221 (1964). This shows that T-$L(1)$ invariance alone forbids $K_1^0\to 2\pi^0$, i.e., for $\mathcal{L}(n)$, $n=8$, 10, $\overline{10}$, 27, and 64. ($n=35$ is of course forbidden by Bose statistics.)

combination of CP invariance and T-$L(1)$ invariance forbids all $K\to 2\pi$ decays.[16]

We now consider corrections due to the breaking of $SU(3)$ by electromagnetic interactions. The electromagnetic current has zero U spin and is therefore T-$L(1)$–invariant. Thus its only effect is to introduce admixtures of the $U=1$ components of $\mathfrak{L}_{(10)}$, $\mathfrak{L}_{(10)}$, and $\mathfrak{L}_{(64)}$ into $\mathfrak{L}(K\to 2\pi)$. The specific forms of these representations are:

$$\begin{aligned}\mathfrak{L}_{(10)}\equiv T_{\alpha\beta\gamma}=&\sum_{\alpha\beta\gamma\text{ c.p.}}\epsilon_{\alpha\mu\nu}\{L_\lambda[27]_{\beta\gamma}{}^{\mu\lambda}\\&+(18/5)L_\beta{}^\mu D_\gamma{}^\nu+(18/5)L_\gamma{}^\mu D_\beta{}^\nu\},\\ \mathfrak{L}_{(\overline{10})}\equiv T^{\alpha\beta\gamma}=&\sum_{\alpha\beta\gamma\text{ c.p.}}\epsilon^{\alpha\mu\nu}\{L_\mu{}^\lambda[27]_{\mu\lambda}{}^{\beta\gamma}\\&+(18/5)L_\mu{}^\beta D_\nu{}^\gamma+(18/5)L_\mu{}^\gamma D_\nu{}^\beta\},\\ \mathfrak{L}_{(64)}\equiv T_{\alpha\beta\gamma}{}^{\lambda\mu\nu}=&\sum_{\substack{\alpha\beta\gamma\text{ c.p.,}\\ \lambda\mu\nu\text{ c.p.}}}L_\alpha{}^\lambda[27]_{\beta\gamma}{}^{\mu\nu}-\text{trace terms},\end{aligned}\tag{15}$$

where c.p. means "cyclic permutations," and the Hermitian correction to $\mathfrak{L}(K\to 2\pi)$ is

$$\begin{aligned}&g_{(10)}T_{122}+g_{(10)}{}^*T^{122}+g_{(\overline{10})}T^{133}\\&\quad+g_{(\overline{10})}{}^*T_{133}+g_{(64)}T_{211}{}^{311}+g_{(64)}{}^*T_{311}{}^{211}.\end{aligned}\tag{16}$$

CP invariance and T-$L(1)$ invariance require

$$g_{(n)}=-g_{(n)}{}^*,\quad n=10,\,\overline{10},\,64,\tag{17}$$

and

$$g_{(64)}=g_{(64)}{}^*,\quad g_{(10)}=-g_{(\overline{10})}{}^*,\tag{18}$$

respectively. [Note that the negative sign in Eq. (18) arises from the antisymmetric tensor $\epsilon_{\alpha\mu\nu}$ used in the definition of $\mathfrak{L}_{(10)}$ and $\mathfrak{L}_{(\overline{10})}$, Eq. (15).] The correction due to electromagnetic interactions is therefore

$$i|g_{(10)}|\{T_{122}+T^{133}-T^{122}-T_{133}\}.\tag{19}$$

Because $\mathfrak{L}_{(10)}$ and $\mathfrak{L}_{(\overline{10})}$ are totally symmetric in the octets L, M, and N, we may identify L with the initial K meson and M and N with the final-state π mesons. The Hamiltonian in Eq. (19) is then proportional to

$$\begin{aligned}2K_1{}^0[K^+K^-+K^-K^+-\pi^+\pi^--\pi^-\pi^+]\\-K^+[\pi^-\pi^0+\pi^0\pi^+].\end{aligned}\tag{20}$$

It is interesting to note that $K_1{}^0\to 2\pi^0$ is still forbidden[15] and that the amplitudes for $K\to 2\pi$ satisfy the sum rule[17]

$$\begin{aligned}&2M(K^+\to\pi^+\pi^0)\\&\quad=M(K_1{}^0\to\pi^+\pi^-)-\sqrt{2}M(K_1{}^0\to\pi^0\pi^0).\end{aligned}\tag{21}$$

This sum rule follows from the fact that $\mathfrak{L}_{(10)}$ and $\mathfrak{L}_{(\overline{10})}$ contain $\Delta T=\frac{1}{2}$ and $\frac{3}{2}$ but no $\Delta T=\frac{5}{2}$.[17]

The forbiddenness of $K_1{}^0\to 2\pi^0$ under T-$L(1)$ invariance can be seen more directly. Under the Weyl reflection $2\leftrightarrow 3$, $K_1{}^0$ changes sign, but π^0 and η transform into linear combinations of themselves. These linear combinations can be expressed in terms of two objects, $\pi^{0\prime}$ and η', which are, respectively, odd and even under $2\leftrightarrow 3$; $\pi^{0\prime}$ is the $U_3=0$ member of a U-spin triplet π', and η' is a U-spin singlet.[18] T-$L(1)$ invariance forbids both $K_1{}^0\to\eta'\eta'$ and $K_1{}^0\to\pi^{0\prime}\pi^{0\prime}$, but it allows $K_1{}^0\to\pi^{0\prime}\eta'$. Since $K_1{}^0$ and $\pi^{0\prime}$ are members of the same U-spin triplet,[18] say π', any nonderivative coupling $\pi_1'\pi_2'\eta'$ with total U spin equal to one must be antisymmetric under the exchange of π_1' and π_2'. However, only symmetric couplings are allowed, and so $K_1{}^0\to\pi^{0\prime}\eta'$ is also forbidden. It follows that the nonderivative coupling $K_1{}^0\to\pi^0\pi^0$, which is a linear combination of $K_1{}^0\to\eta'\eta'$, $\pi^{0\prime}\pi^{0\prime}$, $\pi^{0\prime}\eta'$, must be forbidden.

It is clear from this analysis that electromagnetic corrections to the current×current hypothesis are not sufficient to account for the smallness of the ratio R_K [see Eq. (6)]. We therefore turn to the medium-strong $SU(3)$-breaking interaction. If it transforms as the $Y=T=0$ member of an octet, it can be written as

$$\begin{aligned}S_3{}^3&=\tfrac{1}{2}(S_3{}^3+S_2{}^2)+\tfrac{1}{2}(S_3{}^3-S_2{}^2)\\&=\tfrac{1}{2}S_1{}^1+\tfrac{1}{2}(S_3{}^3-S_2{}^2).\end{aligned}\tag{22}$$

The first term of Eq. (22) transforms in exactly the same way as the electromagnetic current and will yield the type of corrections discussed above. The second term, however, has U spin equal to one[18] and breaks the T-$L(1)$ symmetry; in fact when combined with $\mathfrak{L}_{sv}$ [see Eq. (4)]; it will produce corrections to the effective Hamiltonian $\mathfrak{L}(K\to 2\pi)$ that are antisymmetric under the Weyl reflection $2\leftrightarrow 3$ [T-$L(2)$ invariance]. Thus the medium strong interaction can give rise to the terms

[16] This fact has previously been observed by Itabashi, Ref. (15); however, his subsequent claim that the T-$L(2)$-invariant interaction of Eq. (14) also forbids $K\to 2\pi$ is not correct. The error is to be found in the principle of his argument. Itabashi observes that $\mathfrak{L}(K\to 2\pi)$ is an admixture of the first two components of a U-spin vector $(L^{(-)}, L^{(+)}, L^{(0)})$. He rotates $\mathfrak{L}(K\to 2\pi)$ into $L^{(0)}$ and $K\to 2\pi$ into $K'\to 2\pi'$, and then uses the CP invariance of $L^{(0)}$ to show that $\langle K'|L^{(0)}|2\pi'\rangle=0$; from this he concludes that the original matrix element for $K\to 2\pi$ must also vanish. In order for this conclusion to be valid, the rotation operator must commute with CP (B. Sakita, private communication). For $L^{(-)}$ the rotation operator is $T^{(-)}\equiv\exp[\frac{1}{4}\pi(A_3{}^2-A_2{}^3)]$ and for $L^{(+)}$ it is $T^{(+)}\equiv\exp[\frac{1}{4}i\pi(A_3{}^2+A_2{}^3)]$, where $A_\nu{}^\mu$ is a generator of $SU(3)$; and from the definition of CP in Eq. (12), we see that $[T^{(-)},CP]=0$, but $[T^{(+)},CP]\neq 0$. Thus Itabashi's argument is correct when $\mathfrak{L}(K\to 2\pi)\equiv L^{(-)}$, but not when $\mathfrak{L}(K\to 2\pi)\equiv L^{(+)}$. Finally we note that $L^{(-)}$ points along the U_1 axis and is symmetric under $2\leftrightarrow 3$, while $L^{(+)}$ points along the U_2 axis and is antisymmetric. Another way of making this point is to note: (a) that T-$L(1)$ invariance is necessary to forbid $K\to 2\pi$; and (b) that an interaction with U spin different from zero cannot be simultaneously T-$L(2)$-invariant and T-$L(1)$-invariant (see Ref. 6).

[17] E. C. G. Sudarshan, Syracuse University report, NYO-3399-41, 1965 (unpublished); T. Das and K. Mahanthappa, University of Pennsylvania Report, 1965 (unpublished).

[18] A detailed discussion of the Weyl reflection $2\leftrightarrow 3$ is given in A. J. Macfarlane, E. C. G. Sudarshan, and C. Dullemond, Nuovo Cimento **30**, 845 (1963); and in the appendix of Ref. 6.

of Eq. (14) as well as similar contributions from $\mathcal{L}_{(10)}$, $\mathcal{L}_{(\overline{10})}$ and $\mathcal{L}_{(64)}$.

If we assume that the major correction from $S_3{}^3$ is octet term of Eq. (14) together with a small admixture of the 27-plet, the effective Hamiltonian will be proportional to

$$\begin{aligned}|g_{(8)}|K_1{}^0(\pi^+\pi^-+\pi^-\pi^++\pi^0\pi^+) \\ +|g_{(27)}|\{\tfrac{1}{5}K_1{}^0(6\pi^+\pi^-+6\pi^-\pi^+-4\pi^0\pi^0) \\ +K^+(\pi^-\pi^0+\pi^0\pi^-)\} \\ \times|g_{(27)}|/|g_{(8)}|\ll 1. \qquad (23)\end{aligned}$$

The sum rule of Eq. (21) is again satisfied, but $K_1{}^0\to 2\pi^0$ is no longer forbidden. The ratio of the two coupling constants can be determined from the experimental value of R_K [Eq. (6)].

Our last point concerns the use of nonderivative coupling in $\mathcal{L}(K\to 2\pi)$. If we were to use derivative coupling instead, the amplitudes for $K\to 2\pi$ would be proportional to the K-π mass difference. In the limit of exact $SU(3)$, this mass difference vanishes, and with it all the decay amplitudes.[19] Thus the qualitative consequences of derivative coupling are accounted for when we consider corrections induced by $SU(3)$ breaking strong interactions in the nonderivative coupling scheme.

3. APPLICATION TO HYPERON DECAYS

In order to determine the consequences of the current $\times$current hypothesis for nonleptonic hyperon decay, we shall write the effective Hamiltonian for

$$X\to Y+\pi \qquad (24)$$

as a linear combination of scalar and pseudoscalar couplings. CP invariance then leads to an Hermitian interaction of the form[6]

$$\begin{aligned}(\mathcal{L}X\to Y\pi)\equiv[\bar{\psi}_X(A+B\gamma_5)\psi_Y]\varphi_\pi \\ +[\bar{\psi}_Y(-A+B\gamma_5)\psi_X]\varphi_\pi{}^+, \qquad (25)\end{aligned}$$

where A and B are real coupling constants and correspond to the parity violating and parity conserving parts of $\mathcal{L}(X\to Y\pi)$, respectively.

Let us now suppose that $\mathcal{L}(X\to Y\pi)$ is an admixture of the octet and 27-plet and that it is T-$L(1)$-invariant. Parity-conserving decays will then depend upon eight independent constants and parity-violating ones upon five; the difference between these numbers is due to the negative sign of A in the second term of Eq. (5). Since there are only seven observable decay modes, we can derive two sum rules for pv decays, but none for pc decays.

The $SU(3)$ structure of the pv part, of $\mathcal{L}(X\to Y\pi)$ consists of three octets terms and two 27-plets. It is

$$\begin{aligned}&f_1[D_\lambda{}^3\pi_2{}^\lambda+D_\lambda{}^2\pi_3{}^\lambda-D_3{}^\lambda\pi_\lambda{}^2-D_2{}^\lambda\pi_\lambda{}^3] \\ &\quad+f_2[F_\lambda{}^3\pi_2{}^\lambda+F_\lambda{}^2\pi_3{}^\lambda-F_\lambda{}^2\pi_3{}^\lambda-F_2{}^\lambda\pi_\lambda{}^3]+f_3\{[10]_2{}^3\pi+[\overline{10}]_2{}^3\pi+[10]_3{}^2\pi+[10]_3{}^2\pi+[\overline{10}]_3{}^2\pi\} \\ &\quad+h_1\{[10]_{21}{}^{3\lambda}\pi_\lambda{}^1+[10]_{21}{}^{1\lambda}\pi_\lambda{}^3+[\overline{10}]_{2\lambda}{}^{31}\pi_1{}^\lambda+[\overline{10}]_{1\lambda}{}^{31}\pi_2{}^\lambda+[\overline{10}]_{3\lambda}{}^{21}\pi_1{}^\lambda+[\overline{10}]_{1\lambda}{}^{21}\pi_3{}^\lambda+[10]_{31}{}^{2\lambda}\pi_\lambda{}^1 \\ &\quad+[10]_{31}{}^{1\lambda}\pi_\lambda{}^2-\text{trace terms}\}+h_2\{[27]_{21}{}^{3\lambda}\pi_\lambda{}^1+[27]_{21}{}^{1\lambda}\pi_\lambda{}^3-[27]_{2\lambda}{}^{31}\pi_1{}^\lambda-[27]_{1\lambda}{}^{31}\pi_2{}^\lambda \\ &\qquad-[27]_{3\lambda}{}^{21}\pi_1{}^\pi-[27]_{1\lambda}{}^{21}\pi_3{}^\pi+[27]_{31}{}^{2\lambda}\pi_\lambda{}^1+[27]_{31}{}^{1\lambda}\pi_\lambda{}^2\}, \qquad (26)\end{aligned}$$

where D and F denote the usual octet couplings of baryons and antibaryons, and $[X]_{\alpha\beta}{}^{\lambda\mu}$ denote the 10, $\overline{10}$, and (27), respectively. The symbol π represents the meson octet and

$$[X]_2{}^3\pi\equiv[X]_{2\beta}{}^{3\alpha}\pi_\alpha{}^\beta. \qquad (27)$$

The trace terms associated with h_1 are equal to one fifth of the expression associated with f_3. By direct calculation, we obtain two sum rules from Eq. (26):

$$A(\Lambda_-{}^0)+\sqrt{2}A(\Lambda_0{}^0)=\sqrt{2}A(\Xi_0{}^0)-A(\Xi_-{}^-) \qquad (28)$$

$$\begin{aligned}2\{-\sqrt{3}A(\Sigma_0{}^+)+A(\Lambda_-{}^0)+2A(\Xi_-{}^-)\} \\ =A(\Lambda_-{}^0)+\sqrt{2}A(\Lambda_0{}^0)-\sqrt{3}/\sqrt{2}\{\sqrt{2}A(\Sigma_0{}^+) \\ +A(\Sigma_+{}^+)-A(\Sigma_-{}^-)\}. \qquad (29)\end{aligned}$$

The first one, Eq. (28), relates the $\Delta T=\frac{3}{2}$ amplitude in Λ decay to the corresponding amplitude in Ξ decay, and was first derived by Suzuki.[12] The second sum rule is a generalization of the Lee-Sugawara triangle,[9] Eq. (5), and appears to be new. If we impose the $\Delta T=\frac{1}{2}$ rule, both sides of Eq. (28) are identically zero, and Eq. (29) reduces to Eq. (5). To understand why this happens, we note that the combination of T-$L(1)$ invariance and the $\Delta T=\frac{1}{2}$ rule forces $\mathcal{L}(X\to Y\pi)$ to transform as an octet[20] (i.e. $h_1=h_2=0$) and reproduces the conditions under which Gell-Mann[8] derived the Lee-Sugawara triangle.

4. CONSEQUENCES OF CURRENT COMMUTATION RELATIONS

To conclude this discussion, we use the results of the previous paragraph to analyze the recent work of Sugawara[13] and Suzuki.[14] These authors have applied current commutation relations to the current$\times$current interaction and have derived four sum rules for pv decays. Two of them are the $\Delta T=\frac{1}{2}$ rule predictions for Λ and Ξ decay, the third is a "pseudo-$\Delta T=\frac{1}{2}$ rule" for Σ decay, and the fourth is closely related to the Lee-Sugawara triangle.[9] What we wish to point out here is that these results are special cases of Eqs. (28), (29) and

[19] S. Okubo, Ref. 9.

[20] S. P. Rosen, Phys. Rev. **135**, B1041 (1964).

that they follow from one simple constraint upon $\mathcal{L}(X \to Y\pi)$.

The essential feature of the Sugawara-Suzuki calculation is that, in the effective Hamiltonian they derive from current commutators, the baryon-antibaryon system is coupled to 8(27) whenever the effective Hamiltonian transforms as 8(27). For pv decays this is identical to saying that the baryon-antibaryon system is never coupled to a decuplet, i.e. in Eq. (26)

$$f_3 = h_1 = 0. \tag{30}$$

Two of the remaining terms, f_1 and f_2, are octets and satisfy the $\Delta T = \frac{1}{2}$ rule; the third, h_2 is a 27-plet and although it includes a $\Delta T = \frac{3}{2}$ component, it nevertheless gives rise to the $\Delta T = \frac{1}{2}$ in Λ and Ξ decay. To show this, we note that the h_2 term is invariant under the transformation

$$RX_\mu{}^\lambda R^{-1} = -X_\lambda{}^\mu, \tag{31}$$

where $X_\mu{}^\lambda$ is any octet; from this it follows that the h_2 amplitudes satisfy[21]

$$\begin{aligned} A(\Lambda_-{}^0) &= +A(\Xi_-{}^-) \\ A(\Lambda_0{}^0) &= -A(\Xi_0{}^0) \end{aligned} \tag{32}$$

and give rise to the $\Delta T = \frac{1}{2}$ sum rules through Eq. (28). Notice that the h_1 term is invariant under

$$R' X_\mu{}^\lambda R'^{-1} = +X_\lambda{}^\mu \tag{33}$$

and it yields relations like Eq. (32) except that the signs are reversed in this case Eq. (28) becomes identity.

Because the system $\bar{n}\Sigma^+$ has isospin $T = \frac{3}{2}$, the amplitude $A(\Sigma_+{}^+)$ is independent of f_1 and f_2 and must be proportional to h_2 when Eq. (30) holds. In addition, f_1 and f_2 satisfy the $\Delta T = \frac{1}{2}$ rule, and so the expression $\sqrt{2}A(\Sigma_0{}^+) - A(\Sigma_-{}^-)$ is also proportional to h_2. Explicit calculation yields a relation

$$\sqrt{2}A(\Sigma_0{}^+) - A(\Sigma_-{}^-) = A(\Sigma_+{}^+), \tag{34}$$

which differs from the $\Delta T = \frac{1}{2}$ rule prediction only in the sign of $A(\Sigma_+{}^+)$. If we substitute Eq. (34) and the $\Delta T = \frac{1}{2}$ rule for Λ decay into Eq. (29) we obtain Suzuki's sum rule

$$\sqrt{\tfrac{3}{2}}A(\Sigma_-{}^-) - A(\Lambda_-{}^0) = 2A(\Xi_-{}^-). \tag{35}$$

We note that $A(\Sigma_+{}^+) \approx 0$ if $\mathcal{L}(X \to Y\pi)$ is dominated by its octet components and that (35) reduces to the Lee-Sugawara triangle. The sum rule (35) can be derived for very general transformation properties of $\mathcal{L}(X \to Y\pi)$ provided only that the baryon-antibaryon system is coupled to an octet.[22]

Finally, it is amusing to note that if we carry over to pc decays, the coupling of the baryon-antibaryon system to 8 and 27, only, then using the properties (a), (b), and (c) we find that there is one sum rule among the seven decay amplitudes. This is a giant Lee-Sugawara sum rule which relates the deviation from Lee-Sugawara triangle to the deviation from $\Delta T = \frac{1}{2}$ rule in Λ, Σ, and Ξ decays:

$$2\Delta(L\cdot S) = \sqrt{3}\Delta(\Sigma) - \Delta(\Lambda) - 2\Delta(\Xi), \tag{36}$$

where

$$\begin{aligned} \Delta(L\cdot S) &= \sqrt{3}B(\Sigma_0{}^+) - B(\Lambda_-{}^0) - 2B(\Xi_-{}^-), \\ \Delta(\Sigma) &= B(\Sigma_0{}^+) - (B(\Sigma_-{}^-)/\sqrt{2}) + (B(\Sigma_+{}^+)/\sqrt{2}), \\ \Delta(\Lambda) &= B(\Lambda_-{}^0) + \sqrt{2}B(\Lambda_0{}^0), \\ \Delta(\Xi) &= B(\Xi_-{}^-) - \sqrt{2}B(\Xi_0{}^0). \end{aligned}$$

[21] See Ref. 6 for the sign convention for baryon and meson states.

[22] S. P. Rosen, Phys. Rev. 143, 1388 (1966).