

mesons within the framework of the $SU(2) \times SU(2)$ algebra used in this paper. We therefore appeal to the crude and ridiculously large upper limit

$$\Gamma(\phi^0 \rightarrow \pi^0 + \gamma) / \Gamma_{\text{total}} < 10\%$$

to estimate that the calculated value of $\Gamma(\omega^0 \rightarrow \pi^0 + \gamma)$ can be decreased by no more than 15% from the value predicted in Eq. (49).

A by-product of the calculation is obtained when either side of the Eq. (43) is equated to the appropriate coefficient in $V_{\rho}^{\alpha\beta}(t)$ [defined in Eq. (4)]. Since

$$\begin{aligned} V_{\rho}^{\alpha\beta}(t) &= \int d^3x \langle p', a | [\mathcal{F}_0^{\alpha}(\mathbf{x}, 0), \mathcal{F}_{\rho}^{\beta}(0)] | p, b \rangle \\ &= \langle p', a | [T^{\alpha}, \mathcal{F}_{\rho}^{\beta}(0)] | p, b \rangle \\ &= i\rho_{\alpha\beta\gamma} \langle p', a | \mathcal{F}_{\rho}^{\gamma}(0) | p, b \rangle, \quad (51) \end{aligned}$$

we obtain an expansion in powers of t for the pion form factor. The term linear in t yields an expression for the

pion charge radius in terms of the rate $\Gamma(\omega^0 \rightarrow \pi^0 + \gamma)$, a result obtained by Cabibbo and Radicati.¹²

In conclusion, it should be stressed that a number of unproved assumptions have been made in the derivation of our result. Some of these have to do with the nature of singularities of commutators of currents, and they are certainly at variance with what is found in perturbation theory. Calculations such as this one may perhaps be viewed as encouraging to the point of view that the assumptions are correct. Other assumptions have to do with the states which are taken to "saturate" the sum rules. The structure of matrix elements which have been investigated is such as to suggest that if the coupling constants do not grow with mass, then the higher mass and higher spin-state contributions, in addition to yielding sum rules for $\Gamma(f' \rightarrow \pi + \gamma)$ in terms of $\Gamma(f^0 \rightarrow 2\pi)$, say, will not significantly alter the numbers so far obtained.

I would like to acknowledge useful discussions with Dr. D. Geffen and Dr. J. Meyer.

¹² N. Cabibbo and L. Radicati, Phys. Letters **19**, 697 (1965).

Sum-Rule Calculation of the Isovector Form Factor*

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A class of sum rules due to Fubini is rederived from a point of view which clarifies the assumptions made about the singularities of field operators. A calculation of the isovector magnetic moment is performed in the isobar approximation. The result $F_1^V(0) = \frac{1}{2}$ emerges as a model-independent consistency condition, but the calculated value $F_2^V(0) = 3.64$ is in disagreement with the experimental value.

In a recent paper Fubini¹ presented a new method of obtaining sum rules of interest in strong-interaction physics. It is the purpose of this note to present a set of assumptions which lead to an alternative derivation of the Fubini result. Sum rules for the isovector form factors of the nucleon are derived and their properties are discussed.

We consider the matrix element $T_{\mu}^{\alpha\beta}(p', k; p, q)$ defined by

$$\begin{aligned} T_{\mu}^{\alpha\beta}(p', k; p, q) &= -i \int dx e^{ik \cdot x} \theta(x_0) \\ &\quad \times \langle p' | [j_{\mu}^{\alpha}(x), \phi^{\beta}(0)] | p \rangle. \quad (1) \end{aligned}$$

Here $j_{\mu}^{\alpha}(x)$ is a current operator, $\phi^{\beta}(0)$ is scalar or a pseudoscalar field operator, and the indices α, β may be

isospin or $SU(3)$ labels, so chosen that the operators $j_{\mu}^{\alpha}(x)$ and $\phi^{\beta}(0)$ are Hermitian. The matrix element $T_{\mu}^{\alpha\beta}(p', k; p, q)$ may be decomposed into invariant functions. For example, if the particles described by the state vectors $|p\rangle$ and $|p'\rangle$ have spin zero, then the most general form of the matrix element is

$$\begin{aligned} T_{\mu}^{\alpha\beta}(p', k; p, q) &= P_{\mu} A_1^{\alpha\beta}(v, t) \\ &\quad + Q_{\mu} A_2^{\alpha\beta}(v, t) + \Delta_{\mu} A_3^{\alpha\beta}(v, t). \quad (2) \end{aligned}$$

We have used the conventional notation

$$\begin{aligned} P_{\mu} &= \frac{1}{2}(p_{\mu}' + p_{\mu}), \\ Q_{\mu} &= \frac{1}{2}(k_{\mu} + q_{\mu}), \\ \Delta_{\mu} &= p_{\mu}' - p_{\mu} = q_{\mu} - k_{\mu}, \\ t &= \Delta^2, \\ v &= P \cdot Q. \end{aligned} \quad (3)$$

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¹ S. Fubini, Nuovo Cimento (to be published).

Let us now consider

$$\begin{aligned}
 k^\mu T_{\mu}^{\alpha\beta}(\mathbf{p}', k; \mathbf{p}, q) &= -i \int dx \left(-i \frac{\partial}{\partial x_\mu} e^{ikx} \right) \theta(x_0) \langle \mathbf{p}' | [j_\mu^\alpha(x), \phi^\beta(0)] | \mathbf{p} \rangle, \\
 &= - \int_{x_0=-\infty}^{\infty} d^3x e^{ikx} \langle \mathbf{p}' | [j_0^\alpha(x), \phi^\beta(0)] | \mathbf{p} \rangle, \\
 &\quad + \int dx e^{ikx} \theta(x_0) \langle \mathbf{p}' | [\partial^\mu j_\mu^\alpha(x), \phi^\beta(0)] | \mathbf{p} \rangle, \\
 &\quad + \int dx e^{ikx} \delta(x_0) \langle \mathbf{p}' | [j_0^\alpha(x), \phi^\beta(0)] | \mathbf{p} \rangle. \quad (4)
 \end{aligned}$$

We shall be interested in the limit $\nu \rightarrow \infty$. Under those circumstances the first term on the right gives no contribution.² The second term will vanish if the current is conserved. If the current is not conserved, we shall assume that it is partially conserved, i.e. that the divergence of the current is a less singular operator than the current itself. Thus the quantity

$$\int dx e^{ikx} \theta(x_0) \langle \mathbf{p}' | [\partial^\mu j_\mu^\alpha(x), \phi^\beta(0)] | \mathbf{p} \rangle,$$

may be expected to satisfy unsubtracted dispersion relations, and in the limit $\nu \rightarrow \infty$ it will be dominated by the last term in (4), which is given by

$$\begin{aligned}
 &\int dx e^{ikx} \delta(x_0) \langle \mathbf{p}' | [j_0^\alpha(x), \phi^\beta(0)] | \mathbf{p} \rangle \\
 &= \int d^3x \langle \mathbf{p}' | [j_0^\alpha(\mathbf{x}, 0), \phi^\beta(0)] | \mathbf{p} \rangle \\
 &= \langle \mathbf{p}' | [Q^\alpha, \phi^\beta(0)] | \mathbf{p} \rangle, \quad (5)
 \end{aligned}$$

and is a function of $(\mathbf{p}' - \mathbf{p})^2$ independent of ν . We therefore obtain, as a result of our assumptions about the nature of the current operators, that

$$\lim_{\nu \rightarrow \infty} k^\mu T_{\mu}^{\alpha\beta}(\mathbf{p}', k; \mathbf{p}, q) = \langle \mathbf{p}' | [Q^\alpha, \phi^\beta(0)] | \mathbf{p} \rangle, \quad (6)$$

for all t . It appears that for a conserved current

$$k^\mu T_{\mu}^{\alpha\beta}(\mathbf{p}', k; \mathbf{p}, q) = \langle \mathbf{p}' | [Q^\alpha, \phi^\beta(0)] | \mathbf{p} \rangle, \quad (7)$$

should hold for all values of ν as well, but we shall see that the ν dependence of this sum rule is spurious.

For spinless particles it follows from (2) that

$$\begin{aligned}
 k^\mu T_{\mu}^{\alpha\beta}(\mathbf{p}', k; \mathbf{p}, q) &= (\nu - \frac{1}{2} P \cdot \Delta) A_1^{\alpha\beta}(\nu, t) \\
 &\quad + (Q^2 - \frac{1}{2} Q \cdot \Delta) A_2^{\alpha\beta}(\nu, t) + (Q \cdot \Delta - \frac{1}{2} \Delta^2) A_3^{\alpha\beta}(\nu, t).
 \end{aligned}$$

² If the commutator is expanded in usual fashion by the insertion of a complete set of states, contributions are only possible from single particle states. These, however, must have four-momentum $(P \pm Q)$, i.e. $(\text{mass})^2 = \mathbf{p}^2 + Q^2 \pm 2\nu$, and there are no such states for large ν .

The consequences of crossing symmetry may be deduced from the formal identity

$$T_{\mu}^{\alpha\beta}(\mathbf{p}, -k; \mathbf{p}', -q)^* = T_{\mu}^{\alpha\beta}(\mathbf{p}', k; \mathbf{p}, q). \quad (8)$$

If, for simplicity we assume that the particles have isospin $\frac{1}{2}$, then we may write

$$A_i^{\alpha\beta}(\nu, t) = \delta_{\alpha\beta} A_i^{(+)}(\nu, t) + \frac{1}{2} [\tau_\alpha, \tau_\beta] A_i^{(-)}(\nu, t), \quad (9)$$

and the crossing relations are

$$\begin{aligned}
 A_1^{(\pm)}(\nu, t) &= \pm A_1^{(\pm)}(-\nu, t)^*, \\
 A_2^{(\pm)}(\nu, t) &= \mp A_2^{(\pm)}(-\nu, t)^*, \\
 A_3^{(\pm)}(\nu, t) &= \mp A_3^{(\pm)}(-\nu, t)^*.
 \end{aligned} \quad (10)$$

If the $A_i^{(-)}(\nu, t)$ are assumed to obey unsubtracted dispersion relations, then

$$A_1^{(-)}(\nu, t) = \frac{2\nu}{\pi} \int_0^\infty dv' \frac{a_1^{(-)}(\nu', t)}{\nu'^2 - \nu^2 - i\epsilon}, \quad (11)$$

and

$$A_i^{(-)}(\nu, t) = - \frac{2}{\pi} \int_0^\infty dv' \frac{\nu' a_i^{(-)}(\nu', t)}{\nu'^2 - \nu^2 - i\epsilon}, \quad (12)$$

for $i=2, 3$. Hence Eq. (6) yields

$$\begin{aligned}
 & - \frac{2}{\pi} \int_0^\infty a_1^{(-)}(\nu', t) d\nu' \\
 &= [\text{isovector part of } \langle \mathbf{p}' | [Q^\alpha, \phi^\beta(0)] | \mathbf{p} \rangle], \quad (13)
 \end{aligned}$$

which is just Fubini's result.

We now proceed to apply the technique to the deduction of a sum rule for the isovector form factor $F_2^v(0)$. To do this, we take the states $|\mathbf{p}\rangle$, $|\mathbf{p}'\rangle$ to describe single nucleon states. The most general form for $T_{\mu}^{\alpha\beta}(\mathbf{p}', k; \mathbf{p}, q)$ is given by

$$\begin{aligned}
 T_{\mu}^{\alpha\beta}(\mathbf{p}', k; \mathbf{p}, q) &= i\bar{u}(\mathbf{p}') \gamma_5 (\gamma_\mu A_1^{\alpha\beta}(\nu, t) + [\gamma_\mu, \mathbf{Q}] A_2^{\alpha\beta}(\nu, t) \\
 &\quad + P_\mu (A_3^{\alpha\beta}(\nu, t) + \mathbf{Q} A_4^{\alpha\beta}(\nu, t)) \\
 &\quad + Q_\mu (A_5^{\alpha\beta}(\nu, t) + \mathbf{Q} A_6^{\alpha\beta}(\nu, t)) \\
 &\quad + \Delta_\mu (A_7^{\alpha\beta}(\nu, t) + \mathbf{Q} A_8^{\alpha\beta}(\nu, t))) u(\mathbf{p}), \quad (14)
 \end{aligned}$$

where we use the notation $\mathbf{Q} = \gamma_\mu Q^\mu$. If $j_\mu^\alpha(x)$ is taken to be the isospin current, so that

$$\partial^\mu j_\mu^\alpha(x) = 0,$$

and

$$\int d^3x j_0^\alpha(\mathbf{x}, 0) = T^\alpha, \quad (15)$$

we obtain, after some straightforward manipulations, two identities

$$\begin{aligned}
 A_1^{(-)}(\nu, t) + \nu A_4^{(-)}(\nu, t) \\
 + (Q^2 - \frac{1}{4} t) (\frac{1}{2} A_6^{(-)}(\nu, t) + A_8^{(-)}(\nu, t)) = 0, \quad (16)
 \end{aligned}$$

and

$$\begin{aligned}
 mA_1^{(-)}(\nu, t) + 2\nu A_2^{(-)}(\nu, t) + \nu A_3^{(-)}(\nu, t) \\
 + (Q^2 - \frac{1}{4} t) (\frac{1}{2} A_5^{(-)}(\nu, t) + A_7^{(-)}(\nu, t)) = G(t). \quad (17)
 \end{aligned}$$

Here $G(t)$ is defined by

$$\langle p' | [T^\alpha, \phi^\beta(0)] | p \rangle = i\bar{u}(p')\gamma_5\frac{1}{2}[\tau_\alpha, \tau_\beta]u(p)G((p'-p)^2), \quad (18)$$

so that

$$G(t) = (\sqrt{2}gK_{NN\pi}(t))/(m_\pi^2 - t), \quad (19)$$

and we have taken $k^2=0$, so that

$$Q \cdot \Delta = Q^2 + \frac{1}{4}t. \quad (20)$$

The constant g in (19) is the pion-nucleon coupling constant and $K_{NN\pi}(t)$ is the pion-nucleon vertex function, normalized so that

$$K_{NN\pi}(m_\pi^2) = 1. \quad (21)$$

It is generally believed that $K_{NN\pi}(t)$ is a slowly varying function of t for small t , and we shall approximate $K_{NN\pi}(0)$ by unity when the question arises.

It follows from (8) and the assumption of unsubtracted dispersion relations that

$$A_i^{(-)}(\nu, t) = \frac{2\nu}{\pi} \int_0^\infty \frac{d\nu' a_i^{(-)}(\nu', t)}{\nu'^2 - \nu^2 - i\epsilon} \quad i=2, 3, 4, \quad (22)$$

and

$$A_i^{(-)}(\nu, t) = \frac{2}{\pi} \int_0^\infty \frac{d\nu' \nu' a_i^{(-)}(\nu', t)}{\nu'^2 - \nu^2 - i\epsilon} \quad i=1, 5, 6, 7, 8. \quad (23)$$

We will now show that the ν dependence of the sum rules (16) and (17) is spurious.³ It follows from (16)

and (17) that

$$a_1^{(-)}(\nu, t) + \nu a_4^{(-)}(\nu, t) + (Q^2 - \frac{1}{4}t) \times [\frac{1}{2}a_6^{(-)}(\nu, t) + a_8^{(-)}(\nu, t)] = 0, \quad (24)$$

and

$$m a_1^{(-)}(\nu, t) + 2\nu a_2^{(-)}(\nu, t) + \nu a_3^{(-)}(\nu, t) + (Q^2 - \frac{1}{4}t) [\frac{1}{2}a_5^{(-)}(\nu, t) + a_7^{(-)}(\nu, t)] = 0. \quad (25)$$

Thus Eq. (16) may be rewritten in the form

$$\frac{2}{\pi} \int_0^\infty d\nu' \frac{\nu' a_1^{(-)}(\nu', t)}{\nu'^2 - \nu^2} + \frac{2\nu^2}{\pi} \int_0^\infty \frac{d\nu' a_4^{(-)}(\nu', t)}{\nu'^2 - \nu^2} + \frac{2}{\pi} \int_0^\infty \frac{d\nu'}{\nu'^2 - \nu^2} [-\nu' a_1^{(-)}(\nu', t) - \nu'^2 a_4^{(-)}(\nu', t)] = 0,$$

i.e.,

$$\int_0^\infty a_4^{(-)}(\nu', t) = 0. \quad (26)$$

This is just the result which is obtained by letting $\nu \rightarrow \infty$ in (16). We use (25) to rewrite (17) in the form

$$-\frac{2}{\pi} \int_0^\infty d\nu' (2a_2^{(-)}(\nu', t) + a_3^{(-)}(\nu', t)) = G(t), \quad (27)$$

which is again equivalent to letting $\nu \rightarrow \infty$ in (17).

The absorptive parts $a_i^{(-)}(\nu, t)$ are determined from the isovector coefficients of γ_μ , $[\gamma_\mu, \mathbf{Q}]$, $P_\mu, \dots, \Delta_\mu \mathbf{Q}$ in the expansion of

$$\begin{aligned} t_\mu^{\alpha\beta}(p', k; p, q) &= -\frac{1}{2} \int dx e^{ikx} \langle p' | [j_\mu^\alpha(x), \phi^\beta(0)] | p \rangle \\ &= -\frac{1}{2} (2\pi)^4 \sum_n \{ \delta(P_n - p' - k) \langle p' | j_\mu^\alpha(0) | n \rangle \langle n | \phi^\beta(0) | p \rangle + \text{crossed term} \} \\ &= -\frac{1}{2} (2\pi)^4 \sum_n \{ \delta(P_n - p' - k) \langle p' | j_\mu^\alpha(0) | n \rangle \langle n | j_\mu^\beta(0) | p \rangle / (m_\pi^2 - q^2) + \text{crossed term} \}. \end{aligned} \quad (28)$$

The single-particle terms are of the form

$$\begin{aligned} &-\frac{(2\pi)^4}{2} \int d^4p'' \frac{2M}{(2\pi)^3} \delta(p''^2 - M^2) \delta(p'' - p' - k) \sum_{\text{spins}} \langle p' | j_\mu^\alpha(0) | p'' \rangle \frac{\langle p'' | j_\mu^\beta(0) | p \rangle}{m_\pi^2 - q^2} + \text{crossed term} \\ &= -\frac{1}{2} \pi \delta(\nu - \frac{1}{2}(M^2 - P^2 - Q^2)) \sum_{\text{spins}} 2M \langle p' | j_\mu^\alpha(0) | p' + k \rangle \langle p' + k | j_\mu^\beta(0) | p \rangle (m_\pi^2 - q^2)^{-1} + \text{crossed term}. \end{aligned} \quad (29)$$

When the single particle contribution comes from a nucleon state then $M=m$. With the current matrix element written in the form ($k^2=0$):

$$\langle p' | j_\mu^\alpha(0) | p'' \rangle = \bar{u}(p') \tau_\alpha [\gamma_\mu (F_1^V(0) + F_2^V(0)) - ((p_\mu'' + p_\mu')/2m) F_2^V(0)] u(p''), \quad (30)$$

³ I am indebted to Professor Fubini for this simple argument. The same result is obtained if one notes that the $a_i^{(-)}(\nu, t)$ can be expressed in terms of the absorptive parts of the photoproduction amplitude which can be expressed in terms of only four amplitudes, $A^{(-)}(\nu, t), \dots, D^{(-)}(\nu, t)$ of G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

and with

$$\langle p'' | j_\mu^\beta(0) | p \rangle = ig\bar{u}(p'') \tau_\beta \gamma_5 u(p) K_{NN\pi}(q^2), \quad (31)$$

a straightforward calculation yields the following *single-particle terms*:

$$\begin{aligned} A_1^{(-)}(\nu, t) &= 0, \\ A_2^{(-)}(\nu, t) &= -\frac{1}{2} G(q^2) (F_1^V(0) + F_2^V(0)) \nu / (\nu m^2 - \nu^2), \\ A_3^{(-)}(\nu, t) &= -G(q^2) F_1^V(0) \nu / (\nu m^2 - \nu^2), \\ A_4^{(-)}(\nu, t) &= -(G(q^2)/m) F_2^V(0) \nu / (\nu m^2 - \nu^2), \end{aligned} \quad (32)$$

where

$$\nu_m = \frac{1}{2}(Q^2 - \frac{1}{2}t) = \frac{1}{4}(q^2 - t). \quad (33)$$

The sum rules thus take the form

$$\frac{G(q^2)F_2^V(0)}{m} - \frac{2}{\pi} \int_{\text{continuum}} d\nu' a_4^{(-)}(\nu', t) = 0, \quad (34)$$

and

$$G(q^2)(2F_1^V(0) + F_2^V(0))$$

$$- \frac{2}{\pi} \int_{\text{continuum}} d\nu' (2a_2^{(-)}(\nu', t) + a_3^{(-)}(\nu', t)) = G(t). \quad (35)$$

$G(q^2)$ has a pole at $q^2 = m_\pi^2$, but so do the terms under the integral, so that there is no obvious difficulty with that point. It is far from clear that the integral term has a pole at $t = m_\pi^2$, but then the one-dimensional dispersion relations which we use may only be valid for a range of values of t . If we restrict ourselves to $t = q^2$, then we see from (24) and (25) that

$$2a_2^{(-)}(\nu', t) + a_3^{(-)}(\nu', t) = - (ma_1^{(-)}(\nu', t)/\nu') = ma_4^{(-)}(\nu', t). \quad (36)$$

Thus the sum rules (34) and (35) become

$$\frac{G(t)F_2^V(0)}{m} + \frac{2}{\pi} \int_0^\infty d\nu' \frac{a_1^{(-)}(\nu', t)}{\nu'} = 0, \quad (37)$$

$$\frac{1}{m} G(t)(2F_1^V(0) - 1 + F_2^V(0))$$

$$+ \frac{2}{\pi} \int_0^\infty d\nu' \frac{a_1^{(-)}(\nu', t)}{\nu'} = 0,$$

i.e., we obtain

$$G(t)F_2^V(0) = - \frac{2m}{\pi} \int_0^\infty d\nu' \frac{a_1^{(-)}(\nu', t)}{\nu'}, \quad (38)$$

and

$$F_1^V(0) = \frac{1}{2}. \quad (39)$$

It is interesting that the correct value of the isovector charge form factor emerges as a consistency condition.

We conclude with a calculation of the isobar contribution to the right-hand side of (38). We set $t = q^2 = 0$. The $a_i^{(-)}(\nu', 0)$ are obtained from

$$- \frac{1}{2}\pi \sum_{\text{resonant states}} \delta(\nu - \frac{1}{2}(M_{\text{res.}}^2 - m^2)) \sum_{\text{spins}} (2M_{\text{res.}}/m_\pi^2) \langle \langle p' | j_\mu^\alpha(0) | (p' + k)_{\text{res.}} \rangle \rangle \langle \langle (p' + k)_{\text{res.}} | j_\mu^\beta(0) | p \rangle \rangle. \quad (40)$$

If we specialize to $\alpha = 3, \beta = 1, 2$ and take $|p\rangle$ to be a neutron state, $|p'\rangle$ to be a proton state, then⁴

$$\langle p'' | j_\pi^{(+)} | p \rangle = (3)^{-1/2} (\lambda_1/m_\pi) \bar{u}_\sigma(p'') q^\sigma u(p), \quad (41)$$

where $\lambda_1 = 2.2$ is the coupling leading to an isobar width of 125 MeV. We also use

$$\langle p' | j_\mu^{(3)}(0) | p'' \rangle = (iC/m_\pi) \bar{u}(p') \gamma_5 (\delta_\mu^\lambda \mathbf{k} - \gamma_\mu k^\lambda) u_\lambda(p''), \quad (42)$$

with $C = 0.345$ determined by Gourdin and Salin⁴ from the photoproduction of the isobar state. The coefficients of γ_μ , etc. are obtained with the help of

$$\sum_{\text{spins}} u_\lambda(p'') \bar{u}_\sigma(p'') = 1/2M^* (g_{\lambda\sigma} - \frac{1}{3}\gamma_\lambda \gamma_\sigma - (2p_\lambda'' p_\sigma''/3M^{*2}) - (p_\lambda'' \gamma_\sigma - p_\sigma'' \gamma_\lambda/3M^*) (\mathbf{p}'' + M^*)), \quad (43)$$

where M^* is the isobar mass. The result of the calculation is that

$$(F_2^V(0))_{\text{isobar}} = \frac{\lambda_1 C}{6^{1/2} g} \frac{4}{3} \left(\frac{m}{m_\pi} \right) \left(\frac{m + M^*}{m_\pi} \right) \times \left(1 + \frac{M^{*2} - m^2}{4M^{*2}} \right) = 3.64 \quad (44)$$

This result is in rather poor agreement with the experimental value $F_2^V(0) = 1.85$. In contrast to the threshold-type of sum rule used by Fubini *et al.*,⁴ the isobar does not seem to yield the major contribution to the form factor in this sum rule, even though the expression in (44) reduces to that of Fubini *et al.* in the limit $m/M^* = 1$. An estimate of the effect of the next resonance shows no major change in the result, so that pending a clearer understanding of the higher energy contributions, or a formulation which shifts the weight to lower energies in the sum rule, we must conclude that the new sum rules are not as useful as the old ones.

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⁴ We have taken the parameters from S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento (to be published), who reevaluated them following the analysis of M. Gourdin and Ph. Salin, Nuovo Cimento **27**, 193 (1963).