ing the parameters completely in our model. Even if one breaks the crossing-symmetry property, the two bootstrap equations never give any more useful conditions than a relation which merely relates the symmetry-breaking factors built in. Furthermore, using crossing symmetry rather than destroying it seems to be the reasonable approach, in the sense that the bootstrap equations then can result in a nontrivial relation between the bootstrap parameters. The bootstrap equations, when crossing symmetry is used, do not permit any C.D.D. zeros and hence exclude the possibility of having a positive scattering length in our model.

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Sum Rule Connecting Mesonic and Photonic Matrix Elements and the Rate for $\omega^0 \to \pi^0+\gamma$ †

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The Gell-Mann current-commutation relations are used to obtain a relation between matrix elements involving vector currents and matrix elements involving axial currents, in the infinite energy limit. This relation leads to a set of sum rules. One of these is used to calculate the rate for $\omega^0 \to \pi^0+\gamma$ in terms of the ρ-meson width and the axial-current renormalization constant, under the assumption that only states with spin and parity $0^\pm$ and 1^- need be taken into account. The result, $\Gamma(\omega^0 \to \pi^0+\gamma)=1.2$ MeV, is in reasonable agreement with experiment.

INTRODUCTION

THE remarkable calculation of the axial-current renormalization constant by Weisberger[1] and Adler[2] may be ranked with the determination of the Yukawa coupling constant from the forward scattering dispersion relations as one of the major successes of field theory in the domain of the strong interactions. This calculation, based on the equal-time current commutation relations of Gell-Mann,[3] has stimulated further exploitation of these relations in a number of sum rules. When certain assumptions concerning the dominant contributions to the sum rules are made, relations between different matrix elements are obtained. In all of the calculations published so far, little use has been made of the symmetry between the vector currents and the axial currents. Thus, in the calculation of the axial-current renormalization constant only matrix elements of the pion field operator appear, with the commutation relations providing a scale. Similarly, the calculations of the magnetic moment of the nucleons[4,5] relate these to matrix elements of the isovector current alone. What has been missing so far is the analog of the old photoproduction or form-factor[6] calculations of dispersion theory, in which, using certain assumptions concerning the intermediate states, photonic matrix elements were calculated in terms of pionic ones. It is the purpose of this paper to suggest a method of using the commutation relations for such problems, and to illustrate it by a calculation of the rate for the process $\omega^0 \to \pi^0+\gamma$.

THE SUM RULE

We begin by considering two matrix elements, one involving a commutator of vector isospin currents, the other a commutator of axial isospin currents:

$$V_{\mu\rho}{}^{\alpha\beta}(p',a,k;\,p,b,q)=-i\int dx\, e^{ikx}\theta(x_0)\times\langle p',a|[\mathfrak{F}_\mu{}^\alpha(x),\mathfrak{F}_\rho{}^\beta(0)]|p,b\rangle \quad (1)$$

and

$$A_{\mu\rho}{}^{\alpha\beta}(p',a,k;\,p,b,q)=-i\int dx\, e^{ikx}\theta(x_0)\times\langle p',a|[\mathfrak{F}_\mu{}^{5\alpha}(x),\mathfrak{F}_\rho{}^{5\beta}(0)]|p,b\rangle. \quad (2)$$

The indices a, b, α, and β refer to the isospin, and we con-

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[1] W. I. Weisberger, Phys. Rev. Letters 14, 1047 (1965).

[2] S. L. Adler, Phys. Rev. Letters 14, 1051 (1965).

[3] M. Gell-Mann, Phys. Rev. 125, 1067 (1962); Physics 1, 63 (1964).

[4] S. Fubini, G. Furlan, and C. Rossetti (to be published).

[5] S. Gasiorowicz, this issue, Phys. Rev. 146, 1071 (1966).

[6] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. 106, 1345 (1957); G. F. Chew, R. Karplus, S. Gasiorowicz, and F. Zachariasen, *ibid.* 110, 265 (1958); P. Federbush, M. L. Goldberger, and S. B. Treiman, *ibid.* 112, 642 (1958).

sider the matrix elements for

$$k^2=q^2=0. \tag{3}$$

If it is assumed that matrix elements involving currents are not very singular, and further, that the divergences of currents are operators less singular than the currents themselves, then it has been shown that[5]

$$\lim_{\nu\to\infty} k^\mu V_{\mu\rho}{}^{\alpha\beta}(p',a,k;\,p,b,q)=\int dx\,\delta(x_0)\times\langle p',a|[\mathfrak{F}_0{}^\alpha(x),\mathfrak{F}_\rho{}^\beta(0)]|p,b\rangle\equiv V_\rho{}^{\alpha\beta}(t), \tag{4}$$

and

$$\lim_{\nu\to\infty} k^\mu A_{\mu\rho}{}^{\alpha\beta}(p',a,k;\,p,b,q)=\int dx\,\delta(x_0)\times\langle p',a|[\mathfrak{F}_0{}^{5\alpha}(x),\mathfrak{F}_\rho{}^{5\beta}(0)]|p,b\rangle\equiv A_\rho{}^{\alpha\beta}(t). \tag{5}$$

Here the standard notation

$$P_\mu=\tfrac{1}{2}(p_\mu'+p_\mu);\quad Q_\mu=\tfrac{1}{2}(k_\mu+q_\mu);\quad \Delta_\mu=p_\mu'-p_\mu=q_\mu-k_\mu$$
$$\nu=P\cdot Q;\quad t=\Delta^2 \tag{6}$$

is used. The Gell-Mann commutation relations imply that

$$V_0{}^{\alpha\beta}(t)=A_0{}^{\alpha\beta}(t). \tag{7}$$

Lorentz invariance demands that therefore

$$\lim_{\nu\to\infty} k^\mu[V_{\mu\rho}{}^{\alpha\beta}(p',a,k;\,p,b,q)-A_{\mu\rho}{}^{\alpha\beta}(p',a,k;\,p,b,q)]=0. \tag{8}$$

This relation may be used to derive sum rules which connect photonic matrix elements to mesonic ones.

The simplest case which is of interest is one for which $|p',a\rangle$ and $|p,b\rangle$ represent one-pion states. In this case, the most general form of the matrix elements is given by

$$\begin{aligned}V_{\mu\rho}{}^{\alpha\beta}&(p',a,k;\,p,b,q)\\ &=[P_\mu P_\rho A_1{}^{\alpha\beta}(\nu,t)+P_\mu Q_\rho A_2{}^{\alpha\beta}(\nu,t)\\ &\quad+P_\mu\Delta_\rho A_3{}^{\alpha\beta}(\nu,t)+Q_\mu P_\rho A_4{}^{\alpha\beta}(\nu,t)+Q_\mu Q_\rho A_5{}^{\alpha\beta}(\nu,t)\\ &\quad+Q_\mu\Delta_\rho A_6{}^{\alpha\beta}(\nu,t)+\Delta_\mu P_\rho A_7{}^{\alpha\beta}(\nu,t)+\Delta_\mu Q_\rho A_8{}^{\alpha\beta}(\nu,t)\\ &\quad+\Delta_\mu\Delta_\rho A_9{}^{\alpha\beta}(\nu,t)+g_{\mu\rho}A_{10}{}^{\alpha\beta}(\nu,t)].\end{aligned} \tag{9}$$

The quantities $(A_i{}^{\alpha\beta}(\nu,t))_{ab}$ may be written in the form

$$(A_i{}^{\alpha\beta}(\nu,t))_{ab}=F_i(\nu,t)(\delta_{a\alpha}\delta_{b\beta}+\delta_{a\beta}\delta_{b\alpha})+G_i(\nu,t)(\delta_{a\alpha}\delta_{b\beta}-\delta_{a\beta}\delta_{b\alpha})+H_i(\nu,t)\delta_{ab}\delta_{\alpha\beta}. \tag{10}$$

The crossing relation

$$V_{\mu\rho}{}^{\alpha\beta}(p,\,b,\,-k;\,p',\,a,\,-q)^*=V_{\mu\rho}{}^{\alpha\beta}(p',a,k;\,p,b,q) \tag{11}$$

implies the following symmetries:

$$\begin{aligned}&F_i^*(-\nu,\,t)=F_i(\nu,t),\\ &G_i^*(-\nu,\,t)=-G_i(\nu,t),\quad i=1,5,6,8,9,10;\\ &H_i^*(-\nu,\,t)=H_i(\nu,t),\\ &F_i^*(-\nu,\,t)=-F_i(\nu,t),\\ &G_i^*(-\nu,\,t)=G_i(\nu,t),\quad i=2,3,4,7.\\ &H_i^*(-\nu,\,t)=-H_i(\nu,t),\end{aligned} \tag{12}$$

Thus, if the scalar functions $F_i(\nu,t)$, $G_i\,(\nu,t)$ and $H_i(\nu,t)$ can be represented by subtraction-free dispersion integrals, these must have the form

$$\begin{aligned}&F_i(\nu,t)=\frac{2\nu}{\pi}\int_0^\infty d\nu'\frac{f_i(\nu',t)}{\nu'^2-\nu^2-i\epsilon},\\ &G_i(\nu,t)=\frac{2}{\pi}\int_0^\infty d\nu'\frac{\nu' g_i(\nu',t)}{\nu'^2-\nu^2-i\epsilon},\quad i=2,3,4,7;\\ &H_i(\nu,t)=\frac{2\nu}{\pi}\int_0^\infty d\nu'\frac{h_i(\nu',t)}{\nu'^2-\nu^2-i\epsilon},\\ &F_i(\nu,t)=\frac{2}{\pi}\int_0^\infty d\nu'\frac{\nu' f_i(\nu',t)}{\nu'^2-\nu^2-i\epsilon},\\ &G_i(\nu,t)=\frac{2\nu}{\pi}\int_0^\infty d\nu'\frac{g_i(\nu',t)}{\nu'^2-\nu^2-i\epsilon},\quad i=1,5,6,8,9,10;\\ &H_i(\nu,t)=\frac{2}{\pi}\int_0^\infty d\nu'\frac{\nu' h_i(\nu',t)}{\nu'^2-\nu^2-i\epsilon}.\end{aligned} \tag{13}$$

It follows that the only nonvanishing terms in (4) and (5) when $\nu\to\infty$, are

$$\lim_{\nu\to\infty} k^\mu V_{\mu\rho}{}^{\alpha\beta}(p',a,k;\,p,b,q)=-\frac{2}{\pi}\bigg[P_\rho(\delta_{a\alpha}\delta_{b\beta}-\delta_{a\beta}\delta_{b\alpha})\int_0^\infty d\nu' g_1{}^V(\nu',t)+(\delta_{a\alpha}\delta_{b\beta}+\delta_{a\beta}\delta_{b\alpha})\times\bigg(Q_\rho\int_0^\infty d\nu' f_2{}^V(\nu',t)+\Delta_\rho\int_0^\infty d\nu' f_3{}^V(\nu',t)\bigg)+\delta_{ab}\delta_{\alpha\beta}\bigg(Q_\rho\int_0^\infty d\nu' h_2{}^V(\nu',t)+\Delta_\rho\int_0^\infty d\nu' h_3{}^V(\nu',t)\bigg)\bigg] \tag{14}$$

for the vector commutator (hence the superscript V), with a similar result for the axial-current commutator. The sum rule which will be studied in the next section is the one obtained by equating the coefficients of $P_\rho(\delta_{a\alpha}\delta_{b\beta}-\delta_{a\beta}\delta_{b\alpha})$ in the vector and axial terms, i.e.

$$\int_{-\infty}^{\infty} d\nu'[g_1{}^V(\nu',t)-g_1{}^A(\nu',t)]=0. \tag{15}$$

CALCULATION

Our calculation will take into account the lowest lying boson states in the two matrix elements. For the vector-current matrix element, the contributions will come from intermediate one-pion and ω-meson states.[7] Since the isovector part of the vector current is being considered, G parity excludes the ρ-meson contribution. The $T=0$ scalar meson (σ) contribution[8] vanishes because of parity conservation. For the axial-current commutator matrix elements, it is the σ meson and the ρ meson which contribute, whereas the pion and ω meson contributions vanish.

In all cases the absorptive part of the matrix element is obtained from

$$-\tfrac{1}{2}(2\pi)^4 \textstyle\sum_n \{\delta(P_n-p-q)\langle p',a|\mathfrak{F}_\mu{}^\alpha|n\rangle \times\langle n|\mathfrak{F}_\rho{}^\beta|p',b\rangle+\text{crossed terms}\}. \quad (16)$$

The single-particle contributions are of the form

$$-\tfrac{1}{2}(2\pi)^4\int\frac{dp''}{(2\pi)^3}\delta(p''^2-m_n{}^2)\delta(p''-P-Q) \times\sum_{\substack{\text{spins,}\\ \text{isospins}}}\langle p',a|\mathfrak{F}_\mu{}^\alpha|p'',\cdots\rangle\langle p'',\cdots|\mathfrak{F}_\rho{}^\beta|p,b\rangle$$
$$=-\tfrac{1}{2}\pi\delta(\nu-\tfrac{1}{2}(m_n{}^2-P^2-Q^2)) \times\sum_{\substack{\text{spins,}\\ \text{isospins}}}\langle p',a|\mathfrak{F}_\mu{}^\alpha|p''\cdots\rangle\langle p,b|\mathfrak{F}_\rho{}^\beta|p''\cdots\rangle^*. \quad (17)$$

The delta function disappears in the integral in Eq. (15), so that we need only concern ourselves with

$$\sum_{\substack{\text{spins,}\\ \text{isospins}}}\langle p',a|\mathfrak{F}_\mu{}^\alpha|p''\cdots\rangle\langle p,b|\mathfrak{F}_\rho{}^\beta|p'',\cdots\rangle^*. \quad (18)$$

Pion Contribution

We write

$$\langle p',a|\mathfrak{F}_\mu{}^\alpha|p'',c\rangle=-ie_{a\alpha c}(p_\mu'+p_\mu'') \quad (19)$$

so that

$$\textstyle\sum_c\langle p',a|\mathfrak{F}_\mu{}^\alpha|p'',c\rangle\langle p,b|\mathfrak{F}_\rho{}^\beta|p'',c\rangle^*$$
$$=e_{a\alpha c}e_{b\beta c}(p'+p'')_\mu(p+p'')_\rho$$
$$=[\delta_{\alpha\beta}\delta_{ab}+\tfrac{1}{2}(\delta_{a\alpha}\delta_{b\beta}-\delta_{a\beta}\delta_{b\alpha})-\tfrac{1}{2}(\delta_{a\alpha}\delta_{b\beta}+\delta_{a\beta}\delta_{b\alpha})] \times(4P_\mu P_\rho+2P_\mu Q_\rho-P_\mu\Delta_\rho+\cdots). \quad (20)$$

Thus the coefficient of interest is

$$g_{1\pi}{}^V(t)=2. \quad (21)$$

ω-Meson Contribution

We write

$$\langle p',a|\mathfrak{F}_\mu{}^\alpha|p'',\lambda\rangle=\delta_{a\alpha}\frac{B}{m_\pi}\epsilon^\gamma(\lambda)e_{\gamma\mu\sigma\tau}p'^\sigma p''^\tau \quad (22)$$

so that with the help of the polarization sum

$$\textstyle\sum_\lambda \epsilon^\gamma(\lambda)\epsilon^\delta(\lambda)=-(g^{\gamma\delta}-p''^\gamma p''^\delta/m_\omega{}^2) \quad (23)$$

and

$$e^\mu{}_{\nu\rho\sigma}e_{\mu\alpha\beta\gamma}=-g_{\nu\alpha}(g_{\rho\beta}g_{\sigma\gamma}-g_{\rho\gamma}g_{\sigma\beta}) +g_{\nu\beta}(g_{\rho\alpha}g_{\sigma\gamma}-g_{\rho\gamma}g_{\sigma\alpha})-g_{\nu\gamma}(g_{\rho\alpha}g_{\rho\beta}-g_{\rho\beta}g_{\sigma\alpha}) \quad (24)$$

we obtain

$$\textstyle\sum_\lambda\langle p',a|\mathfrak{F}_\mu{}^\alpha|p'',\lambda\rangle\langle p',a|\mathfrak{F}_\rho{}^\beta|p'',\lambda\rangle^*$$
$$=(B^2/m_\pi{}^2)[\tfrac{1}{2}(\delta_{a\alpha}\delta_{b\beta}+\delta_{a\beta}\delta_{b\alpha})+\tfrac{1}{2}(\delta_{a\alpha}\delta_{b\beta}-\delta_{a\beta}\delta_{b\alpha})]$$
$$\times[g_{\mu\rho}(p\cdot p''p'\cdot p''-p'\cdot pp''^2)-P_\mu P_\rho(p''-p)$$
$$\times(p''-p')+P_\mu Q_\rho(-\tfrac{1}{2}(m_\omega{}^2-m_\pi{}^2)-\tfrac{1}{2}t)$$
$$+\tfrac{1}{4}P_\mu\Delta_\rho(m_\omega{}^2-m_\pi{}^2)+\cdots]. \quad (25)$$

Thus the coefficient of interest is

$$g_{1\omega}{}^V(t)=\tfrac{1}{4}(B^2/m_\pi{}^2)t. \quad (26)$$

We next turn to consideration of the absorptive part of the axial-current commutator matrix elements.

σ-Meson Contribution

Here the matrix element of interest is $\langle p',a|\mathfrak{F}_\mu{}^{5\alpha}|p''\rangle$. We write it in the form

$$\langle p',a|\mathfrak{F}_\mu{}^{5\alpha}|p''\rangle=i\delta_{a\alpha}(M_1p_\mu'+M_2p_\mu''). \quad (27)$$

The assumption of a partially conserved axial current (PCAC)[9] leads to a relation between the two constants. If we define the coupling constant $g_{\sigma\pi\pi}$ by

$$\langle p',a|j_\pi{}^\alpha|p''\rangle\equiv\delta_{a\alpha}m_\pi g_{\sigma\pi\pi}, \quad (28)$$

then PCAC leads to

$$M_1+M_2=2m_\pi Cg_{\sigma\pi\pi}/(m_\sigma{}^2-m_\pi{}^2), \quad (29)$$

where C is defined by

$$\partial^\mu\mathfrak{F}_\mu{}^{5\alpha}(x)=Cm_\pi{}^2\phi_\pi{}^\alpha(x) \quad (30)$$

and has the value

$$C=MF_A/g_{NN\pi}, \quad (31)$$

where M is the nucleon mass, $g_{NN\pi}$ is the pion-nucleon coupling constant, and F_A is the axial-current renormalization constant. It should be noted that with our normalization of states, the matrix element (28) leads to the following expression for the width of the σ meson:

$$\Gamma_\sigma/m_\pi=3\frac{g_{\sigma\pi\pi}{}^2}{4\pi}\frac{m_\pi}{8m_\sigma}\left(1-\frac{4m_\pi{}^2}{m_\sigma{}^2}\right)^{1/2}. \quad (32)$$

[7] Actually the φ-meson contribution should also be included. We shall argue later that this is small.

[8] The evidence for the existence of a $T=0$ scalar meson is still inconclusive. It may perhaps be viewed as a convenient parametrization of a strong S-wave attraction in the pion-pion system.

[9] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960). As is the case in all applications of PCAC, one of the pions is off the mass shell ($q^2=k^2=0$); the correction factor $K_{NN\pi}(0)$ which is expected to be close to unity is ignored in what follows.

Equation (29) implies that

$$\langle p',a|\mathfrak{F}_\mu^{5\alpha}|p''\rangle = i\delta_{a\alpha}\left(\frac{2m_\pi C g_{\sigma\pi\pi}}{m_\sigma^2-m_\pi^2}p_\mu'+M_2(p_\mu''-p_\mu')\right). \quad (33)$$

Consequently

$$\langle p',a|\mathfrak{F}_\mu^{5\alpha}|p''\rangle\langle p,b|\mathfrak{F}_\rho^{5\beta}|p''\rangle^* = [\tfrac{1}{2}(\delta_{a\alpha}\delta_{b\beta}+\delta_{a\beta}\delta_{b\alpha})+\tfrac{1}{2}(\delta_{a\alpha}\delta_{b\beta}-\delta_{a\beta}\delta_{b\alpha})] \times\left[\left(\frac{2m_\pi C g_{\sigma\pi\pi}}{m_\sigma^2-m_\pi^2}\right)^2 P_\mu P_\rho+\frac{2m_\pi C g_{\sigma\pi\pi}}{m_\sigma^2-m_\pi^2} \times\left(\frac{1}{2}M_2-\frac{m_\pi C g_{\sigma\pi\pi}}{m_\sigma^2-m_\pi^2}\right)P_\mu\Delta_\rho+\cdots\right]. \quad (34)$$

Thus the σ-meson contribution to g_1^A is given by

$$g_{1\sigma}^A(t)=\tfrac{1}{2}[2m_\pi C g_{\sigma\pi\pi}/(m_\sigma^2-m_\pi^2)]^2. \quad (35)$$

It should be noticed that this contribution does not contain the unknown constant M_2. It is this fact which motivated our choice of sum rule.

ρ-Meson Contribution

We write

$$\langle p',a|\mathfrak{F}_\mu^{5\alpha}|p'',\lambda,c\rangle = -ie_{a\alpha c}[i\epsilon^\gamma(\lambda)p_\gamma'](K_1p_\mu''+K_2p_\mu') \quad (36)$$

and introduce the constant $\gamma_{\rho\pi\pi}$ by

$$\langle p',a|j_\pi^\alpha|p'',\lambda,c\rangle=-ie_{a\alpha c}[2\epsilon^\gamma(\lambda)p_\gamma']\gamma_{\rho\pi\pi}. \quad (37)$$

With this form of the matrix element, the width of the ρ meson is given by

$$\frac{\Gamma_\rho}{m_\pi}=\frac{\gamma_{\rho\pi\pi}^2}{4\pi}\frac{m_\rho}{12m_\pi}\left(1-\frac{4m_\pi^2}{m_\rho^2}\right)^{1/2}. \quad (38)$$

PCAC leads to the relation

$$K_1+K_2=4C\gamma_{\rho\pi\pi}/(m_\rho^2-m_\pi^2). \quad (39)$$

Thus

$$\langle p',a|\mathfrak{F}_\mu^{5\alpha}|p'',\lambda,c\rangle=e_{a\alpha c}\epsilon^\gamma(\lambda)p_\gamma' \times\left[\frac{4C\gamma_{\rho\pi\pi}}{m_\rho^2-m_\pi^2}p_\mu'+K_1(p_\mu''-p_\mu')\right]. \quad (40)$$

From this we calculate

$$\sum_{c\lambda}\langle p',a|\mathfrak{F}_\mu^{5\alpha}|p'',\lambda,c\rangle\langle p,b|\mathfrak{F}_\rho^{5\beta}|p'',\lambda,c\rangle^* = [\delta_{ab}\gamma_{\alpha\beta}+\tfrac{1}{2}(\delta_{a\alpha}\delta_{b\beta}-\delta_{a\beta}\delta_{b\alpha})-\tfrac{1}{2}(\delta_{a\alpha}\delta_{b\beta}+\delta_{a\beta}\delta_{b\alpha})] \times\left(\frac{(p'\cdot p'')(p\cdot p'')}{m_\rho^2}-p'\cdot p\right)\left[\left(\frac{4C\gamma_{\rho\pi\pi}}{m_\rho^2-m_\pi^2}\right)^2P_\mu P_\rho +\frac{4C\gamma_{\rho\pi\pi}}{m_\rho^2-m_\pi^2}K_1P_\mu Q_\rho+\cdots\right]. \quad (41)$$

The coefficient of interest is

$$g_{1\rho}^A(t)=\frac{1}{2}\left(\frac{4C\gamma_{\rho\pi\pi}}{m_\rho^2-m_\pi^2}\right)^2\left[\left(\frac{m_\rho^2-m_\pi^2}{2m_\rho}\right)^2+\tfrac{1}{2}t\right]. \quad (42)$$

Collecting terms from (21), (26), (35) and (42), we obtain what are hopefully the dominant parts of the sum rule (15):

$$2+\frac{1}{4}\frac{B^2}{m_\pi^2}t=2\left(\frac{m_\pi C g_{\sigma\pi\pi}}{m_\sigma^2-m_\pi^2}\right)^2 +2\left(\frac{C\gamma_{\rho\pi\pi}}{m_\rho}\right)^2+\tfrac{1}{4}t\left(\frac{4C\gamma_{\rho\pi\pi}}{m_\rho^2-m_\pi^2}\right)^2. \quad (43)$$

The term independent of the momentum-transfer variable t leads to the relation

$$\frac{1}{F_A^2}=\left(\frac{m_\pi M g_{\sigma\pi\pi}}{(m_\sigma^2-m_\pi^2)g_{NN\pi}}\right)^2+\left(\frac{M\gamma_{\rho\pi\pi}}{m_\rho g_{NN\pi}}\right)^2. \quad (44)$$

This is just the expression for the axial-current renormalization constant obtained by Kawarabayashi, McGlinn and Wada.[10] With the popular choice of parameters

$$m_\sigma=390\text{ MeV},\quad \Gamma_\sigma=90\text{ MeV}, \quad (45)$$

one finds that

$$F_A^2=1.47. \quad (46)$$

The term linear in t leads to a relation between a photonic matrix element $M(\omega^0\to\pi^0+\gamma)$ and a mesonic matrix element, which reads

$$B^2=\left(\frac{4C\gamma_{\rho\pi\pi}m_\pi}{m_\rho^2-m_\pi^2}\right)^2\cong\left(\frac{4F_AMm_\pi}{m_\rho^2}\right)^2\frac{\gamma_{\rho\pi\pi}^2}{g_{NN\pi}^2}. \quad (47)$$

The rate for the decay $\omega^0\to\pi^0+\gamma$ is calculated to be

$$\frac{\Gamma(\omega_0\to\pi^0+\gamma)}{m_\pi}=\frac{e^2}{4\pi}B^2\frac{1}{24}\left(\frac{m_\omega}{m_\pi}\right)^3\left(1-\frac{m_\pi^2}{m_\omega^2}\right)^3. \quad (48)$$

With the choice of parameters $m_\rho=750$ MeV, $\Gamma_\rho=100$ MeV, and $F_A^2=1.39$ we obtain

$$\Gamma(\omega^0\to\pi^0+\gamma)=1.2\text{ MeV}, \quad (49)$$

which is in good agreement with experiment.[11]

If the ϕ-meson contribution is included, Eq. (47) is replaced by

$$B_\omega^2+B_\phi^2=[4C\gamma_{\rho\pi\pi}m_\pi/(m_\rho^2-m_\pi^2)]^2 \quad (50)$$

which replaces (49) by an inequality. There is no way of separating the contributions of the two isoscalar vector

[10] K. Kawarabayashi, W. D. McGlinn, and W. W. Wada, Phys. Rev. Letters **15**, 897 (1965). See also S. L. Adler, Phys. Rev. **140**, B736 (1965).

[11] References to the experiments may be found in A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. **37**, 633 (1965).

mesons within the framework of the $SU(2)\times SU(2)$ algebra used in this paper. We therefore appeal to the crude and ridiculously large upper limit

$$\Gamma(\phi^0 \to \pi^0+\gamma)/\Gamma_{\text{total}} < 10\%$$

to estimate that the calculated value of $\Gamma(\omega^0 \to \pi^0+\gamma)$ can be decreased by no more than 15% from the value predicted in Eq. (49).

A by-product of the calculation is obtained when either side of the Eq. (43) is equated to the appropriate coefficient in $V_\rho^{\alpha\beta}(t)$ [defined in Eq. (4)]. Since

$$\begin{aligned} V_\rho^{\alpha\beta}(t) &= \int d^3x \langle p',a|[\mathfrak{F}_0^\alpha(\mathbf{x},0),\mathfrak{F}_\rho^\beta(0)]|p,b\rangle \\ &= \langle p',a|[T^\alpha,\mathfrak{F}_\rho^\beta(0)]|p,b\rangle \\ &= i\rho_{\alpha\beta\gamma}\langle p',a|\mathfrak{F}_\rho^\gamma(0)|p,b\rangle, \quad (51) \end{aligned}$$

we obtain an expansion in powers of t for the pion form factor. The term linear in t yields an expression for the pion charge radius in terms of the rate $\Gamma(\omega^0 \to \pi^0+\gamma)$, a result obtained by Cabibbo and Radicati.[12]

In conclusion, it should be stressed that a number of unproved assumptions have been made in the derivation of our result. Some of these have to do with the nature of singularities of commutators of currents, and they are certainly at variance with what is found in perturbation theory. Calculations such as this one may perhaps be viewed as encouraging to the point of view that the assumptions are correct. Other assumptions have to do with the states which are taken to "saturate" the sum rules. The structure of matrix elements which have been investigated is such as to suggest that if the coupling constants do not grow with mass, then the higher mass and higher spin-state contributions, in addition to yielding sum rules for $\Gamma(f' \to \pi+\gamma)$ in terms of $\Gamma(f^0 \to 2\pi)$, say, will not significantly alter the numbers so far obtained.

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[12] N. Cabibbo and L. Radicati, Phys. Letters **19**, 697 (1965).

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Sum-Rule Calculation of the Isovector Form Factor*

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A class of sum rules due to Fubini is rederived from a point of view which clarifies the assumptions made about the singularities of field operators. A calculation of the isovector magnetic moment is performed in the isobar approximation. The result $F_1^V(0)=\frac{1}{2}$ emerges as a model-independent consistency condition, but the calculated value $F_2^V(0)=3.64$ is in disagreement with the experimental value.

In a recent paper Fubini[1] presented a new method of obtaining sum rules of interest in strong-interaction physics. It is the purpose of this note to present a set of assumptions which lead to an alternative derivation of the Fubini result. Sum rules for the isovector form factors of the nucleon are derived and their properties are discussed.

We consider the matrix element $T_\mu^{\alpha\beta}(p',k;p,q)$ defined by

$$T_\mu^{\alpha\beta}(p',k;p,q) = -i\int dx\, e^{ikx}\theta(x_0) \times\langle p'|[j_\mu^\alpha(x),\phi^\beta(0)]|p\rangle. \quad (1)$$

Here $j_\mu^\alpha(x)$ is a current operator, $\phi^\beta(0)$ is scalar or a pseudoscalar field operator, and the indices α, β may be isospin or $SU(3)$ labels, so chosen that the operators $j_\mu^\alpha(x)$ and $\phi^\beta(0)$ are Hermitian. The matrix element $T_\mu^{\alpha\beta}(p',k;p,q)$ may be decomposed into invariant functions. For example, if the particles described by the state vectors $|p\rangle$ and $|p'\rangle$ have spin zero, then the most general form of the matrix element is

$$T_\mu^{\alpha\beta}(p',k;p,q) = P_\mu A_1^{\alpha\beta}(\nu,t) + Q_\mu A_2^{\alpha\beta}(\nu,t) + \Delta_\mu A_3^{\alpha\beta}(\nu,t). \quad (2)$$

We have used the conventional notation

$$\begin{aligned} P_\mu &= \tfrac{1}{2}(p_\mu'+p_\mu), \\ Q_\mu &= \tfrac{1}{2}(k_\mu+q_\mu), \\ \Delta_\mu &= p_\mu'-p_\mu = q_\mu-k_\mu, \quad (3) \\ t &= \Delta^2, \\ \nu &= P\cdot Q. \end{aligned}$$

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[1] S. Fubini, Nuovo Cimento (to be published).