TABLE II. Calculated and observed values of electromagnetic mass splittings, in MeV. χ is chosen to be -2.3×10^{-2}.

	Theoretical value			
	Nonvacuon contribution	Vacuon contribution	Total contribution	Experiment
$\Sigma^0-\Sigma^+$	-0.7	4.2	3.5	3.0 ± 0.2
$\Sigma^--\Sigma^0$	1.4	4.2	5.6	4.9 ± 0.1
$p-n$	1.1	-2.8	-1.7	-1.3
$\Xi^--\Xi^0$	1.2	5.8	7.0	6.5 ± 1.0
$\pi^+-\pi^0$	4.9	0	4.9	4.6
K^+-K^0	2.8	-5.8	-3.0	-3.9 ± 0.3

The vacuon contribution to the kaon mass splitting is then given (in MeV) by

$$m_{K^+}-m_{\Sigma^0}=253\chi. \qquad (16f')$$

Nonvacuon contributions to the electromagnetic mass splittings of baryons and mesons have been calculated by Coleman and Schnitzer,[2] and Socolow,[3] respectively. Table II shows theoretical and experimental results[8] for the six electromagnetic mass splittings, using the estimates of the nonvacuon contributions reported by Socolow[3] and the arbitrary choice $\chi=-2.3\times10^{-2}$. In view of the uncertainties regarding the nonvacuon calculations, which one probably can claim to be correct to within 0.5 MeV in the case of baryons and 1 MeV in the case of mesons, the agreement between theory and experiment should be regarded as satisfactory.

We conclude this note with a brief comparison of our calculation with that of Socolow.[3] He based his calculation on the original "tadpole" model of Coleman and Glashow and found that the electromagnetic kaon mass splitting is too small in magnitude compared with experiment. We have about a 20% larger *vacuon* contribution to the kaon mass splitting than Socolow has. Half of this additional contribution is due to the modification of the original "tadpole" model by allowing the effects of field renormalizations to be taken into account. The other half is due to a different way of fixing the coefficient of $\mathcal{Y}(\alpha_\Phi)\chi$ in Eq. (16f); while we fix this coefficient by using the 0^- meson masses *only*, the corresponding coefficient in Socolow's calculation involves the baryon masses as well as the meson masses.

[8] The experimental masses are from A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. 37, 633 (1965).

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Zero-Effective-Range Analysis of Pion-Nucleon Reactions near the η^0 Production Threshold*

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A two-channel scattering matrix, assuming zero effective range, has been fitted to two sets of pion-nucleon phase shifts in the S_{11} channel. One of these, due to Cence, agrees well with the model, and implies a virtual bound state of the η^0-nucleon system at a mass of about 1460 MeV. The predicted η^0 production cross section is in reasonable agreement with experiment.

I. INTRODUCTION

RECENTLY considerable interest has been focused on the nature of the S-wave, isospin-zero pion-nucleon interaction in the vicinity of the η^0-production threshold. The possibility that the interaction may be dominated by a nearby pole in the scattering amplitude fits into several theoretical proposals. For example, such a pole might be identified with a member of a 70^- $SU(6)$ multiplet, as suggested by Gyuk and Tuan[1]; or with an $L=1$ excited configuration of a three-quark system, as in the model discussed by Dalitz.[2] The availability of extensive phase-shift analyses in the relevant energy range and recent measurements of the η^0-production cross section make possible a detailed investigation of this problem. There have now been published two such studies. One, due to Uchiyama-Campbell,[3] attempts a two-channel zero-effective-range fit to the experimental η^0-production cross sections and the phase shifts of Auvil, Donnachie, Lea, and Lovelace[4] (ADLL). Hendry and Moorhouse[5] have included both the effects of other open channels and the energy dependence due to nonzero effective range; and have treated the phase shifts of Bransden, Moorhouse, and O'Donnell[6] as well as the ADLL set.

More recently, a new set of pion-nucleon phase shifts

* Supported in part by the U. S. Atomic Energy Commission.
[1] I. P. Gyuk and S. F. Tuan, Phys. Rev. Letters 14, 121 (1965).
[2] R. H. Dalitz, invited paper presented at the Oxford International Conference on Elementary Particles, 1965.

[3] F. Uchiyama-Campbell, Phys. Letters 18, 189 (1965).
[4] P. Auvil, A. Donnachie, A. T. Lea, and C. A. Lovelace, Phys. Letters 12, 76 (1964). Later referred to as ADLL.
[5] A. W. Hendry and R. G. Moorhouse, Phys. Letters 18, 171 (1965).
[6] B. H. Bransden, R. G. Moorhouse, and P. J. O'Donnell, Phys. Letters 11, 339 (1964); also Phys. Rev. 139, B1566 (1965).

has been reported by Cence.[7] These differ considerably from other published values in several channels, including the S_{11}. In this paper, we will apply the zero-effective-range model to the Cence phase shifts, comparing the results with a similar treatment of the ADLL values.

Both the ADLL and Cence analyses are energy-independent parametrizations of a large amount of experimental data, and both are consistent with assuming negligible inelasticity below the η^0 threshold. Thus they lend themselves well to a two-channel approximation in which both channels contain only two particles. A clear and complete discussion of the effective-range analysis of the two-channel, two-body scattering amplitude may be found in the paper of Frazer and Hendry.[8] A summary of the basic results will be given here.

Let $T(s)$ be the 2×2 matrix of transition amplitudes for the coupled pion-nucleon and η-nucleon S_{11} channels, where s is the square of the total energy in the center-of-mass reference frame. We normalize $T(s)$ so that for s between zero and the next inelastic threshold, unitarity of the S matrix implies

$$\operatorname{Im}(T^{-1}) = -\rho(s). \tag{1}$$

$\rho(s)$ is a diagonal matrix containing only kinematic quantities. We will take as its components

$$\rho_{ij} = \delta_{ij} \times k_i (s_2/s)^{1/2};\ s > s_i$$
$$\quad 0 \quad ;\ s < s_i. \quad i = 1,2$$

In this expression, k_1 and k_2 are the pion and η^0 center-of-mass momenta, and s_1 and s_2 are the pion-nucleon and η-nucleon thresholds. The factor $s_2^{1/2}$ has been included to reproduce the nonrelativistic normalization for s near s_2.

Now if we define the quantities

$$K_i(s) = k_i (s_2/s)^{1/2}$$

with the convention that k_i is positive imaginary for $s < s_i$, then

$$\operatorname{Re} K_i(s) = \rho_i(s) \quad \text{for} \quad s > 0.$$

Thus, letting K be the 2×2 diagonal matrix with components

$$K_{ij} = K_i \delta_{ij}$$

the relation (1) implies

$$\operatorname{Im} M(s) = 0; \quad s > 0,$$

where

$$M(s) = T^{-1}(s) + iK(s).$$

This means that $M(s)$ is free of the threshold singularities and its components may be expanded in power series in s. Performing the matrix inversion we find that

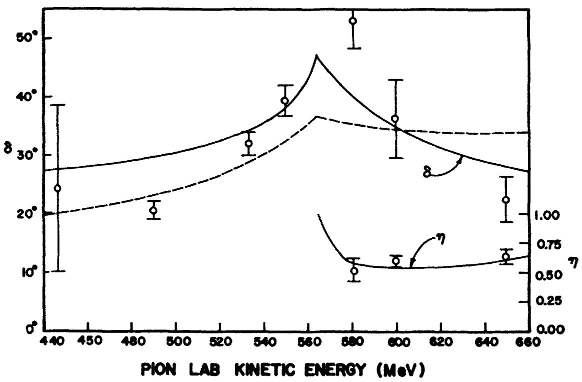

FIG. 1. Zero-effective-range fit to S_{11} phase shifts and absorption parameters of ADLL (Ref. 4). Dashed curve is fit of Ref. 3.

the elements of $T(s)$ have the form

$$T_{11}(s) = (M_{22} - iK_2)/D(s),$$
$$T_{12}(s) = -M_{12}/D(s), \tag{2}$$
$$T_{22}(s) = (M_{11} - iK_1)/D(s),$$

with

$$D(s) = (M_{11} - iK_1)(M_{22} - iK_2) - M_{12}^2.$$

The zero-effective-range approximation consists of putting the M_{ij} equal to real constants, and gives a three-parameter formula for the coupled pion-nucleon and η-nucleon amplitudes which we expect to be valid in some neighborhood of the η-production threshold.

II. ZERO-EFFECTIVE-RANGE FIT TO THE PHASE SHIFTS

Using

$$T_{11}(s) = (\eta e^{2i} - 1)/2iK_1,$$

we can apply (2) to compute the pion-nucleon S_{11} phase shift δ and absorption parameter η in terms of the three real parameters M_{11}, M_{12}, and M_{22}. Best values of these parameters were obtained by making a least-squares fit of the predicted δ and η to those given by the ADLL and Cence analyses. The results are summarized in Figs. 1 and 2.

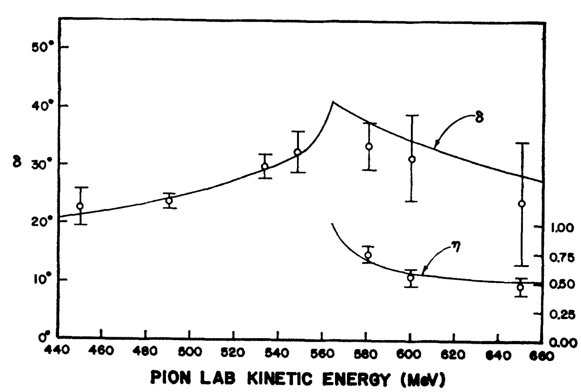

FIG. 2. Zero-effective-range fit to S_{11} phase shifts and absorption parameters of Cence (Ref. 7).

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[7] R. J. Cence, Phys. Letters **20**, 306 (1966).

[8] W. R. Frazer and A. W. Hendry, Phys. Rev. **134**, B1307 (1964).

The analysis of Uchiyama-Campbell[3] is almost identical in spirit to that reported here, but gave rather different results for the ADLL phases than we obtained. This appears to be due to several factors. First, we have been content here to attempt a fit to the phase shifts in an energy range extending about 100 MeV above and below threshold, whereas Uchiyama-Campbell works with double this range. The latter choice of range produces a fit which is at its worst in the threshold region, as shown by the dotted line in Fig. 1, where the analysis should have its greatest validity. Secondly, in fitting the absorption parameter we have used the values provided by the phase-shift analyses, in the expectation that these values and the phases are an internally consistent set of parameters. Although the potential ability to relate the absorption directly to the experimental η-production cross sections is a significant one, it hinges on several additional pieces of information; namely, a precise knowledge of the branching ratios of η^0 decays (which is required to compute the total cross section from the observations which see only the 2γ decay mode), and the assumption that η^0 production is predominantly S wave over the energy range considered. While we will have more to say later about the production cross section, it seemed to us better to tackle these questions after a fit had been obtained.

As is seen in Figs. 1 and 2, only the Cence analysis is fit by the zero-effective-range model with a reasonable χ^2. The best fit to the ADLL phases, using the parameters[9]

$$M_{11}=1202 \quad \mathrm{MeV}/c\,,$$
$$M_{22}=248 \quad \mathrm{MeV}/c\,,$$
$$M_{12}=460 \quad \mathrm{MeV}/c\,,$$

has a χ^2 of 13.6, even excluding the point at 490 MeV. For the Cence phases, we find that

$$M_{11}=4041 \quad \mathrm{MeV}/c\,,$$
$$M_{22}=1384 \quad \mathrm{MeV}/c\,,$$
$$M_{12}=2215 \quad \mathrm{MeV}/c\,,$$

provide a fit with $\chi^2=4.3$. This is statistically quite good. However, it is not possible to use this as a criterion for choosing the Cence phase shifts over those of ADLL, since the poor fit to the latter can be interpreted as due to neglect of energy dependence and other inelastic channels. In agreement with the other studies of this problem, we found that modest energy dependence is in fact inadequate to significantly improve the ADLL fit. However, Hendry and Moorhouse[5] did get excellent fits by including the approximate effects of other inelastic channels.

In a refinement of the Cence fit, we took into account the fact that the phase shift and absorption parameter errors are correlated by using the off-diagonal elements of the Cence error matrix.[10] The resultant χ^2 was about 1.0 higher, but we found no significant change in the best-fit parameters.

[9] Because of the normalization, the elements of M have the dimensions of momentum.

III. PHYSICAL IMPLICATIONS OF THE FITS

One of the most important physical features of the fits obtained above is the existence of a nearby pole in the elastic-scattering amplitude. As shown by Frazer and Hendry,[8] poles in the scattering amplitudes of the two-channel zero-effective-range model can occur only on sheets II and IV of the Riemann surface, the topology of which is given by their Fig. 4. Furthermore, the poles occur in complex conjugate pairs on a given sheet. For our purposes, the sheet structure is sufficiently summarized by the mnemonic triangle:

$$\begin{array}{c} \mathrm{I} \\ \mathrm{II}\quad \mathrm{III} \\ \mathrm{I}\quad \mathrm{IV}\quad \mathrm{I} \\ \mathrm{II}\quad \mathrm{III}\quad \mathrm{II}\quad \mathrm{III} \end{array}$$

In this figure, if we identify a subtriangle, say

$$\begin{array}{c} A \\ B\quad C \end{array}$$

then B is the sheet found by continuing from A through the branch cut between s_1 and s_2, while C is the sheet found through the cut above s_2. A pole on sheet II below the η nucleon threshold would correspond to the the existence of an η-N bound state, unstable against decay into $\pi+N$. It would manifest itself as a genuine elastic resonance in the π-N scattering amplitude. A pole on sheet IV, on the other hand, is a "virtual state" of η and N. It can manifest itself only in the form of threshold cusp effects as are seen in Figs. 1 and 2. The reason is that any path in the Riemann surface which leads from sheet I (the physical sheet) to the sheet-IV pole must wind around the branch point at s_2; hence s_2 is the physical value of s which comes closest to the pole position on the Riemann surface.

Defining the mass M of the virtual state as the square root of the pole position in the s plane, we find

$$M=1489+i15 \text{ MeV}$$

for the ADLL phase shifts, and

$$M=1461+i68 \text{ MeV}$$

for the Cence phase shifts. It is interesting that these two analyses, despite their obvious differences, imply rather similar pole positions. In both cases, the poles are on sheet IV. The π-N scattering amplitude on this sheet is

$$T_{11}^{\mathrm{IV}}=T_{11}-(2iK_2T_{12}^2)/(1+2iK_2T_{22})\,,$$

where T_{11}, T_{12}, and T_{22} are the physical sheet ampli-

[10] R. J. Cence (private communication).

tudes. This formula was used to calculate $|T_{11}^{IV}|$ for the ADLL and Cence fits; the result verified the sheet location of the poles.

Of course, the pole position obtained from the ADLL analysis cannot be taken seriously, since the fit is so poor. The Hendry and Moorhouse[5] analysis gives a pole on sheet III about 20–30 MeV above the η-nucleon threshold. Since sheet III lies directly through the η-nucleon cut, the pole represents a true S-wave resonance. This pole position also explains the difficulty in fitting the ADLL phases, since the zero-effective-range model can produce poles only on sheets II and IV. The other inelastic channels evidently provide a mechanism for the migration of a sheet II or sheet IV pole through the cut onto sheet III.

A second physical feature of the fits is their predictions for the η^0-production cross section in the reaction

$$\pi^- + p \to \eta^0 + n.$$

In the approximation that the production is purely S wave and that other inelastic channels may be neglected, we have

$$\sigma = \tfrac{2}{3}\pi \lambda\!\!\!^{-2}(1-\eta^2)/k_1^2, \quad (3)$$

where η is the S_{11} absorption parameter. Since the neglect of other open channels is clearly not justified for the ADLL phases, we present here only the prediction of our Cence fit, which is given in Fig. 3. The comparison with experiment requires a knowledge of the branching ratio

$$f = \Gamma(\eta \to 2\gamma)/\Gamma(\eta \to \text{all decays})$$

since only $f\sigma$ is measured experimentally. The solid curve in Fig. 3 corresponds to $f=\frac{1}{3}$, in agreement with the most recent data of the Berkeley-Hawaii collaboration,[11] while the dashed curves represent $f=0.30$ and 0.36. The experimental points are from the Berkeley-Hawaii[12] and the Brown-Brandeis-Harvard-M.I.T.-Padova[13] experiments. It is seen that the predicted cross section is in reasonable agreement with experiment, except for being somewhat high near threshold. It is likely that in the neighborhood of the threshold competition from other open channels depresses the η-production below that predicted by a model which neglects such channels.

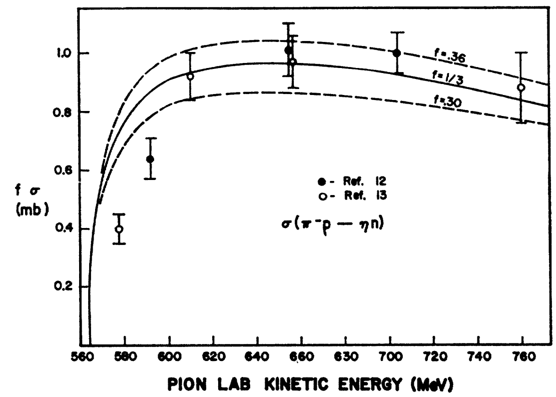

FIG. 3. Predicted and experimental values of the η^0-production cross section. f is the branching ratio for $\eta \to 2\gamma$.

IV. CONCLUSIONS

The study reported here makes possible an interpretation in physical terms of the substantial disagreement of the Cence and ADLL phase shifts for the S_{11} channel. The former can be understood very nicely in terms of a two-channel zero-effective-range model; the latter cannot. Physically, the reason for this lies in the fact that the Cence phase shifts indicate a virtual bound state of the η-nucleon system; while those of ADLL seem to require a resonant state, which cannot occur in the simple model. Just how different these two analyses are to be regarded depends on one's point of view. If we accept the virtual bound state as a "particle" on equal footing with an inelastic resonance, then Cence and ADLL merely provide different predictions as to the mass; the former giving a value of about 1460 MeV, and the latter, about 1510 MeV.[14] This interpretation represents the author's view. Further refinements of the phase-shift analyses, and the possible elimination of some of the existing solutions, should permit a more accurate determination of the mass of this state. It is hoped that the currently planned measurements of charge-exchange polarization may be of some value in resolving the phase-shift ambiguities, since the existing solutions make rather different predictions for this experiment.

ACKNOWLEDGMENT

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[11] V. Z. Peterson (private communication).

[12] W. B. Richards and C. B. Chiu, Bull. Am. Phys. Soc. 10, 702 (1965).

[13] Brown-Brandeis-Harvard-M.I.T.-Padova (Italy) Collaboration, Phys. Rev. Letters 13, 15 (1964).

[14] It is of some interest to note that 1460 MeV lies in the range considered in Ref. 1 as input values for the mass of the η-nucleon state.