Broken $U(3)$ Symmetry and Semileptonic Weak Interactions*

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A phenomenological field-theoretic model of $U(3)$-symmetry breakdown is proposed. It allows a unified treatment of both mass renormalization and field renormalization of the phenomenological fields associated with particles. Specifically, we have calculated the renormalization effects on the semileptonic weak coupling constants. Based on our model, it is shown that the empirically observed damping factor for $|\Delta Y|=1$ transitions *cannot* be explained by symmetry-breaking effects and therefore the Cabibbo angle is intrinsic to weak-interaction theory. From our calculation, we expect a 20% renormalization effect on *all* $|\Delta Y|=1$ axial-vector coupling constants for baryon leptonic decays. This renormalization effect is supported by the present experimental information about the G_A/G_V ratio of lambda beta decay. To make a sharp test of our model, more accurate experimental data concerning the G_A/G_V ratios of the baryon leptonic decays are required. Our model also reproduces the Ademollo-Gatto theorem on the nonrenormalization of the $|\Delta Y|=1$ vector transitions and Okubo's speculated mass formulas for vector mesons.

I. INTRODUCTION

SYMMETRY considerations based on the group $U(3)$ have been successful in bringing considerable order to the experimental data concerning strong, electromagnetic and weak interactions. Since this symmetry is only approximate, it is important to estimate the effects of those interactions that break the U_3 symmetry. Among these symmetry-breaking interactions, the medium strong interaction that preserves $U(2)$ symmetry is the most important one. Although the symmetry-breaking effects on particle masses are generally considered to be well understood, it has not been very clear as to how seriously the symmetry-breaking interactions would affect the coupling constants for the various strong and weak processes.[1] We propose, in this paper a phenomenological field-theoretical model of $U(3)$-symmetry breakdown, which allows a unified treatment of both the mass renormalization and the coupling-constant renormalization. Specifically, we shall calculate the renormalization effects on the semileptonic weak coupling constants. The essential ingredients of our calculation are:

(i) Particles, stable and unstable, *can* be described by phenomenological fields within the framework of conventional Lagrangian field theory.[2]

(ii) $U(3)$-symmetry breakdown is dominated by the nonvanishing vacuum expectation value of the scalar field S_{33}.[3,4]

(iii) Semileptonic weak processes are dominated by the poles[5] $\rho^{\pm}$, $\pi^{\pm}$, $K^{*\pm}$, and $K^{\pm}$.

II. MODEL OF $U(3)$-SYMMETRY BREAKDOWN

Our phenomenological field-theoretical model is based on the idea that $U(3)$-symmetry breakdown is dominated by the nonvanishing vacuum expectation value of the scalar field S_{33}.[3,4] In this model, we assume that the phenomenological Lagrange function for the octuplet of 0^- mesons Φ_{ab}, the nonuplet of 1^- mesons U_{ab} and the octuplet of $\frac{1}{2}^+$ baryons Ψ_{ab} is of the form

$$\mathcal{L}=\mathcal{L}_0+\mathcal{L}_{\text{int}}+\mathcal{L}_{33}\,, \quad (1)$$

where $\mathcal{L}_0$ is the free-field Lagrange function[6]

$$\begin{aligned}\mathcal{L}_0=&\operatorname{Tr}[-\Phi^\mu\partial_\mu\Phi+\tfrac{1}{2}\Phi^\mu\Phi_\mu-\tfrac{1}{2}m_0{}^2\Phi\Phi]\\&+\operatorname{Tr}[-\tfrac{1}{2}U^{\mu\nu}(\partial_\mu U_\nu-\partial_\nu U_\mu)+\tfrac{1}{4}U^{\mu\nu}U_{\mu\nu}-\tfrac{1}{2}m_1{}^2U^\mu U_\mu]\\&-\operatorname{Tr}\bar{\Psi}\left(\gamma^\mu\frac{1}{i}\partial_\mu+M\right)\Psi;\end{aligned} \quad (2)$$

$\mathcal{L}_{\text{int}}$ represents the $U(3)$-symmetric strong interactions, and $\mathcal{L}_{33}$ effectively represents the symmetry-breaking effects resulting from the nonvanishing scalar "vacuon," $\langle S_{33}\rangle\neq 0$. $\mathcal{L}_{33}$ is of the form

$$\begin{aligned}\mathcal{L}_{33}(\Phi,U,\Psi)=&\alpha_\Phi(\Phi^\mu\Phi_\mu)_{33}-\beta_\Phi m_0{}^2(\Phi\Phi)_{33}\\&+\tfrac{1}{2}\alpha_U(U^{\mu\nu}U_{\mu\nu})_{33}-\beta_U m_1{}^2(U^\mu U_\mu)_{33}\\&-\beta_\Psi M(\bar{\Psi}\Psi)_{33}-\beta_\Psi' M(\Psi\bar{\Psi})_{33}.\end{aligned} \quad (3)$$

m_0, m_1, and M are the masses of the pseudoscalar octuplet, vector nonuplet, and baryon octuplet, respectively, in the $U(3)$-symmetry limit. α's and β's are parameters to be determined. In (2), we have assumed that the nine vector mesons are degenerate in the $U(3)$-symmetry limit. This has been justified by

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[1] Because of electrical charge conservation, there is no renormalization of the electric charge vertices.

[2] This approach is advocated by Schwinger. See the series of papers: J. Schwinger, Phys. Rev. **135**, B816 (1964); **136**, B1821 (1964); **140**, B158 (1965).

[3] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964). It is our opinion that the nonvanishing of the vacuum expectation value of a scalar field does not necessarily imply the observability of a scalar particle.

[4] J. Schwinger, Phys. Rev. Letters **13**, 355 (1964). See also J. Schwinger, in *Lectures on Particles and Field Theory, Summer School Proceedings, Brandeis University, 1964* (Prentice Hall, Inc., Englewood Cliffs, New Jersey, 1965), Vol. II.

[5] See, for example, H. T. Nieh, Phys. Rev. **137**, B411 (1965).

[6] The formalism for boson fields is essentially that of Duffin and Kemmer. While Φ and Φ_μ are the five components for the spinless field, U_μ and $U_{\mu\nu}$ are the ten components for the spin-1 field.

Schwinger[7] from $U(6)\times U(6)$ considerations. As a good approximation, we have neglected the possible small mixing effect on the 0^- and $\frac{1}{2}^+$ octuplets by the corresponding singlets. The introduction of the α terms[8] in (3) constitutes the main departure of our theory from that of Coleman and Glashow.[3]

The α terms in the symmetry-breaking Lagrange function $\mathcal{L}_{33}$ can be absorbed into the structure of the free-field Lagrange function $\mathcal{L}_0$ by appropriately redefining the field variables and thus introducing renormalization constants for these fields. For example, $\mathcal{L}_{33}$ contains terms for K and K^*, but not for π and ρ. As a result, the field variables associated with K and K^* are renormalized by $\mathcal{L}_{33}$, while those associated with π and ρ are not. Thus, we redefine the field variables K_μ and $K_{\mu\nu}^*$ according to

$$K_\mu \to (1+\alpha_\Phi)^{-1/2}K_\mu\,, \tag{4}$$

$$K_{\mu\nu}^* \to (1+\alpha_U)^{-1/2}K_{\mu\nu}^*\,, \tag{5}$$

and similarly for other field variables that appear in $\mathcal{L}_{33}$. In order to preserve the kinematical structure of the phenomenological Lagrange function, the structure that characterizes the propagation of physical excitations, we simultaneously make transformations on the field variables K and K_μ^* according to[9]

$$K \to (1+\alpha_\Phi)^{1/2}K\,, \tag{6}$$

$$K_\mu^* \to (1+\alpha_U)^{1/2}K_\mu^*\,, \tag{7}$$

and similarly for others. In (3), there are no α terms for baryons. These terms are excluded by charge conservation. The baryon fields are therefore not renormalized by the symmetry-breaking interactions.

With the transformations (4), (5), (6), (7), etc., the mass terms as well as the $U(3)$-symmetric strong interactions $\mathcal{L}_{\rm int}\,(\Phi,U,\psi)$ are thereby modified to include the first-order symmetry-breaking effects produced by $\mathcal{L}_{33}$. With the help of the pole-dominance model, the symmetry-breaking effects on semileptonic weak coupling constants are also obtained. Once we know the parameters α's and β's, we know all these symmetry-breaking effects quantitatively. To determine the α's and β's, we shall make use of the effective meson-pole model of Schwinger.[7]

III. EFFECTIVE MESON-POLE MODEL

In the effective meson-pole model, we assume that the excitations produced by the current components of the *fundamental* fields can effectively be represented by phenomenological fields, with appropriate Lorentz transformation properties, that are associated with the observed 0^- and 1^- mesons. If we further make the assumption that the electromagnetic and weak currents (at the fundamental level) are members of these current components, the fundamental electromagnetic and weak interactions can be transcribed into the form of direct coupling of the electromagnetic field A_μ and the charged vector field Z_μ with the appropriate phenomenological meson fields. Then, through the coupling of these mesons with other strongly interacting particles, we obtain a form-factor description of the various electromagnetic and weak processes. The Goldberger-Treiman-type relations are also obtained.

The correspondence between the current components and the phenomenological fields associated with the 0^- and 1^- mesons, in the $U(3)$-symmetry limit, has been established by Schwinger[7] from $U(6)\times U(6)$ considerations. For our purpose, we just list those for the vector and axial-vector currents:

$$V_{ab}{}^\mu \leftrightarrow \frac{m_1{}^2}{-g}U_{ab}{}^\mu\,, \tag{8}$$

$$A_{ab}{}^\mu \leftrightarrow \frac{m_1}{g}\Phi_{ab}{}^\mu\,, \tag{9}$$

where m_1 is the degenerate vector meson mass in the $U(3)$-symmetry limit, and g is a dimensionless constant. Based on the *kinematical* $U(6)\times U(6)$ transformation, Schwinger[7] has also obtained the following $U(3)$-symmetric trilinear meson couplings:

$$\mathcal{L}_{\rm int}(MMM)=ig\,{\rm Tr}\Phi^\mu[U_\mu,\Phi] -i\frac{g}{m_1{}^2}{\rm Tr}\tfrac{1}{2}U^{\mu\nu}[\Phi_\mu,\Phi_\nu] +ig\,{\rm Tr}\tfrac{1}{2}U^{\mu\nu}[U_\mu,U_\nu]+\cdots\,, \tag{10}$$

where we have omitted those terms which we shall not make use of. From (8) and the field equation

$$m_1{}^2U^\mu+\partial_\nu U^{\mu\nu}=ig[\Phi,\Phi^\mu]+ig[U_\nu,U^{\mu\nu}]\,, \tag{11}$$

one can easily show that the charges defined by

$$Q_{ab}=-\int d\sigma_\mu V_{ab}{}^\mu(x)=-\int(d\mathbf{x})V_{ab}{}^0(x) \tag{12}$$

are, in terms of the phenomenological meson fields, given by

$$Q_{ab}=\int(d\mathbf{x})(i[\Phi,\Phi^0]+i[U_k,U_k{}^0])_{ab}\,, \tag{13}$$

which are exactly the total charge operators anticipated for 0^- and 1^- particles.

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[7] J. Schwinger, Phys. Rev. **140**, B158 (1965).

[8] The introduction of the α_Φ term was suggested to me by Professor Schwinger in 1964, in an attempt to provide a better account for the 1+8 pseudoscalar meson masses. This suggestion initiated the present investigation. I am deeply indebted to Professor Schwinger.

[9] It may be helpful to point out that the simultaneous transformations (4) and (6) preserve the equal-time commutation relation for the field variables $\bar{K}$ and K_0. It is in this sense that the field is *re*normalized.

IV. CHARGE CONSERVATION AND ITS CONSEQUENCES

In terms of the interactions at the fundamental level, the strong and electromagnetic interactions are invariant under independent phase transformations of each unitary component of the fundamental fields. Therefore, to the extent that weak interactions are neglected, there are the conservation laws:

$$\partial_\mu V_{11}{}^\mu = \partial_\mu V_{22}{}^\mu = \partial_\mu V_{33}{}^\mu = 0. \qquad (14)$$

The corresponding conserved charges are Q_{11}, Q_{22}, and Q_{33}. We shall use the conservation law $\partial_\mu V_{33}{}^\mu = 0$ to prove that the parameter β_U in (3) is zero:

$$\beta_U = 0.$$

The proof is based on the effective meson-pole model described in Sec. III.

The vector-meson nonuplet, in the $U(3)$ limit, is

$$U_{ab} = \begin{pmatrix} -\dfrac{\rho^0}{2^{1/2}}+\dfrac{\omega}{6^{1/2}}+\dfrac{\phi}{3^{1/2}} & \rho^+ & K^{*+} \\ -\rho^- & \dfrac{\rho^0}{2^{1/2}}+\dfrac{\omega}{6^{1/2}}+\dfrac{\phi}{3^{1/2}} & K^{*0} \\ \bar{K}^{*-} & \bar{K}^{*0} & -\dfrac{2}{6^{1/2}}\omega+\dfrac{1}{3^{1/2}}\phi \end{pmatrix}. \qquad (15)$$

It then follows from (1), (2), and (3) that for ρ and K^* we should make the transformations

$$\rho_{\mu\nu} \to \rho_{\mu\nu}, \qquad \rho_\mu \to \rho_\mu, \qquad (16)$$

$$K_{\mu\nu}{}^* \to (1+\alpha_U)^{-1/2} K_{\mu\nu}{}^*, \quad K_\mu{}^* \to (1+\alpha_U)^{1/2} K_\mu, \qquad (17)$$

and the masses are given by

$$m_\rho{}^2 = m_1{}^2, \qquad (18)$$

$$m_{K^*}{}^2 = (1+\alpha_U)(1+\beta_U) m_1{}^2. \qquad (19)$$

For ω and ϕ, we have

$$(\mathcal{L}_0+\mathcal{L}_{33})_{\omega,\phi} = \mathcal{L}_0(\omega,\phi) + \tfrac{1}{2}\alpha_U\left(\frac{1}{3^{1/2}}\phi - \frac{2}{6^{1/2}}\omega\right)^{\mu\nu} \times\left(\frac{1}{3^{1/2}}\phi - \frac{2}{6^{1/2}}\omega\right)_{\mu\nu} - \beta_U{}^2 m_1{}^2\left(\frac{1}{3^{1/2}}\phi - \frac{2}{6^{1/2}}\omega\right)^{\mu} \times\left(\frac{1}{3^{1/2}}\phi - \frac{2}{6^{1/2}}\omega\right)_{\mu}. \qquad (20)$$

What we want to do is to put (20) into a form having the structure of a free-field Lagrange function. We first make the orthogonal transformation

$$(\tfrac{1}{3})^{1/2}\phi - (\tfrac{2}{3})^{1/2}\omega \to \phi,$$
$$(\tfrac{2}{3})^{1/2}\phi + (\tfrac{1}{3})^{1/2}\omega \to \omega, \qquad (21)$$

which leaves $\mathcal{L}_0(\omega,\phi)$ invariant. Then, in place of (15) and (20), we have

$$U_{ab} \to \begin{pmatrix} -\dfrac{\rho^0}{2^{1/2}}+\dfrac{\omega}{2^{1/2}} & \rho^+ & K^{*+} \\ -\rho^- & \dfrac{\rho^0}{2^{1/2}}+\dfrac{\omega}{2^{1/2}} & K^{*-} \\ \bar{K}^{*-} & \bar{K}^{*0} & \phi \end{pmatrix} \qquad (22)$$

and

$$(\mathcal{L}_0+\mathcal{L}_{33})_{\omega,\phi} \to \mathcal{L}_0(\omega,\phi) + \tfrac{1}{2}\alpha_U \phi^{\mu\nu}\phi_{\mu\nu} - \beta_U m_1{}^2 \phi^\mu \phi_\mu. \qquad (23)$$

We note that ω is absent from the symmetry-breaking terms in (23). The newly defined, i.e., physical, ω therefore undergoes no renormalization by the symmetry-breaking interactions. Hence,

$$\omega_{\mu\nu} \to \omega_{\mu\nu}, \quad \omega_\mu \to \omega_\mu, \qquad (24)$$

and

$$m_\omega{}^2 = m_1{}^2. \qquad (25)$$

For ϕ, we should further make the transformations

$$\phi_{\mu\nu} \to (1+2\alpha_U)^{-1/2}\phi_{\mu\nu}, \quad \phi_\mu \to (1+2\alpha_U)^{1/2}\phi_\mu, \qquad (26)$$

and the mass is given by

$$m_\phi{}^2 = (1+2\alpha_U)(1+2\beta_U) m_1{}^2. \qquad (27)$$

It is clear that $\mathcal{L}_{33}$ produces no effect on the physical mesons ρ and ω. As a result, the conservation of Q_{11} and Q_{22} cannot tell us anything about the parameters α_U and β_U. But, the conservation of Q_{33} can.

With the transformation (26), the correspondence between $V_{33}{}^\mu$ and $U_{33}{}^\mu$ becomes

$$V_{33}{}^\mu \leftrightarrow \frac{m_1{}^2}{-g}(1+2\alpha_U)^{1/2} U_{33}{}^\mu = \frac{m_1{}^2}{-g}(1+2\alpha_U)^{1/2}\phi^\mu, \qquad (28)$$

and the field equation (11) becomes

$$m_\phi{}^2\phi^\mu + \partial_\nu\phi^{\mu\nu} = (1+2\alpha_U)^{1/2} ig([\Phi,\Phi^\mu]+[U_\nu,U^{\mu\nu}])_{33}, \qquad (29)$$

where $m_\phi{}^2$ is given by (27). In obtaining (29), use has been made of the fact that $[\Phi,\Phi^\mu]_{aa}$ and $[U_\nu,U^{\mu\nu}]_{aa}$ are invariant under the renormalization transformations

(4), (5), (6), (7), etc. From (28) and (29), one obtains

$$Q_{33}=\frac{m_1^2}{m_\phi^2}(1+2\alpha_U)\int d\mathbf{x}(i[\Phi,\Phi^0]+i[U_k,U_k^0])_{33}. \quad (30)$$

Conservation of Q_{33} then implies that

$$\frac{m_1^2}{m_\phi^2}(1+2\alpha_U)=1,$$

or, with the help of (27),

$$\beta_U=0. \quad (31)$$

The result $\beta_U=0$ has the following remarkable consequences:

(i) With $\beta_U=0$, we have the following masses:

$$m_\omega^2=m_\rho^2=m_1^2, \quad (32)$$

$$m_{K^*}^2=(1+\alpha_U)m_\rho^2, \quad (33)$$

$$m_\phi^2=(1+2\alpha_U)m_\rho^2, \quad (34)$$

and consequently the mass relations

$$m_\omega=m_\rho, \quad (35)$$

$$m_\phi^2+m_\rho^2=2m_{K^*}^2. \quad (36)$$

These mass formulas are very well satisfied experimentally. They were first conjectured by Okubo[10] and later derived by Kuo and Yao[11] on the basis of $SU(6)$.

(ii) To the first order, at least, of the symmetry breaking, the renormalization constant for the phenomenological vector meson field is simply related to its mass displacement. This results from the mass relations (33) and (34). We summarize the field renormalization relations in the following:

$$(\rho_{\mu\nu},\rho_\mu)\rightarrow(\rho_{\mu\nu},\rho_\mu), \quad (37)$$

$$(\omega_{\mu\nu},\omega_\mu)\rightarrow(\omega_{\mu\nu},\omega_\mu), \quad (38)$$

$$K_{\mu\nu}^*\rightarrow\left(\frac{m_{K^*}}{m_\rho}\right)^{-1}K_{\mu\nu}^*,\quad K_\mu^*\rightarrow\left(\frac{m_{K^*}}{m_\rho}\right)K_\mu^*, \quad (39)$$

$$\phi_{\mu\nu}\rightarrow\left(\frac{m_\phi}{m_\rho}\right)^{-1}\phi_{\mu\nu},\qquad \phi_\mu\rightarrow\left(\frac{m_\phi}{m_\rho}\right)\phi_\mu, \quad (40)$$

where the unitary contents of the vector $1+8$ are expressed in (22).

We shall now establish the equivalence, within the effective meson-pole model, of the result (39) to the Ademollo-Gatto theorem,[12] which states: To the first order of the symmetry breaking, there is no renormalization of the $|\Delta Y|=1$ weak vector coupling constants.

[10] S. Okubo, Phys. Letters 5, 165 (1963).
[11] T. K. Kuo and T. Yao, Phys. Rev. Letters 13, 415 (1964).
[12] M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1964); C. Bouchiat and Ph. Meyer, Nuovo Cimento 24, 1122 (1964); S. Fubini and G. Furlan, Physics 1, 229 (1964).

In the effective meson-pole model, the weak leptonic currents interact with the weak currents of the strongly interacting particles through the intermediary of the mesons $\rho^\pm$, $\pi^\pm$, $K^{*\pm}$, and $K^\pm$. In particular, $K^{*\pm}$ are responsible for $|\Delta Y|=1$ vector transitions. As a result of (39), the symmetry-breaking effects arising from mass renormalization and field renormalization of K^* cancel with each other. Further, the $|\Delta Y|=1$ vector currents themselves (at zero momentum transfer) undergo no renormalization to the first order of the symmetry breaking. For 0^- and 1^- mesons, a typical $|\Delta Y|=1$ vector current, in the $U(3)$ limit, is $K_\mu\pi-K\pi_\mu$. With the transformations (4) and (6), this current becomes

$$K_\mu\pi-K\pi_\mu\rightarrow\frac{2+\alpha_\Phi}{2(1+\alpha_\Phi)^{1/2}}\times\left[(K_\mu\pi-K\pi_\mu)-\frac{\alpha_\Phi}{2+\alpha_\Phi}(K_\mu\pi+K\pi_\mu)\right]. \quad (41)$$

In the limit of zero momentum transfer, the $K_\mu\pi+K\pi_\mu$ term in (41) drops out. The renormalization constant for the weak $K\pi$ vertex is then seen to be

$$\frac{2+\alpha_\Phi}{2(1+\alpha_\Phi)^{1/2}}, \quad (42)$$

which is of second order in the symmetry-breaking parameter α_Φ. For baryons, since there are no α terms in $\mathcal{L}_{33}$, the currents are not renormalized. We have thus proved the Ademollo-Gatto theorem.

V. SEMILEPTONIC WEAK INTERACTIONS

We assume that the *charged* vector boson field Z_μ plays a fundamental role[13] in weak interactions. Its interaction with the *fundamental* fields is assumed to be of the form

$$e'\bar{Z}^\mu(\mathcal{J}_\mu+J_\mu)+H.\mathrm{c.}, \quad (43)$$

where $\mathcal{J}_\mu$ is the current associated with leptonic fields,

$$\mathcal{J}_\mu=\bar{\nu}\gamma_\mu(1+i\gamma_5)\mu^++\bar{e}^-\gamma_\mu(1-i\gamma_5)\nu, \quad (44)$$

and J_μ is the weak current associated with those fundamental fields that are the basic entities of the strongly interacting world. According to the "universality" idea, the simplest choice for J_μ would be

$$J_\mu=(V_\mu\pm A_\mu)_{12}+(V_\mu\pm A_\mu)_{13}. \quad (45)$$

However, this scheme cannot provide the empirically observed damping of the $|\Delta Y|=1$ semileptonic processes relative to the $\Delta Y=0$ ones. It might have been hoped that the symmetry-breaking interactions could do the job. But, as we shall presently see this is not the case.

[13] For our purpose, it is actually not necessary to assume the Z_μ field; a local current×current picture suffices.

We shall again rely on the effective meson-pole model. From (43), (44), (45), (8), and (9), one can easily see that the renormalization of K_μ,

$$K_\mu \rightarrow (1+\alpha_\Phi)^{-1/2}K_\mu,$$

introduces a factor into the amplitude ratio

$$\frac{A(K^+\rightarrow\mu^++\nu)}{A(\pi^+\rightarrow\mu^++\nu)}:(1+\alpha_\Phi)^{-1/2}. \quad (46)$$

According to (42), the renormalization of the K and K^* fields introduces a factor into the amplitude ratio

$$\frac{A(K^+\rightarrow\pi^0+e^++\nu)}{A(\pi^+\rightarrow\pi^0+e^++\nu)}:\frac{2+\alpha_\Phi}{2(1+\alpha_\Phi)^{1/2}}. \quad (47)$$

We note that

$$\left|\frac{2+\alpha_\Phi}{2(1+\alpha_\Phi)^{1/2}}\right|\geq 1, \quad (48)$$

if[14] $1+\alpha_\Phi>0$. This shows that the $|\Delta Y|=1$ decay $K^+\rightarrow\pi^0+e^++\nu$ is enhanced, instead of being damped, by the symmetry-breaking interactions. If we were to use the value of $1+\alpha_\Phi$, which is about 16, as inferred from the empirical ratio of $A(K^+\rightarrow\mu^++\nu)/A(\pi^+\rightarrow\mu^++\nu)$ according to (46), the enhancing factor for $K^+\rightarrow\pi^0+e^++\nu$ would be

$$\frac{2+\alpha_\Phi}{2(1+\alpha_\Phi)^{1/2}}\approx 2,$$

in violent contradiction to the empirically observed damping factor $\sim\frac{1}{4}$, We therefore conclude that the empirical damping factor for $|\Delta Y|=1$ semileptonic processes *cannot* come from symmetry breaking effects,[15] and consequently the universal scheme (45) *cannot* accommodate experimental results. The immediate alternative is the Cabibbo scheme,[16] in which the damping factor is assumed to be intrinsic to the $|\Delta Y|=1$ weak interactions.

Cabibbo's universal scheme of semileptonic weak interactions consists of the assumption that

$$J_\mu=J_\mu{}^{\text{Cabibbo}}\equiv\cos\theta(V_\mu-A_\mu)_{12}+\sin\theta(V_\mu-A_\mu)_{13}. \quad (49)$$

This scheme has been very successful in correlating experimental data about semileptonic weak processes. A common practice in determining the Cabibbo angle θ from empirical results is to assume that renormalization effects on the semileptonic weak coupling constants, due to the symmetry-breaking interactions, can be neglected. While this may be justifiable for vector transitions, in the light of the Ademollo-Gatto theorem and provided that second and higher order symmetry-breaking effects are negligible, it is on less secure grounds in the case of axial-vector transitions, as was first pointed out by Sakurai.[17] Based on our model we shall estimate the $U(3)$ symmetry-breaking effects on the semileptonic weak coupling constants within the framework of Cabibbo's theory.

VI. LEPTONIC DECAYS OF K AND π

Because of the renormalization factor $(1+\alpha_\Phi)^{-1/2}$, according to (46), the *apparent* Cabibbo angle θ_A determined from comparing the $K^+\rightarrow\mu^++\nu$ and $\pi^+\rightarrow\mu^++\nu$ decay rates is related to the "bare" Cabibbo angle θ by

$$\tan\theta_A=(1+\alpha_\Phi)^{-1/2}\tan\theta. \quad (50)$$

Similarly, according to (47), the *apparent* Cabibbo angle θ_V determined from comparing the decay rates of $K^+\rightarrow\pi^0+e^++\nu$ and $\pi^+\rightarrow\pi^0+e^++\nu$ is given by

$$\tan\theta_V=\frac{2+\alpha_\Phi}{2(1+\alpha_\Phi)^{1/2}}\tan\theta, \quad (51)$$

where approximation[18] has been made to neglect the momentum-transfer dependence of the form factors of these two processes. From (50) and (51), we further obtain[19]

$$\tfrac{1}{2}(2+\alpha_\Phi)=\tan\theta_V/\tan\theta_A. \quad (52)$$

Empirically,[20]

$$\theta_A=0.266\pm0.005, \quad (53)$$

$$\theta_V=0.241\pm0.008, \quad (54)$$

from which we obtain

$$1+\alpha_\Phi=0.80\pm0.09. \quad (55)$$

The "bare" Cabibbo angle θ is then given by

$$\theta=0.240\pm0.008. \quad (56)$$

There are also other ways to estimate the symmetry-breaking parameter α_Φ. One can determine it from the decay width ratio[21] $\Gamma(K^*\rightarrow K+\pi)/\Gamma(\rho\rightarrow 2\pi)$, and

[14] The condition $1+\alpha_\Phi>0$ certainly must hold, since otherwise the symmetry-breaking interactions would change the charge-conjugation property of K^* and ϕ.

[15] This conclusion is also reached by L. K. Pandit and J. Schechter, Phys. Letters **19**, 56 (1965).

[16] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[17] J. J. Sakurai, Phys. Rev. Letters **12**, 79 (1964).

[18] Since the "apparent" Cabibbo angle θ_V is empirically determined under the assumption of constant form factors, the approximation we made is appropriate.

[19] It should be pointed out that we may have omitted some second-order effect in (51), since we have proved the renormalization relation (39) only to the first order of the symmetry breaking. But this omitted second-order effect is expected to be small and will not significantly affect (52) on which we shall base our estimate of $1+\alpha_\Phi$.

[20] These data are quoted by N. Cabibbo and M. Veltman, CERN 65-30, 1965 (unpublished). Recent $\Gamma(K^+\rightarrow\pi^0+e^-+\nu)$ data seem to indicate that θ_V probably should be smaller than 0.24. See S. Oneda and J. Sucher, Phys. Rev. Letters **15**, 927 (1965). If we take $\theta=0.23$, which corresponds to $\cos\theta=0.974$, Eqs. (50) and (53) imply that $1+\alpha_\Phi=0.74$.

[21] The vector-meson decay widths have previously been used to estimate the symmetry-breaking effect by J. J. Sakurai (Ref. 16). He assumed $Z_3(K^*)/Z_3(\rho)\simeq1$. However, according to our result, $Z_3(K^*)/Z_3(\rho)\simeq m_{K^*}{}^2/m_\rho{}^2=1.36$. This explains the incompatibility between Sakurai's result and that implied by the Ademollo-Gatto theorem.

from the G_A/G_V ratios of the baryon leptonic decays. It is our opinion that the baryon leptonic decays offer the best chance for a reliable determination of the parameter α_Φ and a definite test of the usefulness of our model. As we shall see later, the present experimental data concerning $n \to p+e^-+\nu$, $\Lambda \to p+e^-+\nu$, and $\Sigma^- \to \Lambda+e^-+\nu$ indicate that $1+\alpha_\Phi=0.7$ to 0.8 is indeed a good estimate.

VII. ρ AND K^* DECAYS

We shall assume the $U(3)$-symmetric trilinear meson couplings given by (10). With the renormalization of the K and K^* field variables as given by (4), (5), (6), and (7), the decay widths are calculated to be

$$\Gamma(K^* \to K+\pi)=\frac{g^2}{4\pi}\frac{p^3}{m_{K^*}^2}\left[\frac{m_{K^*}}{m_\rho}\frac{3+\alpha_\Phi}{2(1+\alpha_\Phi)^{1/2}}\right]^2, \quad (57)$$

$$\Gamma(\rho \to 2\pi)=\tfrac{4}{3}\frac{g^2}{4\pi}\frac{p^3}{m_\rho^2}(\tfrac{3}{2})^2, \quad (58)$$

where p is the center-of-mass momentum of the daughter particles in each reaction, and use has been made of (32) and (33). In the previous note,[22] we used the data

$$\Gamma(K^*)=50 \text{ MeV}, \quad \Gamma(\rho)=112 \text{ MeV}, \quad (59)$$

and $1+\alpha_\Phi$ was determined to be

$$1+\alpha_\Phi=0.76, \quad (60)$$

which is consistent with (55). In view of the present uncertainty[23] concerning the ρ-meson width and the high sensitivity of $1+\alpha_\Phi$ to the ratio $\Gamma(K^* \to K+\pi)/\Gamma(\rho \to 2\pi)$, the determination (60) should only be regarded as a confirmation check of (55). It may be pointed out that according to our scheme the calculated ratio $\Gamma(\phi \to \bar{K}+K)/\Gamma(\rho \to 2\pi)$ is somewhat larger than the experimental result.

VIII. PSEUDOSCALAR MESON MASSES

Because of the presence of the α terms in $\mathfrak{L}_{33}$, the Gell-Mann–Okubo mass formula for the octuplet of pseudoscalar mesons is slightly modified. With the 0^- octuplet represented by

$$\Phi_{ab}=\begin{bmatrix} -\frac{\pi^0}{2^{1/2}}+\frac{\eta}{6^{1/2}} & \pi^+ & K^+ \\ -\pi^- & \frac{\pi^0}{2^{1/2}}+\frac{\eta}{6^{1/2}} & K^0 \\ \bar{K}^- & \bar{K}^0 & -\frac{2}{6^{1/2}}\eta \end{bmatrix}, \quad (61)$$

$\mathfrak{L}_{33}$ implies the following masses:

$$m_\pi^2=m_0^2, \quad (62)$$

$$m_K^2=(1+\alpha_\Phi)(1+\beta_\Phi)m_\pi^2, \quad (63)$$

$$m_\eta^2=(1+\tfrac{4}{3}\alpha_\Phi)(1+\tfrac{4}{3}\beta_\Phi)m_\pi^2, \quad (64)$$

and consequently,

$$\frac{m_\pi^2+m_\eta^2-4m_K^2}{m_\pi^2}-\tfrac{4}{3}\alpha_\Phi\left[\frac{m_K^2/m_\pi^2}{1+\alpha_\Phi}-1\right]=0. \quad (65)$$

By setting $\alpha_\Phi=0$, (65) reduces to the usual Gell-Mann–Okubo mass formula,

$$m_\pi^2+3m_\eta^2-4m_K^2=0. \quad (66)$$

Empirically, while the left-hand side of (66) is equal to -3.2 (in units of m_π^2), that of (65), with $1+\alpha_\Phi=0.80$, is equal to $+1$. The modified mass formula (65) is therefore in better agreement with the experimental data than the original Gell-Mann–Okubo formula (66). Incidentally, (65) becomes exact if $1+\alpha_\Phi=0.83$.

For the baryon octuplet, since there are no α terms in $\mathfrak{L}_{33}$, the Gell-Mann-Okubo mass formula remains intact.

IX. LEPTONIC DECAYS OF BARYONS

In the pole-dominance model, semileptonic weak interactions are transmitted by $\rho^\pm$, $\pi^\pm$, $K^{*\pm}$, and $K^\pm$. It is clear that the $\Delta Y=0$ semileptonic coupling constants, vector as well as axial vector, are not renormalized by $\mathfrak{L}_{33}$. In view of the Ademollo-Gatto theorem, which we have proved within the effective meson-pole model in Sec. IV, the $|\Delta Y|=1$ vector coupling constants for baryon leptonic decays are practically unaffected. In other words, the *apparent* Cabibbo angle θ_V' for $|\Delta Y|=1$ vector transitions is practically the same as the bare Cabibbo angle θ. Only the $|\Delta Y|=1$ axial-vector coupling constants are expected to be renormalized by the symmetry-breaking interactions.

The axial-vector transitions are transmitted by axial-vector excitations, i.e., $\pi_\mu^\pm$ and $K_\mu^\pm$. As we shall show in the Appendix, Goldberger-Treiman type relations follow from the field equations for the field variables $\pi_\mu^\pm$ and $K_\mu^\pm$. On the basis of Goldberger-Treiman type relations, the renormalization of K_μ,

$$K_\mu \to (1+\alpha_\Phi)^{-1/2}K_\mu$$

implies the replacement

$$\sin\theta \to (1+\alpha_\Phi)^{-1}\sin\theta \quad (67)$$

for $|\Delta Y|=1$ axial-vector transitions of the leptonic decays of the baryons. The symmetry-breaking effects thus appear as a *universal* multiplicative factor for *all* $|\Delta Y|=1$ axial-vector weak vertices of the baryons. This factor can be absorbed as a renormalization of the Cabibbo angle, allowing the "apparent" axial-

[22] H. T. Nieh, Phys. Rev. Letters **15**, 902 (1965).

[23] See, for example, D. O. Caldwell and R. Weinstein, Nuovo Cimento **39**, 991 (1965).

vector vertex functions to be expressed in terms of two reduced matrix elements F and D, as in the $U(3)$ symmetry limit.[16] The corresponding *apparent* Cabibbo angle θ_A' is related to the "bare" Cabibbo angle θ by

$$\tan\theta_A' = (1+\alpha_\Phi)^{-1}\tan\theta. \tag{68}$$

Combining (50) and (68), we further obtain

$$\tan\theta_A' = (1+\alpha_\Phi)^{-1/2}\tan\theta_A. \tag{69}$$

With $1+\alpha_\Phi$ given by (55) and θ_A given by (53), we obtain

$$\theta_A' = 0.296\pm0.02. \tag{70}$$

Willis *et al.*[24] have carried out an analysis of existing experimental data concerning the branching ratios of baryon leptonic decays. They found that the best values of θ and θ_A' are equal, within a few percent, to 0.26. Our results (56) and (70) are not consistent with their finding. However, as the experimental errors on the rates of hyperon leptonic decays are rather large, their conclusions *may* not be reliable. One such indication comes from the experimental G_A/G_V ratio of Λ beta decays. Solution A of Willis *et al.*[25] implies the ratio

$$-\left(\frac{G_A}{G_V}\right)^{\Lambda\to p+e^-+\nu}_{\text{Willis } et\ al.} = F+\tfrac{1}{3}D = 0.68. \tag{71}$$

But, from polarized lambda beta decays Lind *et al.*[26] found that

$$-\left(\frac{G_A}{G_V}\right)^{\Lambda\to p+e^-+\nu}_{\text{Lind } et\ al.} \simeq 1 \tag{72}$$

is most consistent with the experimental data; Barlow *et al.*[27] found that

$$-\left(\frac{G_A}{G_V}\right)^{\Lambda\to p+e^-+\nu}_{\text{Barlow } et\ al.} = 0.9_{-0.3}^{+0.25}. \tag{73}$$

All these data, still rough though they may be, indicate that the conclusions of Willis *et al.* are not final. It is highly desirable that the type of analysis carried out by Willis *et al.* should be repeated when more accurate experimental data become available.

A better test of our model comes from direct comparison with experimental G_A/G_V ratios. In our model, (67) implies that *the G_A/G_V ratio for all $|\Delta Y|=1$ baryon leptonic decays is uniformly "renormalized" by the $U(3)$ symmetry-breaking interactions, with a single renormalization constant $(1+\alpha_\Phi)^{-1}$. Thus,*

$$-\left(\frac{G_A}{G_V}\right)^{\Lambda\to p+e^-+\nu} = (1+\alpha_\Phi)^{-1}(F+\tfrac{1}{3}D),\ \text{etc.}, \tag{74}$$

where F and D are the usual reduced matrix elements. With[28]

$$-\left(\frac{G_A}{G_V}\right)^{n\to p+e^-+\nu}_{\text{expt}} = F+D = 1.18\pm0.02 \tag{75}$$

and the experimental branching ratio[24]

$$B(\Sigma^-\to\Lambda+e^-+\nu) = (0.75\pm0.28)\times10^{-4} \tag{76}$$

as input, F and D are determined to be[29]

$$F=0.38\pm0.17,\quad D=0.88\mp0.15, \tag{77}$$

which are consistent with some of the most recent theoretical considerations.[30] With $1+\alpha_\Phi$ given by (55), we obtain from (74) and (77) that

$$-\left(\frac{G_A}{G_V}\right)^{\Lambda\to p+e^-+\nu}_{\text{theor}} = 0.81_{-0.21}^{+0.27}, \tag{78}$$

which is consistent with (72) and (73). More accurate experimental data, of course, are required to make a definitive test of our model. A reliable determination of the D/F ratio from $\Delta Y=0$ baryon leptonic decays[31] and the G_A/G_V ratios of $|\Delta Y|=1$ hyperon leptonic decays can best serve the purpose. A reliable determination of the parameter $1+\alpha_\Phi$ will also evolve from these data. Judging from the presently available experimental information, we shall accept the estimate $1+\alpha_\Phi\approx0.7$ to 0.8 as a good guideline. In Table I, we express the

[24] W. Willis, H. Courant, H. Filthuth, P. Franzini, A. Minguzzi-Ranzi, A. Seagar, R. Engleman, V. Hepp, E. Kluge, R. A. Burnstein, T. B. Day, R. G. Glasser, A. J. Herz, B. Kehoe, B. Sechi-Zorn, N. Seeman, and G. A. Snow, Phys. Rev. Letters **13**, 291 (1964).

[25] Solution B of Willis *et al.* is discarded on the basis that the present experimental decay rate of $\Sigma^-\to\Lambda+e^-+\nu$ is much more reliable than that of $\Xi^-\to\Lambda+e^-+\nu$. Also, $D/F>1$ is favored by almost every respectable theoretical consideration, the latest being that of D. Amati, C. Bouchiat, and J. Nuyts, Phys. Letters **19**, 59 (1965); and B. Diu and R. P. Van Royen, CERN, unpublished report. With $D/F>1$, (75) implies the inequality $F+\frac{1}{3}D<0.8$.

[26] V. G. Lind, T. O. Binford, M. L. Good, and D. Stern, Phys. Rev. **135**, B1483 (1964). See also R. P. Ely, G. Gidal, G. E. Kalmus, W. M. Powell, W. J. Singleton, C. Henderson, D. J. Miller, and F. R. Stannard, *ibid.* **137**, B1302 (1965).

[27] J. Barlow, I. M. Blair, G. Conforto, M. I. Ferrero, C. Rubbia, J. C. Sens, P. J. Duke, and A. K. Mann, Phys. Letters **18**, 64 (1965).

[28] The value of the nucleon beta-decay G_A/G_V ratio, 1.18±0.02, has been attributed to C. S. Wu.

[29] Use has been made of (56). Solution with negative D is discarded.

[30] M. E. Ebel and P. B. James, Phys. Rev. Letters **15**, 805 (1965); B. Diu and R. P. Van Royer, CERN, unpublished report.

[31] It is more reliable to determine the D/F ratio from $\Delta Y=0$ decays, because the vertices of these decays are (i) free from first-order renormalization by the $U(3)$ symmetry-breaking interactions and (ii) insensitive to the value of θ for small θ. It is for these reasons that we have chosen the $\Delta Y=0$ decays $n\to p+e^-+\nu$ and $\Sigma^-\to\Lambda+e^-+\nu$ for estimating the D/F ratio, the numerical value of which is contained in (77).

TABLE I. G_A/G_V ratios of baryon leptonic decays.

		G_A/G_V	Inputs $1+\alpha_\Phi=0.8$ $F=0.38$ $D=0.80$	Inputs $1+\alpha_\Phi=0.75$ $F=0.38$ $D=0.80$	Inputs $1+\alpha_\Phi=0.8$ $F=0.47$ $(D=\frac{3}{2}F)$
$\Delta Y=0$:	$n\to p+e^-+\nu$	$-(F+D)$	-1.18	-1.18	-1.18
	$\Sigma^0\to\Sigma^++e^-+\nu$	$-F$	-0.38	-0.38	-0.47
	$\Sigma^-\to\Sigma^0+e^-+\nu$	$-F$	-0.38	-0.38	-0.47
	$\Xi^-\to\Xi^0+e^-+\nu$	$-(F-D)$	$+0.42$	$+0.42$	$+0.24$
	$\Sigma^\pm\to\Lambda+e^\pm+\nu$	pure axial-vector			
$\Delta Y=1$:	$\Lambda\to p+e^-+\nu$	$-(1+\alpha_\Phi)^{-1}(F+D/3)$	-0.81	-0.87	-0.89
	$\Sigma^0\to p+e^-+\nu$	$-(1+\alpha_\Phi)^{-1}(F-D)$	$+0.53$	$+0.56$	$+0.30$
	$\Xi^-\to\Sigma^0+e^-+\nu$	$-(1+\alpha_\Phi)^{-1}(F+D)$	-1.40	-1.57	-1.40
	$\Xi^-\to\Lambda+e^-+\nu$	$-(1+\alpha_\Phi)^{-1}(F-D/3)$	-0.14	-0.15	-0.29
	$\Xi^0\to\Sigma^++e^-+\nu$	$-(1+\alpha_\Phi)^{-1}(F+D)$	-1.40	-1.57	-1.40
	$\Sigma^-\to n+e^-+\nu$	$-(1+\alpha_\Phi)^{-1}(F-D)$	$+0.53$	$+0.56$	$+0.30$

G_A/G_V ratios of baryon leptonic decays in terms of the reduced matrix elements F and D, and give their numerical values for a few different choices of the parameters.

Finally, we should like to make a remark about the approximations involved in obtaining our predictions about the G_A/G_V ratios of the baryon leptonic decays. The predictions are based on Goldberger-Treiman type relations, the derivation of which rests on the assumption that the axial-vector transitions of baryons are *entirely* transmitted through the single-particle meson states. Empirically, the Goldberger-Treiman relation (for $\Delta Y=0$ processes) is in error by about 10%, if we identify the pseudovector pion-nucleon coupling constant with the measured one, which, however, may *effectively* include contribution from the pion-nucleon coupling in pseudoscalar form.[32] Whatever the source of the 10% error may be, it seems quite plausible that single-particle intermediate states predominate over the states of the continuum. For definiteness, let us assume that the single-particle meson states are responsible for, say, 90% of the axial-vector transitions of the baryons (in both $\Delta Y=0$ and $|\Delta Y|=1$ channels). If we also assume that no drastic symmetry-breaking effects are introduced by the intermediate states of the continuum, then the renormalization factor $(1+\alpha_\Phi)^{-1}$ should approximately be replaced by the combination

$$\frac{0.9}{1+\alpha_\Phi}+0.1=\frac{1}{1+\alpha_\Phi}+\frac{0.1}{1+\alpha_\Phi}\alpha_\Phi,$$

which, with $1+\alpha_\Phi\sim0.8$, is approximately

$$\frac{1}{1+\alpha_\Phi}-0.03.$$

[32] From our point of view, field equations imply that pseudoscalar pion-nucleon coupling *only* induces an effective pseudoscalar coupling for the nucleon beta decay.

So, the errors inherent in the pole-dominance approximations are expected to affect our prediction of the G_A/G_V ratios by only a few percent, which is anyway tolerated in our determination of the parameter $1+\alpha_\Phi$ from meson decay processes.

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APPENDIX

In this Appendix, we shall derive the Goldberger-Treiman relation from the field equation for the field variable π_μ. For our purpose, we neglect complications due to isotopic spin. The free-field Lagrange function for the pion field is

$$\mathcal{L}_0(\pi)=-\pi^\mu\partial_\mu\pi+\tfrac{1}{2}\pi^\mu\pi_\mu-\tfrac{1}{2}m_\pi{}^2\pi\pi. \quad (A1)$$

We assume that the coupling for $\pi\to l+\nu$ is of the form

$$(f_{\pi l}/m_\pi)\pi^\mu\bar{l}\gamma_\mu(1+i\gamma_5)\nu \quad (A2)$$

and the pion-nucleon coupling is of the pseudovector form.

$$\frac{f_{\pi N}}{m_\pi}\pi^\mu\bar{N}\gamma_\mu i\gamma_5 N, \quad (A3)$$

where N is the nucleon field. It follows from (A1), (A2), and (A3) that the field equation for π_μ is

$$\pi_\mu=\partial_\mu\pi-\frac{f_{\pi l}}{m_\pi}\bar{l}\gamma_\mu(1+i\gamma_5)\nu-\frac{f_{\pi N}}{m_\pi}\bar{N}\gamma_\mu i\gamma_5 N. \quad (A4)$$

By substituting (A4) back into either (A2) or (A3), we obtain a *direct* coupling term

$$-\frac{f_{\pi l}f_{\pi N}}{m_\pi^2}\bar{N}\gamma^\mu i\gamma_5 N\bar{l}\gamma_\mu(1+i\gamma_5)\nu\,, \tag{A5}$$

which, when compared with the axial-vector coupling for the nucleon beta decay in the form

$$\frac{G_A}{2^{1/2}}\bar{N}\gamma^\mu i\gamma_5 N\bar{l}\gamma_\mu(1+i\gamma_5)\nu\,, \tag{A6}$$

gives rise to the relation

$$\frac{G_A}{2^{1/2}}=-\frac{f_{\pi l}f_{\pi N}}{m_\pi^2}\,,$$

or

$$f_{\pi l}=-\frac{m_\pi^2 G_A}{2^{1/2}f_{\pi N}}\,. \tag{A7}$$

This is seen to be the Goldberger-Treiman relation. From our point of view, the *pseudovector* form of the pion-nucleon coupling is essential for the derivation of the Goldberger-Treiman relation.

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Electromagnetic Masses of Pseudoscalar Mesons

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Based on a recently proposed generalization of the "tadpole" model of Coleman and Glashow, the six electromagnetic mass splittings of baryons and pseudoscalar mesons are considered. With the inclusion of the "nontadpole" contributions calculated by Coleman and Schnitzer and by Socolow, the agreement between theory and experiment is satisfactory.

I. INTRODUCTION

WITHIN the framework of the "tadpole" model of Coleman and Glashow,[1] electromagnetic mass splittings of baryons and pseudoscalar mesons have been calculated by Coleman and Glashow,[1] Coleman and Schnitzer,[2] and Socolow.[3] They express six such mass splittings in terms of one free parameter. By properly choosing this parameter, all of the splittings are in good agreement with experiment *except* for the kaon mass splitting, which has the right sign but only about half of the magnitude.[3]

In this note, we shall consider these mass splittings on the basis of a recently proposed generalization[4] of the "tadpole" model of Coleman and Glashow. This generalized "tadpole" model has reproduced[4] some successful results[5] previously obtained from general considerations and is supported by the presently available semileptonic weak decay data.[4] With the knowledge we gained from the weak decays, the six mass splittings, including the kaon splitting, calculated according to the generalized "tadpole" model are in satisfactory agreement with experiment.

[1] S. Coleman and S. L. Glashow, Phys. Rev. **132**, B671 (1964).
[2] S. Coleman and H. J. Schnitzer, Phys. Rev. **136**, B223 (1964).
[3] R. H. Socolow, Phys. Rev. **137**, B1221 (1965).
[4] H. T. Nieh, Phys. Rev. Letters **15**, 902 (1965); Phys. Rev., preceding paper, **146**, 1012 (1966).
[5] We have in mind here the Ademollo-Gatto theorem and the $SU(6)$ mass formulas for vector mesons.

II. GENERALIZED TADPOLE MODEL

In the generalized model, we assume that the $U(3)$-symmetric interaction Lagrange function for the scalar fields S_{ab} is of the form:

$$\mathcal{L}'=d\,\mathrm{Tr}\{\bar{\Psi},\Psi\}S+f\,\mathrm{Tr}[\bar{\Psi},\Psi]S+g\,\mathrm{Tr}\Phi\Phi S+h\,\mathrm{Tr}\Phi^\mu\Phi_\mu S\,, \tag{1}$$

where Φ and Φ_μ are essentially the five components of the spinless field in the Duffin-Kemmer formalism. This interaction Lagrange function is the same as that of Coleman and Glashow[1] except for the h term, which is the added feature of our generalized model. We exploit the possibility that the scalar field S couples not only to the component Φ but also to the components Φ_μ. The nonvanishing vacuum expectation values[6] of S_{33} and S_{11} break the $U(3)$ symmetry and give rise to the following *effective* terms in the Lagrange function for baryons Ψ_{ab} and pseudoscalar mesons Φ_{ab}:

$$\mathcal{L}_{33}=-\beta_\Psi M(\bar{\Psi}\Psi)_{33}-\beta_\Psi' M(\Psi\bar{\Psi})_{33}+\alpha_\Phi(\Phi^\mu\Phi_\mu)_{33}-\beta_\Phi m_0^2(\Phi\Phi)_{33}\,, \tag{2}$$

$$\mathcal{L}_{11}=\chi[-\beta_\Psi M(\bar{\Psi}\Psi)_{11}-\beta_\Psi' M(\Psi\bar{\Psi})_{11}+\alpha_\Phi(\Phi^\mu\Phi_\mu)_{11}-\beta_\Phi m_0^2(\Phi\Phi)_{11}]\,, \tag{3}$$

where M and m_0 are the degenerate masses of the baryon and pseudoscalar meson octuplets, in the $U(3)$

[6] In place of "tadpole," we shall henceforth use the terminology "vacuon," which was suggested by J. Schwinger [Phys. Rev. **136**, B1821 (1964)].