tion to the $\frac{3}{2}^+$ level is six times larger than the intensity of the 668-keV transition to the $\frac{1}{2}^+$ ground state may be taken as favoring the assignment of spin $\frac{5}{2}$ to the 668-keV level, although spin $\frac{3}{2}$ cannot be ruled out on these grounds.

The results for the transitions originating from the 462-keV level are summarized in Table I. In calculating the $E2$ transition probabilities,[9] using the wave functions given by Kisslinger and Sorensen,[1] a value $B(E2)_{0^+\to 2^+}=46\times 10^{-50}e^2$ cm^4 was assumed for the average of the $B(E2)$ values of the neighboring even-even nuclei. For the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ states, the wave functions were taken from Ref. 1, for the $\frac{5}{2}^+$ state, the following no-phonon and one-phonon amplitudes were used[10];

$$C_{\frac{5}{2}00}{}^{\frac{5}{2}}=0.461,\quad C_{\frac{7}{2}12}{}^{\frac{5}{2}}=0.117,\quad C_{\frac{5}{2}12}{}^{\frac{5}{2}}=0.308,$$
$$C_{\frac{3}{2}12}{}^{\frac{5}{2}}=0.226,\quad C_{\frac{1}{2}12}{}^{\frac{5}{2}}=0.653.$$

For the pure $E2$ 462-keV ground-state transition, which is enhanced by a factor of 13 over the single-particle value, the Kisslinger and Sorensen wave functions reproduce the experimental value to within a factor of 2. With respect to the 427-keV transition one is handicapped by the uncertainty concerning the multipole mixing. It appears, however, that in this case the quasiparticle-plus-phonon wave functions[1] underestimate the $E2$ transition probability considerably: Even with the smallest value of δ^2 suggested by experiment,[5] $\delta^2=0.25$, the experimental transition probability exceeds the predicted value by a factor of 7. In this connection it might be pointed out that the transition probability for the 35-keV $M1$ transition involving the same $\frac{3}{2}^+$ first excited state is also rather poorly accounted for[11] by the Kisslinger and Sorensen wave functions.

Any further discussion of the 427-keV transition as, e.g., the retardation of the $M1$ part, is best left to those[5] in better position to judge the uncertainties in the mixing amplitudes. It is our understanding that such a discussion is forthcoming.[12]

TABLE I. Comparison of the experimental $E2$ transition probabilities (column 3) with theoretical estimates. In column 4 the transition probabilities calculated with the quasiparticle-plus-phonon wave functions of Kisslinger and Sorensen[a] are given; olumn 5 lists the Weisskopf[b] estimates.

E_γ (keV)	Spin sequence	$T(E2)_{\text{expt}}$ (sec^{-1})	$T(E2)_{\text{K+S}}$ (sec^{-1})	$T(E2)_{\text{W}}$ (sec^{-1})
427	$\frac{5}{2}^+\to\frac{3}{2}^+$	$[\delta^2/(1+\delta^2)]40\times10^9$	1.1×10^9	0.7×10^9
462	$\frac{5}{2}^+\to\frac{1}{2}^+$	13×10^9	7.0×10^9	1.0×10^9

[a] See Ref. 1.
[b] See, e.g., J. M. Blatt, and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), Chap. 12.

[9] See, e.g., R. A. Sorensen, Phys. Rev. 133, B281 (1964), Eq. (6).
[10] We are indebted to Professor Sorensen for providing us with these amplitudes.
[11] R. A. Sorensen, Phys. Rev. 132, 2270 (1963).
[12] N. J. Stone (private communication).

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Isobaric Analogue States in Heavy Nuclei. III. Tin Isotopes*

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Isobaric analogue resonances have been identified in (p,p) and (p,n) excitation functions on the target isotopes Sn^{112}, Sn^{114}, Sn^{116}, Sn^{117}, Sn^{118}, Sn^{119}, Sn^{120}, Sn^{122}, and Sn^{124}. All the resonances observed on the even isotopes have been analyzed using a Coulomb-plus-single-level formula. Level separations, l-value determinations, spreading widths, spectroscopic factors, and Coulomb displacement energies are discussed. The spectroscopic information from (d,p) on the same targets are compared to the analogue results.

1. INTRODUCTION

THE recent study of isobaric analogue resonances by Moore, Richard, Watson, Robson, and Fox[1] on the isotopes of Mo demonstrates the use of this type of resonance as a spectroscopic tool in heavy nuclei. It is seen that neutron capture states formed in (d,p) stripping compare very well with the resonances formed by proton-induced reactions on the same target. The two types of reaction lead to states with all quantum numbers the same including isobaric spin but differing in T_z by one and differing in energy by the Coulomb displacement energy for a single proton. The purpose of this paper is to extend these studies to the Sn isotopes where the recent (d,p) data of Schneid, Prakash, and Cohen[2] is available for making comparisons between isobaric analogue states.

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[1] C. F. Moore, P. Richard, C. E. Watson, D. Robson, and J. D. Fox, Phys. Rev. 141, 1166 (1966).
[2] E. J. Schneid, A. Prakash, and B. L. Cohen (to be published).

The Sn isotopes are particularly well suited for this study for several reasons: (1) the protons form a closed shell, $Z=50$, making the observed spectrum relatively simple, (2) a large s-wave resonance which is susceptible to very careful resonance analysis occurs for all the Sn isotopes, and (3) there are many stable isotopes of Sn, allowing for the study of systematic trends.

2. THEORY

All of the spectroscopic information obtained in this study comes from the analysis of compound-nucleus resonances observed in proton elastic scattering. The levels of like spin and parity are well separated in the cases studied so that a Coulomb-plus-single-level formula as given by Lane and Thomas[3] has been used. Since the shape of the excitation function near an isolated resonance is almost completely characterized by the orbital angular momentum of the captured proton, the l value for the resonance was determined by inspection. The other resonance parameters E_J, resonance energy, Γ^J, total state width, and ϕ_l, nuclear phase shift are found by a computer search routine.[4]

The spectroscopic factor[1,5] "S_{pp}" can be estimated for each resonant state using the relations

$$S_{\text{pp}}=W_{\text{ext}}/W_{\text{obs}} \tag{2.1}$$

with

$$W_{\text{ext}}=s_p|L_p^0|^2\Gamma_p/P_p \tag{2.2}$$

and $W_{\text{obs}}=\Gamma^J-\Gamma^J{}_{ls}$ as determined from the experimental observations.

$$|L_p^0|^2=(S_p-S_n)^2+P_p^2\,, \tag{2.3}$$

where S_p and P_p are, respectively the proton shift function and penetrability. S_n is the neutron shift function[5] and is evaluated at $E_J-\Delta E_C$ where ΔE_C is the Coulomb displacement energy between isobaric analogue states. The proton strength function for the l wave of interest is s_p. For this study the statistical model result $\pi s\simeq 1/KR$ has been used where K is the proton wave number inside the nucleus (i.e., evaluated at E_J+U_0 where U_0 is the nuclear potential), and R is the nuclear radius.

TABLE I. Target purities and thicknesses.

Target isotope	Percent enrichment	Target thickness (mg/cm^2)
Sn^{112}	70.05	0.105
Sn^{114}	56.85	0.145
Sn^{116}	94.0	0.210
Sn^{117}	89.2	···
Sn^{118}	95.6	0.240
Sn^{119}	89.8	···
Sn^{120}	98.14	0.470
Sn^{122}	88.92	0.365
Sn^{124}	96.0	0.235

[3] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).
[4] C. F. Moore and P. Richard, Florida State University Technical Report No. 8 (unpublished).
[5] D. Robson, Phys. Rev. **137**, B535 (1965).

3. EXPERIMENT

Proton elastic-scattering data were taken as described elsewhere[1] in a scattering chamber with solid-state counters at the laboratory angles of 90°, 125°, and 165° with respect to the beam. The proton beams were obtained from the F.S.U. tandem Van de Graaff accelerator and ranged in energy from 6.2 to 9.4 MeV. The excitation functions were taken in steps of approximately 7 keV and the data points were run for typically 100 μC.

The (p,n) excitation functions were taken with a Hanson-McKibben BF_3 long counter placed at 90° to the beam in a beam port containing no slits in order to reduce background radiation. The beam was focused using a quadrupole lens to a spot usually smaller than $\frac{1}{4}$ in. in diameter.

Targets for this experiment were made of metal oxide layers on supporting carbon backings. The target thicknesses, including the oxygen contribution, are given in Table I along with the isotopic enrichments. The target thicknesses were determined from the elastic scattering data. The carbon backings were typically 25 μg/cm^2 and showed negligible contamination from heavy elements.

4. RESULTS AND CONCLUSIONS

Proton Scattering and Reactions on the Even Isotopes of Sn

Observation of Analogue Resonances

A total of twenty-nine isobaric analogue resonances are observed by proton elastic scattering and (p,n) excitation functions on the even isotopes of Sn (112, 114, 116, 118, 120, 122, and 124). Twenty-six of these levels are seen in proton elastic scattering and are analyzed to obtain the l value of the captured proton and the resonance parameters (resonance energy, total width, and proton partial width). From this information the proton spectroscopic factors "S_{pp}" have been extracted. The Coulomb displacement energy between each analogue pair is also computed.

The energy range for observing analogue resonances for the above mentioned nuclei is shown in Fig. 1. The proton relative center-of-mass energy "$E_p^{\text{c.m.}}$" for observing analogue resonances ranges from 6.2 to 9.4 MeV. It is clearly seen from the figure that the range for observing analogues in each nuclei studied here increases as one goes up in "A" for a given "Z". It is also seen that $E_p^{\text{c.m.}}$ for observing the ground-state analogue increases with increasing "A." This effect is due to the decrease in the neutron binding energy "B_n" of the neutron analogue as one goes to higher mass isotopes since $E_p^{\text{c.m.}}=\Delta E_C-B_n$, where ΔE_C is the

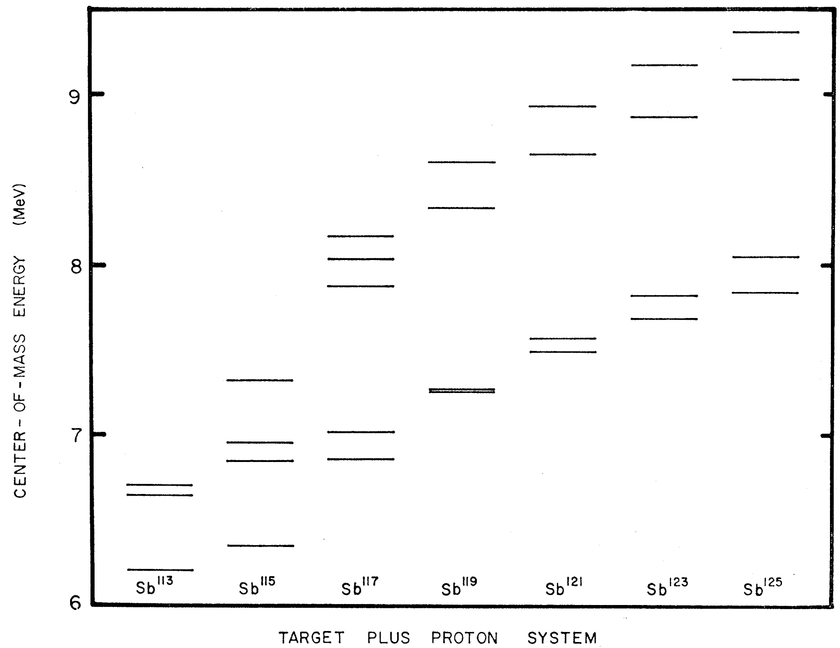

FIG. 1. Energies of bombarding protons for which isobaric analogue resonances were observed with targets of the isotopes of Sn. The compound system (target+proton) is indicated.

Coulomb displacement energy between isobaric analogue pairs and is fairly constant for all the Sn isotopes.

The (p,n) threshold is another important feature to consider which is not shown explicitly in Fig. 1. In the case of Sn^{112} and Sn^{114} the (p,n) thresholds are too high to allow for the observation of analogue resonances whereas in all the other isotopes the (p,n) reaction is very useful in locating the positions of the resonances. However, resonances which are too high above the (p,n) thresholds are lost in the continuous "background" due to the ordinary compound states as is the case of the $E_p^{\text{c.m.}}=9.364$-MeV resonance in Sb^{125} which was not seen by (p,n).

The states seen at the higher excitations are also broadened due to the fact that they are appearing higher up on the Coulomb barrier.

In Fig. 2 the excitation energy above the ground-state analogue resonance is shown for all the nuclei in this study with a comparison to the low-lying levels of the target plus neutron system (neutron analogue) as seen in (d,p) studies.[2] The short lines are the excitation

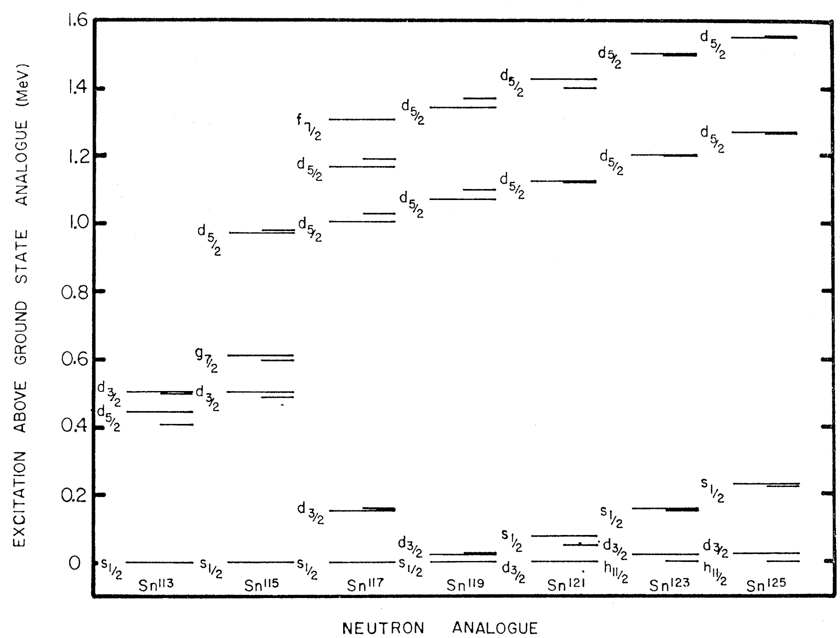

FIG. 2. Comparison of (d,p) results (Ref. 2) and isobaric-analogue-resonance studies.

TABLE II. Some of the relative variations encountered in the study of the isobaric states produced by proton bombardment on the even isotopes of Sn.

Target nucleus	Sn^{112}	Sn^{114}	Sn^{116}	Sn^{118}	Sn^{120}	Sn^{122}	Sn^{124}
B_n neutron binding[a]	8.043	7.525	6.948	6.491	6.159	5.920	5.744
B_n neutron binding[b]	7.729	7.554	...	...	...	5.951	5.731
$E_p^{\text{c.m.}}$ ground state	6.202	6.350	6.869	7.255	7.495	7.664	7.814
B_p proton binding[a]	...	3.720	4.337	5.122	5.769	6.550	7.293
(p,n) threshold[c]	8.029 ± 0.02	6.820 ± 0.05	4.700	3.910	2.220	1.590	0.560
ΔE_C Coulomb displacement energy[a]	14.245	13.875	13.817	13.746	13.654	13.584	13.558
ΔE_C Coulomb displacement energy[b]	13.931	13.904	...	...	...	13.615	13.545
$C=\Delta E_C/(Z/A^{1/3})$[a]	1.349	1.321	1.323	1.324	1.322	1.323	1.329
$C=\Delta E_C/(Z/A^{1/3})$[a]	1.319	1.324	...	...	...	1.326	1.328

[a] Based on data of Damerow *et al.*
[b] Based on (d,p) Q-value measurements.
[c] Taken from *Nuclear Data Sheets* except for Sn^{112} and Sn^{112} (p,n) thresholds.

energies observed by (d,p). The energy resolution for the (d,p) studies of Schneid *et al.*,[2] is 40 keV and of Nealy and Sheline[6] on Sn^{122} and Sn^{124} is 15 keV. The observed resonance energies in proton scattering are correct to ±15 keV so that all the observed levels fall within the experimental errors of the two experiments. Perfect agreement is seen in determining the relative angular momentum of the captured particles.

There are many states seen in the (d,p) studies, the isobaric analogues of which are not seen as proton resonances. These are not seen because they are weakly excited states in (d,p) and thus cannot be expected to have large Coulomb-resonance interference amplitudes in the proton excitation function.

A summary of some of the variations encountered in the study of isobaric analogue resonances in the even isotopes of Sn is shown in Table II.

The magnitudes of the spectroscopic factors S_{pp} and S_{dp} do not compare as well as in the case of the Mo isotopes.[1] However, in some cases, the ratios of the spectroscopic factors agree quite well.

Sn^{113}–Sb^{113}. Figure 3 presents the proton-elastic-scattering data taken with a Sn^{112} target. The solid line in the figure represents a fit to the three observed resonances using the Coulomb-plus-single-level formula. The resonance at 6.202 MeV comes from s-wave proton capture leading to the analogue of the $s_{1/2}$ ground state of Sn^{113} as observed by Schneid *et al.*,[2] in $Sn^{112}(d,p)$. The two higher resonances are both from d-wave capture and were fitted only at 92° and 165°. The extracted resonance parameters and spectroscopic factors are collected in Table III, together with the known $Sn^{112}(d,p)$ results.

Good agreement is seen between the Sn^{112}-plus-neutron states and the Sn^{112}-plus-proton analogues; however, many states are not observed in the latter case. The unobserved resonances are either high-spin states which have a small partial width due to the small proton penetrabilities in the analogue resonances or are weakly excited states in the (d,p) studies, so that the proton elastic scattering data can not rule out these states. The (p,n) Q value in this neutron-deficient Sn isotope was determined to be -8.029 ± 0.03 MeV (see Fig. 4). The previous value taken from mass systematics[7] is -8.28 MeV which is 251 keV higher than our determination. Our measured threshold corresponds to an excitation energy of 1.72 MeV in Sb^{113} relative to the observed $s_{1/2}$ resonance. Due to this threshold the (p,n) reaction was not useful in locating analogue resonances in this isotope.

The s-wave resonance at 6.202 MeV was used to study the effect of introducing a phase shift from nuclear potential scattering. Each angle was fitted separately holding the phase fixed at zero, and then each angle was fitted separately allowing for a search for the best $l=0$ phase shift. The results as shown in Table IV demonstrate that the phases are small and that their consideration does not significantly alter the other resonance parameters. As in the case for the Mo analogue resonances,[1] all phases are set at zero in this

TABLE III. Elastic-scattering parameters for Sb^{113} levels and comparisons to Sn^{113} levels.

$Sn^{112}(p,p)Sn^{112}$						$Sn^{112}(d,p)Sn^{113}$			
$E_p^{\text{c.m.}}$ (MeV)	$E_p^{\text{c.m.}}-6.202$ (MeV)	l_p	Γ_p (keV)	Γ (keV)	S_{pp}	E_{ex} (MeV)	l_n	J^π	S_{dp}
6.202	0.0	0	10.3	40.1	0.50	0.0	0	$\frac{1}{2}+$	1.16
...	...	...	...	...	...	0.07	4	$\frac{7}{2}+$	0.311
6.649	0.447	2	0.48	13.3	0.08	0.41	2	$\frac{5}{2}+$	0.154
6.710	0.508	2	3.5	33.5	0.27	0.50	2	$\frac{3}{2}+$	0.746

[6] C. Nealy and R. K. Sheline, Phys. Rev. **135**, B325 (1964).
[7] *Nuclear Data Sheets* (National Academy of Sciences—National Research Council, Washington 25, D. C.), NRC 60–2–89.

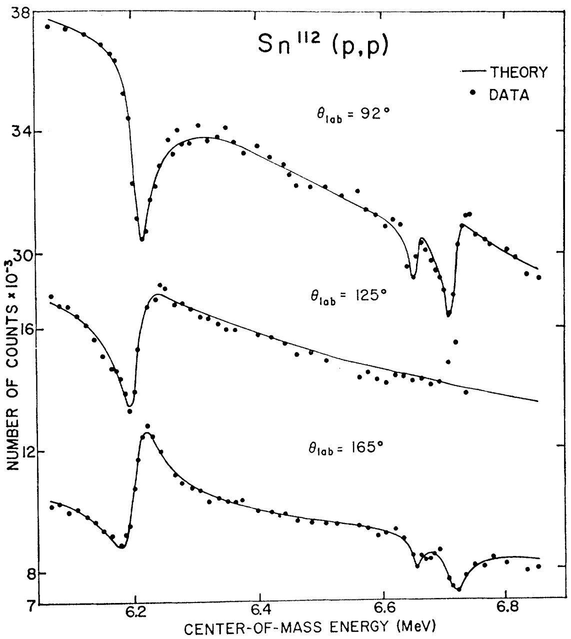

FIG. 3. Proton-elastic-scattering data with a Sn^{112} target. The solid line is a best fit for the three observed resonances using the Coulomb-plus-single-level formula.

study. It is also noted here that when $\phi=0$ (nuclear phase) the ratio of the elastic-scattering normalization coefficients at 92° to 125° to 165° is 1:1.06:1.03 which is evidence that we are seeing Rutherford scattering to within 5%. This is in fact found to be true for the observed analogue resonances of all the Sn isotopes.

The Coulomb energy shift between analogue states is given by $\Delta E_C=B_n+E_p^{\text{c.m.}}(\text{gr. st.})$ where B_n is the neutron binding energy in the target-plus-neutron system. For the Sn^{112} target case using $B_n=8.043$ MeV

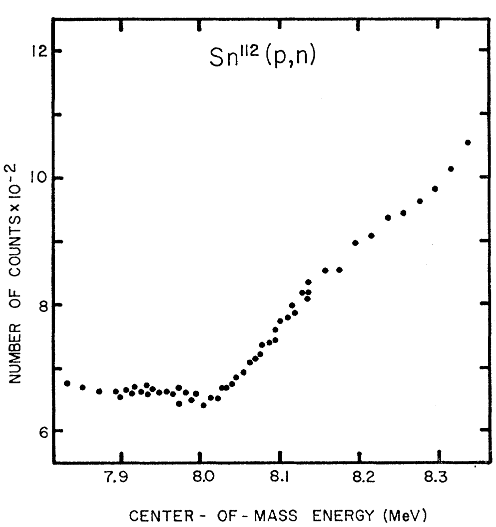

FIG. 4. $Sn^{112}(p,n)$ threshold measurement.

TABLE IV. Variations in resonance parameters of $Sn^{112}(p,p)$ 6.2-MeV resonance phase shift from nuclear potential scattering.

Angle	Γ_p (keV)	Γ (keV)	E_{res} (MeV)	ϕ_0 (radians)	S_{pp}
92°	10.5	42.0	6.208	0.0	0.49
92°	10.2	41.2	6.205	−0.06	0.49
125°	9.8	39.8	6.200	0.0	0.48
125°	9.6	39.4	6.201	0.04	0.48
165°	10.7	38.4	6.198	0.0	0.57
165°	10.1	38.6	6.203	0.14	0.52

taken from Damerow, Ries, and Johnson,[8] the Coulomb displacement energy is 14.245 MeV±0.025. Assuming a spherical charge distribution one gets $\Delta E_C=CZ/A^{1/3}$ where $C=1.349$. The (d,p) Q value leads to the much smaller value for B_n of 7.729 MeV for which $\Delta E_C=13.931$ MeV±0.025 and $C=1.319$. As can be seen from Table II, the value of C is higher for Sn^{112} than for any of the other Sn isotopes using Damerow's data. This problem of the nuclear binding energy in Sn^{112} is discussed by Cohen *et al.*[9] The values of C obtained using the (d,p) data are more consistent.

The magnitude of the spectroscopic factors obtained from proton-elastic scattering differs greatly from the (d,p) spectroscopic factors. It is important to notice however, that by renormalizing either set of spectroscopic factors, the deviations can be reduced to within 20% for the three levels.

Sn^{115}–Sb^{115}. Proton elastic scattering data shown in Fig. 5 on Sn^{114} exhibits analogue resonances over a larger energy range than for Sn^{112}. Four resonances were

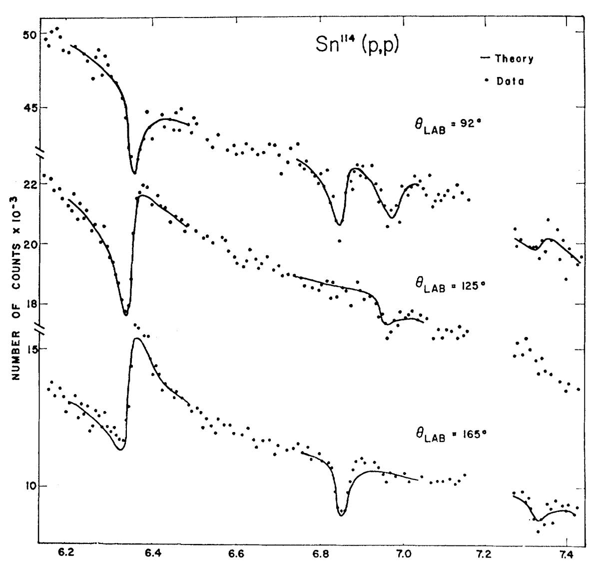

FIG. 5. Sn^{114} proton elastic scattering data. The solid line is a fit to the four resonances identified.

[8] R. Damerow, R. Ries, and W. Johnson, Jr., Phys. Rev. **132**, 1673 (1963).
[9] B. L. Cohen *et al.*, Phys. Rev. **135**, B383 (1964).

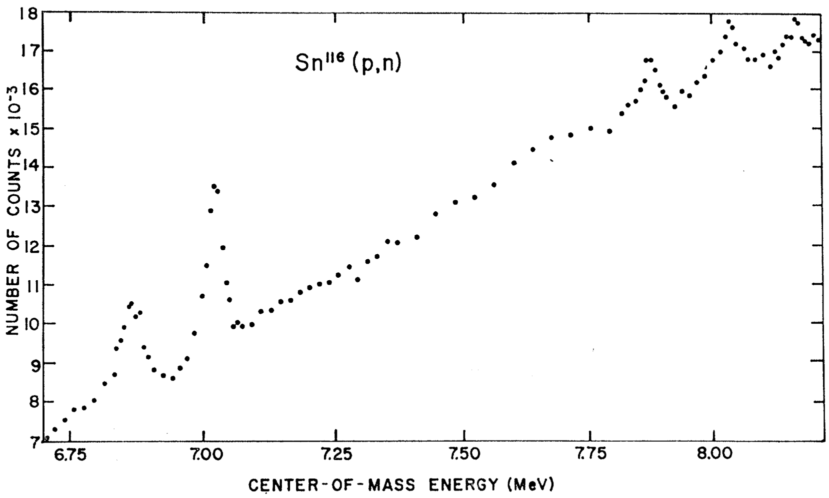

FIG. 6. $Sn^{116}(p,n)$ excitation function.

observed and analyzed for this isotope with the ground-state analogue being observed as an s-wave resonance at 6.35 MeV. The resonances at 6.856 and 6.98 MeV could be fit at 90° as a $d_{3/2}$-$g_{7/2}$ doublet or as a $d_{3/2}$-$d_{5/2}$ doublet. However, at 125° the resonances could only be fitted to a $d_{3/2}$-$g_{7/2}$ doublet. At 165° only the 6.856-MeV level could be fitted assuming a $d_{3/2}$ resonance. There are two comments to be made about the above problem. First, there is a strong $h_{11/2}$ state seen in the (d,p) studies of Schneid *et al.*,[2] which is not seen in the proton-scattering data. This resonance would appear 90 keV above the apparent $g_{7/2}$ resonance and could thus interfere with this resonance to produce a distortion to the normal g-wave shape. The situation is made even more complicated by the observation of a resonance at 7.018 MeV in the $Sn^{114}(p,n)$ excitation function. This resonance is 38 keV above the apparent $g_{7/2}$ state and does not line up with any of the states seen in the (d,p) studies. As can be seen in Fig. 1, this resonance occurs at exactly the same value of $E_p^{c.m.}$ as a resonance observed in Sn^{116}+proton. Since the Sn^{114} target was enriched to only 56.85% and contained 10.62% Sn^{116}, it is very likely that this resonance is due to Sn^{116}. The presence of such an impurity resonance is the most probable cause of the shape analysis difficulty.

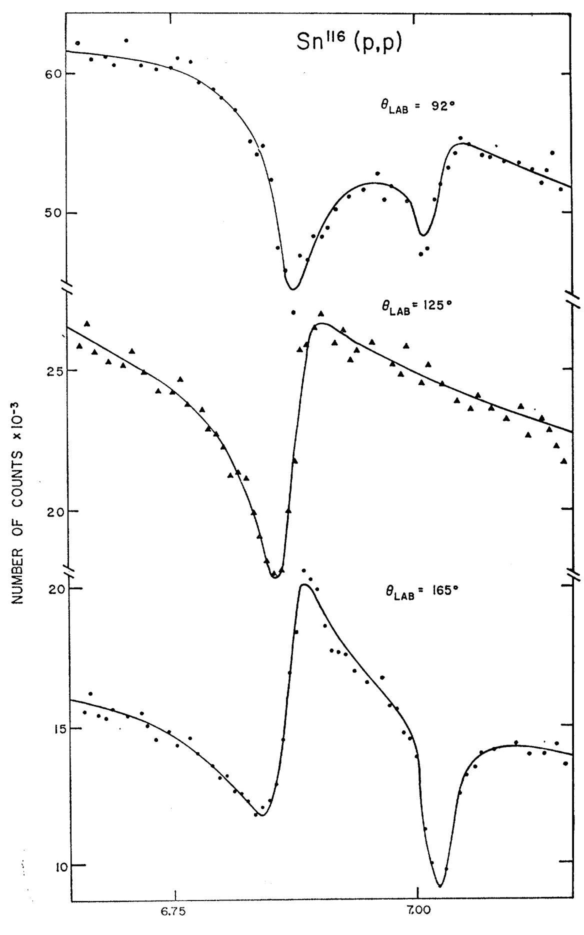

FIG. 7. Sn^{116} proton elastic scattering data and best fit for the two lowest isobaric analogue resonances.

One other resonance was observed at 7.33 MeV. Even though there are poor statistics on this resonance, it was found to be from d-wave proton capture and lines

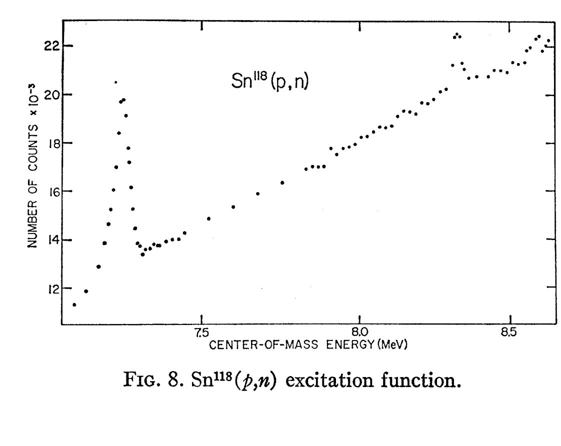

FIG. 8. $Sn^{118}(p,n)$ excitation function.

TABLE V. Elastic-scattering parameters for Sb^{115} levels and comparisons to Sn^{115} levels.

		$Sn^{114}(p,p)Sn^{114}$				$Sn^{114}(d,p)Sn^{115}$			
$E_p^{c.m.}$ (MeV)	$E_p^{c.m.}-6.350$ (MeV)	l_p	Γ_p (keV)	Γ (keV)	S_{pp}	E_{ex} (MeV)	l_n	J^π	S_{dp}
6.35	0.0	0	8	38	0.34	0.0	0	$\frac{1}{2}+$	0.96
6.856	0.506	2	5	36	0.28	0.49	2	$\frac{3}{2}+$	0.619
6.965	0.615	(4)	(3)	(50)	(0.22)	0.60	4	$\frac{7}{2}+$	0.192
7.332	0.982	2	1	50	0.026	0.98	2	$\frac{5}{2}+$	0.117

up very well with a d-wave state observed in the (d,p) studies. The results are summarized in Table V together with a comparison between analogue states.

Sb^{115} was also studied with the $Sn^{114}(p,n)Sb^{114}$ reaction. The $(p.n)$ Q value was found to be 6.82 ± 0.05 MeV which is 470 keV above the ground state analogue. The previous known Q value taken from Selinov *et al.*,[10] is -7.10 MeV which is 280 keV high.

The Coulomb displacement energy for Sn^{115}–Sb^{115} using the neutron-binding energy from the mass data[8] is 13.875 MeV whereas the (d,p) Q-value data[2] gives 13.904 MeV. The two values are within experimental errors and give the constant $C=1.321$ and 1.324 for mass data and (d,p) data respectively.

Sn^{117}–Sb^{117}. The $Sn^{116}(p,n)$ threshold falls below the proton energy for observing the ground-state analogue of Sn^{117}, so that the (p,n) reaction served as a useful means of quickly locating the isobaric analogue resonances in Sb^{117}. Figure 6 presents the excitation function from $E_p^{c.m.}=6.7$ MeV to 8.25 MeV in which five resonances are observed.

Proton-elastic scattering was taken over the same region and exhibits resonance structure at four energies which agree to within 7 keV to the first four resonances seen in the (p,n) excitation function. The first two resonances at 6.869 and 7.022 MeV are identified as the isobaric analogues to the s-wave ground state and d-wave first excited state of Sn^{117}. Figure 7 contains the elastic-scattering data over these two resonances and the theoretical fit for three angles. It was necessary to fit the resonances simultaneously because of their overlap.

A survey of the results is given in Table VI along with a comparison to the (d,p) data.[2] The proton spectroscopic factors are smaller than the (d,p) spectroscopic factors but have similar ratios as in the two previous cases of Sn^{113}–Sb^{113} and Sn^{115}–Sb^{115}. The Coulomb displacement energy is 13.817 MeV giving the empirical constant $C=1.323$.

Sn^{119}–Sb^{119}. The $Sn^{118}(p,n)$ excitation function between 7 and 8.7 MeV yields 3 resonances at $E_p^{c.m.}=7.256$, 8.323, and 8.598 MeV (See Fig. 8). As is evident in the proton-elastic scattering data presented in Fig. 9, the resonance seen at 7.256 MeV in the (p,n) curve is actually an s-, d-wave doublet which is the isobaric analogue to the ground state and first excited state of Sn^{119} as seen in the (d,p) data of Schneid *et al.*[2] The analogue of the $\frac{5}{2}+$ level at 1.10-MeV excitation in Sn^{119} is the only other resonance seen in proton elastic scattering over this energy region. The tabulated results appear in Table VII.

Elastic-scattering resonances in Sb^{119} have recently

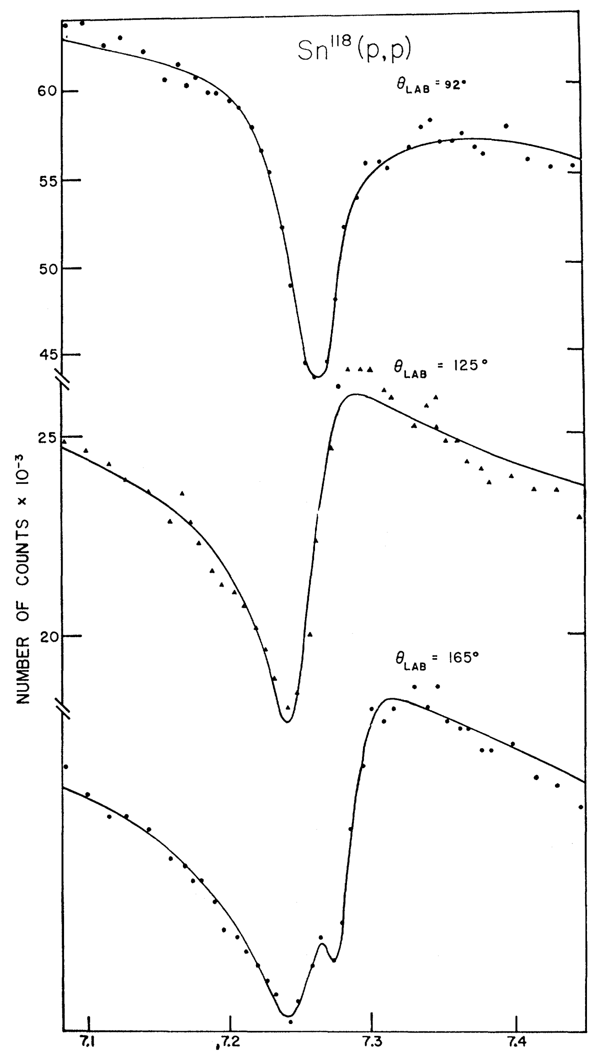

FIG. 9. Sn^{118} proton elastic scattering data and best fit in the region of the lowest energy resonances.

[10] I. P. Selinov *et al.*, Atomnaya Energiya **7**, 547 (1959).

TABLE VI. The observed analogue resonances in Sb^{117} with comparisons to the Sn^{117} states seen by (d,p) studies.

$Sn^{116}(p,n)Sb^{116}$		$Sn^{116}(p,p)Sn^{116}$						$Sn^{116}(d,p)Sn^{117}$			
$E_p^{c.m.}$ (MeV)	E_{ex}^{*a} (MeV)	$E_p^{c.m.}$ (MeV)	E_{ex}^{*a} (MeV)	l_p	Γ_p (keV)	Γ (keV)	S_{pp}	E_{ex} (MeV)	l_n	J^π	S_{dp}
6.862	0.0	6.869	0.0	0	16.5	42	0.58	0.0	0	$\frac{1}{2}^+$	0.647
7.019	0.157	7.022	0.153	2	8.3	37	0.43	0.16	2	$\frac{3}{2}^+$	0.549
7.873	1.011	7.873	1.004	2	1.8	42	0.040	1.03	2	$\frac{5}{2}^+$	0.061
8.035	1.173	8.038	1.169	2	1.4	35	0.035	1.19	2	$\frac{5}{2}^+$	0.033
8.170	1.308	...	...	...	...	...	...	1.31	3	$\frac{7}{2}^-$	0.029

a E_{ex}^* is the excitation above the Sb^{117} ground-state analogue.

TABLE VII. The observed analogue resonances in Sb^{119} with comparisons to the Sn^{119} states seen by (d,p) studies.

$Sn^{118}(p,n)Sb^{118}$		$Sn^{118}(p,p)Sn^{118}$						$Sn^{118}(d,p)Sn^{119}$			
$E_p^{c.m.}$ (MeV)	E_{ex}^{*a} (MeV)	$E_p^{c.m.}$ (MeV)	E_{ex}^{*a} (MeV)	l_p	Γ_p (keV)	Γ (keV)	S_{pp}	E_{ex} (MeV)	l_n	J^π	S_{dp}
7.256	0.0	7.255	0.0	0	17	50	0.38	0.0	0	$\frac{1}{2}^+$	0.586
...	...	7.276	0.021	2	5	32	0.23	0.024	2	$\frac{3}{2}^+$	0.515
8.323	1.067	8.328	1.073	2	3	37	0.063	1.10	2	$\frac{5}{2}^+$	0.084
8.598	1.342	...	...	...	...	...	...	1.37	2	$\frac{5}{2}^+$	0.014

a E_{ex}^* is the excitation above the ground-state analogue.

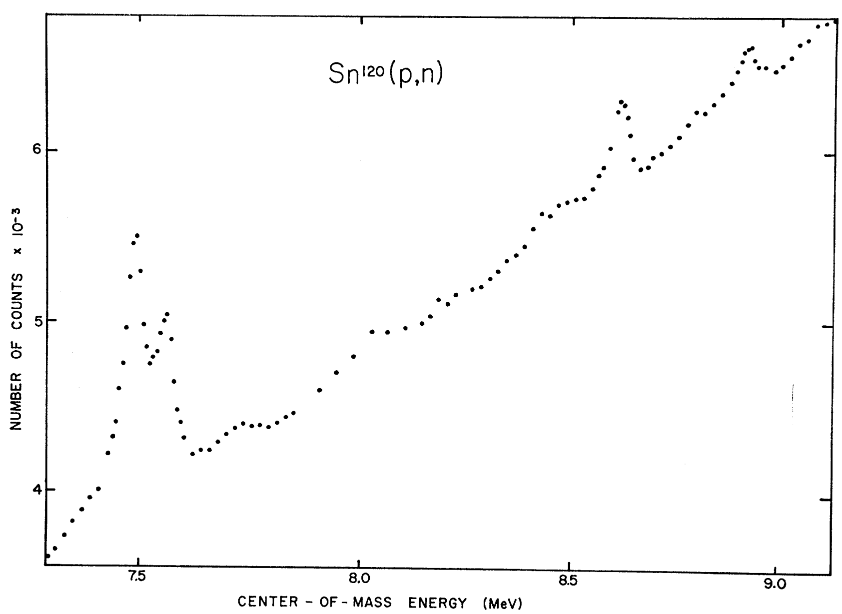

FIG. 10. $Sn^{120}(p,n)$ excitation function.

been reported by Allan *et al.*[11] A single resonance is seen at an energy $E_p^{c.m.}=7.27$ MeV where we see the *s*-*d*-wave doublet. They see resonances at $E_p^{c.m.}=8.34$ and 8.6 MeV in agreement with our work. They see a surprisingly large resonance at $E_p^{c.m.}=11.05$ MeV which is above the Coulomb barrier. They also observe additional isobaric analogue resonances via inelastic-scattering excitation functions to final states in Sn^{118}.

Sn^{121}–Sb^{121}. Four isobaric analogue resonances are observed in the energy region 7.2 to 9.1 MeV by both (p,n) and (p,p) excitation functions on Sn^{120}. Figure 10 presents the (p,n) results where we see that the two lower states are clearly resolved in this case. In contrast to Sn^{119} the ground-state analogue at 7.495 MeV is $\frac{3}{2}^+$ and first-excited-state analogue at 7.570 MeV is $\frac{1}{2}^+$. This is found by performing the Coulomb-plus-single level fit to the proton-elastic scattering data as shown in Fig. 11. One can also deduce this inversion in the order of the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ states by noting the relative amplitudes of the two peaks in the (p,n) curves on Sn^{116} and Sn^{120}.

The results are tabulated in Table VIII with the

[11] D. L. Allan, G. A. Jones, G. C. Morrison, R. B. Taylor, and R. B. Weinberg, Phys. Letters 17, 56 (1965).

TABLE VIII. The observed analogue resonances in Sb^{121} with comparisons to the Sn^{121} states seen by (d,p) studies.

$Sn^{120}(p,n)Sb^{120}$		$Sn^{120}(p,p)Sn^{120}$						$Sn^{120}(d,p)Sn^{121}$			
$E_p^{c.m.}$ (MeV)	E_{ex}^{*a} (MeV)	$E_p^{c.m.}$ (MeV)	E_{ex}^{*a} (MeV)	l_p	Γ_p (keV)	Γ (keV)	S_{pp}	E_{ex} (MeV)	l_n	J^π	S_{dp}
7.487	0.0	7.495	0.0	2	7.5	39	0.25	0.0	2	$\frac{3}{2}^+$	0.43
7.556	0.069	7.570	0.075	0	24	60	0.42	0.05	0	$\frac{1}{2}^+$	0.39
8.618	1.131	8.621	1.126	2	3	53	0.037	1.12	2	$\frac{5}{2}^+$	0.065
8.925	1.438	8.924	1.429	2	1.5	38	0.023	1.40	2	$\frac{5}{2}^+$	0.029

[a] E_{ex}^* is the excitation above the ground state analogue.

appropriate comparisons. From Table II we see that the Coulomb displacement energy is 13.654 for which $C=1.322$.

Sn^{123}–Sb^{123}. The results obtained in the $Sn^{122}+p$ system is very similar to those found in the $Sn^{120}+p$ system. The resonances energies found in the (p,n) excitation function (see Fig. 12) agree to within 5 keV with the (p,p) results. A low-lying doublet is resolved and found to be $\frac{3}{2}^+$ and $\frac{1}{2}^+$ at 7.684 and 7.820 MeV, respectively.

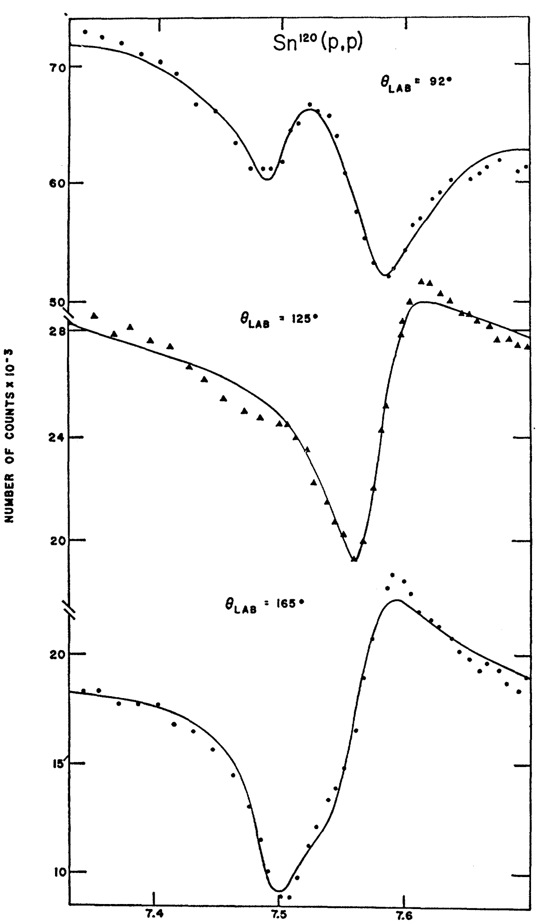

FIG. 11. Sn^{120} proton elastic scattering data and best fit in the region of the lowest energy resonances.

The elastic-scattering fit to this doublet is shown in Fig. 13. In this case however, these states are the first and second excited-state analogues of Sn^{123}. Nealy and Sheline[6] report a state in their (d,p) work 20 keV below the $d_{3/2}$ state which they assign as the $11/2^-$ ground state.

The spectroscopic factors do not agree very well in magnitude or in ratio as they do in the previous cases. Table IX contains the compiled results. The Coulomb displacement energy is found to be 13.615 MeV using the (d,p) Q value[2] and gives $C=1.326$.

Sn^{125}–Sb^{125}. The Sb^{125} results are very similar to the Sb^{123} results in that the $h_{11/2}$ ground state seen by Nealy and Sheline[7] is not seen here and that the low-lying $\frac{3}{2}^+$-$\frac{1}{2}^+$ doublet is seen in (p,n) as shown in Fig. 14 and analyzed in (p,p) as presented in Fig. 15. Only two angles, 125° and 165°, were taken on this particular set of data. Two other resonances are observed in proton-elastic scattering at 9.085 MeV and 9.364 MeV, whereas only a 9.087 MeV resonance is seen in the (p,n) excitation function. S_{pp} for these states are very similar to those obtained in the Sb^{123} case and also disagree with the values of S_{dp}. The analogue comparisons are found in Table X.

Proton Reactions on the odd Isotopes of Sn

Sn^{118}–Sb^{118}. The (p,n) reaction on Sn^{117} was used to identify isobaric analogue resonances in Sb^{118}. By knowing the approximate Coulomb displacement energy

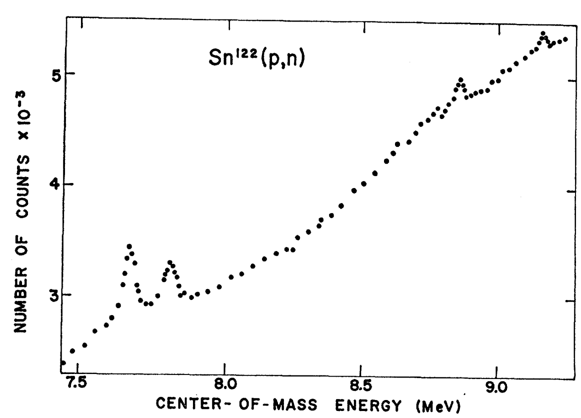

FIG. 12. $Sn^{122}(p,n)$ excitation function.

TABLE IX. The observed analogue resonances in Sb^{123} with comparisons to the Sn^{123} states seen by (d,p) studies.

$Sn^{122}(p,n)Sb^{122}$		$Sn^{122}(p,p)Sn^{122}$						$Sn^{122}(d,p)Sn^{123}$			
$E_p^{c.m.}$ (MeV)	E_{ex}*[a] (MeV)	$E_p^{c.m.}$ (MeV)	E_{ex}*[a] (MeV)	l_p	Γ_p (keV)	Γ (keV)	S_{pp}	E_{ex} (MeV)	l_n	J^π	S_{dp}
...	...	...	...	...	...	...	...	0.0	5	$11/2^-$	...
7.679	0.02	7.684	0.02	2	7	42	0.19	0.02	2	$\frac{3}{2}^+$	0.434
7.816	0.155	7.820	0.158	0	17	59	0.23	0.15	0	$\frac{1}{2}^+$	0.356
8.867	1.205	8.867	1.203	2	7	41	0.11	1.20	2	$\frac{5}{2}^+$	0.062
9.174	1.515	9.169	1.505	2	2	47	0.022	1.49	2	$\frac{5}{2}^+$	0.024

[a] E_{ex}* is the excitation above the ground state analogue.

and by measuring the level spacings between resonances, we were able to adjust the resonances to compare with the spectrum for the neutron analogue states. These comparisons were made with the $Sn^{117}(d,p)$ data of Norris and Moore.[12] The agreement is quite good for the low-lying states as can be seen in Table XI.

The ground-state analogue is found at an energy of 4.44 MeV. The Coulomb displacement energy is thus found to be 13.756 MeV for which $C=1.322$.

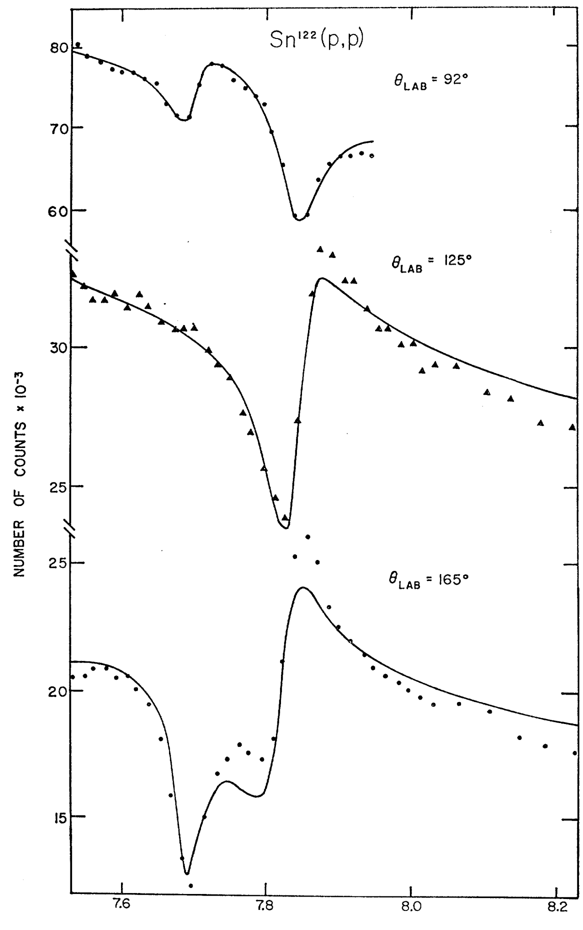

FIG. 13. Sn^{122} proton elastic scattering data and best fit to the low-lying d-, s-wave doublet.

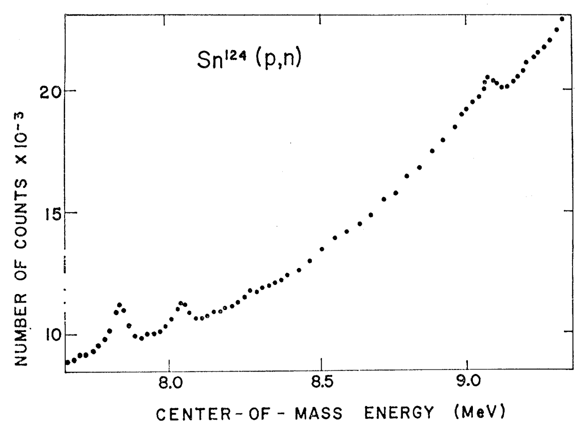

FIG. 14. $Sn^{124}(p,n)$ excitation function.

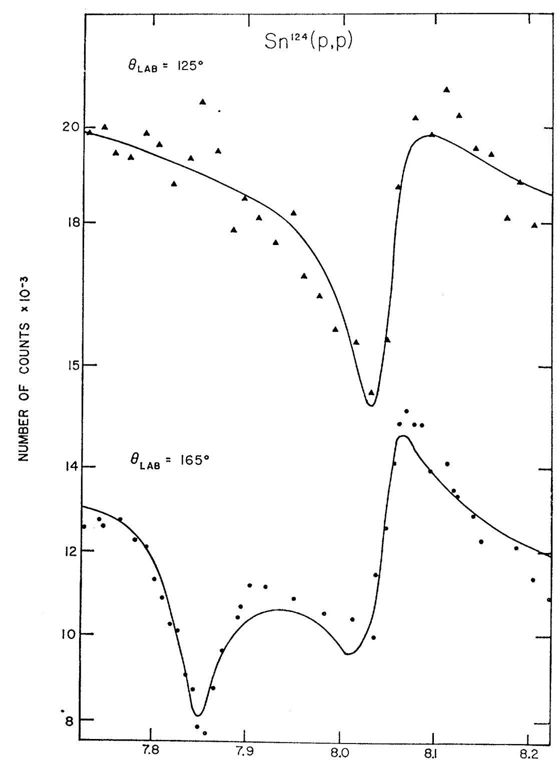

FIG. 15. Sn^{124} proton elastic scattering data and best fit in the vicinity of the low-lying d-, s-wave doublet.

[12] L. R. Norris and C. F. Moore, Phys. Rev. **136**, B40 (1964).

TABLE X. The observed analogue resonances in Sb^{125} with comparisons to the Sn^{125} states seen by (d,p) studies.

$Sn^{124}(p,n)Sb^{124}$		$Sn^{124}(p,p)Sn^{124}$						$Sn^{124}(d,p)Sn^{125}$			
$E_p^{c.m.}$ (MeV)	E_{ex}^{*a} (MeV)	$E_p^{c.m.}$ (MeV)	E_{ex}^{*a} (MeV)	l_p	Γ_p (keV)	Γ (keV)	S_{pp}	E_{ex} (MeV)	l_n	J^π	S_{dp}
...	...	...	...	...	...	...	...	0.0	5	$11/2^-$	...
7.843	0.026	7.840	0.026	2	9	57	0.16	0.026	2	$\frac{3}{2}^+$	0.344
8.043	0.226	8.044	0.230	0	14	45	0.24	0.220	0	$\frac{1}{2}^+$	0.253
9.087	1.270	9.085	1.271	2	4	86	0.025	1.270	2	$\frac{5}{2}^+$	0.039
...	...	9.364	1.550	2	2	56	0.018	1.56	2	$\frac{5}{2}^+$	0.023

[a] E_{ex}^* is the excitation above the ground state analogue.

Sn^{120}–Sb^{120}. Information about the level separations were obtained by $Sn^{119}(p,n)$ as in the case of Sn^{117}. Comparisons were again made to the (d,p) data of Norris and Moore[12] and are shown in Table XI.

The ground-state analogue was observed at an energy $E_p^{c.m.}=4.56$ MeV so that using the Q value of 6.89 MeV, the Coulomb displacement energy is 13.676 with $C=1.322$.

CONCLUSIONS

There is generally very good agreement between the work reported here and the (d,p) reaction results of Schneid, Prakish, and Cohen with respect to l-value assignment and excitation energy. This agreement holds for excitation energies up to 0.6 MeV in the lightest Sn isotope to 1.6 MeV in the heaviest. Unfortunately, this agreement does not extend to the spectroscopic factors calculated from the data. There is rather poor agreement in the absolute magnitudes of the spectroscopic factors although the ratios of spectroscopic factors calculated for different levels in the same nucleus are in fair agreement. This result is in contrast to the work reported[1] for the Mo isotopes where there was good agreement between the isobaric analogue results and the (d,p) reaction results of Hjorth and Cohen.[13]

TABLE XI. Sb^{118} and Sb^{120} resonances with comparisons to the Sn^{118} and Sn^{120} states seen in (d,p).

$Sn^{117}(p,n)$ $E_R-E_R^{g.s.}$ (MeV)	$Sn^{117}(d,p)$ E_{ex} (MeV)	$Sn^{119}(p,n)$ $E_R-E_R^{g.s.}$ (MeV)	$Sn^{119}(d,p)$ E_{ex} (MeV)
0.0	0.0	0.0	0.0
1.23	1.22	1.15	1.17
1.70	1.75	1.79	1.88
2.03	2.05	2.11	2.10
2.33	2.32	...	2.17
...	...	2.30	2.29
...	2.38	...	2.36
2.48	2.49	2.40	2.43
...	2.54	2.58	2.61
2.66	2.67	2.71	2.73
2.73	2.73	2.83	2.81
...	2.81	...	2.84
...	2.84	2.92	2.94
...	2.86	...	3.00
2.89	2.89	...	...

It should be mentioned that the present technique of extraction of the spectroscopic factor S_{pp} from the isobaric-analogue data is relatively model-independent. The approximations that are made have been previously discussed.[1] The technique has the further advantage that each resonance is handled separately and the spectroscopic factor is calculated independently of the identification of other resonances; no summing technique is used.

The Coulomb displacement energies for the Sn isotopes show no dependence on the nuclear parameters except radius; that is, the $A^{1/3}$ dependence seems to dominate. One might expect to see the effects of nuclear deformation in the Coulomb displacement energy for the case of Sn^{112}. However, we believe that the mass data for Sn^{113} is in error so that, if the (d,p)-reaction Q value is used to derive the Coulomb displacement energy for Sb^{113}–Sn^{113}, then there is no observed effect due to nuclear deformation.

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[13] S. A. Hjorth and B. L. Cohen, Phys. Rev. **135**, B920 (1964).